Engineering Journal of the University of Qatar, Vol. 15, 2002, pp. 39-62

STRUCTURE MIXTURE RESPONSE OF BUILDINGS DUE TO SIMULATED EARTHQUAKE LOADING

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ABSTRACT

This paper presents two case studies to evaluate the performance of the structure mixture theory in the earthquake response analysis of buildings. In the first case study, the actual earthquake responses of the two-story reinforced concrete building, which was tested at the University of California at Berkeley, were compared to those obtained using the structure mixture theory. In the second case study, the seven-story reinforced concrete building, which was tested as a part of the US-Japan Cooperative Earthquake Program, was considered. Both case studies showed that the structure mixture theory results were very close to experimental results. The mean differences between the maximum roof displacements obtained experimentally in the first and second case studies and those obtained using the structure mixture theory were found to be equal to 3.5% and 3.2%, respectively.

1. INTRODUCTION

Al-Ansari et. al. (1996) presented a structure mixture theory for the earthquake analysis of two-dimensional structural systems. In this approach, the structural system is considered as mixture of two interacting subsystems: columns and beams/floors. The response of each subsystem is described by its own linear differential operator. The two responses are then subjected to matching boundary conditions which couple them at the subsystem interface. The solution of the coupled equations is carried out using a pair of perturbation series.

The structure mixture technique is an advantageous technique for the earthquake response analysis of two-dimensional buildings because it does not deal with initial conditions and time integration and spectral charts. Moreover, the technique does not require the use of the complete damping matrix for the structure as long as the modal damping ratios are known.

During the past 35 years, a number of experimental studies on the behavior of multi-story buildings under earthquake loading were carried out (Memari et. al. 1999, Filiatrault et.al. 1998, Stephen et. al. 1985, ACI 1985, and Clough and Gidwani 1976). These studies have resulted in a better understanding of the behavior of buildings during earthquakes. The studies' data has been utilized in the development of analytical methods as well as codes of practice for the analysis and design of earthquake-resistant buildings (Filiatrault et.al. 1998, ACI 1995, NBCC 1995, and UBC 1994).

This paper presents a brief description of the structure mixture theory and investigates, using two case studies, its performance in the earthquake response analysis of multi-story buildings. The first case study is concerned with the earthquake responses of the two-story reinforced concrete building, which was tested at the University of California at Berkeley (Clough and Gidwani 1976) while the second is concerned with the seven-story reinforced concrete building, which was tested as a part of the US-Japan Cooperative Earthquake Program (ACI 1985). The simulated earthquake responses of the two experimental buildings were compared to those obtained using the structure mixture theory.

2. STRUCTURE MIXTURE THEORY OVERVIEW

It is always possible to view a building as being composed of two distinct but interlocked structural subsystems . The supporting columns are treated as subsystem 1 while the beams/ floors are treated as subsystem 2. It is possible to represent the effects of subsystem 2 in a relatively simple manner by means of a set of translational (horizontal) and rotational spring stiffeners. A typical one-story building is modeled as two subsystems as shown in Figure 1. The effect of the beam-column interaction is modeled by a set of stiffeners positioned at the interface as shown in Figure 2.

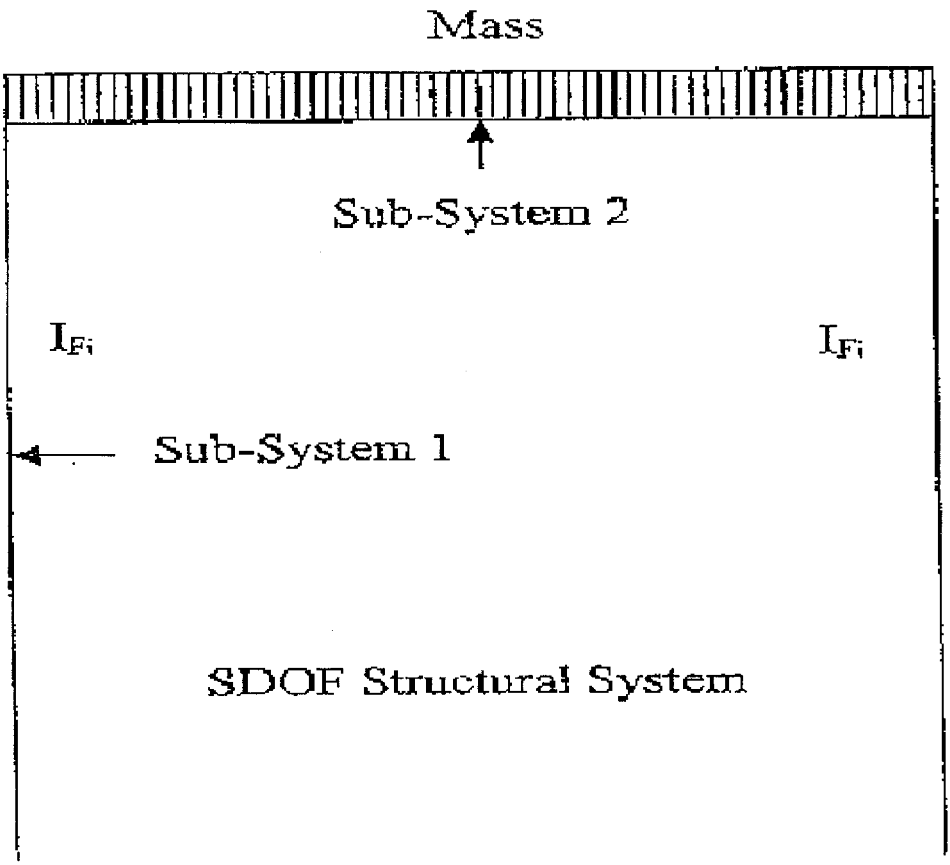


Fig. 1 Structure Mixture Frame

Transverse Dynamic Solution for Subsystem 1 (Columns)

The transverse deflection amplitude of columns is given by the following differential equation:

$$\frac{d^4 y_i}{dx^4} - \kappa_i{}^4 y_i = 0. \qquad (1)$$

The wave number, κ_i, is given by the following equation:

$$\kappa_i = \sqrt{\frac{\Omega}{c_i \, r_{g_i}}} \qquad (2)$$

where c_i = bulk wave speed in the column material; Ω = circular (radian) frequency; and r_{gi} = column radius of gyration. The equation for transverse waves in a column is represented by Eq. 1. It has been chosen because the earthquake loading at the foundation of the structure is predominantly transverse with respect to the columns.

The deflection amplitude of the general column section is given by the following equation (general solution of Eq. 1):

$$y_i = A_i\, e^{\kappa x} + B_i e^{-\kappa x} + C_i \cos(\kappa x) + D_i \sin(\kappa x) \qquad x_{i-1} \le x \le x_i \qquad (3)$$

where the four constants of integration A_i, B_i, C_i, and D_i are determined by the boundary conditions at x_i and x_{i-1} (Al-Ansari et. al. 1996). The general subscript index i represents the column sections between stories in sequence from the ground up (for the Single degree of freedom system under discussion we always have i = 1). For multi-story structures, additional boundary conditions are given at the other interface points x_i (i = 2, 3, ..., n-1) as shown in Figure 2.

