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STRUCTURES FOR NON-HIERARCHICAL ORGANIZATIONS

by

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Abstract

Many of the disadvantages of tree-structured organizations arise from the superior-subordinate relationship on which they are based, and from the unequal distribution of information in the group. Non-hierarchical alternatives are discussed from the point of view of: 1) coordination of some members by others, 2) efficient communcation, 3) rational assignment of parts of the organization to subtasks, and 4) possibilities for organizational growth. Trees are popular because they allow these functions to be performed by a single structure, in a way that each function reinforces the effectiveness of others. A non-hierarchical organization that accomplishes this can be designed based on finite geometrical spaces.

1. Introduction

By a tree-structured organization we will mean one in which every member except one has a unique immediate superior. The exceptional person is "at the top" and has no superior. In theory most formal organizations are trees, since their organization charts show an authority structure that follows this pattern. In fact most are trees only as an approximation -- they contain superior-subordinate relationships but not to the degree that almost everyone is accountable to only one boss.

A non-hierarchically-structured organization is one in which the superior-subordinate relationship does not exist at all. Is it possible to have an organization with a non-hierarchical structure? At first this seems to be a contradiction since with no superiors and subordinates, there would be only a homogeneous group of people. This is not true however, and we will see that organizations without hierarchy can be defined.

This paper will discuss three questions:

1) What forms could a non-hierarchically-structured organization have?
2) What advantages do tree structures have that make them so popular?
3) What organizational tasks and environments favour one type of structure over the other?

The second and third questions are subsidiary to the first, in the sense that we can discuss the advantages of trees only in the context of specific alternatives. As we look for substitutes for trees, we will meet difficulties which will clarify the relative advantages of tree structures in particular environments.

Outline

Problems of trees will be discussed (Section 2), along with the types of situations where non-hierarchical alternatives are likely to succeed (Section 3). The essential problems for any organization are coordination of the members, communication among the members, and the logical assignment of members to tasks. These will be dealt with in turn in the succeeding three sections. There are two ways coordination can be achieved in an organization:

1) by an outside party who coordinates the activities of those doing the job (Section 4)

or 2) by the doers themselves, through voluntarily adjusting to each other. This requires an organizational structure that allows them to be informed of the relevant information, and leads to our investigation of efficient communication structures (Section 5).

Section 6 will look at modularity, which is the existence of efficient substructures in organizations, allowing those parts to be assigned to subtasks. Section 7 will discuss the possibility of growth in each type of design. Section 8 will discuss how trees integrate the various functions that an organization must perform, and Section 9 will show how the same functions can be integrated non-hierarchically using organizations based on finite geometries, or projective spaces.

2. Problems of tree structures

Why should we look for alternatives to trees? The general reason is that in many contexts they do not accomplish the organization's goals efficiently, and also have negative side effects. Many of the problems stem from the superior-subordinate relationship and the consequent division of labour between front-line work and administration. At the bottom, the operating staff do the specific tasks of the organization, while at the top the strategic planners make the long-range decisions. It is ironic that those having the least contact with the daily activities of the organization make the most important choices. Those at the bottom are excluded from policy-making, which produces a sense of alienation from their work and a lack of identification with the organization's goals.

The superior-subordinate relationship also supports a system of unequal rewards, which take the form of large differences in power, status and remuneration. In some corporations those at the bottom are paid less than one-fiftieth as much as those at the top. This has been criticized as an unjust and inefficient distribution of the benefits of work, and one that has negative consequences for society as a whole.

A further difficulty is that trees lead to compartmentalization of the organization's members. Communication between different branches is low, and the different sections tend to work at cross-purposes. This is exacerbated by the fact that each department performs specialized tasks and has specialized knowledge different from the others. Often the work at the bottom is so narrow that it is extremely monotonous and unsatisfying.

Past solutions to the problems of trees

These problems have been widely discussed, and two general classes of solutions have been suggested. The first leaves the tree structure basically intact, but adds features to try to lessen its difficulties. Some examples are the human relations movement (Likert, 1961), or proposals to flatten organizations (i.e., decrease the number of intermediate managers), which has the goal of putting the strategic planners and operating staff into more direct contact. Another suggestion is to decentralize decision-making (Jennergren, 1980), in order to increase the workers' commitment to the organization's goals and improve the quality of decisions. Systematically adding communication links between departments has been proposed under the scheme of matrix organizations (Galbraith, 1973), and of liaison people who connect specific departments.

Solutions in the second class are more basic in that they eliminate the tree altogether and replace it with a group of equal members. These are "collectives", or "participatory democracies", in which the formal structure of the organization is no longer defined by a set of roles, but by procedures that allow the group to function efficiently in an egalitarian way, such as rules for job rotation and group decision-making. Inequalities in influence and specialization of skill or knowledge are regarded as harmful, and are specifically avoided. Organizations of this type have multiplied in the last twenty years and include food cooperatives, medical and legal clinics, alternative schools, and political action groups (Mansbridge, 1973, 1976; Rothschild-Whitt, 1979; Swidler, 1979; Freeman, 1972).

This paper shares the spirit and goals of the second group of authors, with the difference that the organizations described here give their members specific structural roles.

3. Range of application of non-hierarchical structures

These designs are not intended to replace trees in all situations. We believe they would work best to the extent that some or all of the following conditions hold:

1) The organization is small.

2) The environment is unpredictable, the task is complex and calls for innovative solutions.

3) The members are motivated by the goals of the organization rather than money or power.

4) The members understand and have a personal commitment to non-hierarchical structures.

Regarding point 1), most of the structures presented here involve more intense interaction than occurs in trees, where members must deal only with those immediately underneath in their span of control, which is usually deliberately limited. To avoid spending too much time on internal exchanges, the non-hierarchical organizations we envision would have no more than about fifty members.

If a tree organization is operating in the variable environment specified in point 2), it tends to respond too slowly. Information about changes in the environment is either stifled on the way up, or else overloads the person at the top. Frequently organizations in this type of environment abandon the tree structure and become "adhocracies", to use a term introduced by Toffler (1970) and expanded upon by Mintzberg (1979). These groups have flexible structures that shift in response to the demands of the task, and tend to arise in fast-changing technical fields. Communication is emphasized over control mechanisms or standardization of jobs. An administrative superstructure usually exists, but spends more of its time communicating with other organizations than controlling the staff. Adhocracies share many of the characteristics of collectives and are one type of organization to which the non-hierarchical structures presented here could apply.

Regarding points 3) and 4), it is helpful that the members have personal values and operating styles consistent with the absence of hierarchy. In traditional organizations, inequality is used as an incentive, and therefore an egalitarian structure will be less motivating to some individuals. The attitude of sharing the organization's goals is very much a feature of adhocracies and collectives, and therefore makes non-hierarchical structures more practical for these groups.

None of the schemes to be described is complete in the sense of defining a total working entity. Each is only the skeleton of an organization, but our aim is to develop the models to the point where their basic form is clear, and they can be evaluated as an alternative to trees.

4. Exogenous coordination

If two members are left on their own, they will act in incompatible ways, and therefore it is made the duty of a certain third member to coordinate the two. This could mean such things as dividing resources between departments, assigning people to tasks, planning members' future activities to make sure they are compatible, or arbitrating a conflict.

Coordination in trees

In any formal organization there should be some rule of deciding who the third person will be. If the organization has a tree structure, one model is as follows: the coordinator must have authority over both, and therefore should be <u>above</u> <u>both</u> in the tree. Also, the coordinator should be the <u>lowest</u> person who is above both, that is, the coordinator should be the least common superior of the two coordinates (Friedell, 1967).

