STRUCTURED DESIGN OF MICROELECTROMECHANICAL SYSTEMS

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ABSTRACT

In order to efficiently design complex microelectromechanical systems (MEMS) having large numbers of multi-domain components, a hierarchically structured design approach that is compatible with standard IC design is needed. A graphical-based schematic, or structural, view is presented as a geometrically intuitive way to represent MEMS as a set of interconnected lumped-parameter elements. An initial library focuses on suspended-MEMS technology from which inertial sensors and other mechanical mechanisms can be designed. The schematic representation has a simulation interface enabling the designer to simulate the design at the component level. Synthesis of MEMS cells for common topologies provides the system designer with rapid, optimized component layout and associated macro-models. A synthesis module is developed for the popular folded-flexure micromechanical resonator topology. The algorithm minimizes a combination of total layout area and voltage applied to the electromechanical actuators. Synthesis results clearly show the design limits of behavioral parameters such as resonant frequency for a fixed process technology.

INTRODUCTION

Microelectromechanical systems (MEMS) are sensor and actuator systems made from microelectronic batch fabrication processes. New developments span across such diverse applications as inertial navigation systems, digital mirror displays, DNA analysis systems, and RF distributed sensor networks. These systems incorporate truly mixed technology, integrating combinations of digital and analog electronics, mechanical structures, electromechanical actuators, and fluidic chambers.

MEMS has existed as a technical field since the early 1980's. Past research has primarily focused on developing new process technologies to support specific applications. As stable fabrication processes have emerged, many research efforts have shifted to system design of increasing complexity with future systems containing hundreds or even thousands of mixed-domain components. There is a growing demand for CAD tools to support rapid design of systems involving physical interactions between mechanical, electrostatic, magnetic, thermal, fluidic, and optical domains. The design needs are similar to those driving advances in analog and microwave system CAD. As is the case with pure analog design, the existence of hierarchical cell design methodologies, mixed-technology simulators, layout synthesis tools, and design-rule checking will enable MEMS engineers to build larger systems and allow them to concentrate on higher-level design issues.

One relatively mature design area is surface-micromachined suspended MEMS, as exemplified by the recent success of commercial microaccelerometers for automotive airbag deployment [1][2] and digital mirror displays for high-fidelity video projection [3]. The availability of accumulated design expertise, stable fabrication services, and electromechanical CAD modeling tools has made the suspended-MEMS technology a suitable candidate for initial development of structured design tools for MEMS. Our discussion of structured design will be restricted to suspended MEMS, however the concepts should apply to other technologies as they mature.

We begin with an overview of the MEMS design process, followed by a our view of structured design in a MEMS environment. Next, we describe our approach for MEMS cell synthesis, where we detail the results of our synthesis approach and discuss the layout synthesis results. Finally, we present verification of our synthesis results using finite element simulation.

CURRENT MEMS DESIGN PRACTICE

Typically, MEMS engineers begin design of a new component with a rough sketch and very basic equations to ensure feasibility. This stage usually leads directly to a physical layout, as shown in Fig. 1 for the case of a micromechanical resonator. In many cases, the layout is sent to fabrication with little if any verification, frequently resulting in non-functional devices. The designer has two analysis choices: numerical simulation (*e.g.,* finite-element analysis), and behavioral simulation. Tools exist for both kinds of analysis, however each method has drawbacks which researchers are working to eradicate.

Numerical simulation involves self-consistent mechanical finite-element analysis coupled with electrostatic boundary element analysis. Tools that cater to the MEMS community are available from several companies [4][5][6]. Unfortunately, the simulation time of these tools is prohibitively slow for tight iterative design. Faster algorithms and machines will alleviate this problem, however numerical simulation alone will not be able to handle large MEMS design problems. Numerical simulation has massive numbers of variables due to the discretization of the solid model. Interpretation of the simulation results requires domain specific expertise and is tedious. Therefore, numerical simulation is primarily seen as critically important for MEMS modeling and verification, analogous to the role that electronic technology CAD plays with device and interconnect modeling.

