Structure and Performance Evaluation of a Replicated Banyan Network Based ATM Switch

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Abstract

Banyan networks are commonly used as interconnection structures in ATM switches. This paper is concerned with the replication technique which was applied to the standard banyan networks. In this paper, we apply this technique to the Plane Interconnected Parallel Network (PIPN) which is a switch introduced previously as a better banyan based interconnection structure. The normalized throughput of unbuffered and buffered replicated PIPN is analyzed analytically under uniform traffic model. We apply the simulation technique to verify the analytical results under the uniform traffic model and to study the performance of different heterogeneous traffic models. The performance is shown to increase significantly when the replicated PIPN is used which supports the idea of using this switch as a new high performance ATM switch.

Index Terms – ATM switching, banyan networks, replicated networks.

1. Introduction

Due to the high transmission capacity offered by fiber optics, many applications that require high bandwidth have emerged [1]. The creation of a network that provides high bandwidth services to the users is needed. The challenge is to build large switches that can operate at the high data transfer rates and meet the performance requirements.

Banyan networks are commonly used in multistage ATM switches because of their high degree of parallelism, self-routing, modularity, constant delay for all input-output port pairs, in-order delivery of cells, and suitability for VLSI implementation [2-8]. However, in banyan networks, there is only one path between each input and output port pair, and the edges of such a path are not dedicated. This means that other communicating pairs may share some links of a path connecting an input-output port pair.

The concern of this paper is to design a new high performance ATM switch. The switch combine the technique of replication with the plane interconnected parallel network (PIPN). The replication technique was applied on the banyan networks to enhance their performance [2, 3]. PIPN was introduced in [4] as a better banyan based interconnection structure. By combining the two techniques, we take the advantage of replication which provides multiple paths from each input to each output, thus decreasing the effect of conflict between cells, and the advantage of PIPN which gives better performance under heterogeneous traffic over the standard banyan network.

The outline of this paper is as follows: In section 2, we describe the basic structure and operation of the Replicated PIPN switch. In section 3, we discuss its performance under uniform and heterogeneous traffic models. We conclude the paper in section 4.

2. The replicated PIPN switch

2.1. Background

A cell switch is a box with N inputs and N outputs which routes the cells arriving at its inputs to their requested outputs. The general cell switch architecture is shown in Fig. 1. Several architectural designs have emerged to implement this switch. They may be classified into three categories: the shared-memory type, the shared-medium type, and the space-division type. Both shared-memory and shared-medium suffer from their strict capacity limitation, which is limited to the capacity of the internal communication medium. Any internal link is N times faster than the input link and it is usually implemented as a parallel bus. This makes such architectures more difficult to implement as N becomes large. Fig. 2 shows the shared-medium and shared-memory architectures.

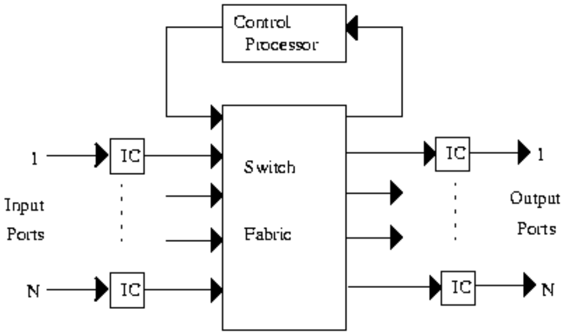


Fig. 1. General cell switch architecture

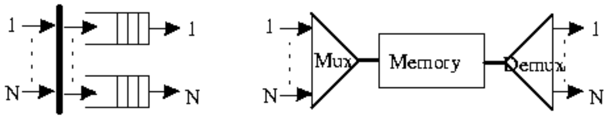


Fig. 2. Shared medium and shared memory architectures

The simplest space-division switch is the crossbar switch, which consists of a square array of NxN crosspoint switches, one for each input-output pair as shown in Fig. 3. As long as there is no output conflicts, all incoming cells can reach their destinations. If, on the other hand, there is more than one cell destined in the same time slot to the same output, then only one of these cells can be routed and the other cells may be dropped or buffered. The major drawback of the crossbar switch stems from the fact that it comprises N^2 crosspoints, and therefore, the size of realizable such switches is limited. For this reason, alternative candidates for space division switching fabrics have been introduced. These alternatives are based on a class of multistage interconnection networks called banyan networks [1].