This is not to say that the least common superior will make the decision completely independently. If the issue is non-routine and touches wider questions, it may be referred to someone higher up. That notwithstanding, for typical problems the primary responsibility rests with the least common superior.

The least common superior is easy to locate using the organization chart -- one simply starts at the two members, and follows the two chains of command upward until they meet.

The idea of going to the least common superior is reflected in the organizational norm of "Don't go over the boss's head." If a conflict involving two members of the same division is appealed directly to the president, for example, this undermines the authority of the division head, and also uses the time of the president inefficiently. The members should be reluctant to take their complaint there, and the president should be reluctant to hear it. The one exception is for disputes involving the rights of people vis-à-vis the organization, e.g., the case of a division head who assigns tasks that were not part of the member's job description. However, for normal administrative matters, authority rests with the least common superior.

We will represent represent general individuals (variables) by small letters, x, y, z, etc., and specific individuals (constants) by numbers, 1, 2, 3, etc. It will be assumed that the number of members is finite. Following Friedell (1967) the operator * will mean the coordinator of two members, i.e., x*y will be the individual whose duty it is to coordinate x and y.

Each organization will have its own operator *, and the latter can be specified by a "multiplication table" showing the value of x*y for every pair x and y. A sample tree with its multiplication table is shown in Figure 1.

FIGURE 1 HERE

What are the general properties of the operator * for a tree organization? First of all, there is always some member responsible for coordinating any two.

<u>T1</u>. For all x, y, there exists a z such that $x*y = z$.

Also, * is idempotent.

<u>T2</u>. $x*x = x$.

That is, people are responsible for making their own actions mutually compatible.

Thirdly, the operator is symmetrical.

<u>T3</u>. $x*y = y*x$.

Next, imagine a situation in which a member enters a dispute with one of his or her own superiors. The boss will have the privilege of deciding, by definition of the relationship of superiority. This is what makes trees hierarchical. It is expressed in the next axiom. (Note that $x*y$ is a superior of both x and y.)

<u>T4</u>. $x*(x*y) = x*y$

Next, * is associative.

<u>T5</u>. $x*(y*z) = (x*y)*z$.

This can be interpreted most easily by conjoining it with T3. The two imply that $x*(y*z) = (x*y)*z = (y*x)*z = y*(x*z) = \ldots$, so that the order of appearance of individuals in the conflict is irrelevant. The identity of the coordinator will depend only on the set of people who require coordination.

A final property is that there exists a single chain of command. If x is a subordinate of two people, here represented as x*y and x*z, then one of the two must be over the other.

T6. (x*y)*(x*z) = either x*y or x*z.

These six axioms are necessary and sufficient to characterize a tree. They can also be shown to be independent of each other, in that it is possible to define organizations that follow any subset.

(To show that they imply a tree-structure, define: x is a superior of y if and only if x*y = x. Then if x satisfies T1 to T5, the superiority relationship will be a partial order (Birkhoff, 1968). To show that it is also a tree, note that by T6 no one can have two immediate superiors. There must be at least one greatest member, i.e., someone with no superior, since the structure is a finite partial order. But there can be no more than one since if there were two, who would resolve their conflicts as required by T1? Therefore there is exactly one greatest member, and the organization is a tree.)

Coordination in non-hierarchical structures

To design a non-hierarchical organization we must specify its operator *, which is equivalent to filling in the multiplication table. There is an enormous number of possibilities of course, so we must explicate the concept of "non-hierarchical" and find tables consistent with that idea.

As in the case of trees we require that the function be completely defined, symmetrical and idempotent. These were the tree axioms T1, T2 and T3, and the new ones will be identical but labelled S1, S2 and S3.

A new assumption is that the coordinator is never one of the two parties in conflict:

S4. $x*y \neq x$ and $\neq y$.

This contrasts sharply with T4. Just as it was T4 that made trees hierarchical by allowing one person to consistently rule another, it is S4 that makes the present organizations non-hierarchical, since for no pair of members can one always prevail.

A common problem for non-hierarchical groups is that small disagreements tend to expand and involve the whole membership. Group meetings become tediously long, debating matters that are relevant to only a few people (Mansbridge, 1973; Kanter, 1972). The organization needs a barrier that catches the smaller problems before they spread to waste time and cause division.

Many multiplication tables are consistent with S1 to S4, but most of them have this objectionable feature, that a two-person conflict tends to grow. For example, suppose that x and y require coordination, that $x*y = z$, and that x is unhappy with z's decision. Individual x now has a conflict with z, and it seems plausible that x would pursue this and therefore go to $x*z$, say, w. If w decides against x, then x has a conflict with x, etc., or alternatively, if w supports x, then the conflict will involve w and y, and also w and z. The disagreement percolates through the whole organization.

Trees solve this problem very elegantly. If x is not content with $x*y$'s decision, then x could complain to $x*(x*y)$, but what would be gained? Since $x*y$ is a superior of x, the decider of the second conflict will also be $x*y$, as follows from T4, who will of course decide in his or her own favour. In a tree, appealing a decision is useless.

We need to impose a requirement on * that limits the spread of conflict, achieving the same end that T4 does for trees. One way is to add the following:

<u>S5</u>. x*(x*y) = y.

Given S5 there will be no reason for x to press a complaint past its initial hearing. If x complains against y, define z = x*y, and assume that z supports y. It would be fruitless for x to take the matter to x*z, since the latter person is y, and it is already clear that y and z are in agreement.

We must now ask if S1 through S5 are contradictory -- perhaps it is impossible to satisfy all five. In fact it is possible, and the first step in showing this is to make a further inference about the structure of *. Any operator on pairs of variables, finite in number, can be represented by a list of ordered triples, where, for example, the operator value on x and y yielding z appears in the list as (x,y,z). The operator * has the special property that if (x,y,z) appears on the list, so will (x,z,y) and all other orderings of the three, on account of S3 and S5. Thus * can be represented by a list of <u>unordered</u> triples.

The list gives a rule for finding the coordinator of any pair of members -- we look up the unique triple containing both x and y and the third element will be the coordinator x*y. Since every pair of members appears in exactly one triple, the rule will always work. An example of such a list is given in Figure 2.

FIGURE 2 HERE

The property that a pair occur in exactly one triple implies that the lists exist only for certain numbers of elements. This was demonstrated by Kirkman (1847), who showed that n must lie in the set (1,3,7,9,13,15,21,...), or in general, n must give a remainder of 1 or 3, after division by 6. These lists of triples are Steiner triple systems (Hall, 1967; Lindner and Rosa, 1980), and the mathematical entities of abstract sets and operators satisfying S1 through S5 have been termed Steiner quasigroups, or squags (Bruck, 1968; Quackenbush, 1976). It seems logical to call organizations satisfying S1 through S5, Steiner organizations.

An example of a multiplication table for a seven-member Steiner organization is shown in Figure 2. It is conceptually easier to represent the structure as a list of triples, as shown there. An even clearer representation is to draw the individuals as points and indicate membership in a triple by joining the three points with a line or curve. This is the "organization chart" of a Steiner organization.

Another way to look at a Steiner organization is as a collection of interlocking three-person committees, each of which functions by majority vote. Clearly the outcome of any two-person conflict will be the same whether it is decided by vote of the three or by asking the third member to arbitrate.