Behavioral simulation can be accomplished using many different commercial tools, such as SPICE [7][8], MATLAB [9][10], Saber [11], and Spectre [12]. Some groups [13][14] have started to construct geometric parameterized cell libraries of MEMS components to support behavioral simulation. Unfortunately, the construction of behavioral models for MEMS components is completely manual, requiring specific device expertise which is often lacking in a system designer. Model construction requires numerical simulation and, therefore, cannot be placed in the iterative design loop.

No rapid design process is available today for MEMS. Only one or two CAD iterations involving simple functional simula-
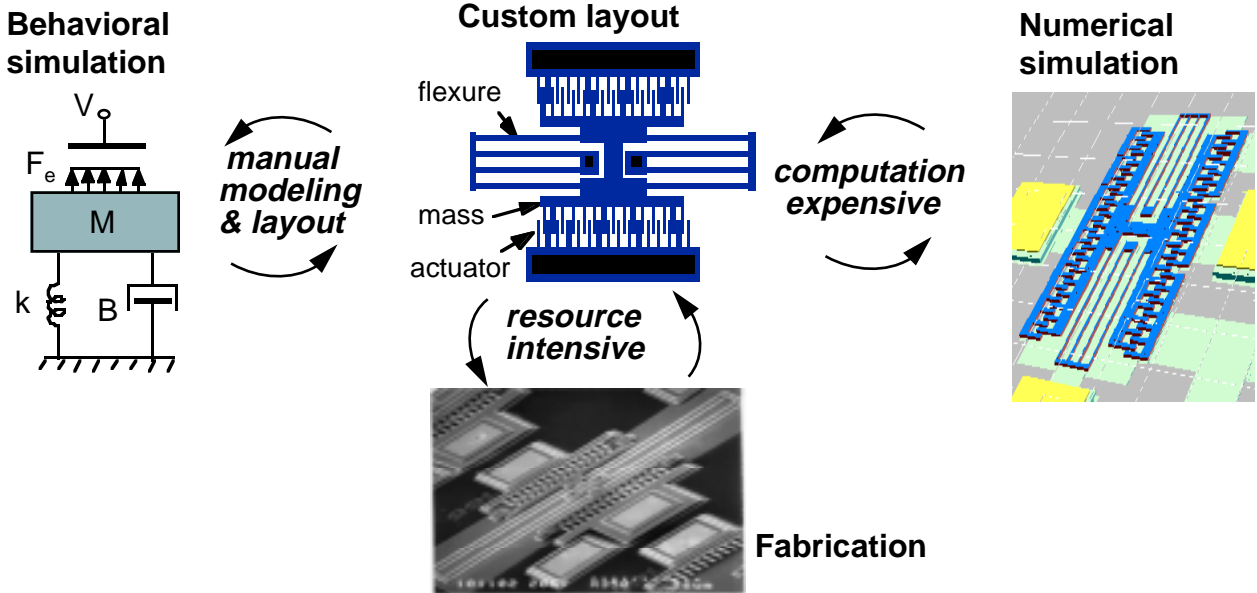
Fig. 1. Schematic depicting the current state of MEMS design using a micromechanical resonator example. Computer-aided design starts with a custom layout. Presently, all three choices of design iteration are slow and expensive.

tions are usually attempted during prototype design. As a result, fabrication replaces simulation in the iterative loop. This is very expensive, since fabricated prototypes often do not meet performance specifications and, sometimes, are not even functional. Full verification of designs requires months of effort, and design optimization is not realistic in all but the simplest of cases. These problems inhibit growth in this field, particularly in the use of MEMS for low-cost, low-volume application specific sensors, integrated on the same chip as attendant electronic information processing and communication capabilities.

THE MICRORESONATOR

The microresonator shown in Fig. 2 will be used to describe our approach to MEMS component design. The specific resonator topology was first described and analyzed by Tang [15]. It is used in resonator oscillators, in filters, and as a mechanical characterization test structure to measure Young's modulus of thin films. The central shuttle mass suspended by two folded-beam flexures forms a mechanical mass-spring-damper system. The folded flexure is a popular design choice for the suspension because it is insensitive to buckling arising from residual stress in the polysilicon film. Instead of buckling, the beams expand outward to relieve the stress in the film.

The resonator can be fabricated via a surface-micromachining fabrication service such as the Multi-User MEMS Process service (MUMPs) from MCNC [19]. In this process, the resonator components are made entirely from a homogeneous, conducting, 2 μm-thick polysilicon film. The movable microstructure is fixed to the substrate at only two anchor points, which also act as electrical vias. The 2 μm spacer gap, g, above the substrate is formed by wet etching a sacrificial oxide film under the structure.