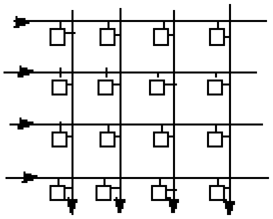


Fig. 3. Crossbar architecture

A banyan network constructed from 2x2 switching elements (SE) consists of $n = \log_2 N$ stages (N is assumed to be a power of 2). Banyan networks have many desirable properties: high degree of parallelism, self-routing, modularity, constant delay for all input-output port pairs, in-order delivery of cells, and suitability for VLSI implementation. Their shortcoming remains blocking and throughput limitation. Blocking occurs every time two cells arrive at a switching element and request the same output link of the switching element. The existence of such conflicts (which may arise even if the two cells are destined to distinct output ports) leads to a maximum achievable throughput which is much lower than that obtained with the crossbar switch. An 8x8 banyan network is shown in Fig. 4.

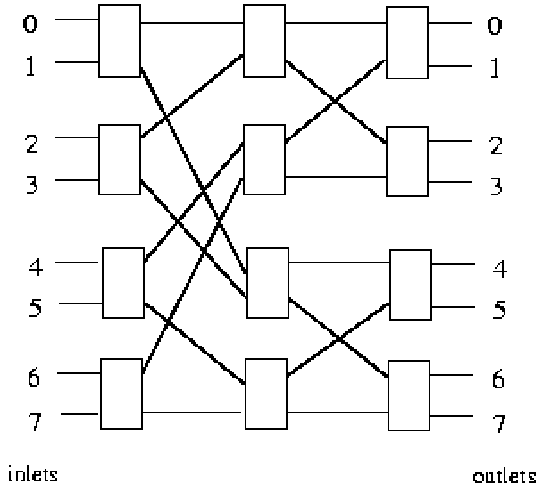


Fig. 4. A banyan network

To overcome the performance limitations of banyan networks, various performance enhancing techniques have been introduced [2, 3, 6, 7]. These techniques have been widely used in designing ATM switches [2-8].

One of the performance enhancement techniques of banyan networks is the replication technique [2, 3]. Using the replication technique, we have $R = 2^r$ ($r = 1, 2, \ldots$) parallel subnetworks. Each of these subnetworks is a banyan network. Two techniques are used to distribute the incoming cells over the R subnetworks. In the first technique, input i of the switch is connected to input i of each subnet by a 1-to-R demultiplexer. The demultiplexer forwards the incoming cells randomly across the subnetworks. Similarly, each output i of a subnet is connected to the output i of the switch through a R-to-1 multiplexer. If more than one cell arrive at the multiplexer, one of them is selected randomly to be forwarded to the output port and the others are discarded. This technique is called randomly loading parallel networks (Rn) [2, 3]. Fig. 5 shows a 4x4 randomly loaded banyan network constructed from two 4x4 banyan networks.

The second technique groups the outputs of the switch and assigns each group to one of R truncated subnetworks. The i^{th} input of the switch is connected to the i^{th} input of each subnet through a 1-to-R demultiplexer. The demultiplexer forwards incoming cells according to their most significant bits of the destination address field. Each truncated subnet has $n-r$ stages. The outputs of each subnet which are destined to the same switch output are connected via a R-to-1 multiplexer to this output. This technique is called selectively loading parallel networks (Sn) [2]. A 4x4 selectively loaded parallel banyan network constructed from two 4x4 truncated banyan networks is shown in Fig. 6.

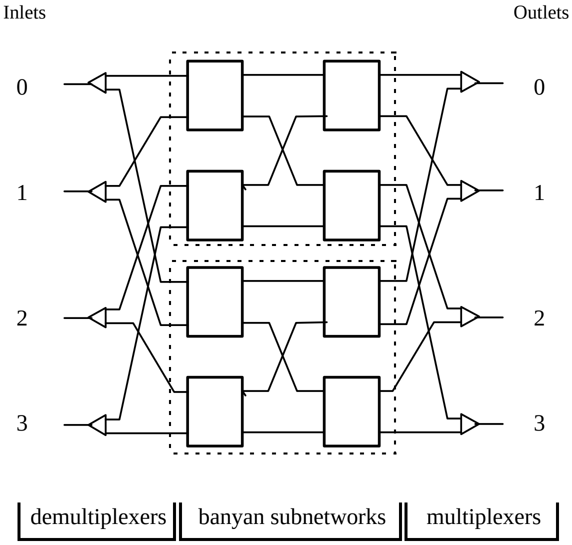


Fig. 5. An 4x4 randomly loaded banyan network constructed form two 4x4 banyan networks

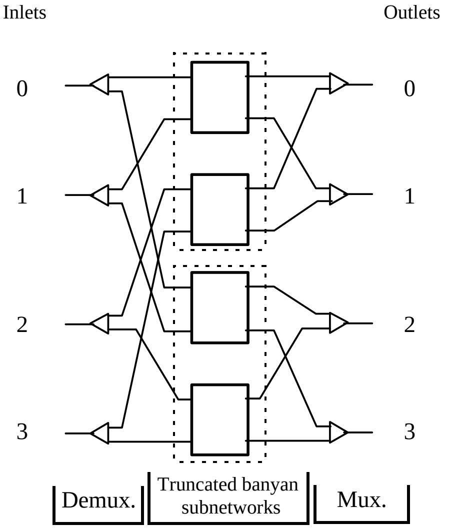


Fig. 6. A 4x4 selectively loaded banyan network constructed form two 4x4 truncated banyan networks

The performance of banyan based switches depends on the applied traffic. As the applied traffic becomes heterogeneous, the performance of banyan based switches degrade drastically even if some performance enhancing techniques are employed.

