Dryden Research Lecture

Structure of Turbulent Shear Flows: A New Look

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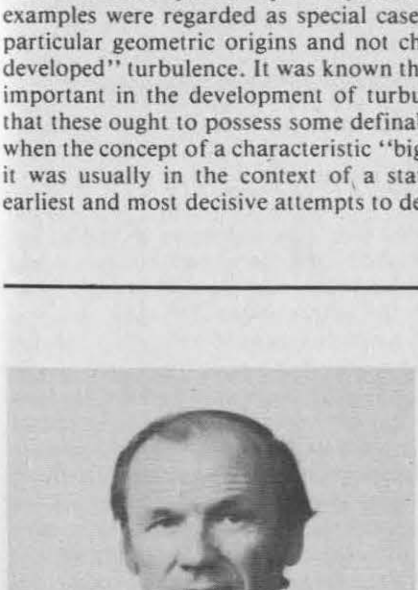
Introduction

THE problem of turbulent flow continues to be an outstanding one in technology and in physics. Of the nine Dryden research lectures so far, four have been on some aspect of the turbulence problem. At meetings such as this one the turbulence problem is always the subject of some sessions and lurks in the background of many others; for example, separated flow, combustion, jet noise, chemical lasers, atmospheric problems, etc. It is continually the subject of conferences, workshops and reviews. In his time Hugh Dryden wrote several reviews of turbulent flow. In reading some of them again, one statement[1] particularly relevant to the present lecture caught my attention: "—it is necessary to separate the random processes from the nonrandom processes. It is not yet fully clear what the random elements are in turbulent flow." Neither is it fully clear what the nonrandom, orderly elements are, but some of them are beginning to be recognized and described.

Generally the picture one has had of turbulence is of chaos and disorder, implicit in the name. Although it was known that organized motion could exist, superimposed on the background of "turbulence," for example, vortex shedding from a circular cylinder up to Reynolds numbers of 10^7, such examples were regarded as special cases closely tied to their particular geometric origins and not characteristic of "well-developed" turbulence. It was known that large structures are important in the development of turbulent shear flows and that these ought to possess some definable features. But even when the concept of a characteristic "big eddy" was explored, it was usually in the context of a statistical quantity. The earliest and most decisive attempts to define the form of such large eddies were made by Townsend and his students.[2,3] In recent years it has become increasingly evident that turbulent shear flows do contain structures or eddies whose description is more deterministic than had been thought, possessing indentifiable characteristics, existing for significant lifetimes, and producing recognizable and important events. More accurate descriptions of their properties, how they fit into the complete description of a turbulent flow, to what extent are they central to its development, and how they can be reconciled with the apparent chaos and disorder, are problems which are becoming of interest to an increasing number of researchers. It is the purpose of this lecture to describe some of these new developments. The discussion will draw largely on experiences from our own laboratory; it is not intended to be a complete survey. Other discussions of these ideas can be found in various recent publications.[4-8]

The Turbulent Mixing Layer

Our own ideas about turbulent flow structure were drastically altered by an investigation of plane turbulent mixing layers which we undertook,[9,10] initially to study the effects of nonuniform density. Figure 1 is a schematic diagram of this class of flows, in which the uniform streams on either side of the mixing layer are characterized by their velocities U_1 and U_2, etc. What is attractive about this configuration is that, of all the flows in the catalog of basic turbulent shear flows, it allows the effects of variation of various parameters such as velocity ratio U_2/U_1, density ratio ρ_2/ρ_1, Mach numbers, etc. to be explored in the simplest way (this remark applies to the concept, not to the actual realization in the laboratory!). The particular case of homogeneous flow



Dr. Roshko is known for his research work in several areas of gas dynamics and fluid mechanics. He has made contributions to problems of separated flow; bluff-body aerodynamics; shock-wave boundary-layer interactions; structure of turbulent wakes and mixing layers; shock-tube technology. He is coauthor with H.W. Liepmann of "Elements of Gas Dynamics," which has been widely used as a textbook during the past twenty years.

He was born in Canada, obtained his undergraduate education at the University of Alberta, from which he received the B.Sc. degree in Engineering Physics in 1945. He received the M.S. degree from Caltech in 1947 and the Ph.D. in 1952. His academic career includes two years of teaching at the University of Alberta and many years at Caltech, where he is Professor of Aeronautics. He is a Fellow of AIAA, the Canadian Aeronautics and Space Institute, and the American Academy of Arts and Sciences; a member of the American Physical Society and the Arctic Institute of North America. He helped organize the Wind Engineering Research Council and is a member of its Executive Board.

Dr. Roshko has been a consultant to government laboratories and various companies, particularly the McDonnell-Douglas Corporation. During 1961-62 he was Liaison Scientist with the U.S. Office of Naval Research in London and in 1969 was an NSF Visiting Scientist in India.

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Index categories: Jets, Wakes, and Viscid Flow Interactions; Boundary Layers and Convective Heat Transfer—Turbulent.

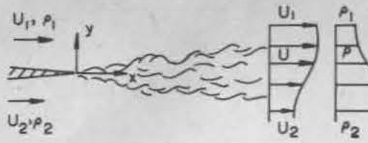

Fig. 1 Plane mixing layer between two streams with velocities U_1 and U_2, densities ρ_1 and ρ_2.

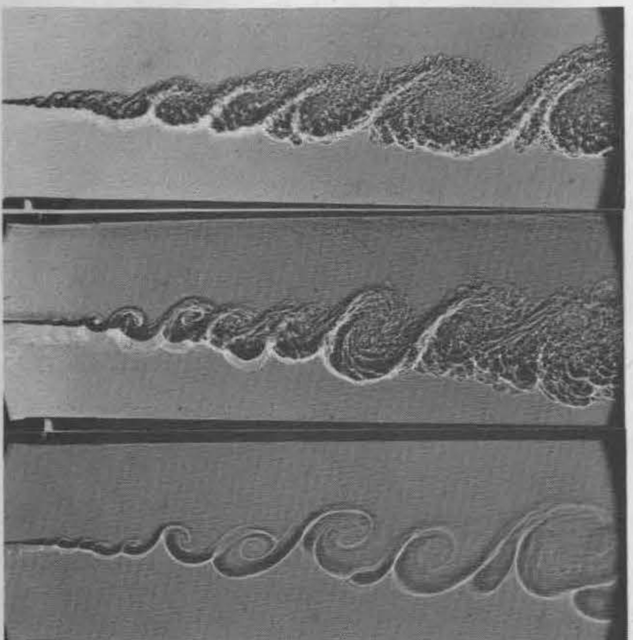

Fig. 2 Mixing layer between helium and nitrogen $U_2/U_1 = 0.38$; $\rho_2/\rho_1 = 7; \rho_1 L_1 L/\mu_1 = 1.2$, 0.6 and 0.3×10^5, respectively, from top to bottom. (L is the width of the picture.)

