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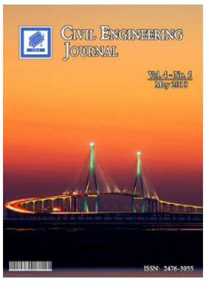

Strut and Tie Modeling for RC Deep Beams under non-Central Loadings

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Abstract

This work aims at presenting detailed procedures companied by numerical examples for analyzing and designing reinforced concrete deep beams that subjected to non-central loadings based on Strut and Tie method (STM). The subjected loadings were moved from the center of the beam span towards the supports reaching the maximum non-centrality could be achieved (after which the beams became 'not deep' from ACI 318M-14 point of view). A total of three deep beams with three different types of loadings were taken into considerations; one concentrated force, two concentrated forces and uniformly distributed load. Every specimen had a cross section of 150 × 400 mm and a total length of 1000 mm. Generally, it was found that moving load from the span center towards one of the supports leads to worth notable decreases in the beam ultimate capacity. Therefore, in the case of one-concentrated force, the ultimate load capacity decreased by 30.2% when left shear span to effective depth ratio (a_L/d) decreased from 1.3 to 0.65. While in the cases of two-concentrated forces or uniformly distributed loading, it was found that changing (a_L/d) ratio from 1.02 to 0.37 led to decrease the deep beam ultimate capacity by 30.5%.

Keywords: RC; Deep beams; STM; Load non-Centrality; Design Procedures.

1. Introduction

Deep beams are members that loaded on one face and supported on the opposite face such that strut-like compression elements can develop between the loads and supports to satisfy (a) or (b) [1]:

(a) Clear span ln does not exceed four times the overall member depth h.

(b) Concentrated loads exist within a distance 2h from the face of the support.

Many investigators have suggested empirical and semi-empirical expressions to determine the ultimate load capacity of conventionally reinforced concrete deep beams [2-4]. Some researchers studied the parameters that affect deep beams [5-10]. Since 2002, the ACI-318 Code procedure is based on empirical equations for the design of deep beams.

According to ACI 318M-14 [1], STM is defined as "a truss model of a structural member of a D-region in such a member, made up of struts and ties connected at nodes, capable of transferring the factored loads to the supports or to adjacent B-regions". Provisions for STM have been taken into considerations for the design purpose. STM complies with the plasticity lower bound theory, which needs that only yield conditions in addition to equilibrium to be satisfied. Plasticity lower bound theory states that if the load has such a value that it is possible to find a distribution of stress corresponding to stresses that keep internal and external equilibrium within the yield surface, then this load will not cause failure of the body. In other words, the capacity of a structure as estimated by a lower bound theory will be less than or equal to the real failure load of the body in question [11].

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Strut and tie model is a very useful tool for analyzing and designing reinforced concrete members in which D-regions exist. The non-centric loading cases are very common in structural engineering, while lack of such studies using STM is obvious. That is why this article investigates modeling in detail the struts and ties in the reinforced concrete deep beams under various non-central loading cases.

2. Strut-and-Tie Method (STM) Design Procedure

An emerging methodology for the design of all types of D-regions is to predict and design an internal truss. This truss is consisting of steel tension ties and concrete compressive struts that are interconnected at nodes, to support the imposed loading through the regions of discontinuity. The STM design procedure includes the general steps summarized below [1]:

- Define the D-region boundaries and determine the imposed sectional and local forces.
- Draw the internal supporting truss, find equivalent loadings, and calculate the truss member forces.
- Choose the reinforcing steel to provide the necessary capacity of the tie and ensure that this tie reinforcement is adequately anchored in the nodal zone (joints of the truss).
- Evaluate the dimensions of the nodes and struts, such that the capacities of these components (nodes and struts) are adequate to carry the values of the design forces.
- Select the distributed reinforcement to guarantee the ductile behavior of the D-region.

It is important to note that both hydrostatic and non-hydrostatic nodes are idealizations of reality. The use of either hydrostatic or non-hydrostatic nodes is an assumption; a design tool intended to provide a simple method for proportioning STM. The classic method of node dimensioning is by node shape arranging so that the applied stresses on all sides of the node are equal. The stress biaxial state in the node is hydrostatic; so, the in-plane stresses are homogeneous, isotropic, and equal to those on the sides. Arranging the node in this shape can be made by sizing the node boundaries so that they become proportional and perpendicular to the forces that acting on them (hydrostatic) [12]. In the case of non-centric loading, there is no symmetry in checking nodes, struts and tie, because the truss formed by loading transferring from the applying nodes to the supports is not symmetric too. In order to recognize specimens designation easily, Table 1 shows the way followed in this designation.