In [4], the PIPN, a new banyan based interconnection structure which exploits the desired properties of banyan networks while improving the performance by alleviating their drawbacks, is introduced. In PIPN, the traffic arriving at the network is shaped and routed through two banyan network based interconnected planes. The interconnection between the planes distributes the incoming load more homogeneously over the network.

PIPN is composed of three main units, namely, the distributor, the router, and the output-port dispatcher as shown in Fig. 7 [4, 5]. The cells arriving at the distributor divides the network into two groups in a random manner: the back plane and the front plane groups. The destination address fields of cells in one of the groups are complemented. The grouped cells are assigned to the router, which is a N/2 x N/2 banyan network. The cells are routed with respect to the information kept in their destination address fields. Due to the internal structure of the router and the modifications in the destination address fields of some cells, an outlet of the router may have cells actually destined to four different output ports. The cells arriving from the outlets of the router are assigned to the requested output ports in the output-port dispatcher [4, 5]. The output-port dispatcher has two different sub-units: the decider and the collector. There is a decider unit for each router output and a collector unit for each output port. There are a total of N deciders and N collectors. The decider determines to which output port an arriving cell will be forwarded and restores its destination address field. Each collector has four inlets and internal buffer to accommodate the cells arriving from four possible deciders.

2.2. The replicated PIPN switch structure

The Replicated PIPN switch applies the replication technique to the PIPN to benefit from the advantage of both techniques. Replication provides multiple paths from each input to each output pair, thus decreasing the effect of conflict between cells. PIPN gives better performance under heterogeneous traffic over the standard banyan network.

A 8x8 Randomly Loaded PIPN for R=2 is shown in Fig. 8. It is shown from the figure that the Replicated PIPN is composed of R PIPN's connected in parallel. No multiplexers are needed at the output of the router as the deciders forward the incoming cells to the collectors' buffers. The structure of the Selectively Loaded PIPN is similar to the structure of the Randomly Loaded PIPN but the routers are truncated (have n-r-1 stages instead of n-1), and the demultiplexer forward cells to subnetworks according to their r most significant bits outputs.

3. Performance evaluation of the replicated PIPN switch

In this section, the throughput of the Replicated PIPN switch is evaluated. Analytical analysis is performed under uniform traffic model. For simulation, a Timed Colored Petri Net [9] is used to model the original and the

