Structured Dirty Paper Coding with Known Interference Structure at Receiver

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Abstract— **The Tomlinson-Harashima precoding is well known for dirty paper coding implementation. Despite its simplicity, THP suffers from a significant performance loss in the low SNR region due to modulo operations. In this paper, we propose new dirty paper precoding scheme by taking advantage of the known modulation structure of interference (e.g., BPSK and QPSK signals). The new method, termed structured DPC (SDPC), outperforms the regular THP with modest changes to the transmitter and receiver. For BPSK and QPSK cases investigated, the SDPC only suffers power loss, which is up to 1.25dB compared with non-interference case, while the regular THP-based scalar dirty paper coding has a typical 4-5 dB capacity loss in the same low SNR regions.**

I. Introduction

Broadcast is a communication scenario where a single transmitter sends independent information to multiple uncooperative receivers. The broadcast channel was first introduced by T. Cover in [1], where an achievable capacity region of the degraded channel is derived by means of superposition coding. In [2] and [3], an inner bound was presented for degraded discrete memoryless broadcast channels. Marton presented an inner bound of a class of nondegraded broadcast channel in [4].

Recently the capacity region of MIMO Gaussian broadcast channel has been studied intensively due to its promises in wireless applications. The sum capacity of multi-user MIMO broadcast channel is well understood - see [5][6][7]. All these results rely on the dirty paper coding (DPC) to achieve the sum capacity of the MIMO Gaussian broadcast channel.

A well-known practical DPC implementation approach is the Tomlinson-Harashima precoding (THP)[8][10], originally developed for the intersymbol interference (ISI) channels. Its simple structure makes THP a very attractive solution for DPC implementation [11]. However it is shown that in high SNR region, THP suffers a 1.53dB capacity loss due to shaping, although the capacity gap can be eliminated by using vector lattice coding with much higher complexity.

In low SNR region, the THP performance loss is even more significant (4-5 dB). The cause of this degradation is quantization, which lead to a modulo loss evident only in low SNR region. The purpose of this study is to reduce the effect of the quantization noise introduced by nonlinear coding. Towards this end, we observe that in some applications, *the interference bear certain structures known to the receiver*. This is often the case in wireless scenario (e.g. downlink) where the interference is simply another modulated signal. Exploiting this know structure, we arrive at a new approach (dubbed structure DPC - SDPC) that reduces the modulo loss without undue complexity..

The paper is organized as the following. In section II, we first go through the background and then formulate the problem for broadcasting with known interference structure. In section III, a SDPC precoding strategy is presented. We study a typical case with low modulation order in the same section. In section IV, the symbol error rate is analyzed and simulation results are presented. Finally, a conclusion is drawn in section V.

II. System Model

Firstly, we review the DPC principle and its implementation.

A. Dirty paper coding

Dirty paper coding was introduced by M. Costa in [9]. In Fig. 1, an AWGN channel is corrupted by an interference s, which is non-causally known to the transmitter. v is the source message. The output of the precoder is given as x, which obeys the transmitter power constraints: $E|x|^2 \leq P_x$.The interference s and the noise are Gaussian i.i.d. The Costa's result shows that the capacity of this channel is the same as if the interference were not present: $C_{DPC} = \frac{1}{2}\log(1 + \frac{P_x}{P_n})$.

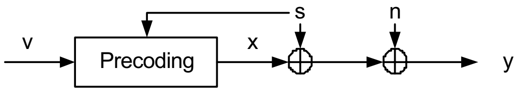

Fig. 1. Dirty paper coding

Following the Costa's results, some extended works have been conducted. [14] shows that the same capacity can be achieved for arbitrary interference distribution, if the noise is Gaussian. In [13], it is shown that the result still holds for arbitrary interference sequence, if a common random dither information is shared by transmitter and receiver.

B. Implementation of Dirty Paper Coding

In high SNR regime, Dirty Paper Coding can be approximated by Tomlinson-Harashima precoding (THP). THP

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was originally introduced in the context of ISI channel [8][10]. The basic idea is illustrated in Fig 2. As shown, the system intends to send the message v from the transmitter to the receiver through an AWGN channel, which is corrupted by interference s. The TH precoder involves two stages. In the first stage, the interference s is subtracted directly from the source v to compensate the interference in the channel. But the power of $v - s$ may exceeds the transmitter power constraint. A modulo operator is applied to sustain the power constraints at the transmitter. At the receiver, another modulo operation is performed to recover the intended message v.

