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Structure of a Viscoplastic Theory

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STRUCTURE OF A VISCOPLASTIC THEORY

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ABSTRACT

The general structure of a viscoplastic theory is developed from physical and thermodynamical considerations. The flow equation is of classical form. The dynamic recovery approach is shown to be superior to the hardening function approach for incorporating nonlinear strain hardening into the material response through the evolutionary equation for back stress. A novel approach for introducing isotropic strain hardening into the theory is presented, which results in a useful simplification. In particular, the limiting stress for the kinematic saturation of state (not the drag stress) is the chosen scalar-valued state variable. The resulting simplification is that there is no coupling between the two state variables in the flow equation, and there is no coupling between dynamic and thermal recovery terms in each evolutionary equation. The derived theory of viscoplasticity has the structure of a two-surface plasticity theory when the response is plasticlike, and the structure of a Bailey-Orowan creep theory when the response is creeplike.

I. INTRODUCTION

The development of power systems with greater thermodynamic efficiency makes the need for accurate analytical representations of inelastic deformation a necessity. These mathematical models must be capable of accurately predicting short-term plastic strain, long-term creep strain, and the interactions between them. Multiaxial, cyclic, and nonisothermal

conditions are the norm, not the exception. This formidable task has received considerable attention over the past three decades, resulting in an emerging field of continuum mechanics called viscoplasticity.

The theoretical development of viscoplasticity has its origin with the works of STOWELL [1957], PRAGER [1961], and PERZYNA [1963], whose theories did not contain evolving internal state variables. The field blossomed in the 1970's when rapid advances in computing technology enabled accurate solutions to be obtained readily. Internal state variable theories began to appear in the models of GEARY & ONAT [1974], BODNER & PARTOM [1975], HART [1976], MILLER [1976], PONTER & LECKIE [1976], CHABOCHE [1977], KRIEG et al. [1978], and ROBINSON [1978]. Theoretical refinements have continued to occur throughout the 1980's in the models of STOUFFER & BODNER [1979], VALANIS [1980], WALKER [1981], SCHMIDT & MILLER [1981], CHABOCHE & ROUSSELIER [1983], ESTRIN & MECKING [1984], KREMPL et al. [1986], LOWE & MILLER [1986], and ANAND & BROWN [1987]. Reviews on various aspects of viscoplasticity have been written by PERZYNA [1966], WALKER [1981], CHAN et al. [1984], LEMAITRE & CHABOCHE [1985], and SWEARENGEN & HOLBROOK [1985]. Although this listing is by no means complete, it is representative of the work that has been done in viscoplasticity and of the attention that it has received.

The purpose of this paper is to systematically develop the thermodynamically admissible structure of a unique theory of viscoplasticity (given in eqn (20)). This theory is unique in how it accounts for isotropic hardening; otherwise, it is consistent with the current state of the art. Its development was most strongly influenced by the viscoplastic theories of PONTER & LECKIE [1976] and CHABOCHE [1977]. Ponter and Leckie's theory is based on a Bailey-Orowan theory of creep (BAILEY [1926] and OROWAN [1947]), whereas Chaboche's

theory is based on a two-surface theory of plasticity (DAFALIAS & POPOV [1975] and KRIEG [1975]). Others have also formulated viscoplastic theories based on those of Ponter and Leckie (e.g., ROBINSON [1978]) and Chaboche (e.g., WALKER [1981]).

The concept of internal state variables, as it applies to polycrystalline metals, and the thermodynamic constraints placed on the theoretical formulation are discussed in the next section. In Section III, an Ω-form theory of viscoplasticity is derived for kinematic hardening materials. Two approaches to introduce nonlinear strain hardening into the theory are compared in Section IV. The dynamic recovery approach is preferred over the hardening function approach for realistic modeling of material behavior. A novel approach for incorporating isotropic hardening into the theory is presented in Section V, wherein the limit stress for the kinematic saturation of state is considered to be the independent state variable.

II. INTERNAL STATE

This paper addresses isothermal material behavior of viscoplastic materials, such as polycrystalline metals. Each material element is considered to be isotropic and to carry no stress in its initial virgin condition. But while each material element deforms homogeneously, it may lose some or all of its material symmetries. Small strains, displacements, and rotations are considered to compose the deformation of each material element. The formulation of this section is based on the works of Onat and his colleagues (e.g., GEARY & ONAT [1974], ONAT [1986], and ONAT & LECKIE [1988]).

The changing internal structure of a material element is characterized by the pair $(\underset{\sim}{\sigma},\underline{S})$, where $\underset{\sim}{\sigma}$ is the state of stress and $\underline{S}$ is the state of a finite number of internal state variables: that is, $\underline{S} = \{\underline{s}_1,\underline{s}_2,\ldots,\underline{s}_n\}$. The

stress accounts for the elastic (or reversible) changes in internal structure, while the internal state variables account for the inelastic (or irreversible) changes in internal structure. These inelastic changes influence the future response of the material element.

The behavior observed in two similar experiments for the class of materials of interest is given in Table 1. The stress history of experiment 2 differs from that of experiment 1 by a constant rotation Q of the respective internal forces, where the superscript T denotes its transpose. Here $\underset{\sim}{\varepsilon}$ is the state of strain, $\underline{S}_0$ represents the virgin state of the internal state variables, and t is time. The operator P_Q is defined by $P_Q\underline{S} = \{P_q\underline{s}_1, P_q\underline{s}_2, \ldots, P_q\underline{s}_n\}$ where P_q are linear tensor transformations appropriate to the rank of tensor on which they operate. The condition of an isotropic virgin state $\underline{S}_0 = P_Q\underline{S}_0$ is satisfied in these experiments.