$(\rho_2 = \rho_1)$ at low Mach number, along with other incompressible flows such as wakes, jets and boundary layers, had long been a part of the classical literature of turbulent shear flow. No hint of other than a classical "turbulent" picture existed.

It was therefore rather astonishing when shadow pictures of the flow revealed the presence of well-defined large structures (Fig. 2) which have the appearance of breaking waves or rollers or vortices. In this example, the Reynolds number varies from a low value at which fine-scale turbulence is largely absent to a higher value at which it is present throughout the flow,superimposed on the large-scale, two-dimensional motion. What is significant is that the measured mean properties of the flow, the velocity and density profiles, spreading rate, etc. are the same for all three cases. The mean flow is controlled by the large, organized structures which, it may be seen, are not affected by the small-scale turbulence appearing at the higher values of Reynolds number.

That the large-scale organized phenomena are not peculiar to the large-density difference is shown by Fig. 3, which is a sequence of shadowgraphs of a flow in which the density is uniform. Optical visualization was made possible by using different gases, of the same density, on the two sides of the mixing layer: nitrogen on the high-speed side and a mixture of helium and argon on the low-speed side. (The imaging method used to obtain the pictures in a high-speed framing camera did not resolve the fine-scale structure.) The Reynolds number is 8.5×10^5, comparable to the highest values in previous, well-known investigations of turbulent mixing layers.[11-13] Thus these flows, with organized large structure, correspond in every way to what have been classically called turbulent mixing layers. Their mean velocity fields, the Reynolds shear stresses calculated from them, and the corresponding growth rates[10-14] agree with what was already known about these flows. In fact, they are the same flows, they are turbulent, but the existence of the organized structures had not previously been recognized. Still another example obtained in our laboratory,[15] in which the vortex structures were observed with dye visualizations, was a mixing layer in a water channel at Reynolds number of 3×10^6.

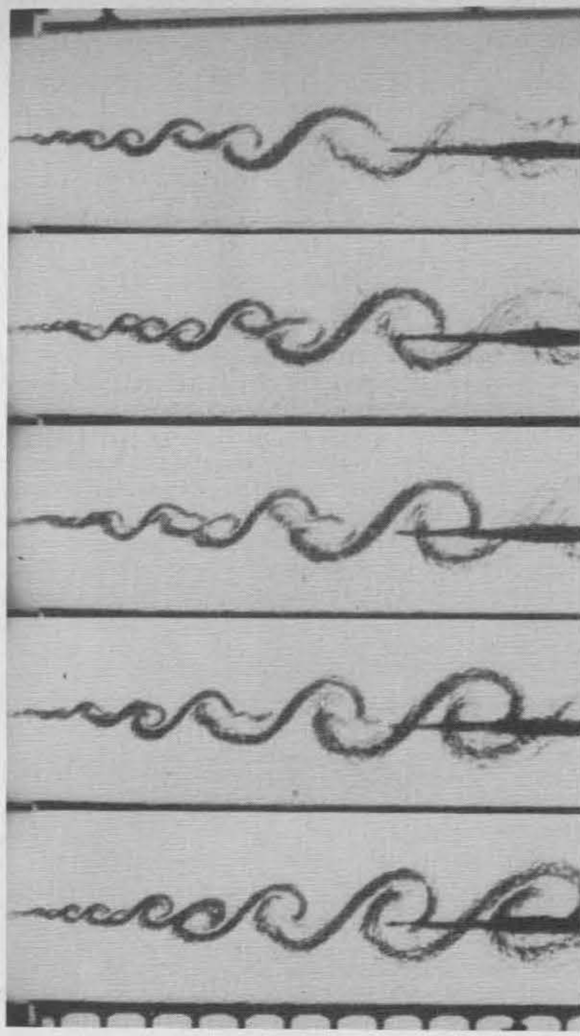

Fig. 3 Mixing layer between nitrogen and a helium -argon mixture of the same density. $\rho_1 U_1 L/\mu_1 = 8.5 \times 10^5$. From top to bottom: frame numbers 1, 6, 9, 13, and 19 of a sequence.

We emphasize these points because typical initial reaction to these pictures (including our own) has tended to be one of skepticism as to whether the flow is really turbulent, whether Reynolds number is high enough, etc. Although these flows had been studied for many years and the presence of the coherent structures not suspected, once the structures are known to be there it is rather easy to find them! It is also interesting that they were being recognized in other investigations[5,16] carried on at about the same time as our own;[9] as mentioned earlier, there has been an increasing awareness over many years of the possibility of organized structures in turbulent shear flows.

Given that they do exist, many questions occur: how can they be described; how are they formed; how long do they exist; what do they do? Some aspects of these are discussed in the following paragraphs.

Coherence and Lifetimes

The vortex-like structures on the photographs in Figs. 2 and 3 can readily be identified from frame to frame on high-framing-rate motion pictures and their progress along the mixing layer can be plotted against time (frame number) on an x-t diagram. A portion of such a diagram is shown in Fig. 4.

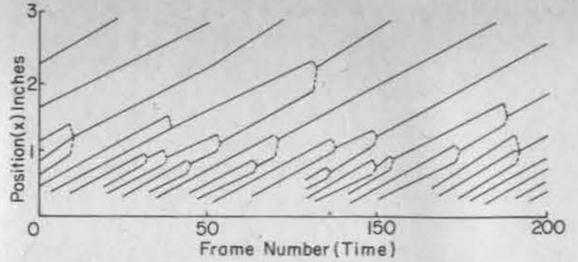

Fig. 4 x-t diagram of eddy trajectories in the mixing layer of Fig. 2a. $\rho_1 U_1/\mu_1 = 1.7\times 10^4$ per inch.