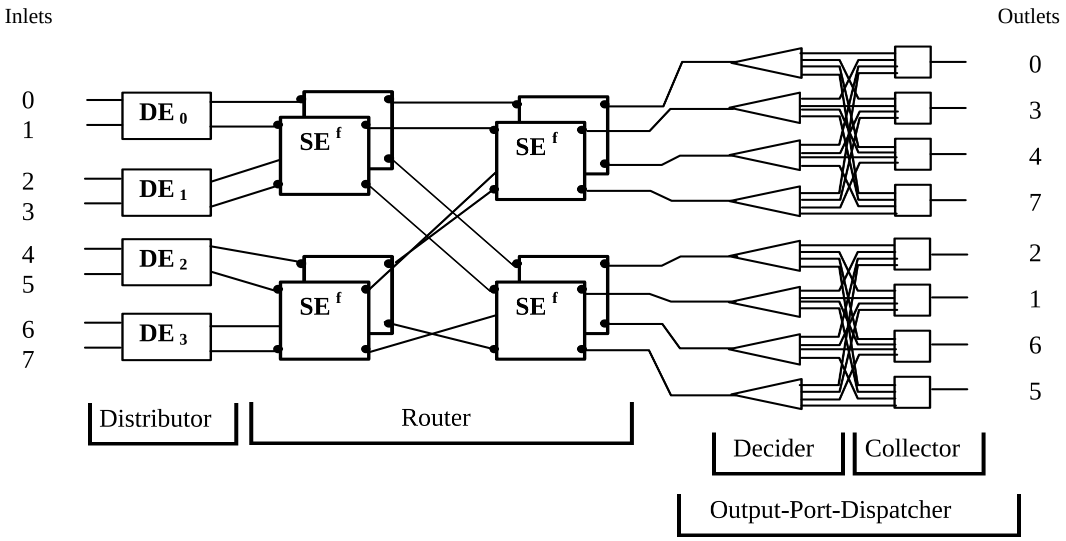


Fig. 7. Complete structure of an 8x8 PIPN

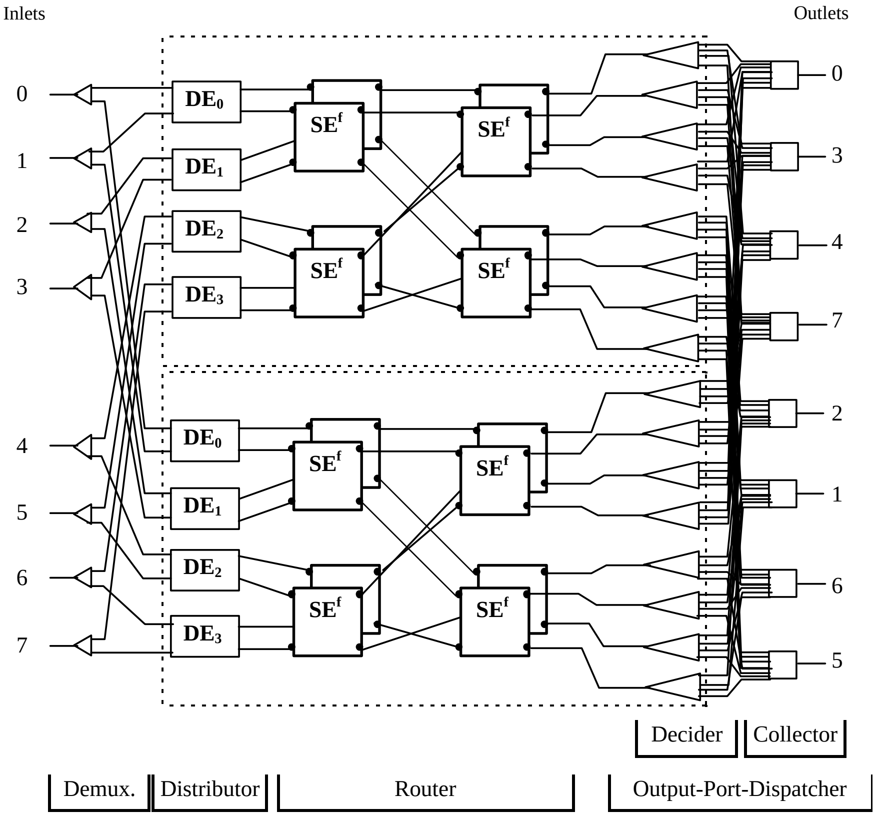


Fig. 8. Complete structure of an 8x8 randomly loaded PIPN for R=2

Replicated PIPN. A Petri Net model for an 4x4 banyan network is given in Fig. 9. It is shown from the figure that the structure of the Petri Net model is the same as that of the switches with each SE modeled as shown in figure.

3.1. Replicated PIPN performance under the uniform traffic model

In this section, we study the performance of the Replicated PIPN switch under uniform traffic model. In this model, cells are equiprobably destined to any output port. Thus, the load at the outlets of all SE's in the same stage will be the same.

The throughput of banyan networks, under uniform traffic model, was given in [2, 3, 5]. The performance of the original PIPN with sufficiently large buffer was studied in [4, 5]. The performance of Replicated banyan networks, both randomly and selectively loaded, was studied in [2, 3].

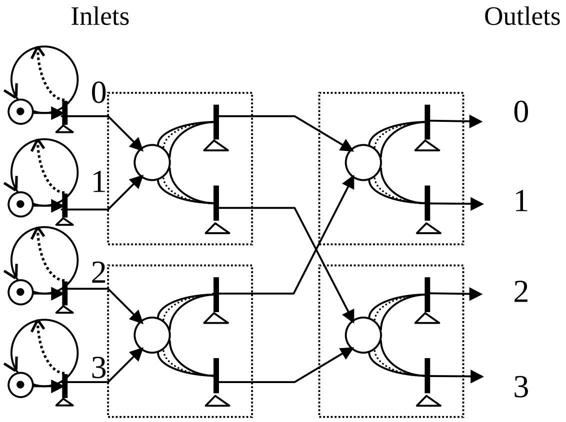


Fig. 9. A Petri Net model for a 4x4 banyan network

In the following subsections, we perform buffer-dimensioning analysis for the original PIPN and study the performance of the unbuffered and buffered Replicated PIPN.