Horizontal Response of Subsystem 2 (beams/floors).]

Considering both damping and support motion, the normalized deflection amplitude at the interface node is given by the following equation (Al-Ansari et. al. 1996):

$$\tilde{\delta}_i = \frac{R_i\, r_i^2}{\sqrt{(1-r_i^2)^2 + (2\,\xi\, r_i)^2}}\ \eta_i \tag{4}$$

where ω_i = natural frequency, Ω = forcing frequency, r_i = frequency ratio (Ω/ω_i), ξ = damping ratio of the frame, and F_i = force amplitude at the interface node i. η_i is given by the following equation:

$$\eta_i = (A_i - A_{i+1})e^{\phi_i} - (B_i - B_{i+1})e^{-\phi_i} + (C_i - C_{i+1})\sin\phi_i - (D_i - D_{i+1})\cos\phi_i \tag{5}$$

where $\phi_i = \kappa_i\, x_i$

The ratio of the column stiffness to the total frame stiffness, R is given by the following equation:

$$R = \frac{I_i}{I_{F_i}} \tag{6}$$

Rotational Response of Subsystem 2 (beam/floor)

Considering system damping and support motion, the rotation normalized amplitude at the interface node i is given by the following equation (Al-Ansari et. al. 1996):

$$\Phi_i = \frac{R_i\, r_i^2}{\sqrt{(1-r_i^2)^2 + (2\xi r_i)^2}}\ \hat{\eta}_i \tag{7}$$

where $\hat{\eta}_i$ is given by the following equation:

$$\hat{\eta}_i = (A_i - A_{i+1})e^{\phi_i} + (B_i - B_{i+1})e^{-\phi_i} - (C_i - C_{i+1})\sin\phi_i - (D_i - D_{i+1})\cos\phi_i \tag{8}$$

Perturbation Procedure

The perturbation procedure (Gillette 1991 and 1993) starts with the determination of the zero-order solution constants A_i^0, B_i^0, C_i^0, and D_i^0. The success of the perturbation procedure is highly dependent on the chosen zero-order (initial) solution. In the current work, the zero-order solution was obtained using the loading case shown in Figure 3.

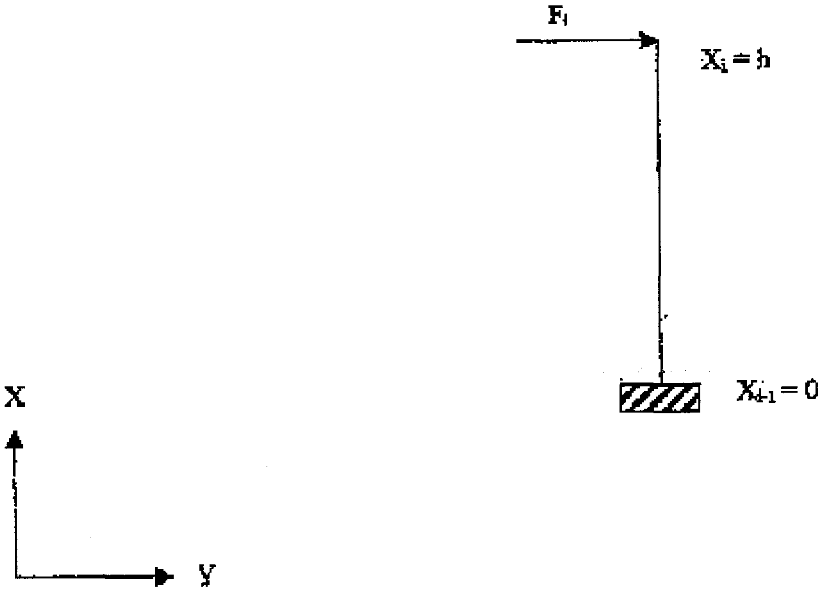


Fig. 3 Free end loading model

The selected loading case reflects an acceptable earthquake design which recommends that the beam-column connections be rigid and that flexural hinges form in beams rather than in columns.

This loading case leads to the following boundary conditions at the interface nodes:

$$\text{At point } x_{i-1} \qquad y_i = \delta_{i-1} = 1 \qquad \frac{1}{\kappa_i}\frac{dy_i}{dx} = 0$$

$$\text{At point } x_i \qquad EI_{F_i}\frac{d^3y_i}{dx^3} = 0 \qquad EI_{F_i}\frac{d^2y_i}{dx^2} = 0 \tag{9}$$

Applying these boundary conditions to Eq. 3 yields the zero-order constants A_i^0, B_i^0, C_i^0, and D_i^0 (Al-Ansari et. al. 1996):

The computed zero-order constants A_i^0, B_i^0, C_i^0, and D_i^0 are then used to determine the first-order responses of both subsystems (i.e., columns and beams). The first-order response $y_i^{(1)}$ of all columns are computed using Eq. 3. After that, the horizontal responses of the beam are determined using Eq. 5 while the rotational responses are determined using Eq. 7.

In the next step, the first-order constants A_i^1, B_i^1, C_i^1, and D_i^1 are computed. These constants are then used to compute the second-order responses $y_i^{(2)}$ of all columns.

The same computational procedure is repeated until the convergence of the perturbation series is reached.

3. PROGRAM FLOWCHART

A computer program has been written in Quick Basic to implement the structure mixture theory procedure. The program computational algorithm is shown in Figure 4.

The first step of the program consists of reading the input data. The second step of the program involves the computation of the structure natural frequencies and corresponding mode shapes. The computation is performed as follows:

1) Assemble the structure mass and stiffness matrices.

2) Calculate the natural frequencies and corresponding mode shapes using Jacobi's method.

The third step of the program includes the following computation tasks for each forcing frequency:

1. The wave numbers, κ_i, is computed using Eq. 2.

2. The zero-order solution constants A_i^0, B_i^0, C_i^0, and D_i^0 are computed.

3. The first-order responses $y_i^{(1)}$ of the column sections are computed using Eq. 3. The horizontal and rotational responses of the beams are then computed using Eqs. 5 and 7, respectively.