Table 1. Specimens designation way

Letter	Meaning
B	Conventional Deep Beam
1F	Subjected to 1-concentrated Force
2F	Subjected to 2-concentrated forces (2F), which means actually (2*0.5P)
U	Subjected to Uniformly Distributed load

3. Three Loading Cases

3.1. One Non-Central Concentrated Force

Figure 1 shows the principal stress paths and the assumed truss under the non-central 1-concentrated force in the simply supported deep beam (B.1F). The geometry should be conformed to the deep beam definition ($l_n \leq 4h$) [1]. Moreover, the minimum web reinforcement ratios for both horizontal and vertical ones should be 0.0025 with, the maximum spacing of d/5 and not more than 300 mm [1]. Finally, checking nominal shear strengths at each node face of the nodes A, B and C, the diagonal strut (idealized bottle shapes AB and BC), in addition to the tie AC.

To analyze the deep beam with one concentrated force, the steps shown in Figure 2 may be followed. A detailed numerical application example is illustrated in Table 2 and Figure 3.

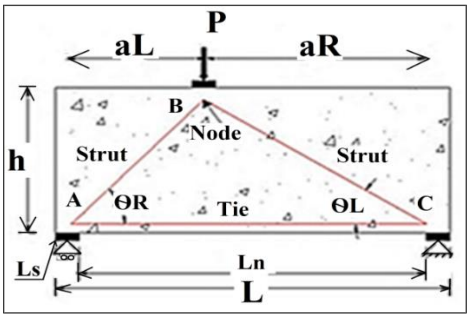


Figure 1. The principal stress paths and the assumed truss for (B.1F)