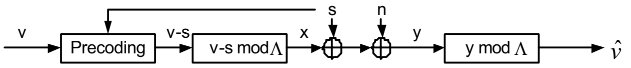

Fig. 2. DPC implementation with THP

Transmitter: The encoder sends:

$$x = (v - s - u) \bmod \Lambda.$$

Receiver: The receiver computes:

$$\hat{v} = (y + u) \bmod \Lambda.$$

u is the common dither shared by the receiver and the transmitter and is uniformly distributed over $(-\frac{\Delta}{2}, \frac{\Delta}{2}]$, which make sure that the channel input x has uniform distribution.

From Fig. 2, it is seen that

$$\begin{aligned} y &= x + s + n + u = (v - s - u) \bmod \Lambda + s + n \\ &= v + n - m\Lambda. \end{aligned}$$

where m is an integer. Then we have

$$\hat{v} = y \bmod \Lambda = (v + n) \bmod \Lambda.$$

The quantizer Λ is chosen to meet the power constraints without causing any ambiguity in v. In the absence of noise, the message v can be fully recovered at the receiver. From the expression of $\hat{v}$, It is clear that quantization introduces extra noise in demodulation. In the following section, the source of the THP performance loss is described.

C. Performance losses of Tomlinson-Harashima precoding

In [15], the capacity of THP channel is shown as:

$$\begin{aligned} I(\hat{v}; v) &= h(\hat{v}) - h(\hat{v}|v) \\ &= h((v + n) \bmod \Lambda) - h(n \bmod \Lambda) \end{aligned}$$

The capacity of THP channel is strictly less than the capacity of the corresponding DPC channel. In [17], it is shown that the THP suffers a shaping loss, a modulo loss and a power loss.

C.1 Shaping loss

As pointed out in [13], the shaping loss is incurred by the input shaping. Assuming the interference s is large, after the modulo operation at the transmitter, the channel input x will be uniformly distributed over $(-\frac{\Delta}{2}, \frac{\Delta}{2}]$. As we know, to achieve the AWGN channel capacity, the channel input must be Gaussian distributed. The uniformly distributed channel input introduces shaping loss. The shaping loss dominates in high SNR regime. In [15], the shaping loss of THP system is proven to be 1.53dB in high SNR regime. Many techniques have been developed to alleviate the effect of the shaping loss in THP system. In [11], it is pointed out that the shaping loss occurs because only the instant information of the interference is used at the precoder. The non-causal information of the interference is not fully explored. In [13][12], lattice and vector quantization methods are introduced to recover the shaping loss in THP system. The key idea in [18] is to make channel input to be uniformly distributed over a high dimensional sphere. Projected into 2-dimention, the input signal has a Gaussian distribution, thus a shaping gain is achieved.

To understand the lattice precoding approach proposed in [11][13], denote $\vec{\Lambda}$ as an n-dimensional lattice and V its fundamental Voronoi region. Also let $\vec{u} \sim Unif(V)$, that is, $\vec{u}$ is a random variable (dither) uniformly distributed over V. The lattice precoding scheme is as follows,

Transmitter: The input alphabet is restricted to V. For any $\vec{v} \subset V$, the encoder sends:

$$\vec{x} = (\vec{v} - \vec{s} - \vec{u}) \bmod \vec{\Lambda}.$$

Receiver: The receiver computes:

$$\hat{v} = (\vec{y} + \vec{u}) \bmod \vec{\Lambda}.$$

For a given lattice $\vec{\Lambda}$, the gap to capacity of a precoding system may be made smaller than $\log 2\pi e G(\vec{\Lambda})$, where $G(\vec{\Lambda})$ is the normalized second moment of $\vec{\Lambda}$. For optimal lattices we have $G(\vec{\Lambda}) \rightarrow 1/2\pi e$, and the gap goes to zero. Also note that when $\vec{\Lambda}$ is one-dimensional, the lattice precoding scheme is simply a scalar quantization and is an extension of Tomlinson-Harashima precoding. In this case, the gap to capacity of a scalar system is $1.53dB$ at high SNR regime.