4. The first-order solution constants A_i^1, B_i^1, C_i^1, and D_i^1 are computed .

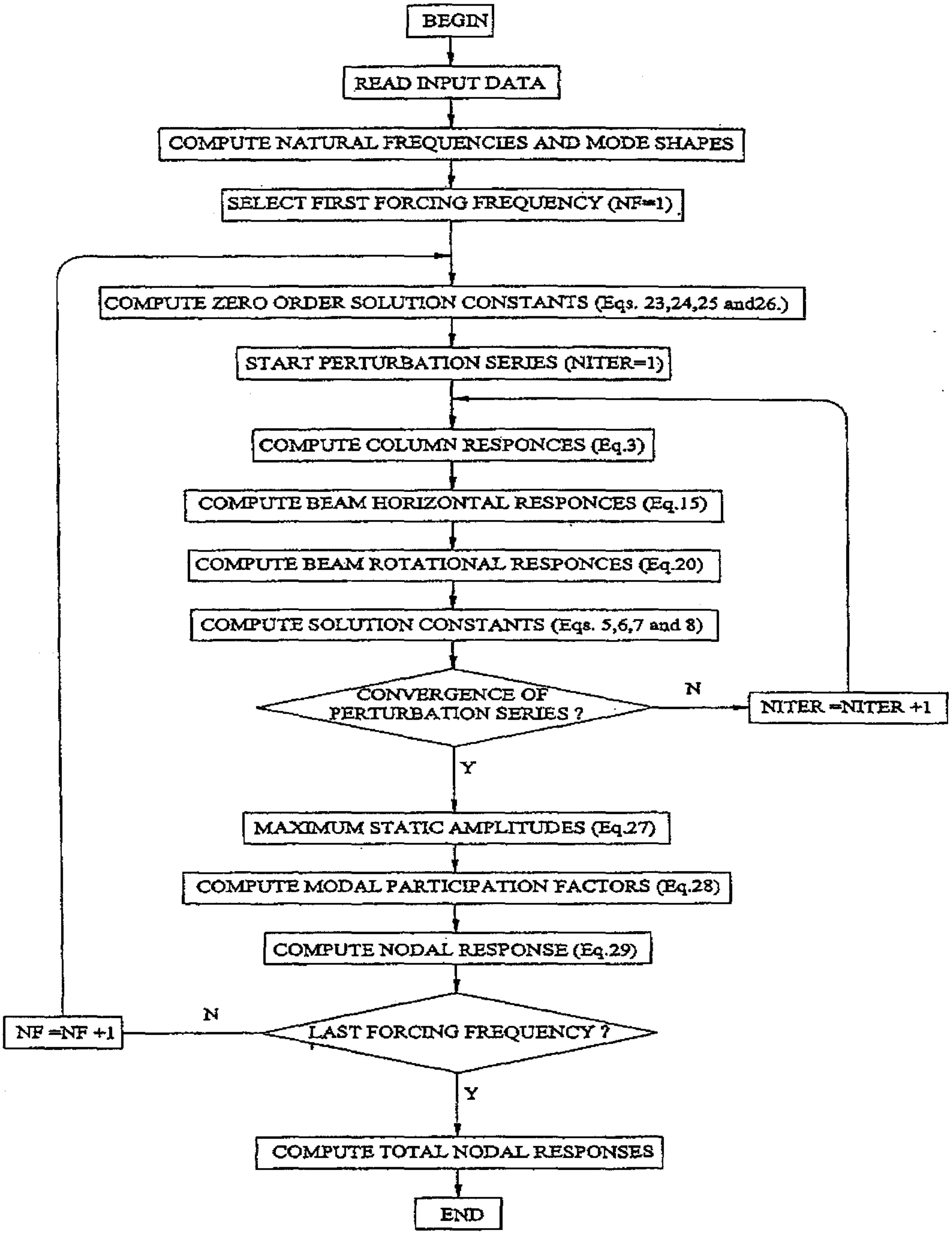


Fig. 4 program flowchart

5. Steps 3 and 4 are repeated until the perturbation series converges.

6. The final nodal displacements δ_i and slopes θ_i are determined at the beam/column interfaces.

7. The maximum amplitudes y_{0i} are computed using the following equation:

$$y_{O_i} = \frac{Fo_i}{K_i\sqrt{1+(2 * r_i * \xi)^2}} \tag{10}$$

where Fo_i = nodal static load, K_i = frame shear stiffness, r_i = frequency ratio, and ξ = damping ratio of the frame.

8. The modal participation factors Γ_i are computed using the following equation:

$$\Gamma_i = \sum_{j=1}^{n} M_j a_{ij} \qquad (i = 1,2,\ldots..n) \tag{11}$$

where n = number of nodes (floors), [M] = structural mass matrix, and [a] = mode shape matrix.

9. The expected nodal deflections are computed using a modified SRSS method (Paz 1991) as follows.

$$u_i = \sqrt{\sum_{j=1}^{n} (\Gamma_i a_{ij} \delta_j)^2} * y_{O_i} * \frac{1}{R_i} \tag{12}$$

10. Finally, the total nodal responses are found by adding the results obtained in Step 9 for all forcing frequencies using the following equation:

$$u_{T_i} = \sum_{j=1}^{NF} u_j \tag{13}$$

where NF = number of forcing frequencies.

4. PROGRAM INPUT DATA

The program input data consists of three parts. The first part contains the following general input information: number of nodes (floors), number of forcing frequencies, maximum number of iterations, and convergence tolerance. The second part of the input data includes the following structural model information: Total building height, story heights, story beam-column stiffness ratios (I_i/I_{Fi}), wave propagation factors (c r_{gi}), story total stiffnesses (E I_i), and floor masses. The third part of the input data includes the forcing frequencies, the corresponding nodal damping ratio (ξ), and the maximum nodal static loads (Fo). Figure 5 shows a sample input data echoed by the program.