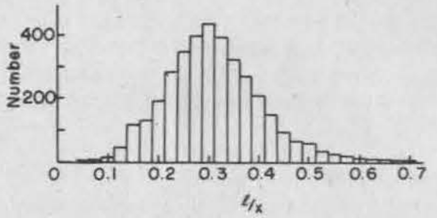

Fig. 5 Distribution of normalized eddy spacings. (Total number of measurements = 3622; total number of eddy pairs ≐ 100.)

Several remarkable results emerge from such observations. The trajectories of individual vortices correspond to the segments of lines (there has been some smoothing) which are nearly all parallel to each other. That is, all the vortices move at nearly constant speed, which is approximately the average of U_1 and U_2 (but with some variation depending on U_2/U_1 and ρ_2/ρ_1). The birth of a new vortex coincides with the demise of two or more old ones, in an interaction process which is only indicated by broken lines on the diagram. We shall presently look in more detail at this interaction; but first it will be useful to discuss some overall features.

It is expected that the scale of any feature of the flow will increase with increasing distance downstream. This follows from the general similarity property of the flow, which requires that all mean length scales be proportional to the distance from the origin. This trend can be seen in the photographs; that is, the sizes of the vortices and their spacings both tend to increase with distance downstream, and both are related to the thickness of the mixing layer. Turning our attention to one of these, the spacing ℓ, we can say that the mean spacing $\bar{\ell}(x)$ must increase with x, *smoothly* because there is nothing special about any particular value of x, and linearly (like the thickness) because of the particular similarity property for this flow. For a mixing layer, similarity requires that $\bar{\ell}$ increase linearly with x. On the other hand, we can see from the x-t diagram that spacings between individual vortices are fairly constant, changing only during the interaction events. Reconciliation between the two apparently contradictory features results from the fact that, in the vortex pairs passing any particular value of x, there is a *distribution* (Fig. 5) of the spacings about a mean value $\bar{\ell}$. An example is shown in Fig. 5; values for different x have been normalized by the distance x from the (apparent) origin to the midpoint between two vortices. The mean value $\bar{\ell}=0.31x$ is also close to the most probable value. Similar results were obtained by Winant and Browand in a mixing layer under quite different conditions.

From the x-t diagram it is also possible to determine the *lifespan* L of each vortex, i.e., the distance traveled from its creation to its absorption into a new one. At any value of x there is also a fairly broad distribution of lifespans; this will be discussed presently.

First it will be interesting and useful to digress into a comparison with correlation measurements that had earlier been used to make inferences about the form and lifetime of the large eddies. The simultaneous measurement of the velocity

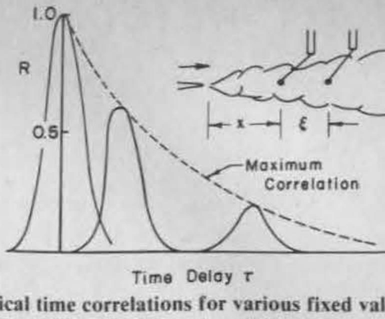

Fig. 6 Typical time correlations for various fixed values of space interval ξ. Maxima occur at values of $\tau=\xi/U_c$.

fluctuation $u(t)$ at two points separated by a distance ξ, say in the streamwise direction, defines the correlation $R(\xi)=\overline{u(x)u(x+\xi)}$, where the bar denotes the average value. By studying such correlations for different directions and different velocity components, Townsend[2] and Grant[3] were able to make some inferences about organized, large-eddy structure in wakes. But it is very difficult to extract a sharp picture of an eddy from such measurements because the well-correlated portion of the signal resulting from the passage of any coherent structures will be degraded by other contributions to the total turbulent signal. Recently the more sophisticated method of conditional sampling[17] has helped to eliminate unwanted parts of the signal. It depends on the possibility of identifying in some way the arrival of the structure to be measured and correlating only while the structure is over the probe. Even with this, the dispersion in scales of the structures passing any point (cf. Fig. 5) will tend to blur the outlines of the structure so defined, unless additional steps are taken to discriminate for size. A variation of what might be called an early form of primitive conditional sampling was the method of space-time correlation,[18] in which the time interval as well as the space interval are varied,

$$R(\xi,\tau)=\overline{u(x,t)u(x+\xi,t+\tau)}$$

Such streamwise, time-delayed correlations gave some of the first evidence for the presence of large eddies moving at a convection velocity U_c, as follows. With $\xi=0$, $R(\tau)$ has a (normalized) maximum value of 1 at $\tau=0$, and drops off on either side as shown in Fig. 6. When one of the measuring points is shifted downstream a distance ξ, the maximum occurs at a later time τ_m which defines a convection velocity $U_c=\xi/\tau_m$. Measurements of this kind in mixing layers were obtained in Refs. 19, 20, and 12.

The maximum value of the shifted correlation is found to decrease with increasing shift, as shown schematically in Fig. 6, which classically has been interpreted as due to decay of the large eddies; the envelope of the set of shifted correlations, that is, the locus of maximum correlation, thus defined the characteristic decay time. From our new point of view, there is a different interpretation; individual eddies do not decay, but their *lifetimes are varied,* and the correlation envelope is in fact the probability $P(\tau)$ for an eddy to survive to an age $\tau=\xi/U_c$. Correspondingly, the probability function for its lifespan to exceed a value $L=U_c\tau$ may be denoted by $P(L)$, or $P(L/\bar{L})$, where $\bar{L}$ is the average lifespan. The measured space-time correlation envelopes for mixing layers[12,19,20] can be fitted by an exponential function

$$P(L)=\exp(-L/\bar{L})=\exp(-\lambda/\bar{\lambda})$$

where $\lambda=L/x=\tau U_c/x$ and $\bar{\lambda}$ can now be interpreted as the average normalized lifetime (and also the most probable value). Values of $\bar{\lambda}$ depend on how the signal was filtered before correlation; values from 0.35 to 0.5 fit the measurements of Refs. 12, 19, 20 on incompressible, homogeneous mixing layers with $U_2=0$. This implies that the average lifespan of an eddy is about $0.4x$ from its point of origin at x.

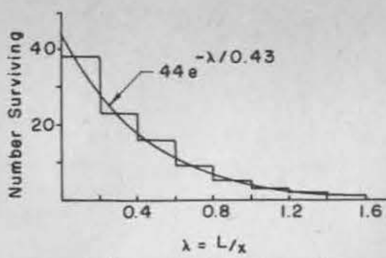

Fig. 7 Survival probability of eddies, from data corresponding to Fig. 4. Sample size = 44.