3.1.1. Unbuffered PIPN. The throughput of an N x N PIPN is achieved by an N/2 x N/2 banyan network. This result is directly related to the number of stages since the traffic is uniform. In an N x N banyan network, there are n stages, however, in an N x N PIPN there are n-1 stages in the router [4, 5]. The throughput of the original PIPN with sufficiently large buffer was studied in [4, 5] and was given by:

$$X_{\text{banyan}} = 1 - \left(1 - \frac{X_{\text{PIPN}}}{2}\right)^2 \qquad (1)$$

(for networks of the same size)

where the throughput of a banyan network at stage i is given in [2, 3, 5] by:

$$X_i = 1 - \left(1 - \frac{X_{i-1}}{2}\right)^2 \qquad \text{for } 1 \le i \le n \quad (2)$$

The throughput of an N x N unbuffered PIPN under uniform traffic can be found as follows:

Let X_{Router} denotes the probability of finding a cell at the output of a router outlet ($X_{\text{Router}} = X_{\text{banyan of size N/2 x N/2}}$). Since each output of the router can have cells destined to four different collectors, the probability of finding a cell at one input of a collector is $X_c = X_{\text{Router}} / 4$. A collector can receive up to four cells at each clock cycle, then assuming buffer size=0 for unbuffered PIPN, the throughput of an N x N unbuffered PIPN is given by:

$$\begin{aligned} X_{\text{unbuffered PIPN}} &= 1 - \left(1 - X_c\right)^4 \\ &= 1 - \left(1 - \frac{X_{\text{banyan of size N/2 x N/2}}}{4}\right)^4 \qquad (3) \end{aligned}$$

where $X_{\text{banyan of size N/2 x N/2}}$ is the throughput of a banyan network with n-1 stages.

Fig. 10 shows the simulation result for the original PIPN under uniform traffic model for various buffer sizes. The simulation and analytical results are consistent for buffer size equal zero. It is shown from the figure that a buffer size of two per each collector is sufficient to achieve performance near infinite buffer. Thus this buffer size is chosen for testing the performance of Replicated PIPN switch under heterogeneous traffic types.

In the following subsections, X will denote the probability of finding a cell at every input of the switch at each slot time. The probability of finding a cell at each input of a subnet at each time slot is $X_{in} = X/R$.

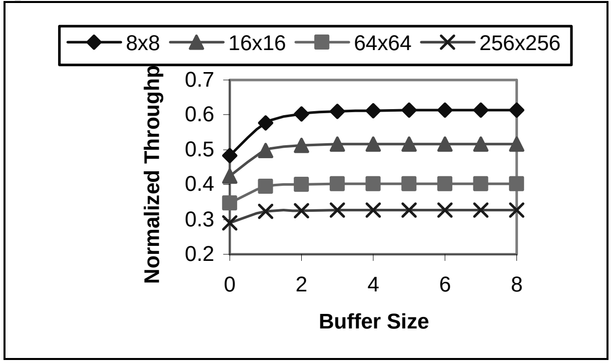


Fig. 10. Effect of buffer size on PIPN performance at full load

3.1.2. Unbuffered replicated PIPN switch. The performance of an NxN unbuffered Replicated PIPN switch with replication degree = R is studied in this section. We study the performance of both randomly and selectively loaded PIPN. The performance of randomly loaded banyan networks was studied in [2] and [3] and was given by:

$$X_{\text{out}} = 1 - (1 - x_n)^R \qquad (4)$$

where x_n is the throughput of a banyan network having n stages with arrival rate equals X_{in}.

The performance of selectively loaded banyan networks was studied in [2] and was given by:

$$X_{\text{out}} = 1 - (1 - x_{n-r})^R \qquad (5)$$

where xn-r is the throughput of a banyan network having n-r stages with arrival rate equals Xin.

A. Unbuffered randomly loaded PIPN switch. Since the router of each subnet of the randomly loaded PIPN switch consists of n-1 stages, the output rate at each router's output link x_{n-1} can be obtained by the recurrence relation (2) with x_0 equals X_{in}. Since each output of the router can have cells destined to four different collectors, the probability of finding a cell at one input of a collector is $X_c = x_{n-1} / 4$. A collector can receive up to four cells at each clock cycle from each subnet, then the throughput of an N x N unbuffered randomly loaded PIPN is given by:

$$X_{\text{unbuffered randomly loaded PIPN}} = 1-\left(1-X_c\right)^{4.R}$$

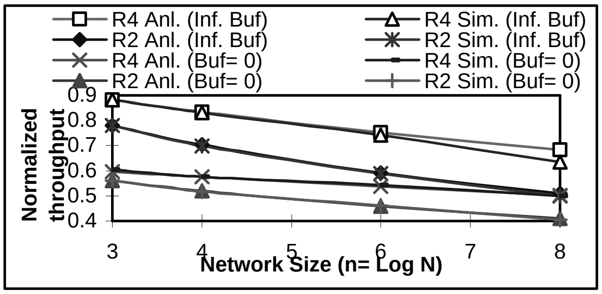
$$\therefore X_{\text{unbuffered randomly loaded PIPN}}$$

$$= 1-\left(1-\frac{X_{\text{banyan with n-1 stages and load= }X_{\text{in}}}}{4}\right)^{4.R}$$