The resonator is driven in the preferred (x) direction by electrostatic actuators that are symmetrically placed on the sides of the shuttle. Each actuator, commonly called a 'comb drive,' are made from a set of interdigitated comb fingers. When a voltage is applied across the comb fingers, an electrostatic force is generated which, to first order, does not depend on x. The suspension is designed to be compliant in the x direction of motion and to be stiff in the orthogonal direction (y) to keep the comb fingers aligned.

A simplified schematic view of the resonator is shown in Fig. 3. The device is represented as an interconnected set of

mixed-domain lumped-parameter elements: the shuttle mass, two folded-flexure springs, and two comb-finger actuators which are displayed as time-varying capacitors. Each of these elements serves both an electrical and mechanical role. A voltage source that drives one actuator is also included in the schematic. Mechanical anchor points are designated by notation similar to electrical ground. In contrast to electrical circuit design, MEMS design at the component level requires a linkage of form and function. For example, the physical attachment point of the springs to the shuttle mass affects functional parameters such as voltage versus displacement characteristics and system modal frequencies. Physical placement parameters must be included in the element models and ultimately linked to the mechanical nodal equations.
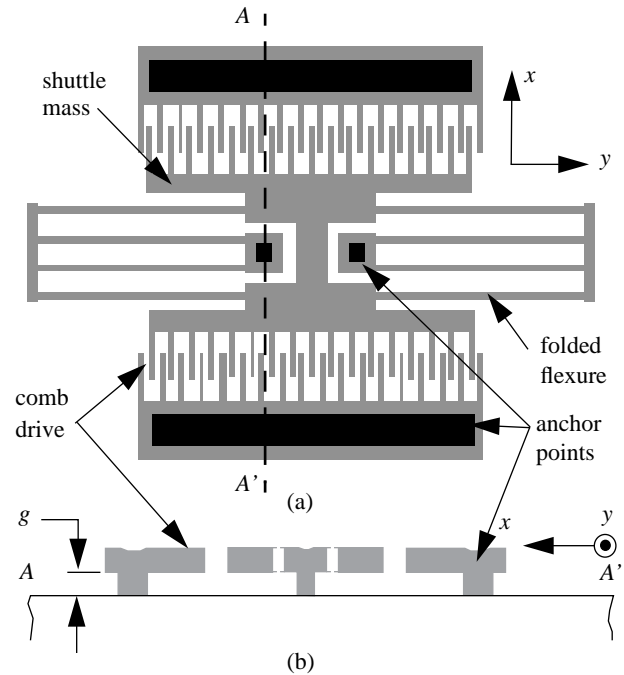

Fig. 2. A folded-flexure comb-drive microresonator fabricated in the MUMPs process. (a) Layout. (b) Cross-section A-A'.
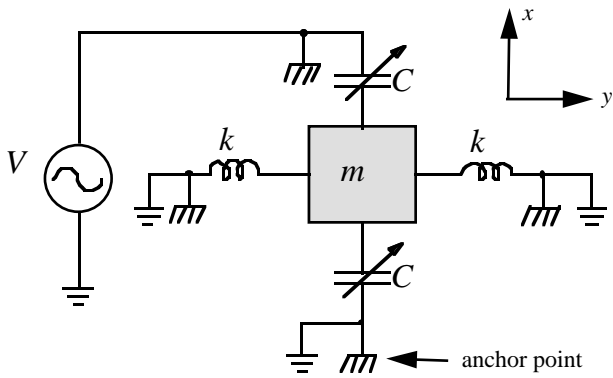
Fig. 3. Mixed-domain schematic of the lateral folded-flexure comb-drive microresonator, including a voltage source, *V,* for comb-drive actuation.

NODAL SIMULATION

At present, MEMS simulation is most often performed using direct numerical simulation (*e.g.,* finite element analysis), or signal flow analysis. Finite element analysis is arduous and time consuming for system design due to its low level of abstraction, and lack of design hierarchy. Previous work on higher-level MEMS structural design has focused on behavioral simulation of individual devices (*e.g.,* microresonators) with abstract macromodels [16], or with eigenmode decomposition using single degree-of-freedom (DOF) elements [17][18].