Steiner organizations do not exhibit the division of administration versus line work found in trees, since everyone performs some coordinating functions. This has its precedents in some existing systems operating in complex environments, where coordination must be handled by those professionals doing the primary work of the organization, because only they have the relevant knowledge. Khandwalla (1976, quoted in Mintzberg, 1979) states of such groups, "the job of coordination is not left to a few charged with the responsibility, but assumed by most individuals in the organization, much in the way members of a well-knit hockey or cricket team all work spontaneously to keep its activities focused on the goal of winning."

So far we have assumed that only two people at a time need coordination, but if more become involved the above system cannot be used. Notice that trees deal with multiperson conflicts very effectively because of assumptions T3 and T5. Any group of people will have a well-specified coordinator. Steiner organizations already satisfy T3 (this is the same as S3), and it is tempting to require T5 also, to solve the problem the same way. However this is a dead end, since from S1 to S5 and T5, we can deduce for any members x and y, $x = y*(y*x) = (y*y)*x = y*x = y*(x*x) = (y*x)*x = y$. This means that a Steiner organization satisfying T5 could have only one member.

Another approach would be to make a list of quadruples, rather than triples, such that every triple appeared in exactly one quadruple, to give a rule for determining the coordinator of three-person conflicts. These structures are Steiner quadruple systems (Lindner and Rosa, 1978), but they can be constructed only if n lies in the set (1,2,4,8,10,16,20, ...), which except for a trivial case is disjoint with the possible sizes of Steiner triple systems, so no organization can have both structures.

For relatively small Steiner organizations a good policy would be to refer conflicts of more than two members to a general meeting. Hopefully these will be infrequent if the organization is basically united on its goals, and if parties are discouraged by group norms from politicking for allies.

Another way of solving the problem of larger conflicts is to refer it to the largest Steiner suborganization containing the parties. This will be discussed in Section 9.

Steiner organizations raise a traditional issue in organization theory, the question of unity of command. A classical doctrine is that an employee should have only one immediate supervisor to avoid conflicting instructions. In a Steiner organization an employee may be asked to follow the decisions of several other members, which may be mutually inconsistent. This may seem a serious flaw, but in fact many organizations that violate the principle of unity of command are able to function quite well by mutual discussion and we would expect the same here.

5. Communication

The focus is now on coordination that does not require a third party, but can be done by the members themselves. They are able to make voluntary adjustments given they have sufficient communication with each other, and therefore the organization should be designed so that this communication is likely to happen.

We will regard an organization as a list of sets. Each set contains individuals, and a set appears on the list if those particular individuals meet as a group. Thus if we have a four-person organization consisting of a boss and three underlings labelled 1, 2, 3 and 4, in which the underlings meet among themselves and also report to the boss, this would be represented by the list (234, 12, 13, 14).

These meetings are regarded as part of the formal structure, not as informal communication patterns. Members are expected to attend them regularly as part of the job.

Authority relationships are ignored in this model, so that if a supervisor and a subordinate meet, this is a symmetric event as far as the list is concerned. For an organization to be non-hierarchical means something slightly different than it did in the previous section. Power relationships tend to develop in a communication network not through formal authority but because one person has more information than another. To achieve an egalitarian organization we will require that each member communicate directly with the same number of others.

Three features of an organization are:

1) n, the number of people in it,

2) m, the maximum number of meetings any member attends,

3) s, the maximum size of any meeting.

We will refer to an <u>(n,m,s)-organization</u>, meaning one that has these three values. For example, the four-person system described above is a (4,3,3)-organization.

Any organization has a fourth feature, which is harder to define:

4) d, communication distance.

Roughly, this means the degree to which the members are far from one another, separated by many communication links. The distance d will depend on n, m and s, and on how well the organization is designed. Several axiomatized definitions of d have appeared in the literature under the title of index of centrality (Sabudissi, 1966; Nieminen, 1974). The exact definition used here will vary through our discussion, according to what is tractable and appropriate, different concepts being distinguished by superscripts: d', d", etc.

Our aim will be to find organizations with an egalitarian structure (all go to meetings of the same number and size), with the required memberships (high enough n), with small and therefore more effective meetings (small s), and with little time spent in meetings (small m), and yet with a low value of d. Structures that minimize d for fixed values of the other three will be called <u>communication-efficient</u> or <u>efficient</u>.

Communication in trees

Trees have a very low meeting size, $s = 2$. We assume that there are no departmental meetings, but that a superior and a subordinate meet in a pair.

They also score well on the <u>average</u> number of meetings per member. If a tree has n people, it will have $n - 1$ communication links, since everyone reports to his or her superior, except one person who reports to no one. The average number of meetings per person will be $2(n - 1)/n$, since each link accounts for two attendances and there are n people in the tree. Although the maximum number of meetings m for any person can be high, the average is low. In fact, there is no way to have a $(n,m,2)$-organization with a lower average without making it disconnected, having some people who are unreachable by others.

Unfortunately, trees sacrifice other qualities: either maximum number of meetings per member, m, or communication distance, d. It is possible to design a very centralized tree by loading up one person with links to everyone else, in the shape of a star. No two people would be more than two steps away from each other, but the value of m would rise to $n - 1$, which is the maximum possible. Alternatively to make the number of contacts almost equal a tree could be shaped like a simple chain. No person would meet with more than two others, but the communication distance, defined as the maximum separation of any two individuals, would be $n - 1$, the maximum possible.

A related difficulty with trees is that communication is precarious, since malfunctioning of one link makes the tree disconnected. On the other hand, communication is predictable, since for any pair of people there is exactly one shortest path between them. This allows the administrators in trees to have more predictable and therefore narrower and relates to the sharp division of labour found in these organizations.

All of these considerations suggest that trees would be most successful when constant and flexible communication is not needed, if the work is especially routine and the environment unchanging. This idea has been supported theoretically and experimentally (Mintzberg, 1979; Christie, Macy and Luce, 1956; Glanzer and Glaser, 1959, 1961). It also explains the fact that many of the proposals for modifying tree structures have been developed in organizations that were induced to innovate and respond quickly to change.

Communication in non-hierarchical organizations: pairwise meetings

Investigating non-hierarchical organizations with s = 2 allows us to compare them directly with trees. If trees are suitable for doing routinized work, these organizations are just the opposite, as will be seen. Workers can go about their own business, but be informed of new developments and activities of their colleagues by meeting in the way prescribed by the network.

For these (n,m,2)-organizations, the smallest non-trivial value of m is 2. The resulting groups are shaped like cycles or "circles", as they are designated in the literature of communication networks. Members interact as if they were sitting in a ring and talking only to their two neighbours. Defining communication distance, d', as the largest number of steps that separate any pair in the group, d' is clearly (n - 1)/2 if n is odd and n/2 if n is even.

Better values of d' can be achieved if we allow m, the number of pairwise interactions per member, to rise to three. These (n,3,2)-organizations can be constructed only if the number of members is even, since the number of links is 3n/2, which must be an integer.

What are the communication-efficient (n,3,2)-organizations? Some examples are shown in Figure 3. They are drawn as undirected graphs, where members are represented as nodes, and a meeting is represented by an edge joining the nodes.

FIGURE 3 HERE

If we insist that d' = 1, i.e., that every pair meet face-to-face, then four people are the maximum possible.

For d' = 2, i.e., when every pair either meets directly or has a common contact, then any even number of members up to ten can be accommodated. The largest organization with d' = 2 has the members arranged in two connected pentagons, (or as drawn in Figure 3, a pentagon and a pentangle, which is equivalent to a pentagon in terms of the structure of its connections.) As a mathematical entity this is called the Peterson graph, and we will borrow the name to call the group the Peterson organization. It is a (10,3,2)-organization and compares favourably with a (10,3,2)-tree, in that its communication distance is no more than one half that of the tree, which has d' = 4 or more.