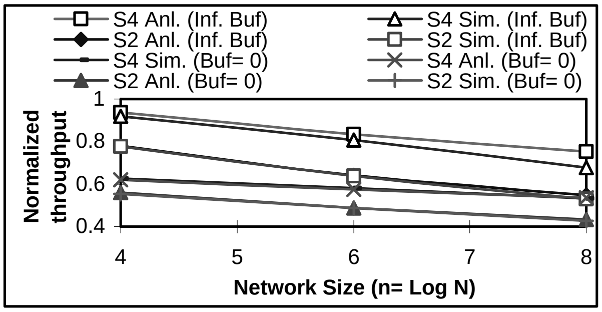
$$= 1-\left(1-X_{\text{unbuffered PIPN}}\right)^{R} \qquad (6)$$

where $X_{\text{unbuffered}}$ PIPN is the throughput of an NxN unbuffered PIPN with load equal Xin.

Fig. 11.a shows the throughput of the unbuffered Randomly Loaded PIPN switch both analytically and by simulation.



(a) Randomly loaded (Rn)



(b) Selectively loaded (Sn)

Fig. 11. Analytical and simulation results for the replicated PIPN under uniform traffic model (full load)

B. Unbuffered Selectively Loaded PIPN Switch. Since the router of each subnet of the randomly loaded PIPN switch consists of n-r-1 stages, The throughput of an NxN unbuffered selectively loaded PIPN is similar to (6) above but with n-r-1 stages instead of n-1.

$$X_{\text{unbuffered selectively loaded PIPN}}$$

$$= 1-\left(1-\frac{X_{\text{banyan with n-r-1 stages}}}{4}\right)^{4.R}$$

$$= 1-\left(1-X_{\text{unbuffered PIPN}}\right)^{R} \qquad (7)$$

where $X_{\text{unbuffered PIPN}}$ is the throughput of a N/RxN/R unbuffered PIPN with load equal X_{in}.

Fig. 11.b shows the throughput of the unbuffered Selectively Loaded PIPN switch both analytically and by simulation.

3.1.3. Buffered replicated PIPN switch. Since there are no loss of cells, the throughput of the randomly loaded Replicated PIPN switch under infinite buffer is exactly R times the throughput of a NxN PIPN with infinite buffer, given in equation (1) .

$$X_{\text{Randomly loaded PIPN with infinite buffer}}$$

$$= R.\,X_{\text{NxN PIPN with infinite buffer}} \qquad (8)$$

Similarly, the throughput of the selectively loaded Replicated PIPN switch under infinite buffer is exactly R times the throughput of a N/RxN/R PIPN with infinite buffer, given in equation (1) .

$$X_{\text{Selectively loaded PIPN with infinite buffer}}$$

$$= R.\,X_{\text{N/RxN/R PIPN with infinite buffer}} \qquad (9)$$

Fig. 11 shows the throughput of the Randomly and Selectively Loaded PIPN switch with infinite buffer both analytically and by simulation. It is shown from the figure that the simulation curve departs from the analytical curve for the infinite buffer case especially for large network sizes. The infinite number of cells in the buffers justifies this difference as a large number of cells remain in the buffer waiting to be transmitted. This effect increases as the network size and replication degree increase.

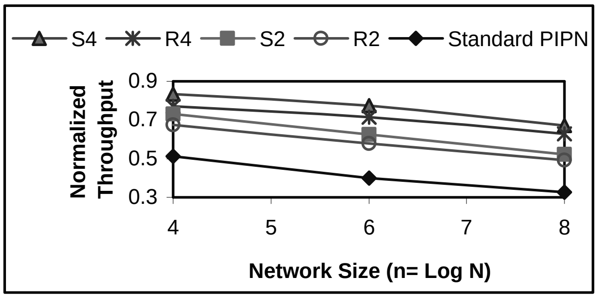


Fig. 12. Performance of the replicated and original PIPN under uniform traffic model (full load)

Also shown in Fig. 12 the throughput of R_2, R_4, S_2, and S_4 Replicated PIPN switches with buffer size equal two cells per collector compared to the original PIPN with infinite buffer. It is shown from the figure that the throughput of the selectively loaded PIPN is better than that of the randomly loaded PIPN. This is expected as the former has fewer stages than the later.