This paper considers materials whose mechanical behavior accepts the general constitutive representation

$$\mathrm{tr}(\underset{\sim}{\varepsilon}) = \frac{1 - 2\nu}{E}\mathrm{tr}(\underset{\sim}{\sigma}) \tag{1a}$$

$$\dot{\underset{\sim}{e}} = \frac{1 + \nu}{E}\dot{\underset{\sim}{S}} + \underset{\approx}{F}(\underset{\sim}{\sigma}, \underline{S}) \tag{1b}$$

$$\dot{\underline{S}} = \underline{G}(\sigma, \underline{S}) \tag{1c}$$

such that

$$\underset{\approx}{F}(\underset{\sim}{Q}, \underline{S}_0) = \underset{\sim}{Q} \tag{1d}$$

$$\underline{G}(\underset{\sim}{Q}, \underline{S}_0) = 0 \tag{1e}$$

where a dot over a variable denotes its time rate of change. The quantity $\underset{\sim}{e} = 1.5\underset{\sim}{\varepsilon} - 0.5\mathrm{tr}(\underset{\sim}{\varepsilon})\underset{\sim}{I}$ is a deviatoric strain, and the quantity $\underset{\sim}{S} = 1.5\underset{\sim}{\sigma} - 0.5\mathrm{tr}(\underset{\sim}{\sigma})\underset{\sim}{I}$ is a deviatoric stress, where $\underset{\sim}{I}$ is the unit tensor.

Both are normalized for uniaxial deformation. Given a Cartesian coordinate system, $\mathrm{tr}(\underset{\sim}{X}) = X_{ii}$ where $\underset{\sim}{X}$ represents any second rank tensor and where repeated indices are summed over. Equation (1a) defines the volume strain, where E and ν are the isotropic elastic constants. According to eqn (1b), the deviatoric strain rate $\dot{\underset{\sim}{e}}$ is the sum of a deviatoric elastic strain rate $\dot{\underset{\sim}{e}}^e = (1 + \nu)\dot{\underset{\sim}{S}}/E$ and an inelastic strain rate $\dot{\underset{\sim}{\varepsilon}}^p = \underset{\approx}{F}(\underset{\sim}{\sigma},\underset{\sim}{S})$ that depends on the present internal state. The inelastic strain $\underset{\sim}{\varepsilon}^p$ is deviatoric by definition, implying plastic incompressibility in agreement with eqn (1a). The internal state variables evolve according to eqn (1c), where $\dot{\underline{S}} = \{\dot{\underline{s}}_1,\dot{\underline{s}}_2,\ldots,\dot{\underline{s}}_n\}$ and $\underline{G} = \{\underline{g}_1,\underline{g}_2,\ldots,\underline{g}_n\}$. The absence of a $\dot{\underset{\sim}{\sigma}}$ term in eqn (1c) implies that sudden changes in the level of stress leaves the internal structure $\underline{S}$ unchanged. Such constant structure experiments provide useful information about the stress dependence of $\dot{\underset{\sim}{\varepsilon}}^p = \underset{\approx}{F}(\underset{\sim}{\sigma},\underline{S})$. The initial conditions of eqns (1d) and (1e) are in agreement with the constraints of Table 1.

As a consequence of thermodynamics, there exists a differentiable real-valued function $\phi(\underset{\sim}{\sigma},\underline{S})$ such that for all deformations of interest

$$\underset{\sim}{\sigma} : \dot{\underset{\sim}{\varepsilon}} \geq \dot{\phi} \tag{2}$$

where $\phi \geq 0$ with the equality holding only in the stress-free virgin state $(\underset{\sim}{0},\underline{S}_0)$ (cf. ONAT & LECKIE [1988]). This equation states that the rate of work done on a material element must equal or exceed the rate of increase in the free energy ϕ of the material element under isothermal conditions. Given a Cartesian coordinate system, $\underset{\sim}{\sigma} : \dot{\underset{\sim}{\varepsilon}} = \sigma_{ij}\dot{\varepsilon}_{ji}$.

The free energy function obtained from eqns (1) and (2) has the form

$$\phi = \frac{1}{2}\,\underset{\sim}{\sigma} : \underset{\approx}{D} : \underset{\sim}{\sigma} + \hat{\phi}(\underline{S}) \tag{3}$$

where $\hat{\phi} \geq 0$ with the equality holding only in the virgin state $(\underset{\sim}{\sigma}, \underline{S}_0)$. The elastic compliance $\underset{\sim}{D}$ is positive definite, and in a reference configuration $D_{ijkl} = [(1 + \nu)/2E](\delta_{ik}\delta_{jl} + \delta_{il}\delta_{jk}) - (\nu/E)\delta_{ij}\delta_{kl}$ where $E > 0$ and $-1 < \nu < 0.5$ for an elastically isotropic material.

The thermodynamic constraint of material dissipativity is determined from eqns (1)-(3) to be

$$\underset{\sim}{\sigma} : \underset{\sim}{F} \geq \frac{\partial \phi}{\partial \underline{S}} \underline{G} \tag{4a}$$

or equivalently

$$\frac{\partial \phi}{\partial \underset{\sim}{\sigma}} : (-\underset{\sim}{C} : \underset{\sim}{F}) + \frac{\partial \phi}{\partial \underline{S}} \underline{G} \leq 0 \tag{4b}$$

where

$$\frac{\partial \phi}{\partial \underline{S}} \underline{G} = \sum_{i=1}^{n} \frac{\partial \phi}{\partial \underline{s}_i} \underline{g}_i = \sum_{i=1}^{n} \pi_i \underline{g}_i \tag{4c}$$

The elastic modulus $\underset{\sim}{C} = \underset{\sim}{D}^{-1}$ exists because $\underset{\sim}{D}$ is positive definite. The thermodynamic force π_i conjugate to the thermodynamic displacement (or internal state variable) $\underline{s}_i$ is defined by $\pi_i = \partial\phi/\partial\underline{s}_i$. Equation (4a) states that the rate of inelastic work done on a material element must equal or exceed the rate of increase in the free energy of the material element at constant stress due to a changing internal structure. Equation (4b) establishes the fact that the free energy ϕ is a Lyapunov function. Since $-\underset{\sim}{C} : \underset{\sim}{F}$ quantifies the rate of stress relaxation when $\dot{\underset{\sim}{\varepsilon}} = \underset{\sim}{0}$, this equation implies that ϕ cannot increase during stress relaxation. Furthermore, the evolution of state is stable in the sense of Lyapunov during stress relaxation.