Systems that incorporate suspended-MEMS technology will have different CAD requirements than individual devices such as a microaccelerometer proof mass. For system-level applications, MEMS behavioral simulation must support a combination of signal-flow analysis and electrical nodal analysis. In this scenario, the MEMS components are black boxes with non-electrical variables treated as input or output states. This is similar to mixed-mode analog simulation with support for non-electrical variables. Aside from the mixed-mode simulation issues, the key bottleneck is rapid development of the MEMS device models.
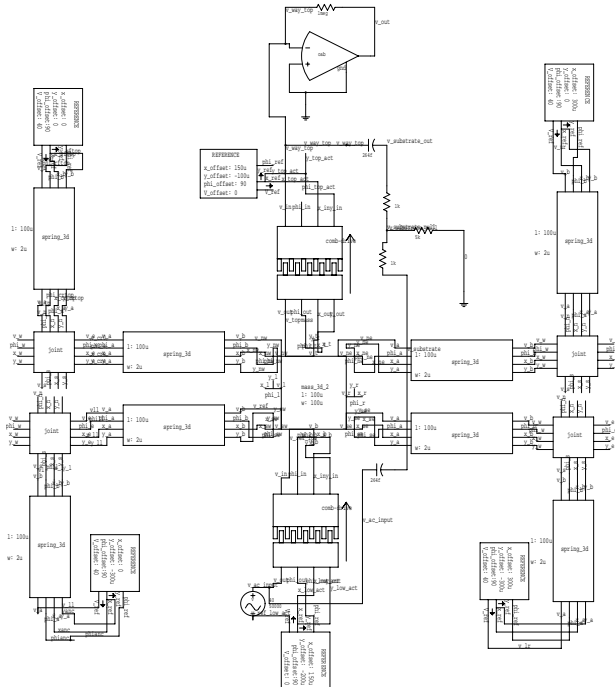

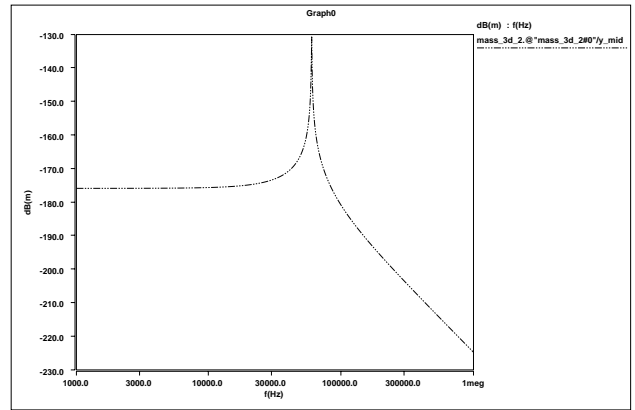Fig. 4. Schematic Representation of a MEMS resonator

Fig. 5. AC analysis of MEMS microresonator

Our nodal simulation approach enables a structured representation for MEMS design using a hierarchical set of MEM components, which can be interconnected in a general way to create more complicated components and systems. The resulting schematic view provides a direct linkage between both physical layout and behavioral simulation, as is the case with standard integrated-circuit design. A key feature is the one to one correspondence of components to layout, which provides an intuitive interface for the designer. Coupling the schematic methodology with existing schematic capture tools that are compatible with electrical circuit analysis enables MEMS design to be quick and efficient.

Furthermore, to get around the bottleneck of MEMS device model development, we are working on a simulation methodology that treats atomic MEMS elements (such as beams and gaps) as the fundamental simulation entities, enabling the simulation of interconnections of beams. To highlight this capability, we show the schematic representation of a 'crab-leg' MEMS resonator in Fig. 4. The crab-leg suspension is a popular MEMS spring device, created by joining two beams at 90°. The a.c. simulation of this microresonator is shown in Fig. 5 indicating that the resonant frequency for that resonator is 40kHz.

Currently, work is progressing on elemental-level simulation, in which fundamental MEMS beam and gap elements are interconnected to create MEMS devices, as well as device-level simulation, in which device macromodels (*e.g.*, comb-drive electrostatic actuator and folded-flexure suspension) are interconnected to simulate a complete system application (*e.g.*, a microresonator oscillator).