It also has the elegant property that the shortest path between two points is always unique. Thus if members adopted a rule that in meetings they would describe only their own activities and those of their immediate contacts, then everyone would know what everyone else was doing, and furthermore everyone would hear the news only once. The property that the shortest path is always unique holds for the 4-member organization of Figure 3, but is otherwise rare in graphs with equal numbers of edges emanating from each node (Hoffman and Singleton, 1960).

To accommodate larger numbers, we must allow d' to go up to three, so that some pairs of people will be three links from each other. Finding the efficient patterns is a difficult problem, which has been investigated in the context of computer networks and solved on a case-by-case basis up to n = 30 (Cerf, Cowan and Mullin, 1974, 1976; McKay and Stanton, 1978). Some of their graphs are given in Figure 3, redrawn to bring out the symmetries. For d' = 3 the organization can have up to 20 members.

(These authors in fact solved the problem for a somewhat stricter criterion of efficiency -- if two graphs were alike in the maximum distance d', then the second, third, etc., greatest distances were examined, and the graph first showing a greater distance was eliminated. It follows that their graphs are some, but not necessarily all of the graphs that are efficient by our definition.)

Efficient structures for larger values of m are hard to find but a few have been described (McKay and Stanton, 1978). One of interest is the Hoffman-Singleton graph (Hoffman and Singleton, 1960; Bondy and Murty, 1976), whose corresponding organization has fifty members, each attending seven pairwise meetings in a pattern that achieves d' = 2.

Communication in non-hierarchical organizations: three-person meetings

Increasing the meeting size from two to three means that each person must attend at least two meetings, except for trivial cases. These structures are therefore (n,m,3)-organizations with $m \geq 3$.

The problem of finding the efficient ones seems difficult but we can make some progress by changing the definition of communication distance. Instead of using d', the greatest number of links between any pair of people, we will define d'' as the greatest number of meetings through which a message must pass to go from one meeting to another.

The requirements that every pair meet face-to-face (d" = 0) and that meetings occur in threes, are satisfied by the Steiner organizations of Section 4 (regarding Steiner organizations not as multiplication tables, but as lists of three-person committees). How many meetings must each person attend? Each must meet once with the other n - 1 people, but a single meeting takes care of two other members, so a person must attend (n - 1)/2 meetings. In summary, Steiner organizations are (n,(n - 1)/2,3)-organizations with d" = 0.

One of their attractive properties is that they allow the coordination and communication functions to be handled in a unitary fashion, since the same list of triples gives the meeting pattern and also specifies the coordinators for conflicts. This is sensible since a coordinator of two members should meet them directly and simultaneously.

Some efficient (n,m,3)-organizations are shown in Figure 4. For n = 7 and 9, the Steiner organizations are shown. The other organization for n = 9 can be constructed in another way, by taking the first (6,3,2)-organization of Figure 3 and substituting a line of three for every node. If two nodes in the original were connected, then two points, one on each of the corresponding lines are identified. Thus there will be 3/2 as many members in the new group, so that the 6-member organization generates the present 9-member one. This operation can be performed for any (n,3,2)-organization. Since single people map into meetings here just as in the definitions of d' and d", then if the original organization was d'-efficient, the new one will be d"-efficient. It is easy to see that d" will equal d'- 1. Figure 4 shows two further designs constructed in this way, a (15,2,3)-organization with d"= 1 generated from the Peterson graph, and a (30,2,3)-organization from the (20,2,3)-organization of Figure 3.

FIGURE 4 HERE

Matrix organizations

The other 9-member organization of Figure 4 is of interest because it has the well-known form of a matrix organization. A (two-dimensional matrix organization is one in which the members can be laid out as the rows and columns of a matrix, and a meeting occurs of every row and also of every column.) The idea has been used frequently in corporations where the rows are the projects on which the member is working (calculators, home computers, electronic watches), and the columns are the functions he or she performs (research and development, manufacturing, advertising). The aim is to set up a logical relationship between the task structure and the communication structure, and allow those doing similar jobs to interact. They are not intended to be non-hierarchical, but have an executive superstructure, as shown in Figure 6. More generally, a p-dimensional matrix organization can be defined. Typically the third dimension corresponds to the geographical area in which the member is working. These p-dimensional matrix organizations are always (n,m,p)-organizations, that is, each person will attend p meetings. The value of m must be at least the p'th root of n, and d'' will equal p - 1.

Matrix organizations do not accommodate the largest number of members for a fixed d''. For example, Figure 4 shows a 15-member organization with d''= 1, as good as the 9-member matrix organization. However, they satisfy a stronger criterion of communication closeness -- if we attend one meeting and want to find out about another, then not only is there someone at the other meeting who is no more than d'' meetings away from someone at ours, but everyone at it is no more than d'' meetings away from someone at ours.

We have surveyed a range of possibilities for non-hierarchical communication structures and illustrated the tradeoffs of meeting demands, membership size and communicative proximity. Almost nothing has been said about the relationship to the task structure, however, and this will be the subject of the next section.

6. Modularity

Modularity is a property of classes of structures rather than individual ones. It means that if we have identified a desirable quality for organizations, we can choose subsets of members, look only at the roles prevailing among them, and find a suborganization with that quality.

It is significant for work organizations because most tasks are not unitary, but can be divided into subtasks, subsubtasks, etc. The basis of the division is that people working on a subtask require more coordination by virtue of the activities required by the subtask (Thompson, 1967). If a certain type of organizational design is efficient for the whole task, often it will be efficient for the subtasks, so that if the organization is in a modular class, the substructures can be assigned to the subtasks and the work can proceed smoothly in the small and in the large.

Trees are excellent examples of modularity since any non-trivial tree contains proper subtrees.

The efficient (n,m,2)-organizations are frequently modular. For example, the Peterson organization contains twelve pentagons as components, which are themselves efficient (n,m,2)-organizations. Likewise in the Hoffman-Singleton graph (Section 5) there can be found no less than 525 Peterson graphs (Benson and Losey, 1971).

Modularity in Steiner or in efficient (n,m,3)-organizations seems less common but some striking examples can be produced one of which will be discussed in Section 9. It involves a 15-member Steiner organization containing fifteen 7-member Steiner organization.

Determining the natural partition of a task is a guide to which of several possible structures should be chosen to accomplish it. For example, if the task comprises two equal parts a structure like the second 14-member organization of Figure 4 would be appropriate, but for a single integrated task the first might be better.

7. Growth

An organization must be flexible enough to adapt if the task grows or shrinks. It would be useful if the organization could grow in a way that it remained in the efficient class, and with a minimum revision of the existing structure.

Types of organizational growth

Three types of growth will be examined: incremental, modular and structural. Incremental growth means growth in small steps by adding new members a few at a time. It is clearly a matter of degree, an intermediate case being a volleyball game, where the membership must grow in even numbers.

The second type of growth is by merger, where two or more existing groups join together. This is modular growth.

The third possibility has an organization remaining constant in membership but increasing in internal structure. Interaction becomes more intense, and the organization becomes more organized. If this can happen with the new structure still in the efficient class, it is termed structural growth.

For either incremental or structural growth, we can ask a further question: Is growth localizable? That is, can it occur with minimum changes in the existing structure? An example of an incrementally but non-localizably growing structure is the American flag. As new states joined it was possible to incorporate the correct number of stars, but all the stars in the past layout had to be repositioned, to the great benefit of flag-makers.