GEARY & ONAT [1974] determined that the set of internal state variables $\underline{S} = \{\underline{s}_1, \underline{s}_2, \ldots, \underline{s}_n\}$ must be composed of irreducible even-rank tensors. Given

that $\underset{\sim}{S}$ is composed of no tensors of rank greater than two (as is the case in this paper), then it must be composed of (a) scalars, α; (b) isotropic tensors, $\underset{\sim}{\alpha} = \alpha\underset{\sim}{I}$; (c) antisymmetric tensors, $\underset{\sim}{\alpha} = -\underset{\sim}{\alpha}^T$; and (d) symmetric and traceless (or deviatoric) tensors, $\underset{\sim}{\alpha} = \underset{\sim}{\alpha}^T$, $\mathrm{tr}(\underset{\sim}{\alpha}) = 0$. Their symmetries distinguish them. Actually, scalars and isotropic second-rank tensors possess the same symmetry and are therefore equivalent. Each antisymmetric tensor is associated with an axial vector; whereas each symmetric and traceless tensor remains unchanged under 180° rotations about its principal directions (cf. BACKUS [1970]). Since the initial state is isotropic (Table 1), all internal state variables other than scalars must vanish at $t = 0$.

The theory developed in this paper considers two internal state variables. They are $\underset{\sim}{S} = (\underset{\sim}{\xi},\zeta)$, where $\underset{\sim}{\xi}$ is a deviatoric tensor of second rank and ζ is a scalar, such that in the virgin state $\underset{\sim}{\xi}(0) = \underset{\sim}{0}$ and $\zeta(0) = \zeta_0 \geq 0$: that is, $\underset{\sim}{S}_0 = (\underset{\sim}{0},\zeta_0)$. The deviatoric tensor accounts for kinematic hardening effects, whereas the scalar accounts for isotropic hardening effects.

The thermodynamic constraint of material dissipativity, eqn (4), therefore becomes

$$\underset{\sim}{\sigma} : \underset{\sim}{F} \geq \frac{\partial\phi}{\partial\underset{\sim}{\xi}} : \dot{\underset{\sim}{\xi}} + \frac{\partial\phi}{\partial\zeta}\dot{\zeta} \tag{5a}$$

or

$$\underset{\sim}{\sigma} : \dot{\underset{\sim}{\varepsilon}}^p \geq \underset{\sim}{B} : \dot{\underset{\sim}{\xi}} + L\dot{\zeta} \tag{5b}$$

where $\underset{\sim}{B} = \partial\phi/\partial\underset{\sim}{\xi}$ is the back stress and $L = \partial\phi/\partial\zeta$ is the limit stress. These are the thermodynamic forces conjugate to the internal state variables $\underset{\sim}{\xi}$ and ζ. The sum $\underset{\sim}{B} : \dot{\underset{\sim}{\xi}} + L\dot{\zeta}$ defines the rate of change in the free energy at constant stress due to a changing internal structure for this viscoplastic theory. In the literature, the back stress $\underset{\sim}{B}$ is also referred to as the

internal stress, the equilibrium stress, the rest stress, or the kinematic stress. The limit stress L is not, however, equivalent in concept to the drag stress (also referred to as the friction stress or the threshold stress) as used in the literature. This point is discussed in Section V.

The dissipative portion of the free energy, $\hat{\phi}(\underset{\sim}{S})$ in eqn (3), is considered to have the following form:

$$\hat{\phi} = \frac{1}{2} H\underset{\sim}{\xi} : \underset{\sim}{\xi} + \frac{1}{2} h\left(\zeta^2 - \zeta_0^2\right) \tag{6}$$

Here $H > 0$, $h > 0$, and $\zeta \geq \zeta_0$ such that the equality holds only in the virgin state $\underset{\sim}{S}_0$; thus, $\hat{\phi} \geq 0$ as required. The linear relationships $\underset{\sim}{B} = H\underset{\sim}{\xi}$ and $L = h\zeta$ result from eqns (5) and (6), where the hardening parameters H and h act like elastic moduli. It follows that the back stress $\underset{\sim}{B}$ is deviatoric. In the virgin state $\underset{\sim}{S}_0$, $\underset{\sim}{B}(0) = \underset{\sim}{0}$ and $L(0) = L_0 = h\zeta_0 \geq 0$.

III. Ω-FORM THEORY

The foundation of an Ω-form theory of viscoplasticity is the normality structure of a potential function Ω. The advantage of this approach is that the choice of two scalar-valued functions of state (i.e., ϕ and Ω) completely defines an elegant mathematical structure for viscoplasticity. The disadvantage is that the theory may be too restrictive. This approach results in a theory where inelastic strain rate strongly influences the evolution of state, as is the case in creep of metals. The ensuing discussion does not address physical justification for the existence of the Ω function; rather, it concentrates on the mathematical consequences of such a function.

The theoretical development of RICE [1971] in modeling the local response of crystallographic slip and the experimental results of BROWN [1970] and ROBINSON [1976] on polycrystalline metals both support the relationship

$$\underset{\sim}{F} = \dot{\underset{\sim}{\varepsilon}}^p = \frac{\partial \Omega}{\partial \underset{\sim}{S}} \tag{7}$$

Implementing the concepts of Rice's crystallographic theory and considering Bailey-Orowan relationships for the evolution of internal state, PONTER & LECKIE [1976] determined that

$$\underset{\sim}{G} = \dot{\underset{\sim}{\xi}} = \frac{\dot{\underset{\sim}{B}}}{H} = -\frac{\partial \Omega}{\partial \underset{\sim}{B}} \tag{8}$$

Only the kinematic variable $\underset{\sim}{\xi}$ shall be considered at this time. Here it is assumed that the potential function $\Omega(\underset{\sim}{S},\underset{\sim}{B}) \geq 0$, that the equality is satisfied only in the stress-free virgin state $(\underset{\sim}{0},\underset{\sim}{0})$, and that the set of all possible surfaces $\{\Omega = \text{constant}\}$ is composed of elements that are convex, that are nested, and that contain the origin $(\underset{\sim}{0},\underset{\sim}{0})$ in state space $(\underset{\sim}{S},\underset{\sim}{B})$. Together, eqns (7) and (8) define a theory of viscoplasticity.