In [16], THP is combined with trellis coding to eliminate the shaping loss in high SNR regime.

C.2 Power loss

In THP, the power loss is caused by the modulo operation at the transmitter. After modulo operation, the channel input signal may have more power than the source. In [16], a Partial-Interference-Presubtraction(PIP) approach is introduced to partially recover the power loss in low SNR regime.

C.3 Modulo loss

The SDPC method primarily addresses the modulo loss in low SNR regime. The modulo loss is caused by the modulo operations at the THP system. Due to noise corruption, the signal at the boundary of the QAM constellation may be folded into the opposite boundary of the constellation, which incurs the error which would not have happened in regular AWGN channel. The modulo loss is significant for low order constellation in low SNR regime (up to 3–4dB).

In the ensuing sections, we describe the SDPC principles in details. We focus our studies in low order constellations (BPSK, QPSK), since the high order QAMs cannot operation in low SNR regime.

D. *System model*

The SDPC is motivated by the following observation. In many scenarios such as vector DSL and wireless downlink, the modulation schemes of each user are commonly broadcasted in the public channel. In other words, each user knows the modulation schemes of all the other users'. Considering two users case (i.e., user A and user B), the transmit signal for user B is seen as an interference at user A. In Fig. 3, v is the signal for user A, while s is actually the signal for user B. It is reasonable to assume that user A knows the modulation scheme of the signal intended for user B. This knowledge allows us to reduce, even eliminate in some case, the modulo loss by taking advantage of the structure information of the interferer s.

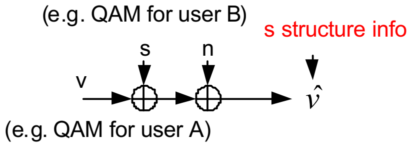

Fig. 3. System model

III. PRECODING WITH INTERFERENCE STRUCTURE KNOWN AT RECEIVER

As we showed above, the THP precoding scheme introduces quantization noise. Can we reduce, even eliminate, such noise caused by the quantization at the receiver? The answer is positive.

In THP system shown in Fig. 4 (a), the receiver assumes no knowledge of the interferer at all. However if s is a QAM interference, the receiver does have prior knowledge of the constellation of y. As a result, we can first de-map y onto its constellation before performing the modulo as in THP. In the meantime, soft information can also be extracted and combined with the modulo results as the input to the successive soft channel decoder.

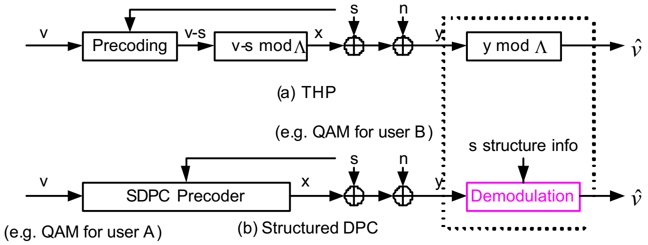

Fig. 4. (a) DPC implementation with THP; (b) DPC implementation with known interferer struture.

Clearly, the constellation of y will be an expanded version of the constellation of v due to the modulo operation at the transmitter. For example, if signal intended for user A and user B are both BPSK modulated, the received signal y may have the constellation shown in Fig 5. The system can be viewed as a 4PAM signal over a AWGN channel. The system performance (symbol error rate) is obviously worse than that of the source v alone (BPSK modulated) over the AWGN channel. Interestingly, as shown in Fig. 5, we can take advantage of the known structure information to rearrange the mapping of y's constellation to achieve the performance of BPSK over AWGN, thereby eliminate the modulo loss. The similar remapping process can be extended to other modulation scheme.

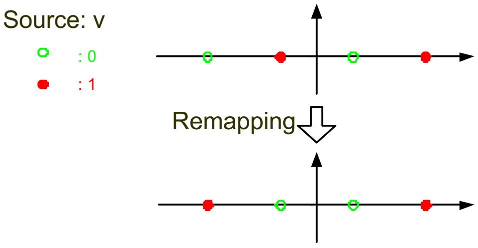

Fig. 5. The possible received constellation of y

The SDPC is further explained by the following two-user example.