To compare these inferences with direct measurements of lifespans from *x-t* diagrams we have used data reported in Ref. 10 to plot the survival distribution in Fig. 7. The data can be fitted by a normalized exponential $P(\lambda) = \exp(-\lambda/0.43)$. (In Ref. 9, the average lifespan was found to be $0.39x$, but x was measured to the midpoint of each lifespan, rather than to the beginning.)

Although no great significance can be placed on the numerical agreement (sample number was limited, U_2/U_1 and ρ_2/ρ_1 were different from those in the cited experiments), the results strongly suggest that the space-time correlation envelope is really a life-expectancy curve.

More quantitative work is needed to show the relation of the old space-time correlation decay curves to the rather different concept of survival probability of coherent structures (which do not decay during their lifetimes; they may even grow!). But it is evident that awareness of the existence of coherent structures can lead to new views of some of the statistical quantities that have been measured. Another such quantity is the energy spectrum of velocity fluctuations. That the presence of coherent vortex structure was not earlier revealed by these spectra is due, in part, to the broad dispersal (Fig. 5) of vortex spacings which produce a correspondingly broad peak in the spectrum; in additon, the velocity energy spectrum receives contributions to small wave numbers from the pairing events which may tend to overlap or submerge the broad peak. That the peaks do exist can be seen in the spectrum measurements in Refs. 12, 13, and 20. For $U_2 = 0$ the peak is not so well separated from other low-wave-number energy as at higher values of U_2/U_1 (Ref. 13). In trying to understand energy spectra, it has always been difficult to explain the sources of the energy contributions to the low wave numbers, which contain most of the energy. Clearly, important scales are furnished by eddy spacings and eddy lifespans, but even larger scales (lower wave numbers) will be introduced by the vortex coalescence events and the resulting disruptions of order along the shear layer; three-dimensional effects on all these may also play a role.

Shear Layer Growth by Vortex Interaction

The concepts of shear layer growth, which can correctly but imprecisely be described as due to turbulent diffusion, are also being changed by the new approach; new insight into the processes at work is obtained once the existence of the coherent structures is realized. The growth of the layer is associated with increase of all mean scales; one of these, $\ell(x)$ was discussed earlier. But from the *x-t* diagram (Fig. 4) it is seen that the spacings between individual pairs of vortices tend to stay constant; a change of scale occurs only at the time of coalescence into larger ones, which increases the spacing between the new ones so formed. Thus the interaction which accomplishes this coalescence must be an important contributor to the growth of the mixing layer.

A detailed description of this interaction process was first given by Winant and Browand[5] who described "pairing" as the dominant mode of interaction and the principal mechanism for growth. In pairing, neighboring pairs of vortices rotate around each other and amalgamate into a larger one. Pictures showing various details of the process at rather low Reynolds number are given in Ref. 5. An example from our laboratory at much higher Reynolds number is shown in Fig. 3, where two vortices just to left of center in the top frame can be followed as they rotate around each other and merge into the single vortex which is just over the tip of the probe in the bottom frame. Following this interacting pair, a triplet of vortices is also going through an amalgamation process. What is interesting is that the Reynolds number of the uniform-density flow in Fig. 3 is comparable to the highest values for turbulent mixing layers that have been investigated in the laboratory.[11,13] The process of growth by vortex pairing is similar to that in the Winant-Browand experiment at a value of Reynolds number about two orders of magnitude lower.

In the flows with large density difference (Fig. 2) individual vortices can be followed over their lifespans (Fig. 4) but the mode of coalescence into larger ones is not quite clear. Pairing by orbiting is not evident. There are suggestions of other modes of amalgamation, e.g. elongation and accretion onto another one.

Entrainment and Mixing

A fundamental property of turbulent shear flow, related to its growth, is the phenomenon of entrainment, that is, the incorporation of nonturbulent, usually irrotational fluid into the turbulent region or, conversely, the diffusion of the turbulent region into the ambient flow. Just how this diffusion occurs and what the entrainment process is had never been quite clear.

On shadowgraphs at high Reynolds number the boundary between turbulent and nonturbulent regions is sharply marked by the presence or absence of fine scales. Ballistic-range shadowgraphs of the wakes of spheres and other projectiles provided especially striking pictures of the sharp interface between the two regions and provoked the question of how it propagates into the nonturbulent fluid. There was a tendency to view this propagation as primarily due to mechanisms at the interface between the two regions, instability of the surface and nibbling on the scale of the smallest eddies being suggested as possibilities. While elements of all these are present, a satisfactory picture was not available, largely because the definite, discrete processes associated with the coherent structures were missing. Bevilaqua and Lykoudis[21] pointed out that the interface convolutions visible on wake pictures are the outer edges of large vortices, which are generally obscured inside the boundaries of the wake and that it is these spinning vortices that ingest nonturbulent fluid into the wake.

Similarly, for a plane mixing layer the entrainment process can now be seen to be an engulfing action of the large coherent eddies. Freestream fluid is drawn in between vortices and ingested into the shear layer where it is made turbulent and digested by the action of the smaller eddies. There is a certain element of semantics in the question of what is turbulent and nonturbulent. Should fluid that finds itself deep within the mixing region but still undigested and irrotational be called nonturbulent? Its role in the exchange of momentum and the extraction of energy from the mean flow has already been accomplished, even though the fluid may be still irrotational.[22] That aspect of the entrainment process which determines the gross, mean characteristics of the flow, the mean velocity profile, the shear stress distribution, mean transport, dissipation rate, etc., is apparently accomplished by this large-scale engulfing action of the large eddies; further "turbulization" by smaller eddies is merely a stage in the dissipation of the energy that has been extracted from the mean flow. Whether there is also some entrainment into the vortices by a nibbling process at the boundaries is not clear.