The thermodynamic constraint of material dissipativity, eqn (5), becomes

$$\frac{\partial \Omega}{\partial \underset{\sim}{S}} : \underset{\sim}{S} + \frac{\partial \Omega}{\partial \underset{\sim}{B}} : \underset{\sim}{B} \geq 0 \tag{9}$$

for the Ω-form theory, and is automatically satisfied because of the properties assigned to the potential function Ω. Comparing eqns (5) and (9) makes it apparent that this Ω-form theory is but a subset of the set of all thermodynamically admissible theories of viscoplasticity that admit $\underset{\sim}{\xi}$ as their only internal state variable.

The specific form of the potential function to be considered is

$$\Omega = \Theta\left[\underline{M}(\Sigma_2) + \underline{N}(B_2)\right] \tag{10}$$

where $\Theta > 0$ is a diffusivity parameter (e.g., an Arrhenius function of temperature) and $\underline{M}(\Sigma_2) \geq 0$ and $\underline{N}(B_2) \geq 0$ are differentiable material functions such that the equalities hold only when their arguments are zero. The notation $X_2 = \sqrt{(2/3)\ \underset{\sim}{X} : \underset{\sim}{X}}$ defines the square root of the second invariant of $\underset{\sim}{X}$ normalized for uniaxial deformation, where $\underset{\sim}{X}$ represents any deviatoric second-rank tensor. The deviatoric tensor defined by $\underset{\sim}{\Sigma} = \underset{\sim}{S} - \underset{\sim}{B}$ is

the effective stress that governs inelastic deformation, sometimes called the overstress. The introduction of third invariants into this theory would bring about higher order effects, which are neglected in accordance with von MISES [1913]. The first invariants are zero because both $\underset{\sim}{S}$ and $\underset{\sim}{B}$ are deviatoric.

The flow and evolutionary equations that result from eqns (7), (8), and (10) are

$$\dot{\underset{\sim}{\varepsilon}}^{p} = \theta \underline{Z}(\Sigma_2) \frac{\underset{\sim}{\Sigma}}{\Sigma_2} \tag{11a}$$

and

$$\frac{\dot{\underset{\sim}{B}}}{H} = \dot{\underset{\sim}{\xi}} = \dot{\underset{\sim}{\varepsilon}}^{p} - \theta \underline{R}(B_2) \frac{\underset{\sim}{B}}{B_2} \tag{11b}$$

where $\underline{Z}(\Sigma_2) = \dot{\varepsilon}_2^p/\theta = 1.5(\partial \underline{M}/\partial \Sigma_2) \geq 0$ is the ZENER & HOLLOMON [1944] parameter and $\underline{R}(B_2) = 1.5(\partial \underline{N}/\partial B_2) \geq 0$ is the thermal recovery function, such that the equalities hold only when their arguments are zero. The flow equation, eqn (11a), is compatible with the kinematic constructs of PRAGER [1949] and ZIEGLER [1959] in the classical theory of plasticity. Likewise, the evolutionary equation, eqn (11b), is compatible with the Bailey-Orowan hypothesis for creep (BAILEY [1926] and OROWAN [1947]); that is, the internal structure evolves as the result of a competitive process between strain hardening due to inelastic deformation and thermal recovery progressing with time. Whenever the thermal recovery term is neglected, the theory becomes one of rate-dependent or dynamic plasticity. The presence of the thermal recovery term gives the theory its viscous characteristics.

Equation (11) satisfies the thermodynamic constraint of material dissipativity, eqn (9), since

$$\frac{\Sigma_2}{B_2} + \frac{\underline{R}(B_2)}{\underline{Z}(\Sigma_2)} \geq 0 \tag{12}$$

is unconditionally satisfied.

Choosing the functional dependence of $\underline{Z}(\Sigma_2)$ to be Σ_2/K (where the drag stress $K > 0$ is a material constant governing strength) results in a theory of viscoplasticity without an explicit yield surface. On the other hand, choosing $\underline{Z}(\Sigma_2)$ to depend on $\Sigma_2/K_y - 1$ (where the threshold stress $K_y > 0$ is a material constant denoting a yield strength, such that $\underline{Z} = 0$ whenever $\Sigma_2 \leq K_y$) results in a theory of viscoplasticity that has an explicit yield surface. Since the function $\underline{Z}(\Sigma_2)$ has vanishingly small values over a finite range in its argument for all known viscoplastic models, these models all possess the property of an implicit yield surface in the predicted response. This implicit yield strength (which is the observed yield in the predicted response) is typically an order in magnitude larger than the explicit yield strength defined by K_y (which is not observable in the predicted response) in those models that incorporate K_y.

IV. NONLINEAR HARDENING

The Ω-form theory of viscoplasticity developed in the previous section produces linear-strain-hardening viscoplastic responses whenever thermal recovery $\theta\underline{R}$ is negligible compared with strain hardening $\dot{\varepsilon}_2^p$. This disagrees with the experimental observation that the rate of strain hardening in metals typically diminishes with increasing deformation to an asymptotic rate in tensile tests at low homologous temperatures. That is, metals exhibit an evanescent strain-memory effect caused by a strain-induced recovery mechanism called dynamic recovery. As in the previous section, only the kinematic state variable $\underset{\sim}{\xi}$ will be considered. Furthermore, this section will address only those deformations where thermal recovery can be neglected.

Two methodologies are commonly used to introduce nonlinear strain hardening into viscoplastic models. The first methodology introduces the evanescent strain-memory effect by adding a dynamic recovery term into the

evolutionary equation for back stress, as advocated by CHABOCHE [1977]. The second methodology incorporates this effect by letting the hardening parameter H in the evolutionary equation for back stress become a hardening function $\underline{H}(\underset{\sim}{B})$, as advocated by PONTER & LECKIE [1976]. Both of these methodologies amend the Ω-form theory of the previous section, in that they both produce viscoplastic responses with nonlinear strain-hardening characteristics.