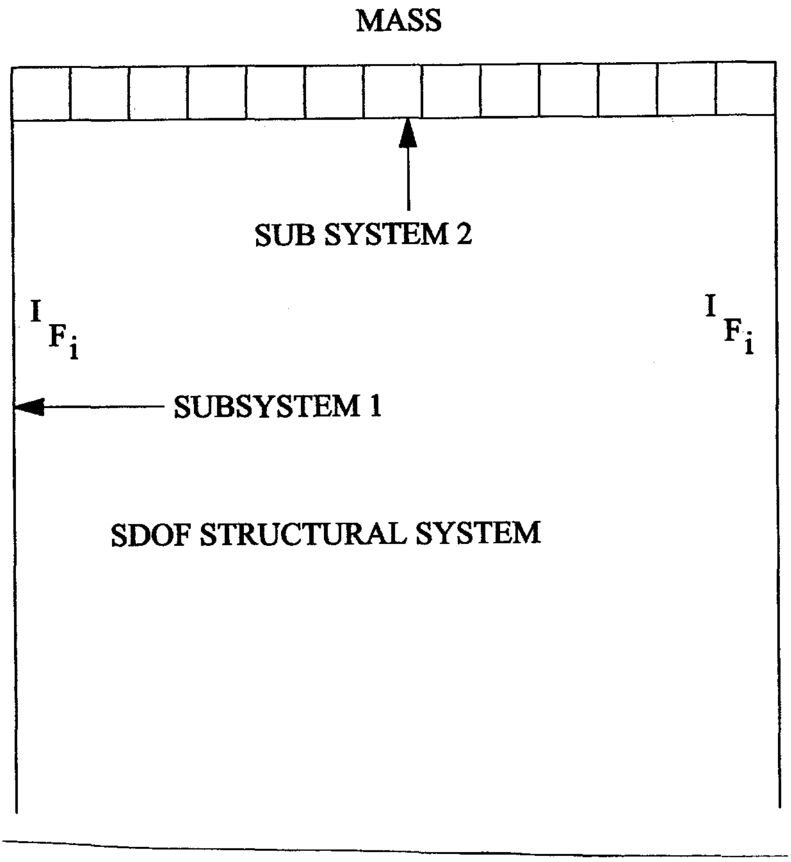


Fig. 5 Program Sample Input Data

5. PROGRAM OUTPUT DATA

The program output data consists of four parts. The first part contains the vertical and torsional expansion parameters. The second part includes the following nodal amplitude information: forcing frequency, number of iterations, node numbers, and nodal amplitudes. The third part includes the static displacement information consisting of forcing frequencies, floor numbers, and static nodal displacements. The fourth part includes the floor displacements for each forcing frequency and the total floor displacements. Figure 6 shows a sample output of the program.