A. *Two users with QPSK constellation*

Let us take a close look at the two-user case. We assume both users are QPSK modulated. For the regular THP case shown in Fig. 2, the received signal $\hat{v} = (v+n) \bmod \Lambda$. The constellation of y is shown in Fig. 6 (a). The dots inside the dashed are the effective constellation of $\hat{v}$. The power loss is about 1.25dB.

The SDPC scheme is shown in Fig. 4 (b). To illustrate the idea, we assume the average power of the interference abides: $|s| = 1.5|v|$. The new scheme is carried out in the following steps:

1. Precoding: we design the precoding strategy as:

$$x = sign(s)((|2v - |v||) - |s|)$$

It can be easily verified that the power loss is about 0.98dB.

2. Decoding: With this precoding strategy, we setup a mapping from source v to y, which is shown in Fig. 6 (b). Now instead of performing modulo at receiver, the new scheme demodulates source v directly from y. Since there is no modulo operation, the quantization noise is eliminated.

The key issue here is that with the structure information of the interference, the receiver shares the knowledge of y's constellation with the transmitter. In this case, y has

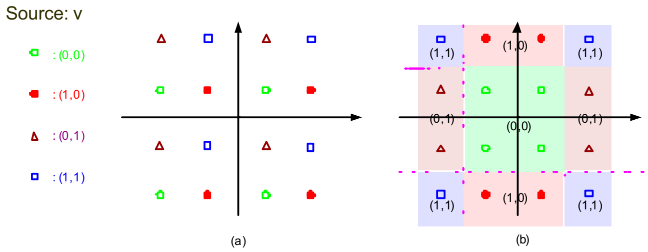

Fig. 6. (a) Effective constellation in regular THP and (b) Constellation for modified THP for QPSK source and interference.

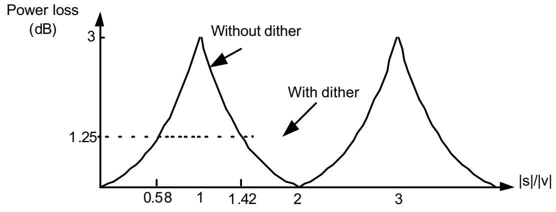

Fig. 7. Dither effect in power loss

a 16QAM like constellation. However, the mapping from source v to y have several possible choices. It is easily to verify that the mapping shown in Fig. 6 (b) yields the best performance, which is comparable to the performance of the QPSK constellation with the same minimum distance.

In low SNR regime, the power loss is another concern. As we showed above, the TH precoder can subtract a uniformly distributed dither in modulo to generate uniform channel input. With the dither, the power loss is about 1.25dB in the BPSK case. Now let us examine what will happen to the power loss without the dither. Since the source and interference signals are both structured and discrete, we can quantify the relation between power loss and the amplitude ratio of interference to source in Fig. 7. We find that the dither in SDPC may worsen the power loss in some case. A simple solution is to decide whether the dither is needed based on the amplitude ratio of interference to source. Thus the power loss of SDPC is upper bounded by that of the corresponding THP system.

In presence of a dither u, the effective interference of the system can be seen as $s+u$. Since u is commonly shared by the transmitter and the receiver, the modulo loss still can be reduced using the SDPC proposed.

B. More than two users case

For more than two users case, things become a little more complicated. The receiver needs to know the structure and strength of all the other users. However once this information is determined, the receiver still can predict the constellation of y. The precoding and decoding procedures are still the same as the two users case.

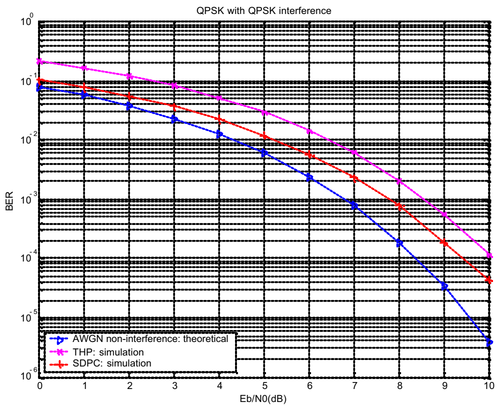

Fig. 8. Ber vs. EbN0 for QPSK source and interference signals

IV. PERFORMANCE ANALYSIS AND SIMULATION RESULTS

In this section, we analyze the symbol error rate of the proposed scheme and compared it with the regular THP precoding.