Thermodynamically admissible viscoplastic theories that contain these two amendments are constructed in such a way that they are identical in form for monotonic and proportional loading conditions. This is done to provide a basis for comparison. These theories predict vastly different responses, however, whenever there is either a reversal or a nonproportionality in the loading direction. The objective of this section is to assess which of these two methodologies for incorporating nonlinear strain hardening into the theory best represents known experimental observations. This is accomplished by presenting the two approaches and then comparing them.

Dynamic Recovery Methodology

The dynamic recovery approach incorporates the evanescent strain-memory effect by introducing a dynamic recovery term into the evolutionary equation for back stress. In particular, the constitutive equations for this viscoplastic theory (with no thermal recovery) are hypothesized to be

$$\dot{\underset{\sim}{\varepsilon}}^{p} = \Theta \underline{Z}(\Sigma_2) \frac{\underset{\sim}{\Sigma}}{\Sigma_2} \tag{13a}$$

and

$$\frac{\dot{\underset{\sim}{B}}}{H} = \dot{\underset{\sim}{\xi}} = \dot{\underset{\sim}{\varepsilon}}^{p} - \frac{\underset{\sim}{B}}{L} \dot{\varepsilon}_2^{p} \tag{13b}$$

where $H > 0$ is the hardening parameter and $L > 0$ is the limit stress (which is considered to be a material constant in this section).

Note that the flow equation, eqn (13a), is identical to the flow equation of the Ω-form theory, eqn (11a). This implies that a flow potential for inelastic strain rate exists for this viscoplastic theory. (See eqn (7).) However, unlike the Ω-form theory, the evolutionary equation in this viscoplastic theory, eqn (13b), cannot be derived from a flow potential for the given flow equation. The Ω-form theory enables the evolution of back stress to be derived from the flow potential by assuming that the internal structure evolves in a Bailey-Orowan manner (cf. PONTER & LECKIE [1976]). Clearly, this is not the case in eqn (13b). This does not imply that the theory of viscoplasticity given in eqn (13) is thermodynamically inadmissible. On the contrary, the thermodynamic constraint of material dissipativity, eqn (5), becomes

$$\frac{\Sigma_2}{L} + \left(\frac{B_2}{L}\right)^2 \geq 0 \tag{14}$$

and it is unconditionally satisfied.

Steady state occurs whenever the internal state variables attain steady values. Under this condition for eqn (13b), $L = B_2$; that is, the limit stress L is the limiting value of back stress at the kinematic saturation of state.

A graphic illustration of the dynamic recovery concept is given in Fig. 1. The limiting state of back stress defines a hypersurface of radius L in state space. Whenever this upper bound is reached, a perfectly plasticlike response is attained. Furthermore, the inelastic strain rate $\dot{\underset{\sim}{\varepsilon}}^p$ becomes coaxial with both the stress $\underset{\sim}{S}$ and the back stress $\underset{\sim}{B}$, and the nested set of flow surfaces $\{\Sigma_2 = \text{constant}\}$ becomes stationary until unloading occurs. Otherwise this set of flow surfaces can translate freely within the bounding surface, as governed by the flow and evolutionary equations.

Hardening Function Methodology

The hardening function approach incorporates the evanescent strain-memory effect by making the hardening parameter H a hardening function $\underset{\sim}{H}(\underset{\sim}{B})$, while retaining the structure of the Ω-form theory. This is accomplished by considering a Legendre transformation of the dissipative free energy $\hat{\phi}(\underset{\sim}{S})$ defined in eqn (3), causing a change in variables; that is,

$$\varphi(\Pi) = \hat{\phi}(\underset{\sim}{S}) - \sum_{i=1}^{n} \pi_i \underset{\sim}{S}_i \tag{15}$$

and therefore $\underset{\sim}{S}_i = -\partial\varphi/\partial\pi_i$ in accordance with eqn (4c). Here $\Pi = \{\pi_1, \pi_2, \ldots, \pi_n\}$ is the set of thermodynamic forces conjugate to the set of internal state variables $\underset{\sim}{S} = \{\underset{\sim}{S}_1, \underset{\sim}{S}_2, \ldots, \underset{\sim}{S}_n\}$.

The particular form of φ to be considered is

$$\varphi = -\int_{\underset{\sim}{0}}^{\underset{\sim}{B}} \left(\int_0^t \frac{\underset{\sim}{B}}{\underset{\sim}{H}(\underset{\sim}{B})} \, dt \right) : d\underset{\sim}{B} \tag{16}$$

giving rise to the desired relationship $\dot{\underset{\sim}{\xi}} = \dot{\underset{\sim}{B}}/\underset{\sim}{H}(\underset{\sim}{B})$, where $\dot{\underset{\sim}{B}}$ is finite and $\underset{\sim}{H}(\underset{\sim}{B}) > 0$ is the scalar-valued hardening function. Note that $\underset{\sim}{H}(\underset{\sim}{B})$ must be independent of stress $\underset{\sim}{\sigma}$ in order for the elastic response to be independent of internal state, as established in eqn (1).

The resulting viscoplastic theory is the Ω-form theory of eqn (11) with H replaced by $\underset{\sim}{H}(\underset{\sim}{B})$. This generalized Ω-form theory is thermodynamically admissible since the constraint of material dissipativity, eqn (12), is unchanged and remains unconditionally satisfied. Here, as in the development of the Ω-form theory of Section III, only the kinematic variable $\underset{\sim}{B}$ is addressed.

A Comparison

To facilitate a comparison between these two methodologies, it is useful to consider the special hardening function

$$\underline{H} = H - H \frac{B_2}{L} \tag{17}$$

where H and L are the same material constants found in eqn (13). Thus, the evolutionary equation for back stress is given by

$$\dot{\underset{\sim}{B}} = H\dot{\underset{\sim}{\varepsilon}}^p - H \frac{\underset{\sim}{B}}{L} \dot{\varepsilon}_2^p \tag{18a}$$

for the dynamic recovery approach and by

$$\dot{\underset{\sim}{B}} = H\dot{\underset{\sim}{\varepsilon}}^p - H \frac{B_2}{L} \dot{\underset{\sim}{\varepsilon}}^p \tag{18b}$$

for the hardening function approach. There is a subtle but significant difference in the last term of these two equations, which is the focus of our attention for the rest of this section. Notice that the hardening function in eqn (17) was chosen in such a way that the dynamic recovery approach and the hardening function approach are nearly equivalent. In fact, they are identical under monotonic and proportional loading conditions. Thermal recovery is being neglected in this section, so as to simplify the discussion.