MEMS CELL SYNTHESIS

System-level design for MEMS requires the use of mixed-technology cells. As is the case with other technologies, the MEMS cells for a particular design may come from libraries of fixed-cells, parameterized cells, or synthesized cells. We believe that MEMS cell synthesis is the best choice, providing a flexible and extensible way to generate cells.

Fixed-cell libraries that adequately cover all possible design space would require an enormous number of cells. For example, microaccelerometer applications span a continuous range of performance specifications. Full-scale acceleration requirements range from 1 G for inertial sensors to greater than 100,000 G for munitions fuzing. Additional specifications, including bandwidth, resolution, sensitivity, linearity, and cross-axis rejection, must be simultaneously satisfied for a specific application. Technology will continue to improve, so that construction of a fixed cell library would be obsolete well before it was completed.
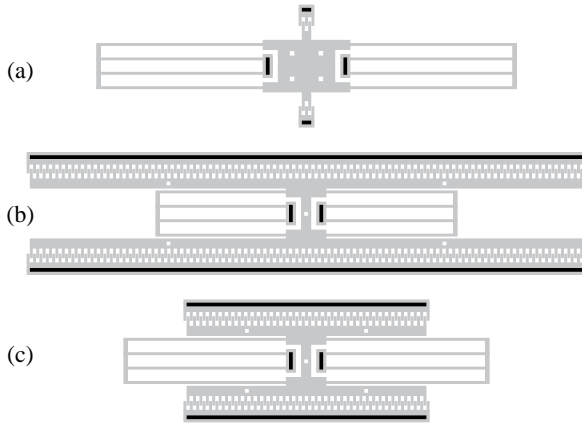
(a)

(b)

(c)

Fig. 6.   20kHz resonators synthesized for three different design objectives: (a) active area, (b) drive voltage and (c) combination of active area and drive voltage

Geometric parameterized cell libraries are a useful first step in applying structured design techniques to MEMS. For example, parameterized cells for the MUMPs process are provided by the Consolidated Micromechanical Element Library (CaMEL) parameterized module generation software [14] from MCNC. The program generates layout in CIF for various electrostatic comb drives and motors, spring suspensions, resonators, hinges, and sliding mechanisms. However, there is no automation to guarantee a specified functionality. In general, the designer must manually evaluate numerous iterations to generate a design which satisfies performance requirements.

Our approach to layout synthesis involves modelling the design problem as a formal numerical synthesis problem, and then solving it with powerful optimization techniques. This philosophy has been successful in analog circuit synthesis [20][21][22], in which synthesizers have generated layout of op-amps, switched-capacitor filters, and other common analog circuits. Although universal building blocks have not been discovered for MEMS, components can be identified which continuously crop up in system designs. In the suspended-MEMS area, reusable topologies include several kinds of accelerometers, gyroscopes, resonators, x-y positioners, and micromirrors. Instead of redesigning these components each time a new system is proposed, engineers will benefit from cell synthesizers which tackle the routine design of frequently-used components.

The process of modelling the design problem involves determining the design variables, the numerical design constraints, and the quantitative design objective. As a starting point, we have developed and tested a synthesis tool for the surface-micromachined resonator topology described previously. The lowest three lateral translational and rotational modes of the mass-spring-damper system are modeled by second-order equations of motion. The vertical mode and other higher order modes are currently not modeled.

All of the design variables are structural parameters of the folded flexure and comb drive elements, with the exception of the comb-drive voltage. Technology-driven design rules constrain the minimum geometries, such as beam widths and minimum spaces between structures. Maximum values of structural parameters are primarily constrained by possible sticking of the structural film to the substrate during sacrificial oxide etching. The functional constraints include resonant frequency, stroke, quality factor, and electromechanical stability.