That there are deep incursions of ambient fluid into the mixing region is suggested by the flow pictures. An even sharper indication is provided by a point measurement from a probe which is sensitive to some passive transportable such as temperature and not to velocity. An example of such an out-

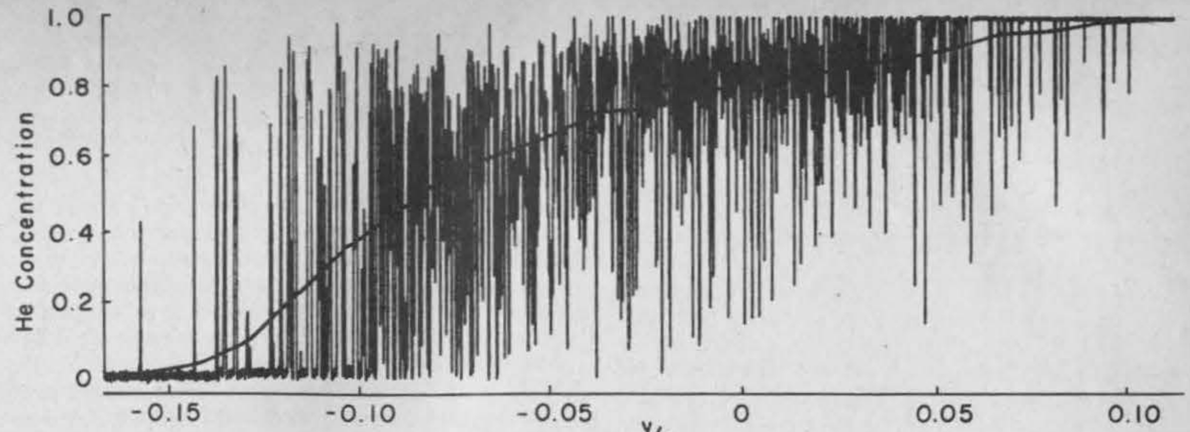

Fig. 8 Computer output of a concentration probe traversing a mixing layer as in Fig. 2a at $\rho_I U_I x/\mu_I = 10^4$. During time of traverse, about 150 eddies have passed by. $C(y)$ is shown by a heavy line.

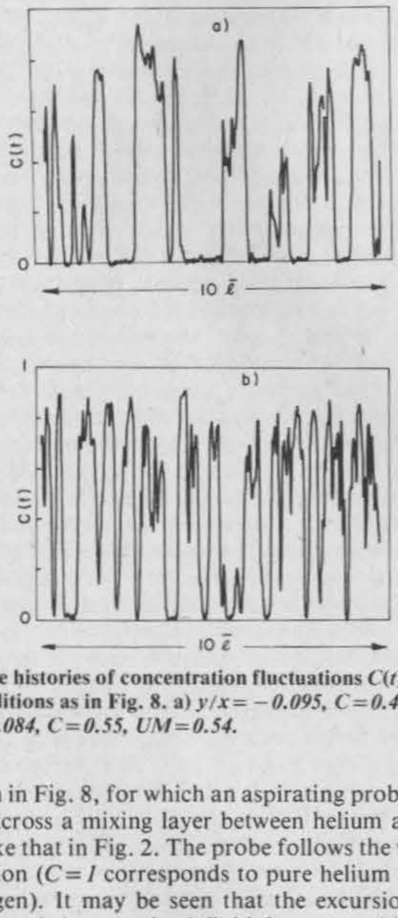

Fig. 9 Time histories of concentration fluctuations $C(t)$ at two values of y/x; conditions as in Fig. 8. a) $y/x = -0.095$, $C = 0.45$, $UM = 0.63$; b) $y/x = -0.084$, $C = 0.55$, $UM = 0.54$.

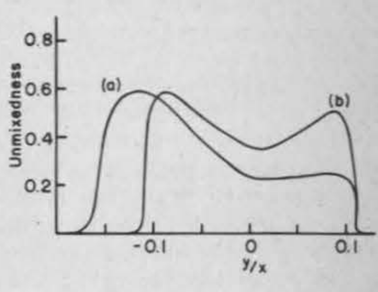

Fig. 10 Distribution of unmixedness factor UM across mixing layers. a) Corresponds to Fig. 2 with $U_I x/\nu_I = 4\times 10^4$. b) Corresponds to Fig. 3 with $U_I x/\nu_I = 3\times 10^5$.

put is given in Fig. 8, for which an aspirating probe[23] has been traversed across a mixing layer between helium and nitrogen streams, like that in Fig. 2. The probe follows the variations in concentration ($C = 1$ corresponds to pure helium and $C = 0$ to pure nitrogen). It may be seen that the excursions of C are very large, and that unmixed fluid from one side penetrates deep into the other side of the mixing region. Intermittency in the flow is strikingly evident, even more so than in a fluctuating velocity signal. This is because velocity influences are long range; i.e., velocity fluctuations can be induced in the nonturbulent or irrotational parts of the fluid by those in the turbulent parts. Intermittency measuring techniques have been devised to separate the turbulent and nonturbulent parts of a velocity signal, but the discrimination occurs naturally for a passive property, which can be changed only by (short-range) intermolecular diffusion.

The question of what happens to portions of fluid that have been entrained into the mixing region from the two sides of the shear layer and how well they are mixed is of interest not only for practical, technical applications but also for further insight into the turbulence structure and mechanisms. Information about this can be obtained by measuring a scalar property such as temperature, density, or species concentration. We can see from Fig. 8. that even in the middle of the mixing layer there are large variations about the local mean value of the concentration of helium in a mixture with nitrogen.

Figure 9 shows the time variation $C(t)$ at two points in the layer obtained from a concentration probe steadily sampling the flow at each of those positions.[24] In Fig. 9a the sampling point is on the nitrogen side of the layer at $y/x = -0.95$, where $\bar{C} = 0.45$ (cf. Fig. 8). The passage of vortices over the probe is indicated by the large pulses (but some vortices are missed). Between vortices, where there is freestream fluid, the signal is steady at $C = 0$; when a vortex passes over the probe, C increases rapidly to a higher value which has some variation in it but not of very fine scale. A little closer to the middle of the mixing layer at $y/x = -0.084$ and $\bar{C} = 0.55$ most of the vortices are being intercepted by the probe. The small pulses embedded in the large pulses correspond either to layers which have been accreted during earlier interactions or to small-scale three-dimensional structure superimposed on the vortices.

Thus the picture of events in the life and interactions of a vortex might be described as follows. In the process of amalgamation, irrotational fluid is ingested and enfolded with the coalescing vortices. The resulting composite structure consists of the two or more new coalesced vortices (plus all previous ones) and the ingested fluid. During its lifetime this structure rotates and strains. At the same time internal mixing is occurring by the action of small-scale turbulence and molecular diffusivity, and the new fluid is digested and incorporated into the structure. There may also be some small-scale turbulent diffusion laterally into the freestream fluid, thus growth of the structure before the next pairing.