The hardening function approach is known to produce too strong of a Bauschinger effect. To overcome this fault, ROBINSON [1978] assigns inequalities to the hardening function $\underline{H}(\underset{\sim}{B})$ so that it becomes discontinuous when crossing specified boundaries in state space. To illustrate this point, it is useful to consider the case of uniaxial loading. Here eqn (18a) reduces to

$$\dot{\beta} = H\dot{\varepsilon}^p - H \frac{\beta}{L} \left|\dot{\varepsilon}^p\right| \tag{19a}$$

for the dynamic recovery approach, and eqn (18b) reduces to

$$\dot{\beta} = H\dot{\varepsilon}^p - H\frac{|\beta|}{L}\dot{\varepsilon}^p \qquad (19b)$$

for the hardening function approach, where $\beta = B_{11}$ and $\dot{\varepsilon}^p = \dot{\varepsilon}^p_{11}$. These equations are equivalent in the loading domain of region I of Fig. 2; however, they differ in the unloading domain of region II. In region II, the terms on the right-hand side of eqn (19a) become additive; whereas the terms in eqn (19b) continue to compete against each other. For this reason, the back stress evolves much faster in region II for the dynamic recovery approach, eqn (19a), than it does for the hardening function approach, eqn (19b), as illustrated in Fig. 2. The slower evolution of the back stress is the cause of the excessive Bauschinger effect exhibited by the hardening function methodology. Equations (19a) and (19b) could be made identical by considering $\underset{\sim}{H}(\underset{\sim}{B}) = H - H(B_2/L)\mathrm{sgn}(\underset{\sim}{B}:\underset{\sim}{\Sigma})$ instead of eqn (17), where sgn() is the signum function. This is analogous to assigning an inequality to the hardening function, as advocated by ROBINSON [1978]. But this particular hardening function is not admissible, since $\underline{H}$ must be independent of stress (recall that $\underset{\sim}{\Sigma} = \underset{\sim}{S} - \underset{\sim}{B}$). Thus, the dynamic recovery approach is preferred over the hardening function approach with regards to its ability to account for the Bauschinger effect.

For nonproportional loading conditions, the dynamic recovery approach, eqn (18a), predicts that the inelastic strain rate $\underset{\sim}{\dot{\varepsilon}}^p$ becomes coaxial with both the stress $\underset{\sim}{S}$ and the back stress $\underset{\sim}{B}$ at the kinematic saturation of state $B_2 = L$ (Fig. 1). On the contrary, no specified coaxiality can be ascertained under nonproportional loading conditions at the kinematic saturation of state for the hardening function approach eqn (18b). The two-

surface theory of plasticity (DAFALIAS & POPOV [1975] and KRIEG [1975]) is based on the experimental observation that a limit surface exists with normality-governed inelastic strain rate (cf. PHILLIPS [1986]). Contained within this limit surface is the actual yield surface, which kinematically translates. The dynamic recovery methodology is consistent with this experimental observation; the hardening function methodology is not. In fact, the dynamic recovery approach is an extension of the two-surface theory from plasticity to viscoplasticity (cf. CHABOCHE & ROUSSELIER [1983]). Thus, the dynamic recovery approach is again preferred over the hardening function approach. This time, the preference is with regards to its ability to account for nonproportional behavior at the kinematic saturation of state in the relative absence of thermal recovery.

It is important to point out that if thermal recovery (i.e., $-\theta H \underline{R}(B_2)\underset{\sim}{B}/B_2$) was considered in eqn (18), then these two methodologies would be equivalent with regards to their ability to account for thermally activated creep behavior under nonproportional loading conditions.

For the reasons just discussed, the dynamic recovery methodology is preferred over the hardening function methodology.

V. ISOTROPIC HARDENING

The current philosophy for incorporating isotropic hardening into an otherwise kinematic theory of viscoplasticity is to normalize the effective stress $\underset{\sim}{\Sigma} = \underset{\sim}{S} - \underset{\sim}{B}$ with a scalar-valued state variable K called the drag stress (cf. WALKER [1981]), such that $\underline{Z}(\Sigma_2/K) = \dot{\varepsilon}_2^p/\theta$. Upon inversion, this relationship becomes $\Sigma_2 = K\underline{Z}^{-1}\left(\dot{\varepsilon}_2^p/\theta\right)$, which finds Σ_2 proportional to K. A similar result is obtained when a yield surface is introduced, such that $\underline{Z}(\Sigma_2/K_y - 1) = \dot{\varepsilon}_2^p/\theta$ where K_y is a scalar-valued state variable called the threshold stress. This is not a particularly desirable situation since there

are an infinite number of pairs $(\underset{\sim}{\Sigma},K)$ (or $(\underset{\sim}{\Sigma},K_y)$) that satisfy this equation for any given value of $\dot{\varepsilon}_2^p/\theta$ (cf. KREMPL et al. [1986]). Which is the correct pair depends on the history and, therefore, on the evolution of internal state. But proper characterization of the evolution of internal state, particularly its thermal recovery aspects, is a difficult task because Σ_2 is proportional to K.