The various organizational structures -- trees, Steiner organizations, matrix and communication efficient organizations -- will now be examined and the same question will be asked for each: Can an organization within the class grow in a convenient way and still remain in the class? The word "convenient" is imprecise, and the answers will be imprecise too. Nevertheless some strong differences will be found.

Incremental and localizable growth

Trees allow incremental and localizable growth without restriction. A new member can be added anywhere as an extra branch, changing only one part of the old structure, the place where the newcomer is added.

The non-hierarchical structures allow incremental growth to a more limited extent. For example, matrix organizations can add a complete function or project team. Note that as in the volleyball game, the right number of individuals must join each time.

The efficient (n,3,2)-organizations of Figure 3 allow for localizable growth in some instances. This is illustrated in Figure 5, where two new members are added to change a (n,3,2)-organization into a (n + 2,3,2)-organization. A relatively small amount of disruption is necessary: two carefully chosen links are disconnected and the new members are inserted into the breaks. It is not always possible to do this, however -- one cannot go from a (16,3,2)-organization to an (18,3,2) one.

FIGURE 5 HERE

Steiner organizations are inflexible in this regard. Incremental growth is possible to a degree in that people must be added in two's or four's, because of the size restrictions on the membership. Localizable incremental growth seems to be ruled out since the Steiner property is a global one and would require changes throughout the existing structure. If we wish to add people without changing old relationships, the best guarantee is that the size must be approximately doubled (Doyen and Wilson, 1973). For example, for any 7-member Steiner organization, we can find a 15-member one that contains it as a proper part.

Modular growth

Modular growth allows organizations to merge or divide without disruption of the subparts. Some of its advantages have been discussed by Simon (1957). It is closely related to the concept of modularity discussed in Section 6, but here the modularity must be of a certain type: the subparts that join will generally have no common members, so the two modules must be disjoint.

Trees allow for perfectly modular growth, since two trees can always be joined to form a third, by making the head of one a subordinate of someone in the other. Modularity is also a feature of p-dimensional matrix organizations since each slice between two rows gives two matrix organizations. For the class of efficient (n,m,2)-organizations, disjoint modularity is common: the Peterson organization comprises two pentagons.

FIGURE 7 HERE

Structural growth

Structural growth involves adding new role relationships. Trees completely disallow this since any additional connection destroys the tree-property.

They are characterized by extreme sparseness in their role relationships and this makes them unadaptable when communication needs increase. If two separate departments must engage in a joint activity they must go up and down the chain of command, increasing information load on the top management and generating complaints about red tape.

The possibility of structural growth occurs in (n,m,3)-organizations as shown in Figure 7. There a (9,2,3)-Steiner organization has meetings added and becomes a (9,4,3)-Steiner organization.

8. Trees as integrated systems

We will now summarize the various features that have been attributed to trees and show how they interrelate. The conclusion will be that they are mutually reinforcing, which helps to explain why trees are effective in some contexts, but also suggests that it would be difficult to eliminate the more troublesome features and retain the others.

Some of the properties of trees are:

1) Exogenous coordination: clear rules based on the superior subordinate relationship, that limit the spread of conflicts of any size.
2) Communication: sparsity of links and predictability of paths.
3) Modularity: the existence of many disjoint subtrees.
4) Growth: the possibility for localizable incremental and modular growth, but no possibility for structural growth.

A unifying link among these features is that tree organizations usually set up a sharp division of labour. This is made possible by 3), the existence of disjoint subtrees, which become the various departments assigned to the subtasks.

The division of labour promotes the superior-subordinate relationship in that it makes it more crucial to integrate the various subtasks. No one has the "total picture" except for the higher administrators, who can demand more status and remuneration for their work.

The limited and predictable communication paths also support the superior-subordinate relationship. Since any non-routine information tends to flow through a higher administrator, the latter gains unique knowledge, and thereby more power.

If greater internal communication is needed, the tree cannot grow structurally and still remain a tree, but it has an alternative, to grow incrementally. In concrete terms, a manager who becomes overloaded can hire an assistant. In the implicit philosophy of tree organizations this is preferable to increasing links among departments. Although the organization might function quite well in the latter plan, from the point of view of those higher up it would be moving out of control.

The administrative elite have a heavy burden of communication and coordination, and it becomes all the more important that their time and attention be protected from unnecessary chores. Many communication paths flow through them and conflicts have a natural tendency to move toward them, and therefore the effective system of limiting conflict referred to in point 1) is necessary.

In summary, the four features listed interact with the division of labour in trees to reinforce one another. This adds to the stability of the trees' structure and helps to explain their popularity.

9. Integrating functions non-hierarchically: projective space organizations.

In past sections, separate theories of coordination, communication and modularity were developed, but for a really effective alternative to trees, these functions must be combined in a single organization. The class to be defined now are Steiner organizations, are communication-efficient, and are highly modular.

An example will be given, in the form of a (15,7,3)-organization, i.e., fifteen persons meet in triplets, each attending seven meetings.

To describe how the meetings are patterned, we imagine that the members are placed on and in a tetrahedron. There is one at each vertex (4), one at the midpoint of each edge (6), one at the middle of each face (6), and one at the centre of the tetrahedron (1). As a step in specifying which triplets meet, we identify ten triangles. Four are the triangles constituting the faces, and the other six are formed by taking a cross-section of the tetrahedron through an edge and the midpoint of the opposite edge. Each of these triangles will contain seven members laid out in the pattern of the 7-member Steiner organization of Figure 2. Meetings are then held for each triangle according to the pattern of the seven-member Steiner organization.

There will be 35 meetings in all:

1 - 2 - 5	2 - 3 - 9	3 - 5 -11	4 -11 -15	6 -13 -14
1 - 3 - 6	2 - 4 - 8	3 - 7 -12	5 - 6 - 9	7 - 9 -15
1 - 4 - 7	2 - 6 -11	3 - 8 -14	5 - 7 - 8	7 -11 -14
1 - 8 -13	2 - 7 -13	3 -13 -15	5 -10 -15	8 - 9 -10
1 - 9 -11	2 -10 -14	4 - 5 -13	5 -12 -14	8 -11 -12
1 -10 -12	2 -12 -15	4 - 6 -12	6 - 7 -10	9 -12 -13
1 -14 -15	3 - 4 -10	4 - 9 -14	6 - 8 -15	10 -11 -13

Every pair meets exactly once so the organization is communication-efficient with d'= 0, and two-person conflicts can be solved by referring them to the third person in the triplet.

The organization is isomorphic to a known mathematical concept, namely PG(3,2), the three-dimensional projective space with three points per line (Coxeter, 1964). Members correspond to points, meetings to lines, seven-person suborganizations to planes, and the entire organization to a three-dimensional space. The structure is similar to a geometrical space in that every pair of points determines a unique line, every two intersecting lines generate a unique plane, and every pair of intersecting planes generates the entire space. "Generates" here means forms by augmenting the two lines or planes with all lines connecting points in the original pair.

As a consequence of its being a geometrical space, the organization is highly modular, the subparts being the one- and two-dimensional subspaces. Every pair of three-person meetings with a common member generates a seven-member organization. It can be calculated that there are fifteen such suborganizations, ten of which were used to describe the construction, and five more of which are hidden in the structure. The membership of the fifteen is:

1	2	3	5	6	9	11
1	2	4	5	7	8	13
1	2	5	10	12	14	15
1	3	4	6	7	10	12
1	3	6	8	13	14	15
1	4	7	9	11	14	15
1	8	9	10	11	12	13

2	3	4	8	9	10	14
2	3	7	9	12	13	15
2	4	6	8	11	12	15
2	6	7	10	11	13	14
3	4	5	10	11	13	15
3	5	7	8	11	12	14
4	5	6	9	12	13	14
5	6	7	8	9	10	15

These suborganizations would be assigned to subtasks, and since each pair of suborganizations intersects in a three-person meeting, the latter would have the responsibility of liaison with respect to the matters of the subtasks.