If two chemical reactants from the respective two sides were brought together inside the mixing layer, the reaction could proceed only if the reactants came into intimate chemical, i.e., molecular contact. Thus it is of interest to determine the extent of molecular mixedness inside the mixing region. A direct measure would be given by the amount of reactant product formed, but such measurements are not yet available for

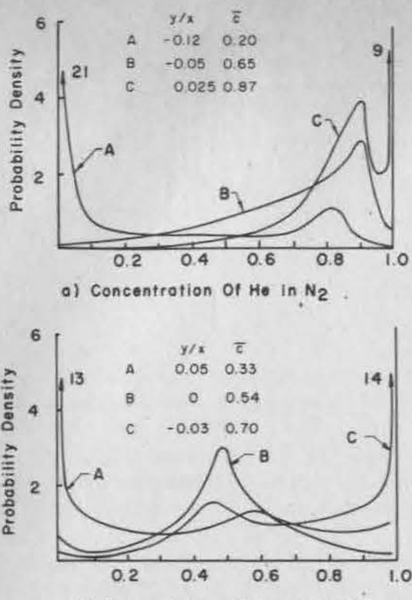

Fig. 11 Probability density distributions of concentration at three values of y/x; conditions a) and b) as in Fig. 10.

mixing layers. Related information can be obtained from measurements of $\bar{C}(y)$ and of the fluctuations around $\bar{C}$. Thus, to define the extent of molecular mixedness M or unmixedness $1\text{-}M$ at any point, various definitions have been introduced. Usually the rms fluctuation is used as in the definition given by $UM \equiv \overline{(C-\bar{C})^2}/\bar{C}(1-\bar{C})$, but this is difficult to measure accurately because the effects of extraneous noise are amplified at small values of $\bar{C}$ or $1\text{-}\bar{C}$. An alternative definition that does not have that difficulty is the unmixedness factor

$$UM \equiv \frac{\int_{T_1} (C-\bar{C})\mathrm{d}t_1 + \int_{T_2} (\bar{C}-C)\mathrm{d}t_2}{(1-\bar{C})T_1 + \bar{C}T_2}$$

where t_1 corresponds to time when $C > \bar{C}$ and t_2 to time when $C < \bar{C}$. With this definition as with the one in the preceding, UM is zero when there is no fluctuation about the mean (i.e., the flow is completely mixed) and has a value of unity when $C(t)$ fluctuates between values of 0 and 1 (i.e., the flow is completely unmixed). The values of UM computed in this way are listed for each case in Fig. 9. The complete distribution of UM across the mixing layer is shown in Fig. 10 for two cases: a) is for a mixing layer between helium and nitrogen ($\rho_2/\rho_1 = 7$) while b) is the case $\rho_2/\rho_1 = 1$ for a mixing layer between nitrogen on one side and helium/argon on the other. It will be noted that for the case of constant density the distribution of UM is nearly symmetrical while for large density difference it tends to be much better mixed on the low-density side. The peaks of unmixedness are evidently connected with the high intermittency at the edge.

Another quantity of interest in connection with chemical reactions is the probability distribution for the concentration at a given point. Examples for the same two cases ($\rho_2/\rho_1 = 7$ and 1, respectively) are shown in Fig. 11 for three different points across the mixing layer. Near the sides, due to intermittency, there is a high probability of seeing the pure gas from that side; in the middle of the layer there is a broad distribution about the peak. Again, the constant-density layer tends to show more symmetry than the one with large difference in density.

Reynolds Number Effects; Transition; Randomness

With increasing Reynolds number the energy-transforming and energy-dissipating scales in a turbulent flow become separated, as more and more small-scale structure becomes available and intervenes between the process of energy extraction and that of viscous dissipation. In fact, the large-scale processes and the resulting mean flow seem to be very little affected by viscosity down to surprisingly small values of Reynolds number. In the sequence of pictures of a mixing layer in Fig. 2 the unit Reynolds number is varied by a factor of 4 (by changes of pressure), the Reynolds number based on U_1, ρ_1, μ_1 and the length of the layer visible in the photographs varies from 0.3 to 1.2×10^5. Based on U_2, ρ_2, μ_2 it is 2.8 times higher. It may be seen that, while there is considerable difference in the turbulent appearance of the flows connected with the small-scale content, the large-eddy structure is similar and the lateral extent of the mixing regions the same. Measurements of the mean-flow properties[14] in these examples do not reveal any significant effects of the Reynolds number. Again, for homogeneous flows, the differences in data collected[10] from various sources seem to be due more to different experimental conditions than to Reynolds number effects. The spreading rates measured by Winant and Browand[5] for $U_2/U_1 = 0.4$ at a Reynolds number of 10^4 agree well with those of Spencer and Jones[13] at a Reynolds number of 10^6. On the other hand the spreading rates measured by Wygnanski and Fiedler[12] and by Liepmann and Laufer,[11] both at high Reynolds number (0.5 and 1.0×10^6, respectively), with $U_2 = 0$, differ by about 30%. Batt[25] has shown that this is connected with the presence or absence of a boundary-layer trip ahead of the shear layer separation from the nozzle wall. In fact all evidence suggests that any important effects of Reynolds number appear indirectly through the initial shear layer conditions and not through direct action of viscosity on the developing turbulent structure. The thickness of the initial mixing layer just after separation and the distribution of vorticity in it depend on the particular nozzle geometry, on the thickness of the splitter plate and on the Reynolds number based on some nozzle dimension. Bradshaw showed some time ago[26] that at least 1000 initial momentum thicknesses are needed for a mixing layer to forget the initial conditions and acquire a local similarity structure. This distance seems surprisingly large in terms of initial thickness but actually corresponds to only 3 or 4 stages of pairing.[15]

Fig. 12 Smoke picture of vortex structure development and pairing in the transition region. Courtesy of P. Freymuth[28].