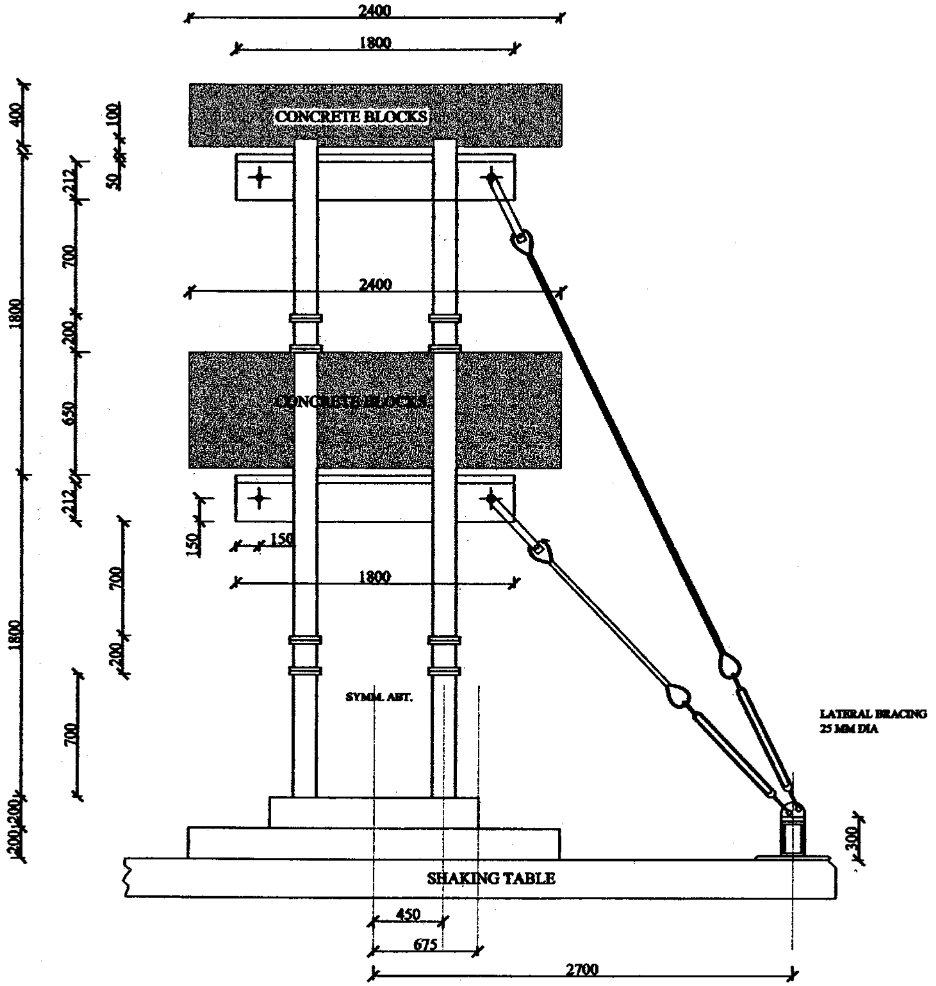


Fig. 6 Program Sample Output

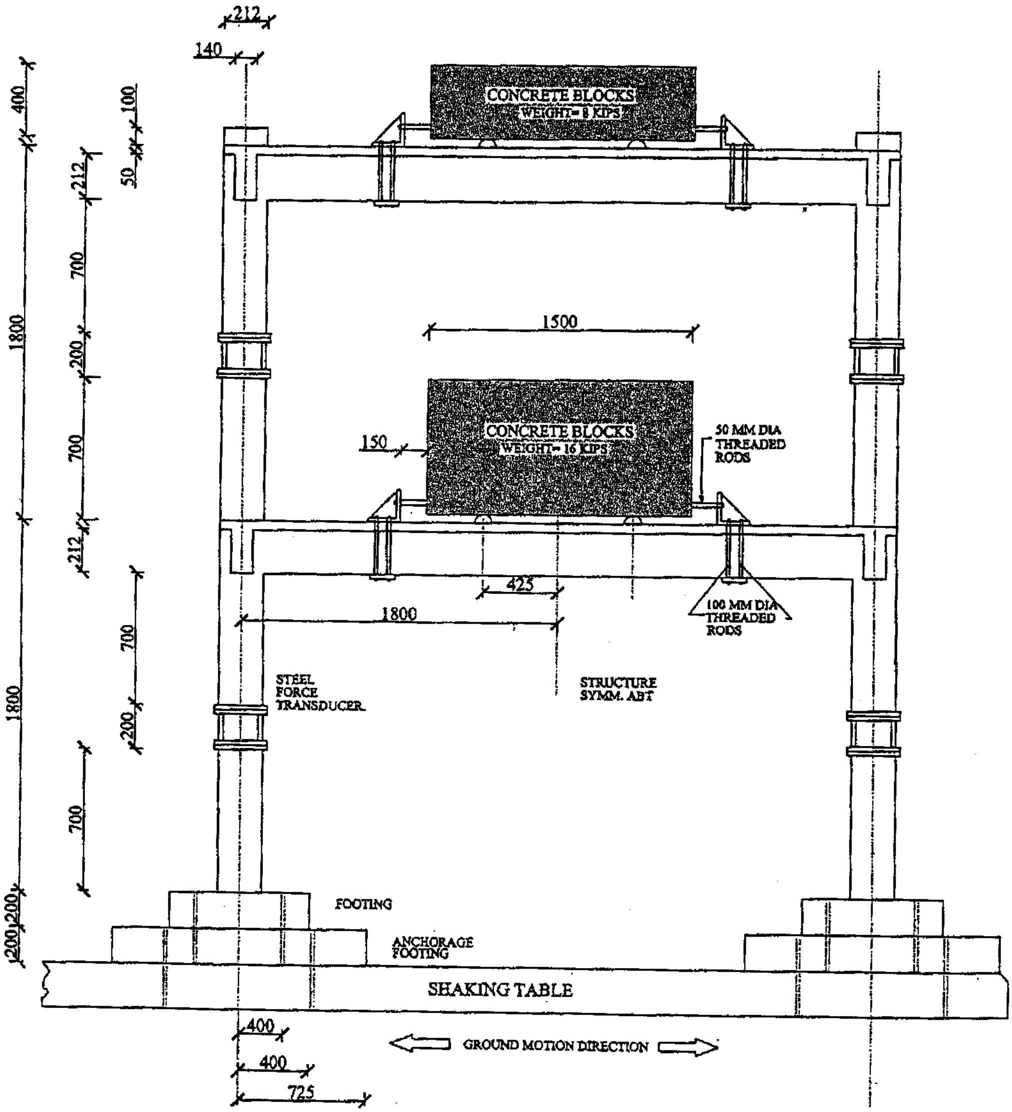
6. CASE STUDY #1

In the first case study, the experimental earthquake responses of a full-size two-story reinforced concrete building were compared to those obtained using the structure mixture theory. The building was tested at the University of California at Berkeley under a National Science Foundation research project grant (Clough and Gidwani, 1976). Figures 7 and 8 show the front and side elevation of the test structure, respectively. Table 1 summarizes the structural model input data. Three simulated earthquakes, labeled W1, W2, and W3 were applied to the test structure. Figure 9 shows the input ground motion used to simulate test W1. Table 2 summarizes the peak input ground acceleration, damping, and dominant forcing frequencies for the tests W1, W2, and W3.

Table 3 summarizes the maximum roof displacements obtained using the structure mixture theory as well as those obtained experimentally. The roof displacements obtained using the structure mixture theory were compared to those obtained experimentally. The average absolute percent difference and the maximum absolute percent difference between the roof displacements obtained using the structure mixture theory and those obtained experimentally are equal to 3.5% and 7.2% (test W2), respectively. This shows that the structure mixture theory results are close to those obtained experimentally.

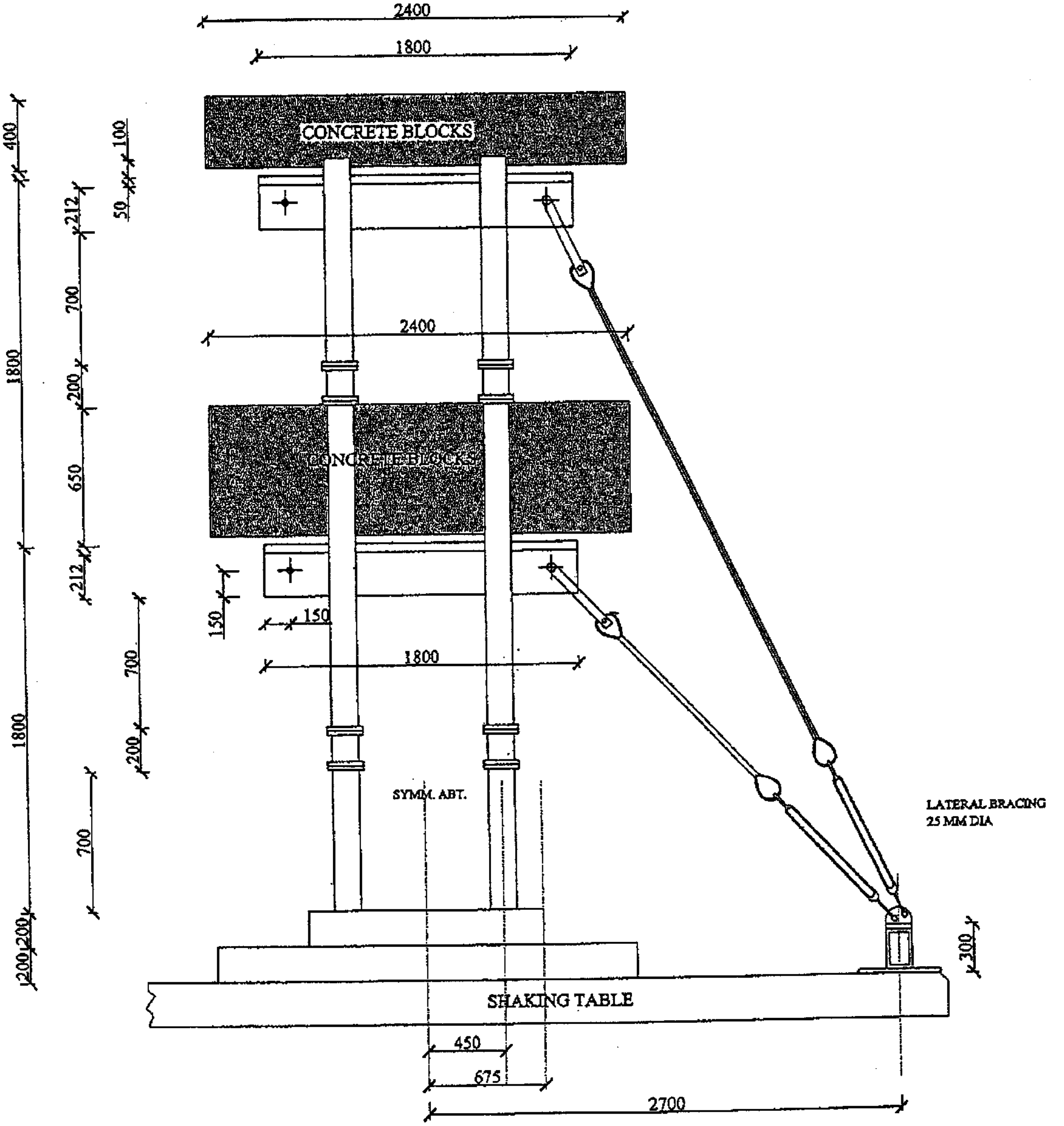
7. CASE STUDY #2

In the second case study, the experimental earthquake responses of a full-size seven story reinforced concrete building were compared to those obtained using the structure mixture theory. The building was tested as part of the US-Japan Cooperative Earthquake Program (ACI, 1985). Figure 10 shows a general plan view of the building while Figure 11 shows a general elevation of the building (frame B). The central frame has a shear wall in the central bay which is continuous from the first through the seventh story. The column dimensions are 500 mm x 500 mm throughout the structure. The dimensions of the girders parallel to the loading direction are 300 mm x 500 mm from the second to the roof level. The shear wall parallel to the loading direction has a thickness of 200 mm. The floor slabs have a thickness of 120 mm throughout the structure. Table 4 summarizes the structural model input data.