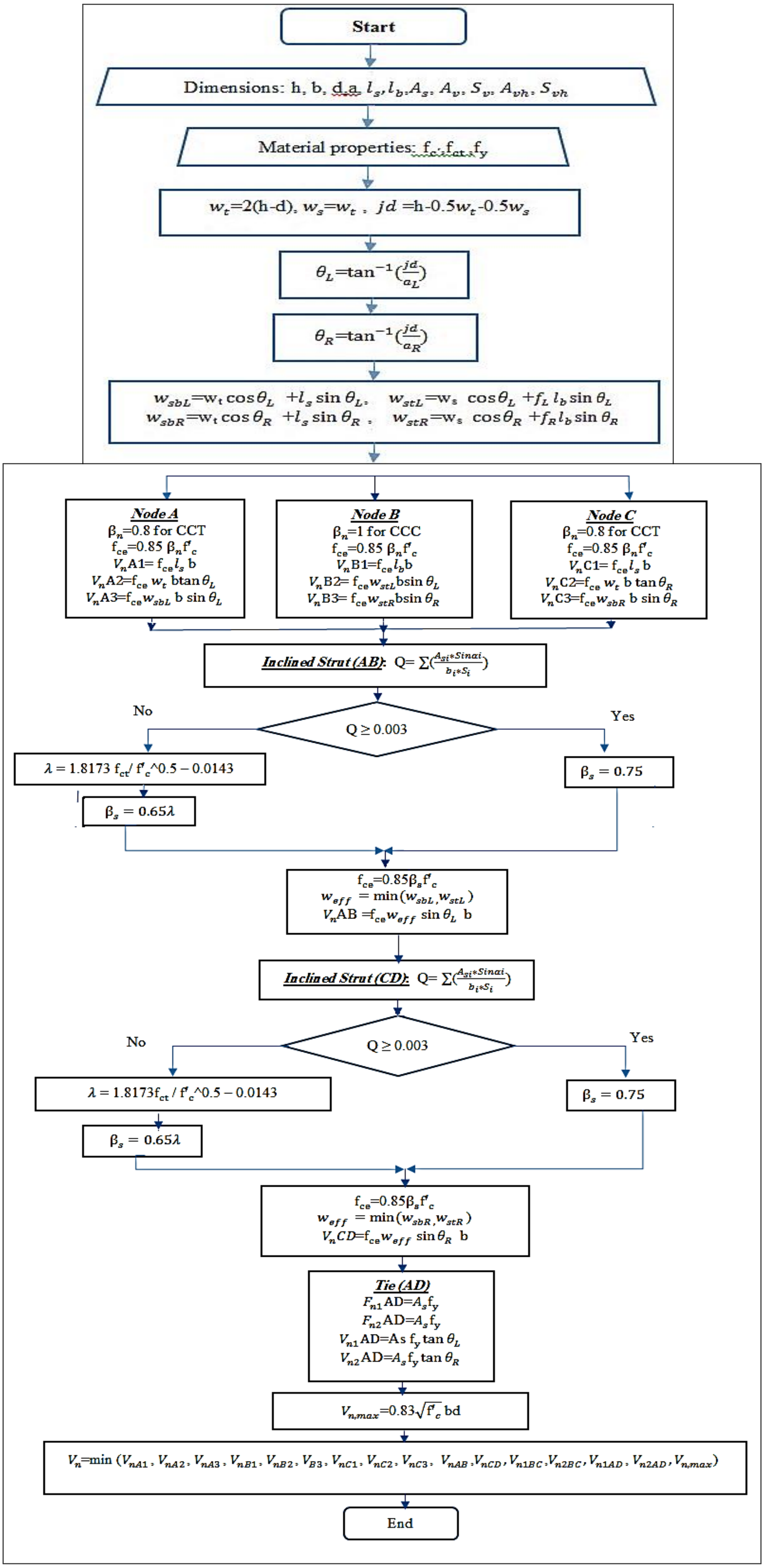


Figure 2. STM Flow chart for light weight and normal weight reinforced concrete deep beams subjected to 1-concentrated force

Table 2. Numerical Example No. 1, (One Non- Central Concentrated Force)

<table>
<tr><th colspan="9">Input data</th></tr>
<tr><td>h=400 mm</td><td>L=1000 mm</td><td>d=361 mm</td><td>a_L =370 mm</td><td>a_R=570 mm</td><td>b_w =150 mm</td><td>f_{ct}=3.9 MPa</td><td>f'c=38 MPa</td><td>Bearing Plates=(60 * 150) mm</td></tr>
<tr><td>f_y=460 MPa</td><td>Vertical web reinf.= ϕ6mm@70mm c/c</td><td>f_{yv} =442 MPa</td><td>Horizontal web reinf.= ϕ6mm@70mm c/c</td><td>f_{yh} =442 MPa</td><td>Bottom and upper covers=25 mm</td><td>Side cover=15 mm</td><td colspan="2">Main longitudinal reinf.= 3ϕ16mm</td></tr>
<tr><th colspan="9">Output data</th></tr>
<tr><td colspan="3">Draw STM of deep beam B.1F, see Figure 4-a.
C and T are required to equilibrate the truss, see Figure 4-b. w_t=78mm, the term w_s is initial unknown. For convenience and simplicity, assuming w_s= w_t gives an error less than 2% due to that w_s is typically ten times smaller than the total deep beam depth h [13, 14].
<u>Node A</u>, Figure 5-a., β_n =0.8 for CCT [ACI 318M-14, Table 23.9.2], [1], f_{ce}=25.84 MPa, V_nA1 =232.56kN, V_nA2 =262.8kN, V_nA3 =249.7 kN</td><td colspan="3"><u>Node B,</u> Figure 5-b., 39.4% of the applied load flows into the right support (f_R) and the other 60.6% is transferred to the left support (f_L).
β_n=1 (CCC) [ACI 318M-14, Table 23.9.2] [1] , f_{ce}=32.3 MPa
V_nB1 = 290.7 kN, V_nB2 = 262.87 kN, V_nB3 = 189.67kN</td><td colspan="3"><u>Node C</u>, Figure 5-c., β_n=0.8 for CCT [ACI 318M-14, Table 23.9.2] [1], f_{ce} =25.84 MPa,
V_nC1 = 232.56 kN, V_nC2 =171.0 kN, V_nC3 =185.95 kN</td></tr>
<tr><td colspan="3"><u>Inclined Strut (AB)</u>, θ_L = α_2=41.0°, α_1 =90°- 41.0°=49°, see Figure 6. Q= 7.59*10^{-3}>0.003 , β_s = 0.75, [ACI 318M-14, Table 23.4.3], [1], f_{ce}=24.225 MPa, w_{eff} = 82.7mm, V_n= 197.15 kN</td><td colspan="3"><u>Inclined Strut (BC)</u> θ_R = α_2=29.5°, α_1 =90°-29.5°=60. 5°, see Figure 6. Q= 7.35*10^{-3}>0.003 → β_s = 0.75 [ACI 318M-14, Table 23.4.3] [1], f_{ce}= 24.225 MPa, w_{eff} = 79.5mm, V_n= 142.25 kN →min</td><td colspan="3"><u>Tie (AC),</u> As=603.18mm^2, V_{n1}AD=241.2 kN, V_{n2}AD= 156.98 kN</td></tr>
</table>

Therefore, maximum nominal shear, [ACI 318-14(9), section 9.9.2.1], [1], $V_{n,max}$=277 kN, Minimum value of Vn =142.25 kN, Ultimate design V_u= ϕVn_{min}=106.68 kN, (ϕ =0.75), Ultimate capacity load of deep beam is Pu = 2*106.68 = 213.37 kN