3.2. Replicated PIPN performance under Type -I traffic model

In Type-I traffic, output ports are grouped. The number of groups is an integer power of two. The ports in the same group have an equal chance of being selected by any incoming cell. However, each group may have a different selection probability. The parameters for the traffic type are selected to create heterogeneous outlet requests. The number of parameters is selected as eight since eight is a reasonable value for the number of outlet groups in the range 16 -256 outlets [4, 5]. In Fig. 13, the normalized throughput for m= (0.3, 0 .02, 0.15, 0.00, 0.20, 0.06, 0.22, 0.05) Type-I traffic with respect to varying incoming load and different network sizes is shown for the R_2, R_4, S_2, and S_4 Replicated PIPN switches, and the original PIPN. The percentage throughput improvement obtained by the Replicated PIPN is shown in Table 1.

It is shown from the figure that the difference between the R_2, and R_4 curves increases as the switch size increase. This is expected as when the switch size increases, the number of stages increases resulting in more contention. R_4 provides more paths for the cells than R_2. The same reason applies to S_2 and S_4.

3.3. Replicated PIPN performance under Type -II traffic model

In Type -II traffic, the inlets and the outlets are both divided into groups. Although the size of input groups is fixed, the output groups have different sizes. Moreover, the selection probability of an output port group varies depending on the input port number that sends the cell [4, 5, 10].

As proved in [10], Type -II traffic represented by more than $\lceil \log_4 N \rceil +1$ parameters on a banyan network can be represented by using $\lceil \log_4 N \rceil +1$ parameters only. Therefore, there is no need to test the performance of the Replicated PIPN under Type -II traffic represented by more than $\lceil \log_4 N \rceil +1$ parameters.

The throughput of the R_2, R_4, S_2, and S_4 Replicated PIPN and original PIPN switches of size 256x256 is evaluated under 19 patterns of Type -II traffic represented by four parameters with incoming load 1.0. The traffic patterns are varied between uniform traffic and the extreme heterogeneous case which is possible under the given traffic type and parameters. The aim is to present the behavior of both the Replicated PIPN and the original PIPN under various traffic patterns. The throughput for all Type-II traffic patterns are shown in Table 3 in appendix A.

The obtained results are summarized in Table 2. The table shows the maximum, minimum, average throughput values, and the standard deviation of each network type under the given traffic set. It is shown from this table that the Replicated PIPN is superior over the original PIPN. When the replication technique is applied to PIPN, it gives high average throughput. However, the randomly loading technique gives the smallest throughput range (Max. – Min.) . This small throughput range is a good indication for the consistency of the switching system as the Randomly Loaded PIPN switch performance does not fluctuate when the applied traffic varies.

The throughput of the selectively loaded PIPN is expected to be better than that of the randomly loaded PIPN as it has fewer stages. However, It is shown from the given Type -II patterns that the selectively loaded PIPN is not always superior over the randomly loaded (patterns 6 -19). Under heterogeneous traffic models, the selectively loading technique may overload some subnetworks, increasing the number of collisions, while leaving other subnetworks lightly loaded.

4. Conclusions

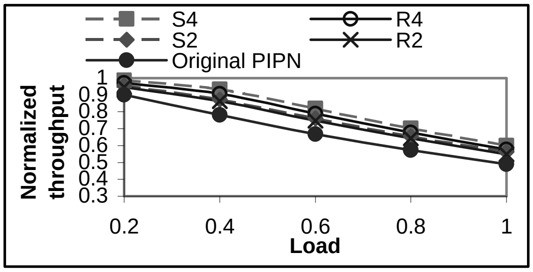
In this paper, a high performance banyan based ATM switch is introduced. The replication technique is applied to the PIPN. The switch uses the replication technique to provide multiple paths between inputs and outputs and uses the PIPN to smooth the heterogeneous traffic models. The existence of more paths between each input -output ports pairs makes the modified switches more reliable than the original PIPN.

The performance of two techniques for distributing cells among the subnetworks of the Replicated PIPN is examined analytically and by simulation. Both analytical and simulation results are coherent. It is shown that the Replicated PIPN gives better performance than the original PIPN under various traffic types. Buffer dimensioning analysis is performed to choose a suitable buffer size.

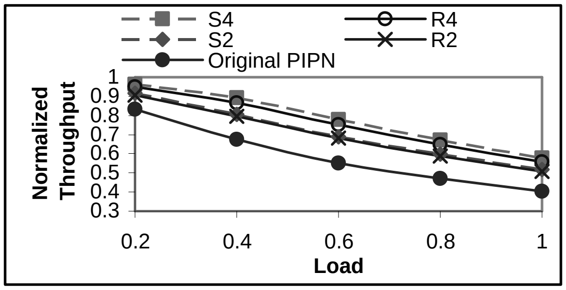
The analysis shows that selectively loading technique is better than the randomly loading technique under uniform traffic model. This is due to the fewer number of stages in the former technique. However, under heterogeneous traffic models, the randomly loading technique becomes better than the selectively loading technique as the second technique may overload some subnetworks while other subnetworks are lightly loaded causing more contention in the overloaded subnetworks while the randomly loading technique distributes incoming cells equiprobably among the subnetworks.