Pertinent to this discussion is the role played by the transition region and its relation to the turbulent layer further downstream. When the boundary layer before separation is laminar, the separated, laminar, free shear layer is unstable and a two-dimensional instability with wavelength proportional to shear layer thickness develops and rapidly amplifies downstream. It is known that even at this early stage the motion is practically independent of viscous effects; the amplification and subsequent nonlinear development can be calculated as an inviscid process, dependent only on the initial thickness and distribution of vorticity.[27] The events in such a transition region are beautifully illustrated in a photograph by Freymuth[28] reproduced in Fig. 12, where several stages in the transition are evident Most remarkable is the development of the wave into an S-shaped pattern, reminiscent of those in turbulent flow (Figs. 2,3), which evolves into two vortices that subsequently rotate around each other in the pairing process described by Winant and Browand; this results in a subharmonic wavelength of twice the initial spacing. At this early stage the scales are still locked into the initial instability wavelength which itself is related to the initial shear layer thickness (this initial instability scale and viscous scale differ by an order of magnitude).

If this perfectly ordered development were to continue downstream, the mixing layer would be rather different from the one that is observed. Only integer subharmonics would be present; the energy spectrum of fluctuations would be concentrated in corresponding discrete bands. But even with only discrete scales present, a broadened spectrum and a smooth increase of *mean* scale along the shear layer would be possible from phase variations in the positions of pairing. Actually, as we have seen, the scales at any position x are broadly distributed around the mean value appropriate to that position. It is apparent that, after the first pairing, any small irregularities or disturbances of the vortices will alter the subsequent development and the process will increasingly depart from a deterministic sequence tied to the initial conditions. For example, any dislocations that occur in the vortex train will have upstream and downstream influence contributing to the randomization of pairing events and dispersal of scales.

What role three-dimensional disturbances play in this evolution of randomness in the large structure is not certain. Two aspects of the problem have to be distinguished, namely the possible development of spanwise instability in the large vortices and the development of smaller scale turbulence such as that visible at the higher values of Re in Fig. 2. It has long been held that one of the attributes of turbulence is three dimensionality, the implication being that the structure is broken down three dimensionally in all scales. Thus Chandrsuda et al.[29] argue that organized structures as in Figs. 2 and 3 are vestiges of the initial laminar instability and would ultimately become "completely three-dimensional." Their smoke-picture views normal to a mixing layer show some manifestations of large spanwise irregularities when $U_2=0$. These include branching, i.e., pairing over only part of the span, and helical pairing. Whether these are indeed preludes to complete breakdown or only more complex interactions of the large structures is yet to be determined. The experience in our laboratory is that in mixing layers with U_2/U_1 not zero the large coherent structures, those that appear well-organized in a cross-sectional view as in Figs. 2 and 3, tend to be organized two dimensionally in a view looking normal to the plane of the layer. Of course, at higher Reynolds number, small-scale instabilities develop and fine-scale turbulence is superimposed on the large structures. A description of the development of small-scale three-dimensional instability and its relation to the two-dimensional organized vortices is given by Konrad and Brown in Ref. 30. A strictly two-dimensional mixing layer can be be calculated in some detail on a large computer, starting from an initial distribution of vorticity.[31,32] Such calculations exhibit the features of large vortex structure formation and growth by coalescence; when the calculations are perfected it will be interesting to see how closely the mean flow properties of such two-dimensional mixing regions compare with those observed in the laboratory.

Coherent Structure in Other Flows

Although for no other flow has the picture emerged so clearly as for the mixing layer, evidence for the existence of organized structures in other turbulent shear flows has also been accumulating, preceding in some cases the discoveries in mixing layers. Of course, examples of organized structure related to particular geometric configurations are familiar. Best known is vortex shedding from cylinders, which occurs up to very high Reynolds number; the organized, periodic motion is superimposed on a background of turbulence or, perhaps more accurately, *vice versa*. Other examples are the organized large-scale oscillation near the end of the potential core in a jet; and oscillation in turbulent flow over a cavity. In such cases the motion of the large organized structures is more deterministic, because phases are still locked into an initial condition, than in fully developed turbulent flows.

In the following we shall briefly mention some of the indications of organized structure in various turbulent shear flows and speculate about how they might differ from those in a mixing layer.

Wakes

The turbulent wake of a cylinder has, near the cylinder, a well-organized, periodic component called vortex shedding, which persists up to the highest Reynolds numbers measured. It was thought that the vortices are obliterated, in some sense, by the turbulence in about 50 diam downstream of the cylinder.[33] Recently, using Prandtl's old technique of flow visualization, it has been found[34] that the vortices persist to much greater distances downstream, at least 300 diam, but are more disorganized than closer to the cylinder. Here again is an example of the usefulness of flow visualization; again it becomes necessary to reinterpret previous results in light of the existence and survivability of coherent structures. It will be recalled that Townsend found that 500–1000 diameters are needed for the wake to achieve similarity, i.e., forget its origins.

We must now ask how this disengagement from initial conditions is achieved, whether by coalescence of the vortices into larger structures. Very suggestive are the flow visualization experiments of Taneda[35] on the wakes of vortex shedding circular cylinders at Reynolds numbers of about 100, i.e., in the transitional range in which irregularity begins to appear in the periodic wake. Taneda observed that the primary vortex street became disorganized at some distance downstream but later reformed with larger scale; this is, of course, reminiscent of the effects of vortex pairing in the mixing layer and suggests that some kind of process of amalgamation is occurring.

The discussion so far implies vortices with axes in the plane of the wake more or less parallel to the cylinder, i.e. a two-dimensional structure. Quite a different picture of coherent structure in a wake was deduced by Grant[3] from correlation measurements 533 diam downstream of a circular cylinder at $R_d = 1300$. He described it as a pair of vortices, side by side and rotating in opposite directions, with axes approximately *normal* to the plane of the wake. This seems quite incompatible with the picture of vortices with axes parallel to the cylinder, but Grant did not say how his vortices terminated. Possibly the two views can be reconciled if in fact they are two mutually perpendicular views of a *vortex loop*, formed by pinching off and joining together of vortices from opposite sides of the street. This is a possibility for wakes and jets, whose underlying mean flow is basically two rows of vortices of opposite sign, and is more complex than the mixing layer, which is one row of vortices of the same sign. Such an array of vortex loops interacting with each other, possibly combining to form larger ones, would appear rather chaotic, and it might be difficult to devise measurements to isolate the form of their structure and interactions. Essentially such a qualitative picture was put forward by Munk[37] over twenty years ago.