A. Two users with QPSK constellation

Assuming that source and interference are both QPSK signals and the signal strength of interference abides $|s| = 1.5|v|$. The constellation of y for regular THP and proposed precoding scheme are shown in Fig. 6. In Fig. 6 (a), the dots inside the dashed are effective constellation points. The symbol error rate for this case is easy to calculate as:

$$P_{THP} = 4 \times P_e = 2\,\mathrm{erf}\,c\left(\sqrt{\frac{E_{b\_THP}}{N_0}}\right).$$

For the SDPC scheme, we map the source into the constellation according to the interference by the following rules:

$$x = sign(s)(|2v - |v|| - |s|)$$

The mapping from source v to y is shown in Fig. 6(b). The symbol error rate for this case is:

$$P_{SDPC} = 2 \times P_e = \mathrm{erf}\,c\left(\sqrt{\frac{E_{b\_SDPC}}{N_0}}\right).$$

In absence of the interference, the symbol error rate for QPSK signal through AWGN channel is $P_{AWGN} = 2 \times P_e = \mathrm{erf}\,c\left(\sqrt{\frac{E_{b\_AWGN}}{N_0}}\right)$. Due to the power loss, under the same transmitter power constrains, we have $E_{b\_THP} \leq E_{b\_SDPC} \leq E_{b\_AWGN}$. So we get $P_{AWGN} \leq P_{SDPC} < P_{THP}/2$. This equation is also valid when source and interference are both BPSK signals.

Simulation studies have been conducted. In our simulation, the source and interference are both QPSK signals. The amplitude of interference signal is chosen to be 1.5 times of that of source signal. The simulation result is shown in Fig. 8. The simulation results match the theoretical results well.

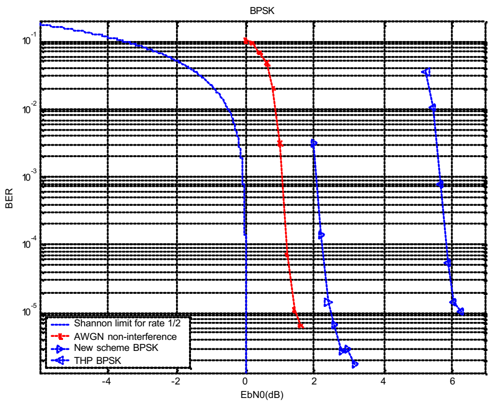

Fig. 9. BER vs EbN0 with rate 1/2 (7,5) turbo code for BPSK source and interference signals

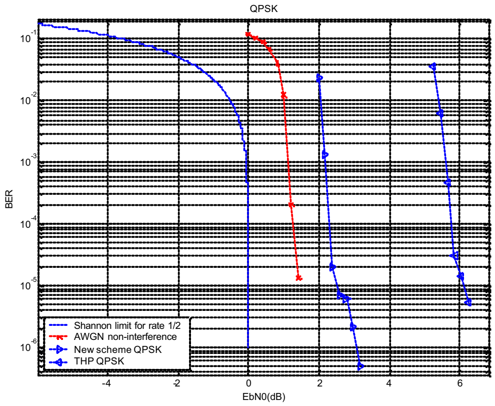

Fig. 10. BER vs. EbN0 with rate 1/2 (7,5) turbo code for QPSK source and interference signals

We also studied the performance of modified THP precoding with channel coding. A rate $1/2$ $(7, 5)$ turbo code is used with the log-map decoding algorithm. The results shows that the SDPC has more than 3dB improvement over the regular THP. The performance loss compared with AWGN channel without interference case is mainly due to the power loss. The BER curves for BPSK and QPSK are shown in Fig.9 and Fig.10 respectively.

V. Conclusions

In this paper, we have studied the scalar dirty paper encoding schemes with interference structure known to the receiver. In low SNR regime, the THP system suffers from a modulo loss and a power loss, which is significant especially for low order constellations. The proposed SDPC recovers most of the modulo loss and yields significant gain over the regular THP in low SNR regime. In BPSK and QPSK cases investigated, more than 3dB improvement is achieved. For the high order constellation, the performance improvement is reduced and the system complexity is increased. This precoding method can be extended into multiuser case.

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