The complete design problem is therefore represented as a constrained non-linear optimization problem, and solved by an off-the-shelf solver [23]. Due to the significant number of local minima in this design problem a gridded-multistart optimization is currently being employed, and we are considering the use simu-
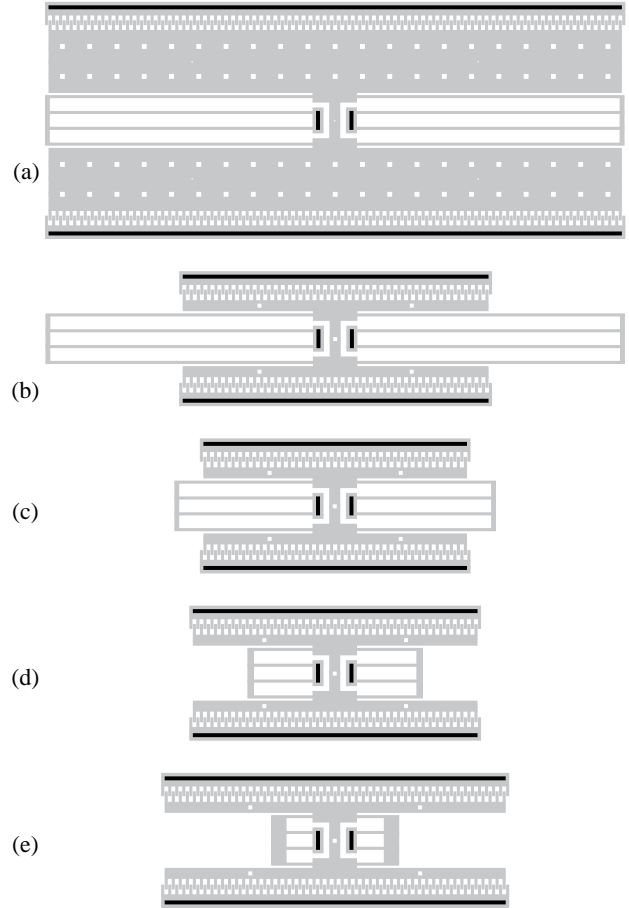

(a)

(b)

(c)

(d)

(e)

Fig. 7.   Layout of five resonators synthesized from specifications. (a) f_r = 3 kHz, (b) 10 kHz, (c) 30 kHz, (d) 100 kHz, (e) 300 kHz.

lated annealing [24] as the optimization engine due to its potential for global optimization in the face of many local minima. The use of a deterministic optimization approach is essential in this early part of this research since it quickly highlights the effects of changes in the macromodels used in during the synthesis. Finally, the gridded-multistart approach is particularly useful in ensuring we exercise the macromodel in the entire design-space to debug these hand-generated macromodels.

For our first attempt at encoding the MEMS component-level design problem as an optimization problem, we initially imported a first-order model of the design objective, area minimization, from VLSI design. In our implementation, we used active area used by the MEMS devices to compute the area being minimized. The resulting designs were clearly impractical from the point of view of a MEMS designer. Essentially, the area minimization was leading to single comb-finger resonators, which while perfectly feasible in terms of operation, were undesirable due to their dependence on high drive voltages to ensure adequate resonant stroke. To highlight the differences we show the layouts synthesized for the 20kHz specification for three different objectives: area minimization, drive voltage minimization, and minimization of a combination of area and drive voltage (Fig. 6).

Five example synthesized resonator layouts are shown in Fig. 7. The design objective is set to minimize a normalized combination of total layout area and drive voltage. Several iterations of the design variables and constraints were necessary to produce synthesized designs that followed manual design convention and common sense. Layout visualization of the structures was instrumental in debugging the equations. Feedback from the synthesis iterations directed our efforts to codify the design variables and

constraints. In many cases, a quick inspection of a synthesized layout was all that was needed to determine errant or missing equations.

As expected, the devices shown in Fig. 7 become smaller with increasing values of resonant frequency. Smaller devices have less mass, and smaller flexures are stiffer. Both effects increase the resonant frequency. These results span the approximate design range for this particular topology and process technology. For high frequency resonators, the mass becomes limited by the lower bounds on the comb drive dimensions to maintain adequate stroke. Very low frequency resonators are limited by the upper bounds imposed on geometry.

MODEL GENERATION AND VERIFICATION

Although approximate, algebraic behavioral equations that predict performance require little computational time and can be integrated with iterative improvement algorithms such as simulated annealing. In this section the accuracy of these equations for performance prediction is discussed.