The mixing layer, with its basic mean vorticity all of one sign, is unique in the catalog of shear flows. Although its vortex lines might develop instabilities, there is not the same possibility for forming three-dimensional structures as in most other shear flows. That is, when the underlying vorticity structure is a double row of vorticity of opposite signs, it becomes possible to form vortex loops or rings. Even the turbulent boundary layer may be considered to have such a double-row structure, namely a row of vorticity and its image in the wall. A vortex loop formed by the joining of opposite vortex lines would now produce, in the real part of the flow, a half loop with its two legs ending on the wall, something like the horseshoe vortex proposed by Theodorsen[38] in 1955 as a basic element of turbulent boundary-layer structure. Some experimental findings and hypotheses that may be pointing toward such a structure are briefly reviewed in the next section.

Turbulent Boundary Layer

Townsend[2] had inferred, from velocity correlation measurements, the presence in turbulent boundary layers of characteristic eddies of finite length with structure elongated in the flow direction. More evidence for this and new impetus to the search for coherent structure was given by the flow-visualization experiments of Kline and Rundstadler.[38] That and succeeding investigations[39,41] have gradually developed a rather complex description of a characteristic, intermittently occurring pattern which is called "bursting." It is a localized, three-dimensional pattern characterized, along with many other manifestations, by sudden and large changes of velocity near the wall. Bursting has been described as the major contributor to turbulence production near the wall; for example, it has been shown[42] that large contributions to the Reynolds stress occur during the bursting period. The cycle of events at any location is intermittent, with broad distribution about a mean period $\bar{T}$.

From the early observations it was thought that bursting phenomena were confined to a region near the wall, but it was later shown[43,44] that the mean period correlates with overall boundary-layer thickness δ, i.e.,

$$\bar{T} \doteq 5\delta / U_I$$

Other measurements, for example of pressure fluctuation on the wall,[45] also pointed to the presence of large-scale structure convecting with a velocity $U_c \doteq 0.8U_I$. These and space-time correlations of fluctuating velocity[17] suggested decay times for the structures which correspond to a travel distance of 10-20δ. If the interpretation of such correlations which we presented earlier is applicable to the boundary-layer structures (i.e., that the structures do not decay but have a distribution of lifespans), then 10-20δ is in fact the lifespan of the longest lived structures and 4-6δ might be an average lifespan.

A picture is gradually emerging[6] that the bursting phenomena are connected with the convection of characteristic, three-dimensional structures on the scale of the boundary-layer thickness.* Whether a bursting event corresponds to the passage of a structure, or to its interaction with another one, or to contributions from both is not clear. A mean lifespan of 4-6δ is consistent with the legth scale $U_c \bar{T} \doteq 4\delta$ which has been measured from the mean bursting period.

A sharp picture of a coherent structure in a turbulent boundary layer has not yet been drawn. To deduce it from measurements of fluctuating velocity or other property, even with the latest computer-aided techniques is still very difficult. Flow-visualization techniques, which were crucial in initial identification, are less helpful in outlining the complete, three-dimensional structure.

An interesting departure has been taken by Coles. Based on the observation that coherent structures were first investigated at low Reynolds numbers, down to the lowest values for which turbulence exists, he has suggested that the basic, coherent structure is the turbulent spot discovered by Emmons,[46] which is found in the transition region of a boundary layer. To test this hypothesis, he and Barker[47] created a synthetic turbulent boundary layer by generating an array of spots in an initially laminar boundary layer. The mean velocity profile of the resulting flow fitted quite well the "standard" profile for a turbulent boundary layer with Reynolds number $R_\theta = 975$. It remains to be determined whether other properties such as turbulent correlations and bursting phenomena are similar to those in natural turbulent boundary layers. The form of the turbulent spot is known from the measurements of Schubauer and Klebanoff.[48] A more precise description has been obtained by Coles and Barker by using conditional sampling techniques on single spots which were produced at controlled rate. They found that it is a horseshoe-shaped vortex with ends on the wall. (With its mirror image below the wall, it forms a vortex loop which is sharply bent at the plane of the wall, the top and bottom of the loop being inclined downstream.) Whether this or some other is indeed the basic coherent structure of a turbulent boundary layer and how it interacts with others are exciting questions still to be investigated.

*Recent additons to the picture were reported at the I.U.T.A.M. Symposium on Structure of Turbulence and Drag Reduction, Washington, D.C., June 7-12, 1976; proceedings to be published in *Physics of Fluids.*

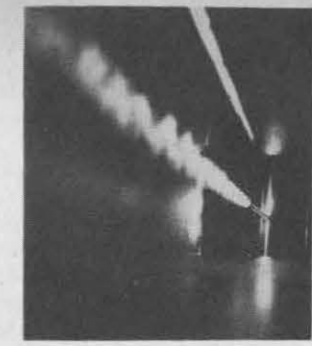

Fig. 13 View upstream in a wind tunnel of a coflowing jet with plane of symmetry illuminated by a vertical sheet of light. Jet diameter $d = 1$ in.; $U_j/U_\infty = 1.5$; $U_j d/\nu = 3500$; $x/d = 80$.

Conclusion

There is little doubt that coherent structures play a central role in the development of the several turbulent shear flows that have been most extensively investigated, namely mixing layers, boundary layers and the early regions of jets and wakes. For the two-dimensional far wake a partial picture of a coherent structure has been described by Townsend and Grant. It is natural to suppose that organized structure is also present in other turbulent shear flows, in each case having a form that is characteristic for that particular flow. Large structures called puffs and slugs are found in the transition region of pipe flow;[49] one wonders whether they provide some indication of a basic, so far hidden structure in fully developed turbulent pipe flow and what the relation may be to those in boundary layers. A spiral vortex structure in Couette flow between two concentric cylinders has been studied by Coles.[50] Evidence for organized structure in the far region of a turbulent round jet is seen in a photograph (Fig. 13) obtained by Higuchi.[51]

From these examples it is evident that experimental investigators interested in turbulent flow have a wide arena. There are also ample challenges for theoretical workers. Computer solutions of nonsteady turbulent flows also can provide valuable insights.[32,52] Understanding of the physical processes actually occurring in turbulent shear flows is indispensable for progress toward an analytical description of them. Even short of that, knowledge of these processes is helpful for understanding and coping with practical problems in which turbulent flow is prominent.

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