The following is a novel approach for incorporating isotropic hardening into an otherwise kinematic theory of viscoplasticity, such that $\underline{Z}(\Sigma_2) = \dot{\varepsilon}^p/\theta$. That is, <u>there is no coupling of the kinematic and isotropic state variables in the Zener-Hollomon parameter</u> $\underline{Z}$. Instead, the coupling is introduced in the evolutionary equation for back stress $\underset{\sim}{B}$. In particular, it is hypothesized that the constitutive equations for this theory of viscoplasticity are

$$\dot{\underset{\sim}{\varepsilon}}^p = \theta \underline{Z}(\Sigma_2) \frac{\underset{\sim}{\Sigma}}{\Sigma_2} \tag{20a}$$

$$\frac{\dot{\underset{\sim}{B}}}{H} = \dot{\underset{\sim}{\xi}} = \dot{\underset{\sim}{\varepsilon}}^p - \frac{\underset{\sim}{B}}{L}\dot{\varepsilon}_2^p \tag{20b}$$

and

$$\frac{\dot{L}}{h} = \dot{\zeta} = \frac{1}{L}\underset{\sim}{\Sigma} : \dot{\underset{\sim}{\varepsilon}}^p - \theta \underline{r}(L) \tag{20c}$$

given that $\theta > 0$, $H > 0$, $h > 0$, $\underline{Z}(\Sigma_2) = \dot{\varepsilon}_2^p/\theta \geq 0$, and $\underline{r}(L) \geq 0$, such that in the stress-free virgin state $\underset{\sim}{B}(0) = \underset{\sim}{0}$, $L(0) = 0$, $\underline{Z}(0) = 0$, and $\underline{r}(0) = 0$. In this theory, the back stress $\underset{\sim}{B}$ and the limit stress L are the internal variables. The theory in eqn (20) is a generalization to the theory in eqn (13), in that the limit stress now evolves.

The flow equation, eqn (20a), is the same flow equation derived in the Ω-form theory, eqn (11a). Therefore, a flow potential for inelastic strain rate (eqn (7)) exists for this viscoplastic theory.

The evolutionary equation for back stress, eqn (20b), is the same as eqn (13b), except that the limit stress is no longer a material constant. At first glance, there appears to be no thermal recovery of the back stress, but this is false conjecture. Thermal recovery has the physical effect of shrinking the limit surface in Fig. 1. This can be modeled in one of two ways. First, the limit stress can diminish because of thermal recovery. Or, second, a separate thermal recovery term can be included in the evolutionary equation for back stress. Both methods lead to thermal recovery in the $-\underset{\sim}{B}$ direction. Both effectively shrink the limit surface in Fig. 1. By allowing L to thermally recover (as in eqn (20)) the thermal recovery of $\underset{\sim}{B}$ becomes implicit. This results in a desirable simplification in the structure of this viscoplastic theory since <u>there is no coupling between dynamic and thermal recovery terms in each evolutionary equation</u>.

The evolutionary equation for limit stress, eqn (20c), is of the Bailey-Orowan form, where work hardening competes against thermal recovery. In concept, the limit stress L differs from the drag stress K (or the threshold stress K_y) of conventional viscoplastic theories. Both are strength parameters. But L is more like an ultimate strength; whereas K is more like a yield strength. In particular, L is a measure of $\underset{\sim}{B}$ at the kinematic saturation of state, where $\dot{\underset{\sim}{\varepsilon}}^p$, $\underset{\sim}{S}$, and $\underset{\sim}{B}$ are all coaxial (Fig. 1). The work hardening term in eqn (20c) $\underset{\sim}{\Sigma} : \dot{\underset{\sim}{\varepsilon}}^p/L \geq 0$ is equivalent to what would be derived from an Ω-form theory (based on that in Section III) that incorporates both a back stress and a drag stress.

The viscoplastic theory of eqn (20) is thermodynamically admissible since the constraint of material dissipativity, eqn (5), becomes

$$\left(\frac{B_2}{L}\right)^2 + \frac{2\underline{r}(L)}{3\underline{Z}(\Sigma_2)} \geq 0 \tag{21}$$

and it is unconditionally satisfied. If one were to choose $\dot{\varepsilon}_2^p$, instead of $\underset{\sim}{\Sigma} : \dot{\underset{\sim}{\varepsilon}}^p/L$, as the measure for isotropic hardening (which is typically done in viscoplastic theories (cf. WALKER [1981])) one would obtain $\Sigma_2/L + (B_2/L)^2 + 2[\underline{r}(L) - \underline{Z}(\Sigma_2)]/3\underline{Z}(\Sigma_2) \geq 0$ as the condition for material dissipativity. But this inequality is only satisfied conditionally, and therefore the choice of $\dot{\varepsilon}_2^p$ is undesirable.

Thermal recovery, as provided in eqn (20), is capable of driving both the back stress and the limit stress to zero, which defines the virgin state. In a neighborhood of the virgin state, L must evolve faster than B_2 if there is to be an evanescent strain-memory effect. This is certainly satisfied through the choice of $\underset{\sim}{\Sigma} : \dot{\underset{\sim}{\varepsilon}}^p/L$ for the hardening rate of L. This choice also implies that thermal recovery of infinite strength is required to keep an initially virgin material in a virgin state during deformation. Such a condition results in a viscoelasticlike fluid response, and probably only exists in the molten state for metals. Thus, in all likelihood, polycrystalline metals are probably not in a virgin state at any initial condition of practical interest (i.e., $L_0 > 0$) as understood within this theoretical framework.

VI. CONCLUDING REMARKS

The structure of a viscoplastic theory has been derived from both physical and thermodynamical considerations. The development began by considering an Ω-form theory, from which the flow equation and an evolutionary equation for back stress were derived. The structure of this evolutionary equation was altered to introduce an evanescent strain-memory effect through the addition of a dynamic recovery term. This approach was deemed superior to that of introducing a nonlinear hardening function. A unique hypothesis was then

considered for isotropic hardening (i.e., the limit stress (not the drag stress) is considered to be the scalar-valued state variable). This led to a desirable simplification in the structure of the theory. That is, there is no coupling between the kinematic and isotropic state variables in the flow equation, and there is no coupling between dynamic and thermal terms in each evolutionary equation. The final result is a theory of viscoplasticity with adequate capabilities for modeling polycrystalline metals. This theory has the structure of a two-surface theory of plasticity when the response is plasticlike, and the structure of a Bailey-Orowan theory of creep when the response is creeplike.

Before the theory can be used to model a material or a class of materials, specific forms for the material functions need to be determined. The intent of this paper, however, has been to focus on the underlying structure of a viscoplastic theory. The development and use of specific viscoplastic models that are compatible with this theory are left for future research.