If a conflict occurs among three or more members belonging to the same 7-member organization, it seems sensible that it should be solved by the suborganization, rather than the entire group. Thus there are three levels of conflict resolution, within groups of three, seven and fifteen, and the intermediate level is a further way to protect the time of the whole group.

The organization has fifteen members, and also fifteen suborganizations. (This is not a coincidence given its structure as a projective space.) Each member can be assigned a unique suborganization for which he or she has the special role of chairperson. Likewise the seven-member subgroups contain seven meetings allowing a one-to-one assignment of chairpeople to three-person meetings. The organization is symmetrical among the members _a priori_, but this 1-1 assignment procedure allows us to preserve the equality after it has begun its task.

The most serious difficulty is that the organization is inflexible in its size -- the next smaller projective-space organization has seven members, and the next larger 31. (The formula for the size with three-person meetings and dimension D is $2^{D+1} - 1$.)

One possible response is to allow fewer than fifteen members but continue to have exactly fifteen "roles" which are defined by the above structure. A member might carry an extra role, and this would be rotated through the group.

This organization and its relatives are quite the opposite of trees in the four aspects described in the last section, but like trees their features are mutually consistent and self-reinforcing. This suggests that in certain circumstances, it would not be the best policy to try to modify a tree little by little. Instead we should start anew with an organization of the type described here.

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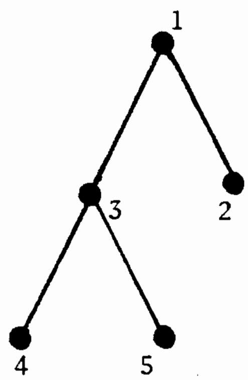


	1	2	3	4	5
1	1	1	1	1	1
2	1	2	1	1	1
3	1	1	3	3	3
4	1	1	3	4	3
5	1	1	3	3	5

Figure 1: Organization chart and multiplication table for a tree.

	1	2	3	4	5	6	7
1	1	3	2	7	6	5	4
2	3	2	1	6	7	4	5
3	2	1	3	5	4	7	6
4	7	6	5	4	3	2	1
5	6	7	4	3	5	1	2
6	5	4	7	2	1	6	3
7	4	5	6	1	2	3	7

123
147
156
246
257
345
367

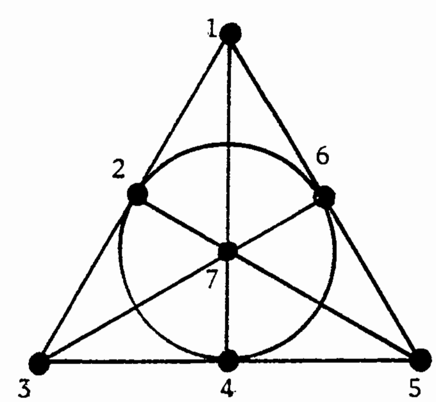
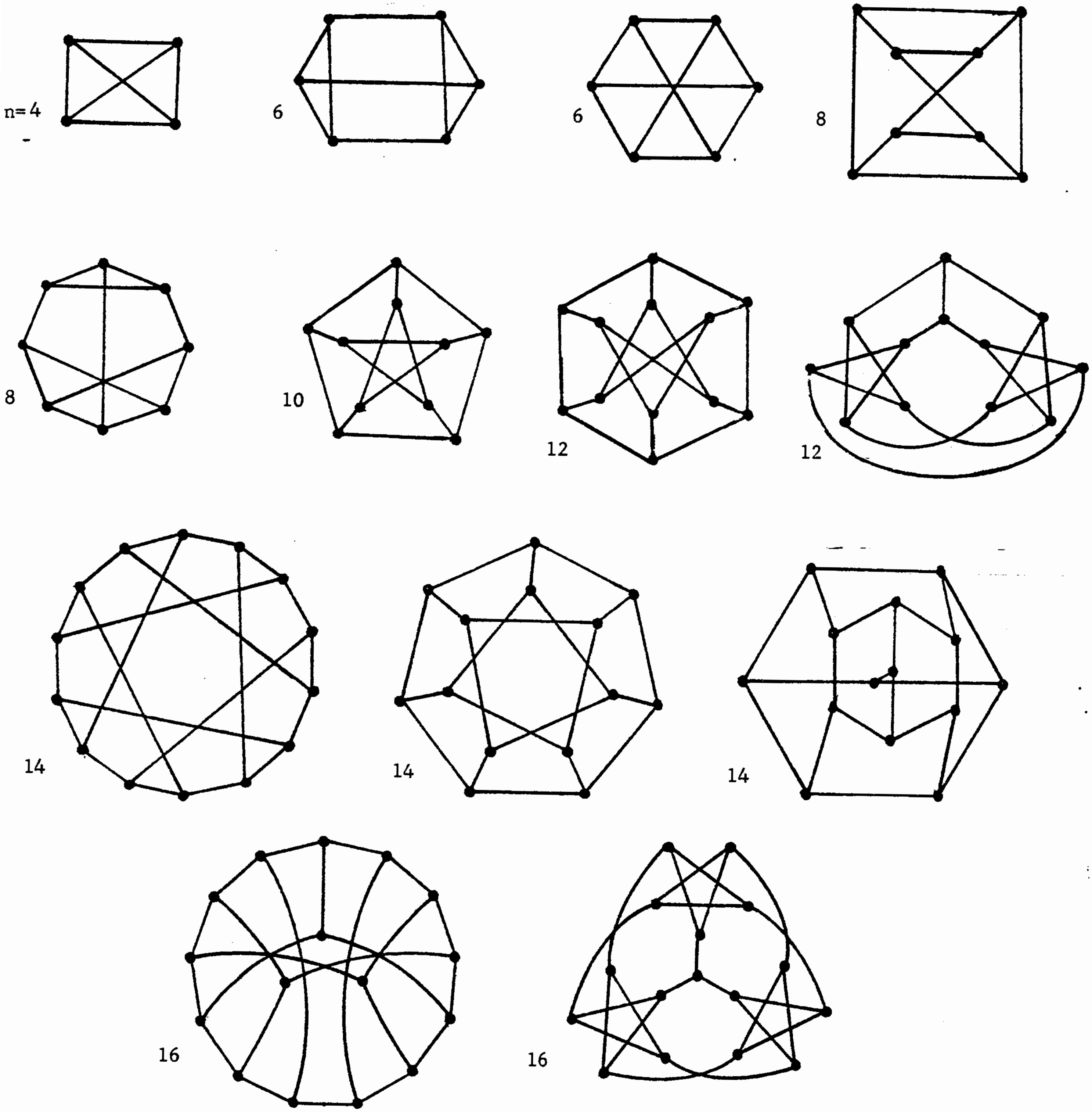


Figure 2: Multplication table, list and chart representations for a seven-member Steiner organization.