The resulting switch has a significant increase in performance under homogeneous and heterogeneous traffic models which supports the idea of using it as a new ATM switch.

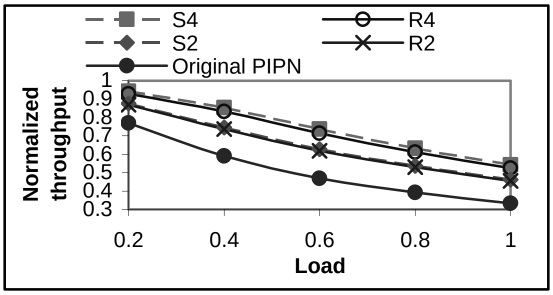
For future work, the performance of the switch can be tested under other arrival traffic models. The implementation aspects of the switches, such as cost and reliability, may be studied in more detail.



(a) N=16



(b) N=64



(c) N=256

Fig. 13. Performance under (0.30, 0.02, 0.15, 0.00, 0.20, 0.06, 0.22, 0.05) Type-I traffic for different network sizes

Table 1. Average percentage throughput improvement for replicated PIPN over the original PIPN

Net. Size	**R_2**	**S_2**	**R_4**	**S_4**
16x16	10.2	12.1	15.5	19.2
64x64	20.0	21.7	30.9	34.8
256 x 256	28.2	29.9	45.3	49.6

Table 2. Summary of performance results for 256x256 replicated PIPN and PIPN

Net. Type	**PIPN**	**R_2**	**S_2**	**R_4**	**S_4**
Min.	0.2571	0.4425	0.3154	0.6121	0.3914
Max.	0.3216	0.4943	0.5298	0.6452	0.6937
Avg.	0.3040	0.4823	0.4094	0.6365	0.5494
Max–Min	0.0645	0.0517	0.2143	0.0330	0.3023
Std. Dev.	0.0201	0.0156	0.0722	0.0097	0.0955

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Appendix A

Here we list the patterns of Type-II traffic model used to compare the performance of the replicated and original PIPN [4, 5].

Table 3. Throughput of 256x256 replicated PIPN and PIPN under various Type-II traffic patterns

No	Type-II Traffic	Original PIPN (thr)	R_2 (thr)	S_2 (thr)	R_4 (thr)	S_4 (thr)
1	(0.12, 0.13, 0.25, 0.50)	0.321455	0.49436	0.52983	0.64185	0.69378
2	(0.05, 0.05, 0.45, 0.45)	0.318803	0.49306	0.52803	0.64381	0.66568
3	(0.05, 0.45, 0.10, 0.40)	0.311484	0.48618	0.51738	0.63853	0.64850
4	(0.45, 0.05, 0.05, 0.45)	0.310813	0.48574	0.52115	0.63896	0.64041
5	(0.00, 0.20, 0.00, 0.80)	0.319086	0.49078	0.47911	0.63695	0.63532
6	(0.45, 0.05, 0.40, 0.10)	0.317029	0.48899	0.41857	0.64071	0.61593
7	(0.40, 0.30, 0.20, 0.10)	0.314419	0.49033	0.41687	0.64202	0.56963
8	(0.30, 0.00, 0.60, 0.10)	0.3175056	0.49177	0.41542	0.64265	0.59706
9	(0.50, 0.25, 0.15, 0.10)	0.307978	0.48532	0.41593	0.64025	0.55118
10	(0.05, 0.45, 0.45, 0.05)	0.318976	0.49070	0.38730	0.64201	0.59481
11	(0.00, 0.00, 0.00 1.00)	0.321693	0.49148	0.35410	0.63361	0.56148
12	(0.25, 0.25, 0.50, 0.00)	0.320476	0.49418	0.35364	0.64522	0.56473
13	(0.70, 0.15 0.10, 0.05)	0.293638	0.47319	0.37188	0.63425	0.48825
14	(0.45, 0.45, 0.05, 0.05)	0.298879	0.48488	0.38102	0.64292	0.46144
15	(0.00, 0.20, 0.80, 0.00)	0.311829	0.49018	0.35228	0.64206	0.51480
16	(0.80, 0.10, 0.06, 0.04)	0.284395	0.46394	0.35225	0.62463	0.46107
17	(0.00, 0.00, 1.00 0.00)	0.291829	0.48364	0.35382	0.64009	0.39243
18	(1.00, 0.00, 0.00, 0.00)	0.257484	0.44259	0.31545	0.61262	0.39147
19	(0.00, 1.00, 0.00, 0.00)	0.257191	0.44362	0.31572	0.61217	0.39152