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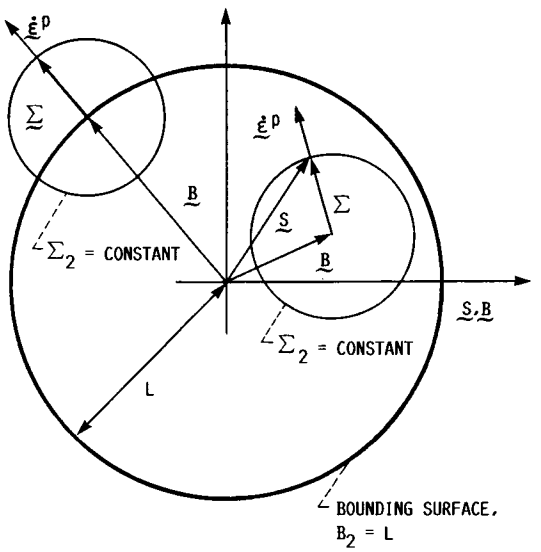
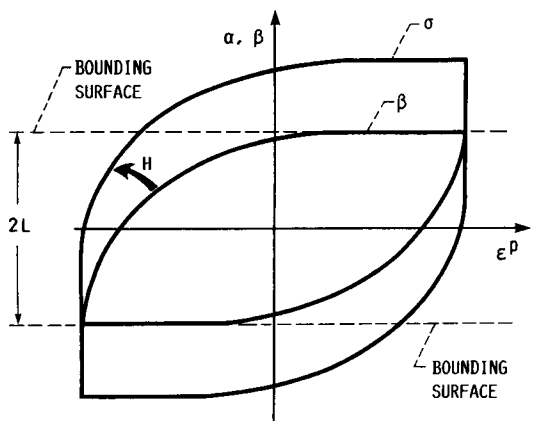
TABLE 1. - STRESS-STRAIN CONSTRAINTS FOR MATERIALS THAT ARE INITIALLY STRESS FREE AND ISOTROPIC

Experiment	Input	Output	Structure
1	$\sigma(0) = 0$ $\sigma(t)$	$\varepsilon(0) = 0$ $\varepsilon(t)$	$\underset{\sim}{S}(0) = \underset{\sim}{S}_0$ $\underset{\sim}{S}(t)$
2	$\sigma'(0) = 0$ $\sigma'(t) = Q\sigma(t)Q^T$	$\varepsilon'(0) = 0$ $\varepsilon'(t) = Q\varepsilon(t)Q^T$	$\underset{\sim}{S}'(0) = \underset{\sim}{S}_0$ $\underset{\sim}{S}'(t) = P_Q\underset{\sim}{S}(t)$

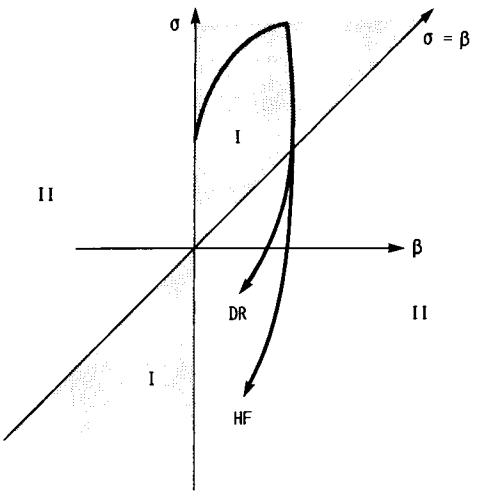


(A) STATE SPACE REPRESENTATION.

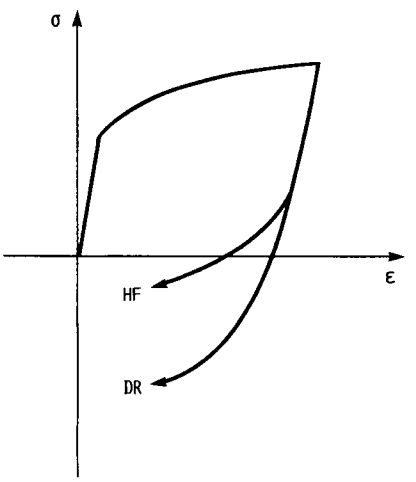


(B) UNIAXIAL REPRESENTATION WHERE $\sigma = \sigma_{11} = S_{11}$, $\beta = B_{11}$, AND $\varepsilon^p = \varepsilon^p_{11}$.

FIGURE 1. - SCHEMATIC REPRESENTATION OF THE DYNAMIC RECOVERY OF BACK STRESS.



(A) STATE SPACE REPRESENTATION. LINE $\sigma = \beta$ REPRESENTS THE ELASTIC DOMAIN GIVEN THAT $Z(\Sigma_2/K)$ SINCE $\dot{\varepsilon}^p = 0$ THERE.



(B) STRESS-STRAIN REPRESENTATION.

FIGURE 2. - SCHEMATIC COMPARISON OF THE DYNAMIC RECOVERY APPROACH DR OF EQN (19A) AND THE HARDENING FUNCTION APPROACH HF OF EQN (19B) FOR TENSILE LOADING AND UNLOADING WHERE $\sigma = \sigma_{11} = S_{11}$, $\beta = B_{11}$, AND $\varepsilon = \varepsilon_{11}$.

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16. Abstract

The general structure of a viscoplastic theory is developed from physical and thermodynamical considerations. The flow equation is of classical form. The dynamic recovery approach is shown to be superior to the hardening function approach for incorporating nonlinear strain hardening into the material response through the evolutionary equation for back stress. A novel approach for introducing isotropic strain hardening into the theory is presented, which results in a useful simplification. In particular, the limiting stress for the kinematic saturation of state (not the drag stress) is the chosen scalar-valued state variable. The resulting simplification is that there is no coupling between the two state variables in the flow equation, and there is no coupling between dynamic and thermal recovery terms in each evolutionary equation. The derived theory of viscoplasticity has the structure of a two-surface plasticity theory when the response is plasticlike, and the structure of a Bailey-Orowan creep theory when the response is creeplike.

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