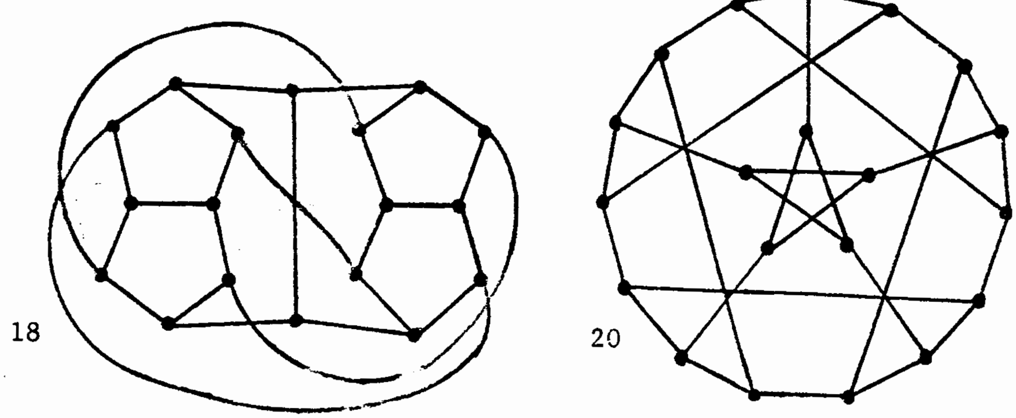


Figure 3:

Some efficient (n,3,2)-organizations.

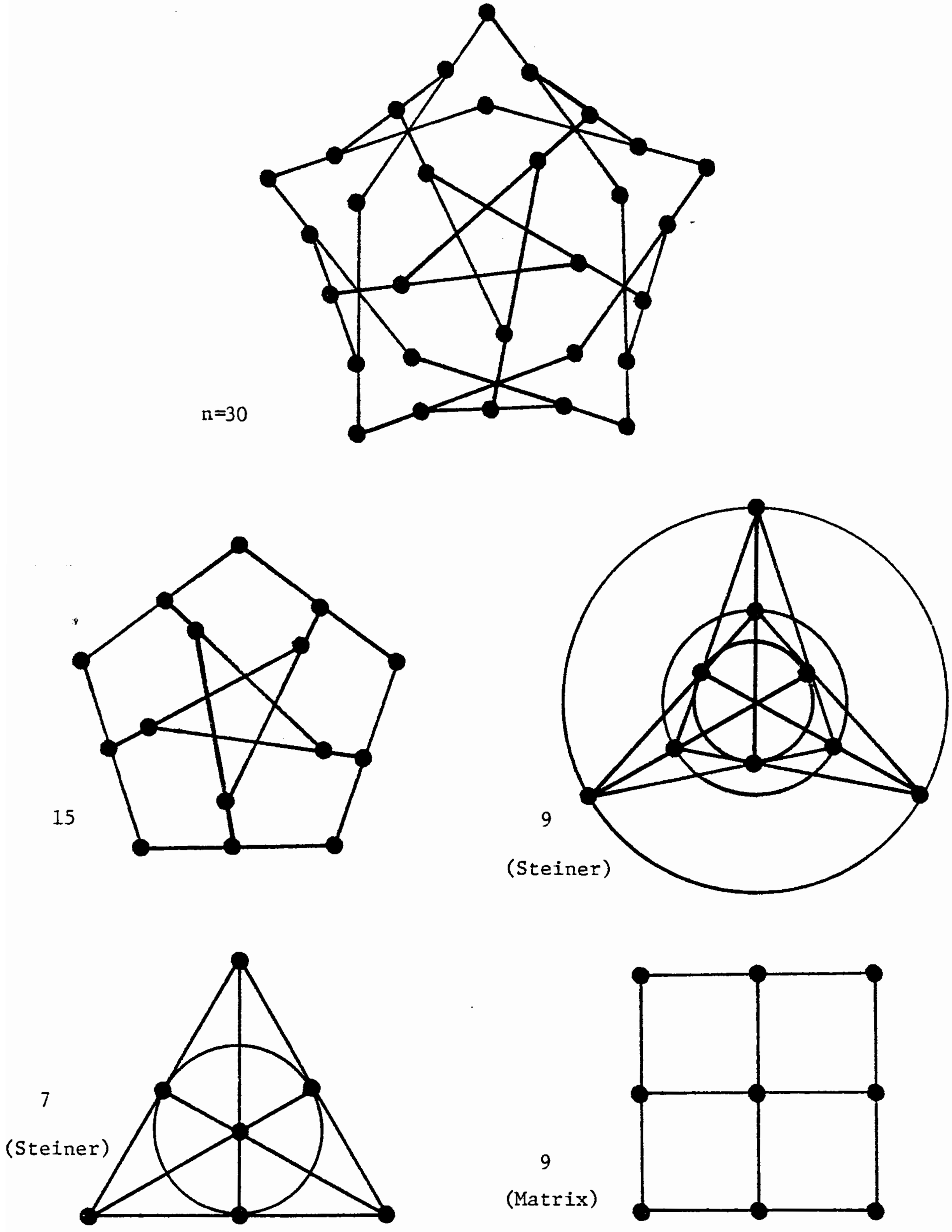
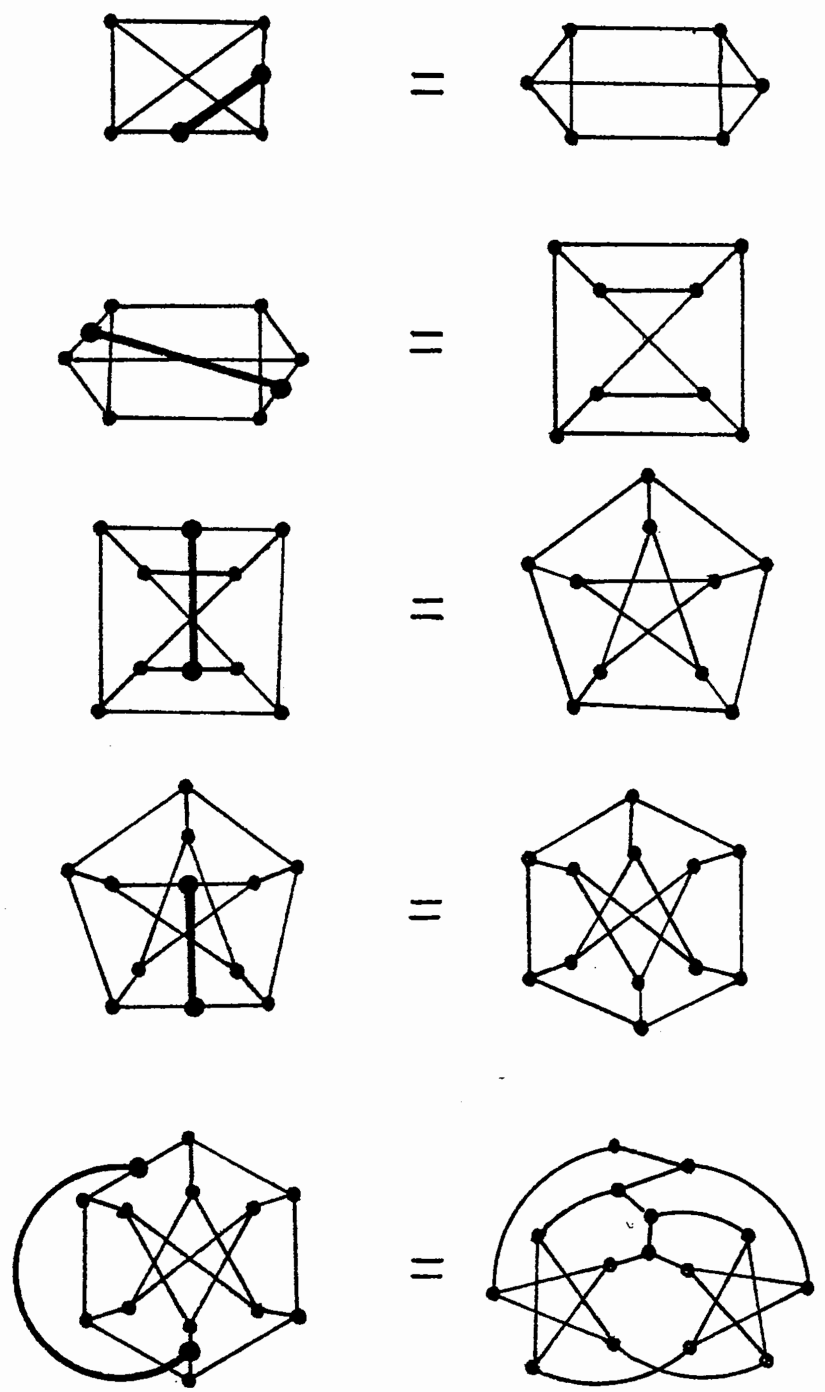


Figure 4: Some efficient (n,m,3)-organizations.