Dimensions are in millimeters

Fig. 7 Two – story building front elevation



Dimensions are in millimeters

Fig. 8 Two – Story building elevation

Table 1. Structural model input data for the two-story building

Floor Number	Floor Height (m)	Floor Mass (kg)	Floor Shear Stiffness (kN/m)	Floor Stiffness Ratio (I/I_F)	Wave Propagation Factor (C rg) (m^2/s)
1	1.8	10010	5347	0.5	173
2	2.2	6160	4213	0.5	173

Table 2. Peak ground acceleration, damping, and forcing frequency for W1, W2, W3

Test	Input Signal	Ground Acceleration (% g)	Percent Damping (%)	Dominant Forcing Frequency (Hz)
W1	TAFT N69W	9.7	2	2.53
W2	TAFT N69W	57.0	2	2.53
W3	TAFT N69W	65.0	2	2.53

Table 3. Max. experimental and computed lateral displacements for Case-Study #1

Test Number	Experimental Maximum Displacements (mm)	Computed Maximum Displacements (mm)
W1	11.0	11.1
W2	69.2	64.2
W3	70.4	72.1

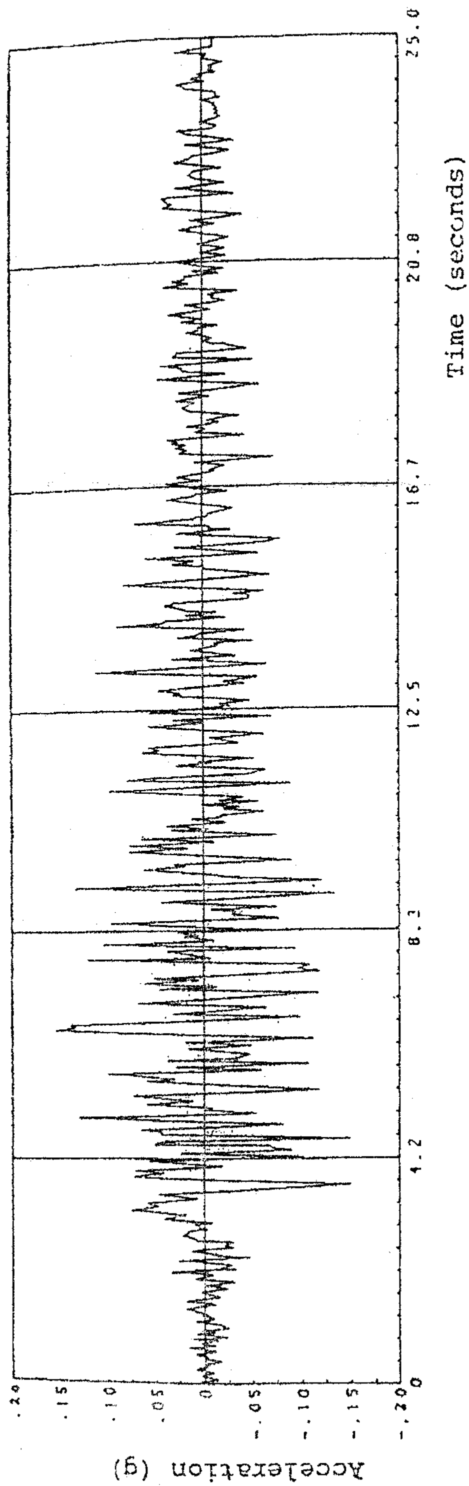
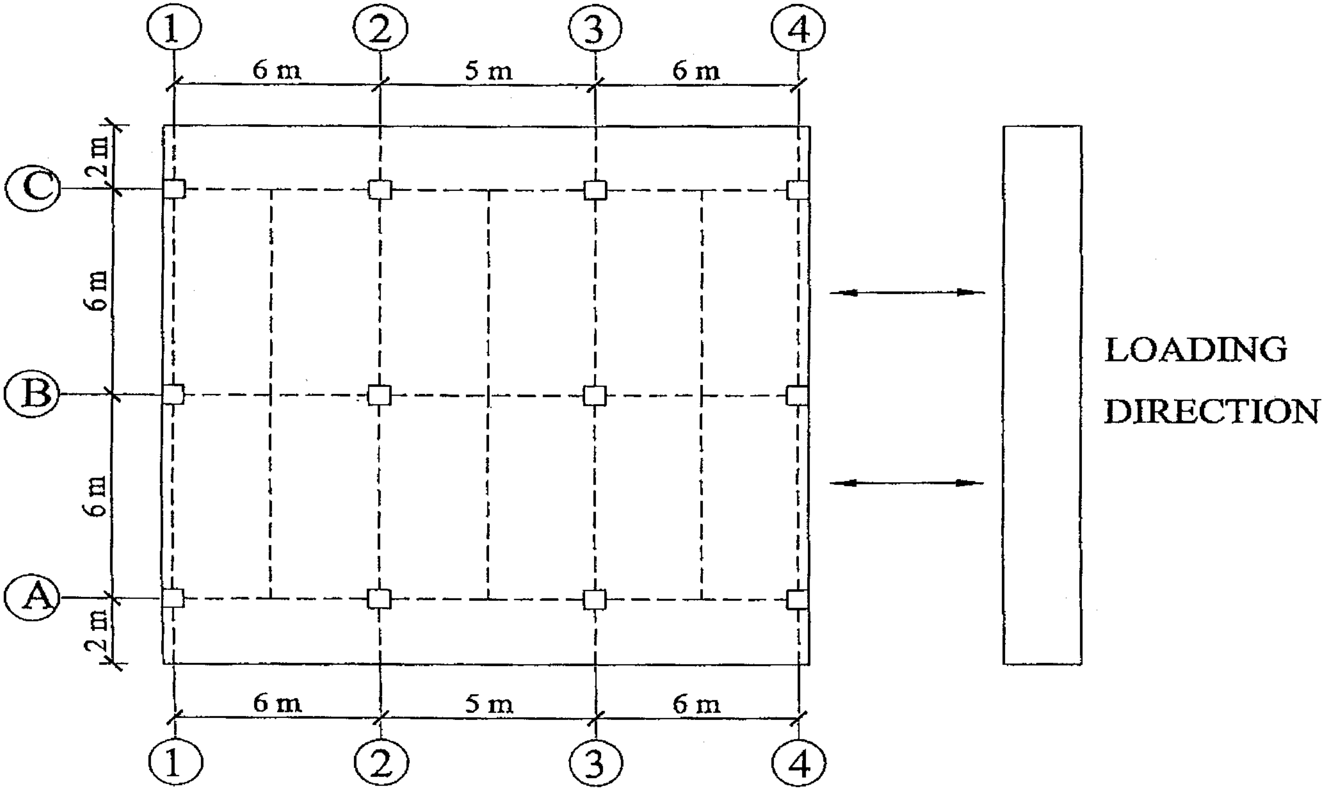


Fig. 9 Input ground motion for test w1



TYPICAL FLOOR PLAN

Fig. 10 Seven –story building plan view.