The MEMS system that is being synthesized is decomposable into a number of common sub-blocks, for example a folded-flexure spring, or a lateral comb-drive electrostatic actuator. Algebraic models for the specific topology of these device-level sub-blocks either exist or can be generated. Performance prediction begins by traversing upward through the hierarchical decomposition, starting with the lowest level sub-blocks and working upward until the performance prediction of the structure is completed. Once the performance of each sub-block has been determined, the information is used to compute the performance of components that are higher up in the hierarchy, eventually leading to the predicted performance for the desired structure [25]. Note that the information flow need not be purely upward in this hierarchy. If two sub-blocks interact, for example in determining an equivalent spring constant, then the analysis equations at the next higher level must iterate on these two sub-blocks until they converge.

The analytic equations for the microresonator are derived from electrical and mechanical principles. The algebraic equations for the folded-flexure resonator spring constants were derived using an energy-based method by solving for the linear force versus displacement of the spring [15]. The comb-drive model is a physics-based model derived from a parallel-plate capacitor plus a fringe field term determined by electrostatic finite element analysis [26]. The resonant frequencies of the microresonator are then derived from these parameters using Rayleigh's Energy method [27]. The parasitic effects on the microresonator are determined from air-damping modeled as Couette flow in the spacer gap [28].

The algebraic model for the synthesized resonators of Fig. 7 were verified using ABAQUS [29], a commercial mechanical finite-element tool. Our simulation setup employed the 8-node biquadratic, reduced integration, plane stress elements, generated directly from the layout. Table 1 lists the simulated performances of the synthesized microresonators, as well as the percentage error of the analytical models with respect to simulated performance values. Each row in the table corresponds to a design specified by the design frequency in the first column, with a ±10% allowable error margin. As can be seen from the table, the percentage errors of the algebraic performance prediction equations compare favorably with the simulated values. At low frequencies, the analytical and simulated values match well, hence the synthesis accurately generates a synthesized layout if feasible. At very high frequencies (greater than 1MHz) the analytical model for the microresonator breaks down because modes that are not modelled become dominant. In addition, the linear models for the spring constant and the resonant frequency fail. Therefore, the synthesis system can generate synthesized results that are actually not feasible. To

Table 1. Simulated results and percentage error with respect to equation-based model

Design f_x±10% [kHz]	k_x [μN/μm]	k_x error [%]	M_x [ng]	M_x error [%]	f_x [kHz]	f_x error [%]
3	0.19	0.17%	430	0.6%	3.38	2.2%
10	0.19	0.16%	50	10.1%	9.93	8.3%
30	1.45	0.68%	41	0.3%	29.8	9.5%
100	16.7	0.29%	41	9.3%	98.7	8.7%
300	181.3	1.54%	23	13.9%	271.2	0.5%

eliminate this, further work is needed in improving the range of validity of the models, or this range of validity has to be coded as a constraint into our synthesis system.

CONCLUSIONS

Structured design methods for suspended MEMS promise to shorten the development cycle to days, and enable design of more complex systems comprised of hundreds to thousands of micromechanical elements. Identification of reusable hierarchical representations of MEMS components is a critical first step in advancing toward a structured design methodology and in leveraging existing CAD tools.

A mixed-domain schematic representation will enable rapid exploration and analysis of the design space for MEMS components. Many existing suspended-MEMS designs can be partitioned into discrete elements and devices, such as beam springs, plate masses, and electrostatic actuators, that are modeled as lumped-parameter elements. Conversely, new components can be created by connecting together these lumped elements. The development of component-level simulation capability that can simulate novel interconnections of these MEMS elements and devices is critical for shortening the MEMS design cycle.

MEMS cell synthesis is a powerful tool for building common components that can then be used in larger systems. Our work on layout synthesis of microresonators has shown that a key prerequisite for synthesis is a set of lumped-parameter models that adequately link device behavior with physical design variables. The use of algebraic models of the MEMS components instead of a numerical simulation is essential in controlling the computation time required to generate synthesized results via an iterative improvement algorithm.

Finally, we envision a MEMS design environment in which the expert MEMS designer can rapidly iterate on ideas for MEMS designs, in the same integrated environment where a system-level designer can use synthesized and custom-made MEMS components to develop monolithic mixed-technology chips for low-cost, low-volume commonplace applications. Such a design environment is essential for designs in which sensors need to be integrated on the same chip as attendant electronic information processing capability.

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