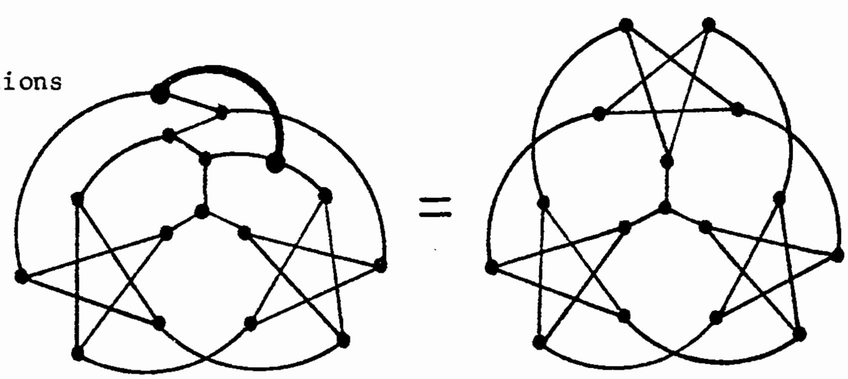


Figure 5:

Incremental growth in (n,3,2)-organizations

New members indicated by heavier dots.

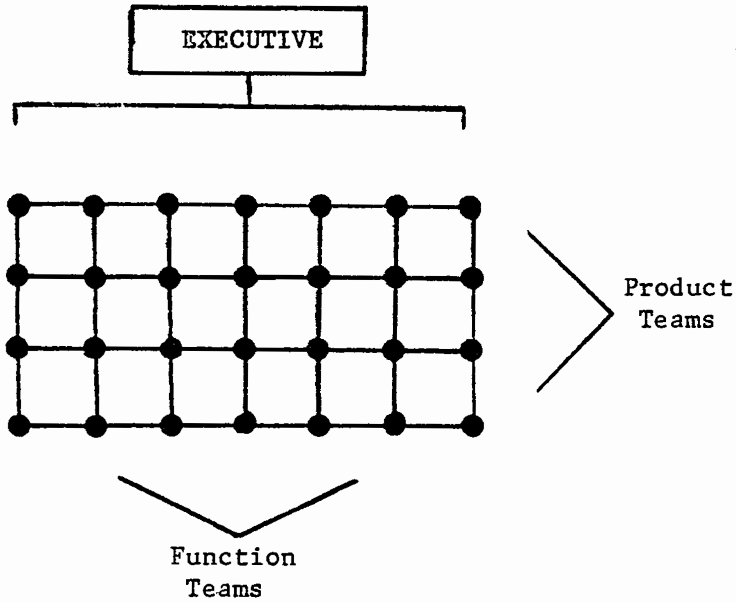


Figure 6. Matrix organization

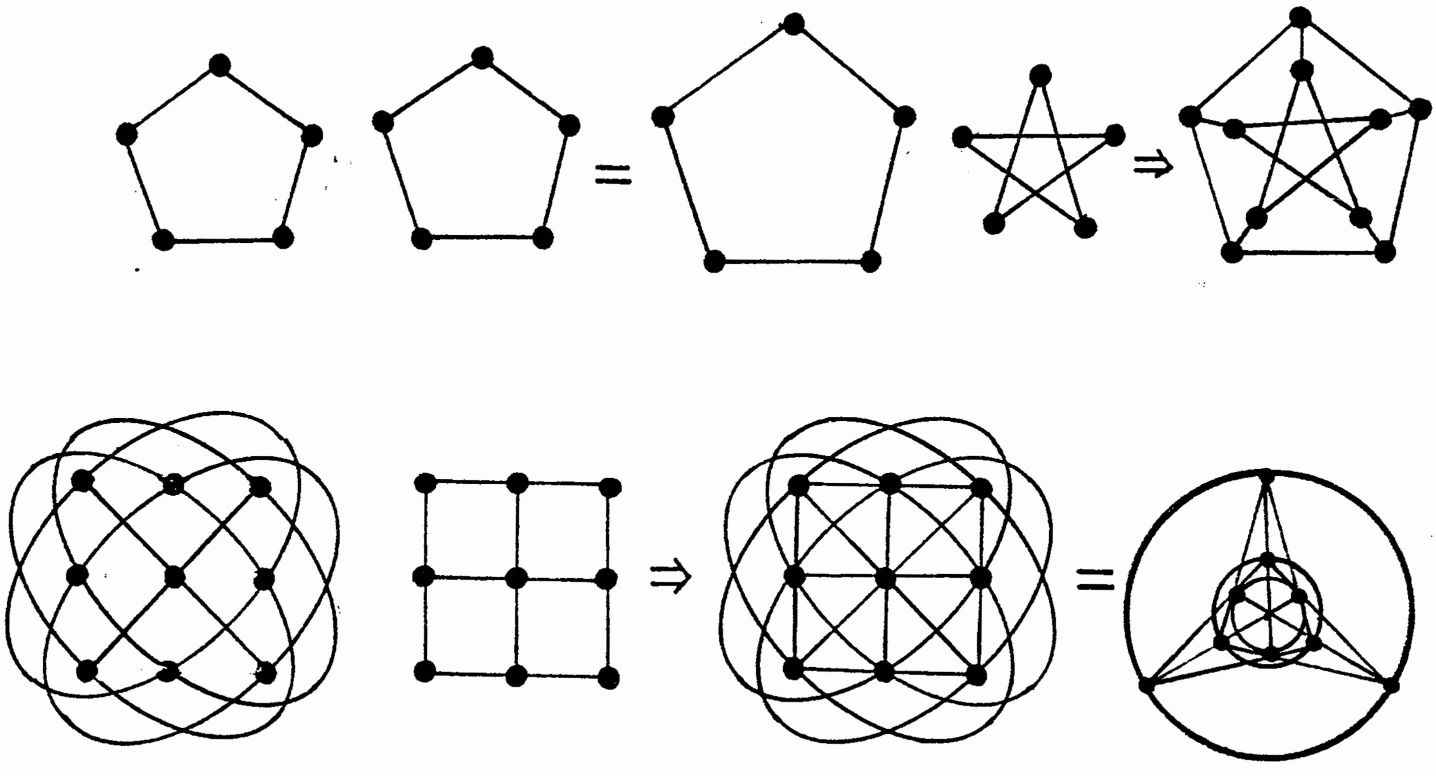


Figure 7. Modular growth: two (5,2,2)-organizations become a (10, (10,3,2)-organization. Structural growth: a (9,2,3)-matrix organization adds meetings to become a (9,4,3)-Steiner organization