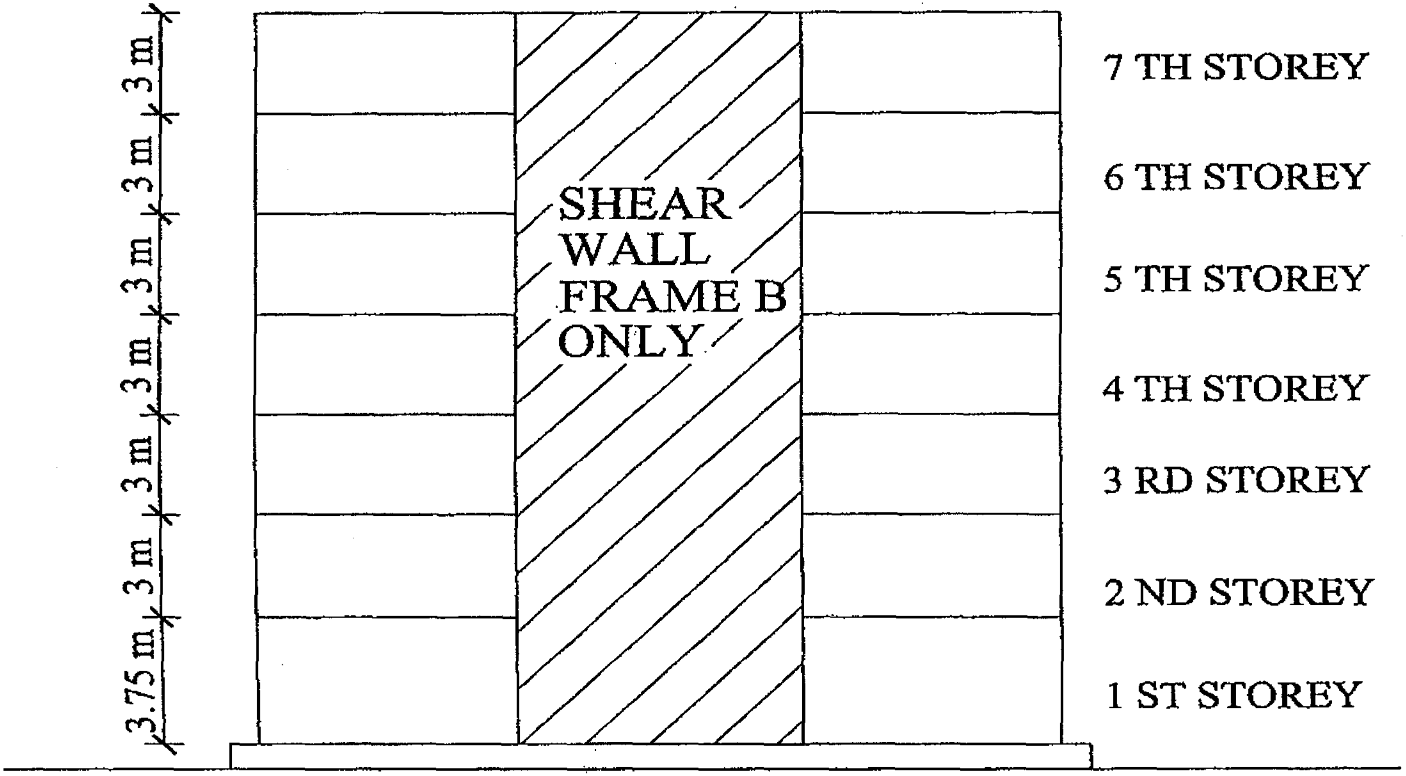


Fig.11 Seven-story building elevation

Table 4. Structural model input data for seven-story building

Floor Number	Floor Height (m)	Floor Mass (kg)	Floor Shear Stiffness (kN/m)	Floor Stiffness Ratio (I/I_F)	Wave Propagation Factor (C rg) (m^2/s)
1	3.75	182700	222400	0.08	447
2—6	3.00	177300	439700	0.08	447
7	3.00	136000	439700	0.08	447

A simulated ground motion test, labeled SPD-1, was applied to the test structure. Figure 12 shows the test input ground motion while Figure 13 shows the fast Fourier transform of the test input ground motion acceleration . Table 5 summarizes the peak input ground acceleration, damping, and forcing frequencies for the test.

Table 6 summarizes the maximum roof displacement for the building obtained using the structure mixture theory as well as that obtained experimentally. The roof displacement obtained using the structure mixture theory was compared to the one obtained experimentally. The absolute percent difference between the roof displacement obtained using the structure mixture theory and that obtained experimentally is equal to 3.2%. This shows that the structure mixture theory result is close to the experimental result.

8. CONCLUSION

Two case studies were conducted to evaluate the performance of the structure mixture theory in the earthquake response analysis of buildings. In the first case study, the actual earthquake responses of a two-story reinforced concrete building, which was tested at the University of California at Berkeley, were compared to those obtained using the structure mixture theory. In the second case study, the actual earthquake responses of a seven-story reinforced concrete building, which was tested as a part of the US-Japan Cooperative Earthquake Program, was considered. In both case studies, the experimental results were found to be close to those obtained using the structure mixture theory. The mean differences between the maximum roof displacements obtained experimentally in the first and second case

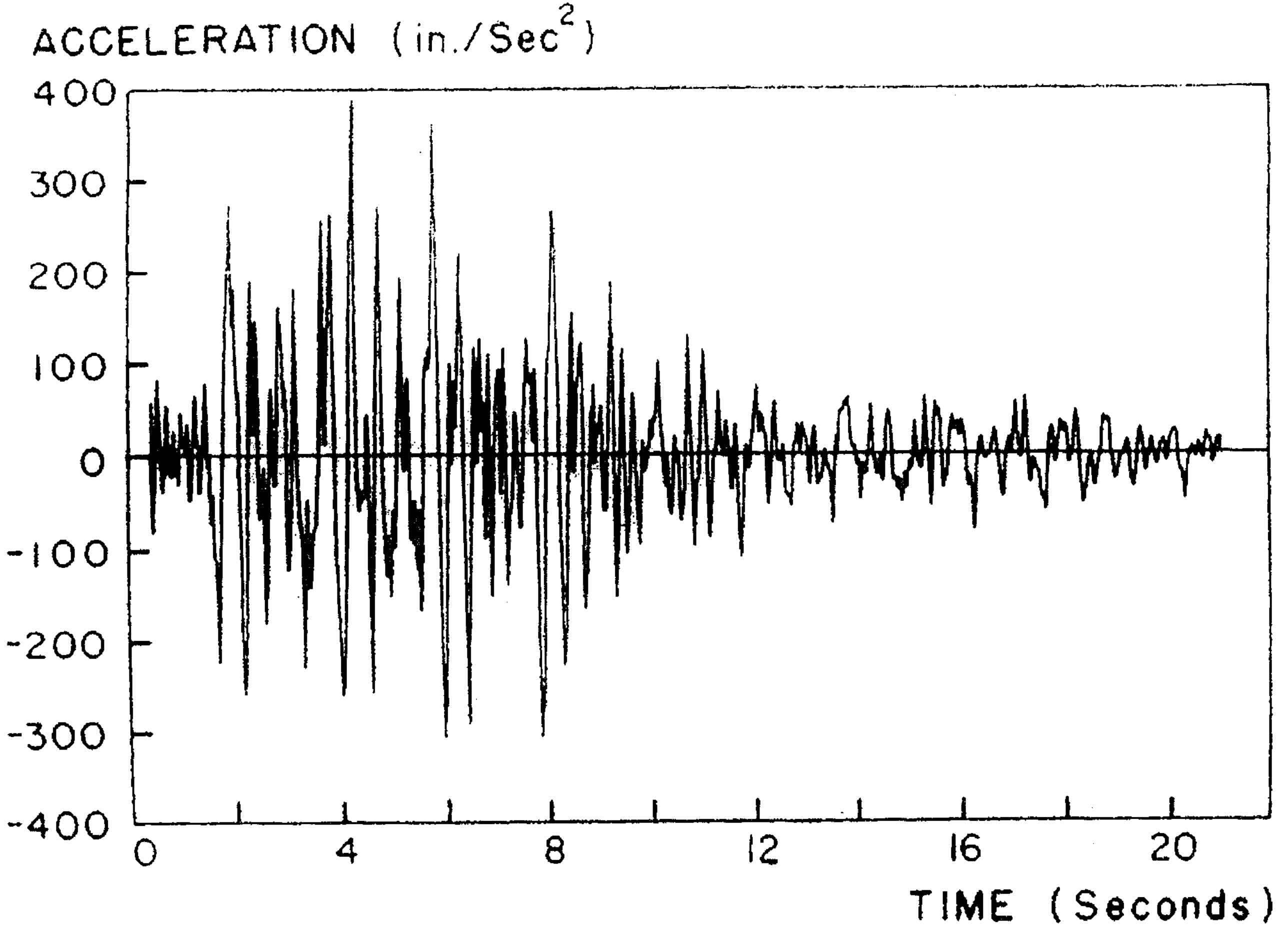


Fig. 12 Acceleration records for test SPD-1 (normalized to peak of 1.0 g)

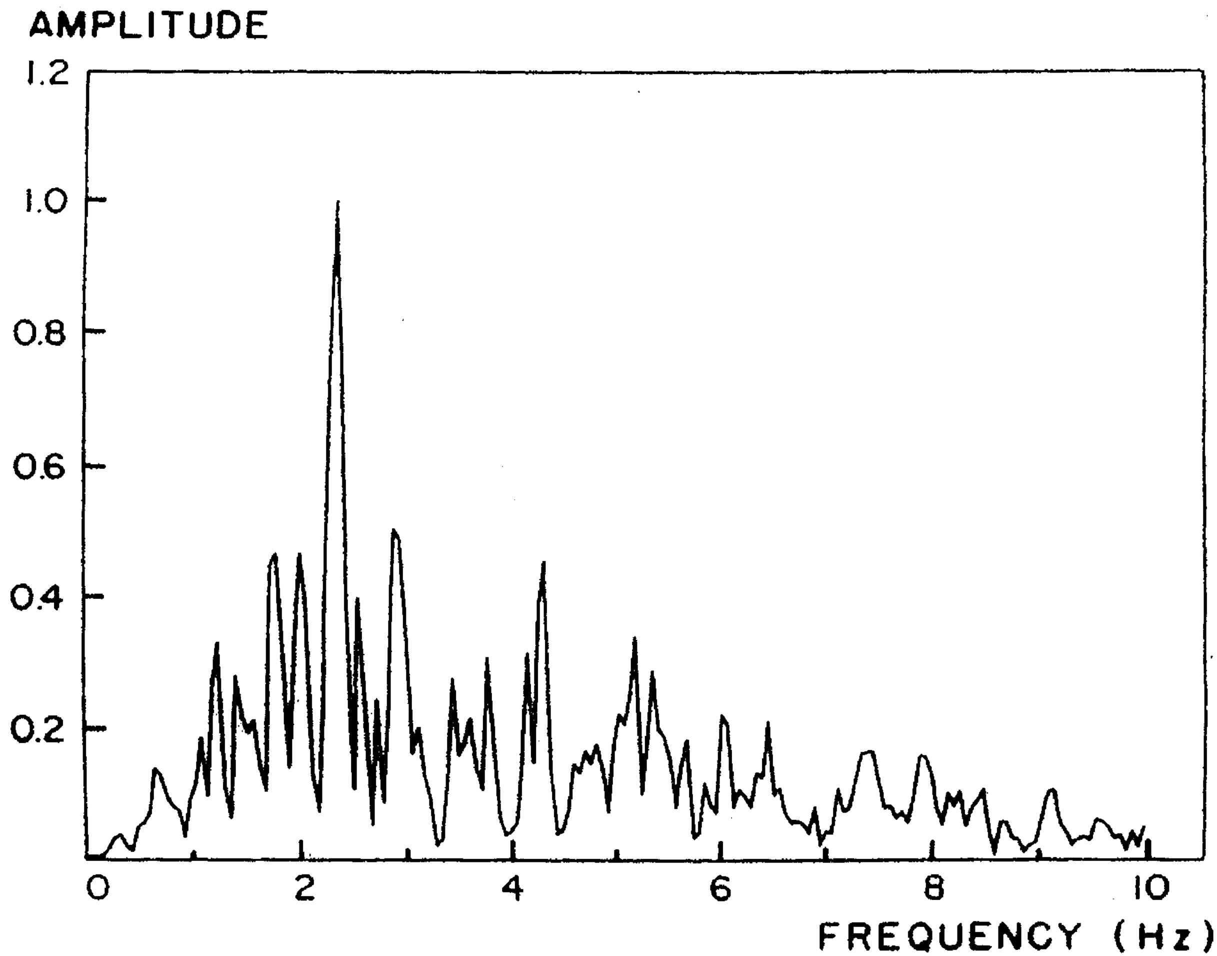


Fig. 13 Fourier amplitude spectra of acceleration records

Table 5. Peak ground acceleration, damping, and forcing frequency for SPD-1.

Test	Input Signal	Ground Acceleration (% g)	Percent Damping (%)	Dominant Forcing Frequency (Hz)
PSD-1	Miyagi-Oki	2.40	2.1	1.82

Table 6. Max. Experimental and computed lateral displacements for Case-Study #2

Test Number	Experimental Maximum Displacements (mm)	Computed Maximum Displacements (mm)
SPD-1	2.5	2.56

studies and those obtained using the structure mixture theory were found to be equal to 3.5% and 3.2%, respectively. These results indicate that the performance of the structure mixture theory is adequate in the earthquake analysis of multi-story buildings.

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