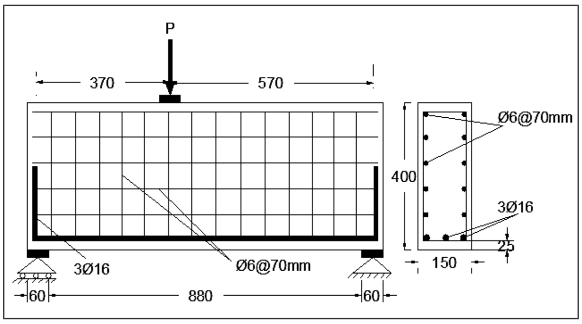
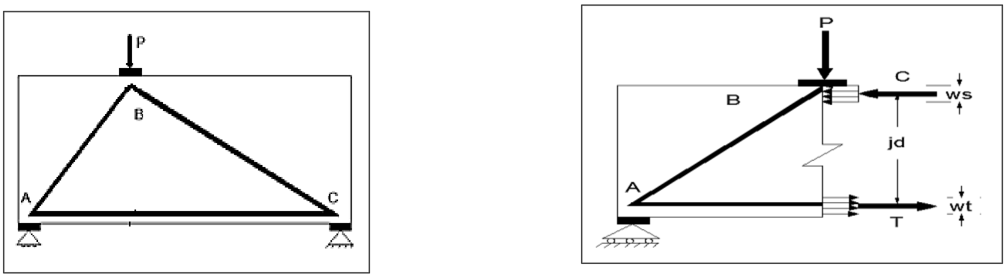


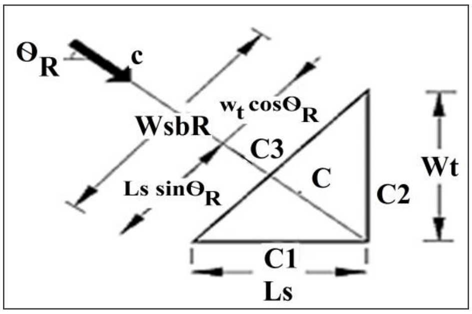
Figure 3. Details of (B.1F), all dimensions in (mm)



a): STM for (B.1F) **b):Free body diagram for the left half of (B.1F)**

Figure 4. Strut – Tie model for 1-concentrated force loaded beam (B.1F)

a): Faces of support nodal zone A, (B.1F) **b): Faces of load nodal zone B, (B.1F)**



c):Faces of support nodal zone C, (B.1F)

Figure 5. Nodes in 1-concentrated force loaded beam (B.1F)

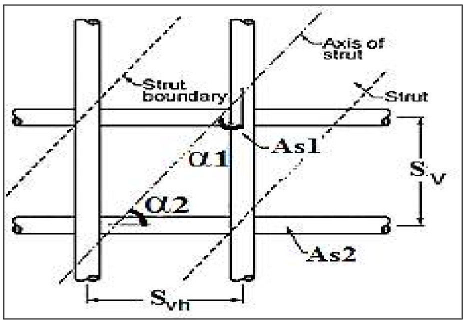


Figure 6. Reinforcement crossing strut AB for (B.1F)

2.2. Two Non-Central Concentrated Forces

Figure 7 shows the principal stress paths and the assumed truss under the non-central 2-concentrated forces in the simply supported deep beam (B.2F). According to the shear provisions of the ACI 318M-14 design code, same as in the case of 1-concentrated force, the geometry should be conformed to the deep beam definition ($l_n \leq 4h$) [1]. In addition, the minimum web reinforcement ratios for both horizontal and vertical ones should be 0.0025 with the maximum spacing of d/5 and not more than 300 mm [1]. Finally, checking nominal shear strengths at each face of the nodes (A, B, C and D), horizontal strut (uniform cross section BC), and the diagonal strut (bottle shape AB and CD) in addition to the tie (AD).

To analyze the deep beam with two concentrated forces, the steps shown in Figure 8 may be followed. A detailed numerical application example is shown in Table 3 and Figure 9. The STM draw of deep beam B.2F is shown in Figure 10-a. Strut BC and tie AD are required to equilibrate the truss. These strut and tie form a force couple shown in Figure (10-b).

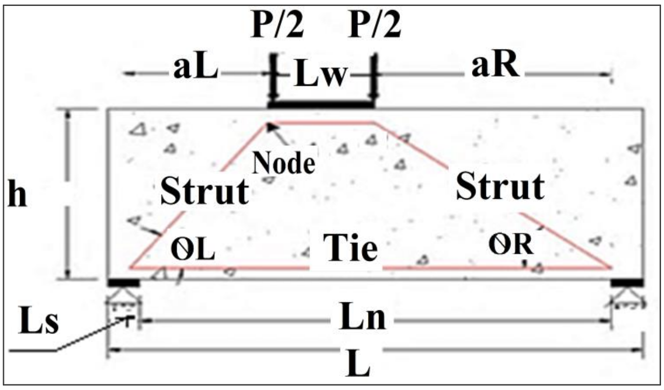


Figure 7. The principal stress paths and the assumed truss for (B.2F)

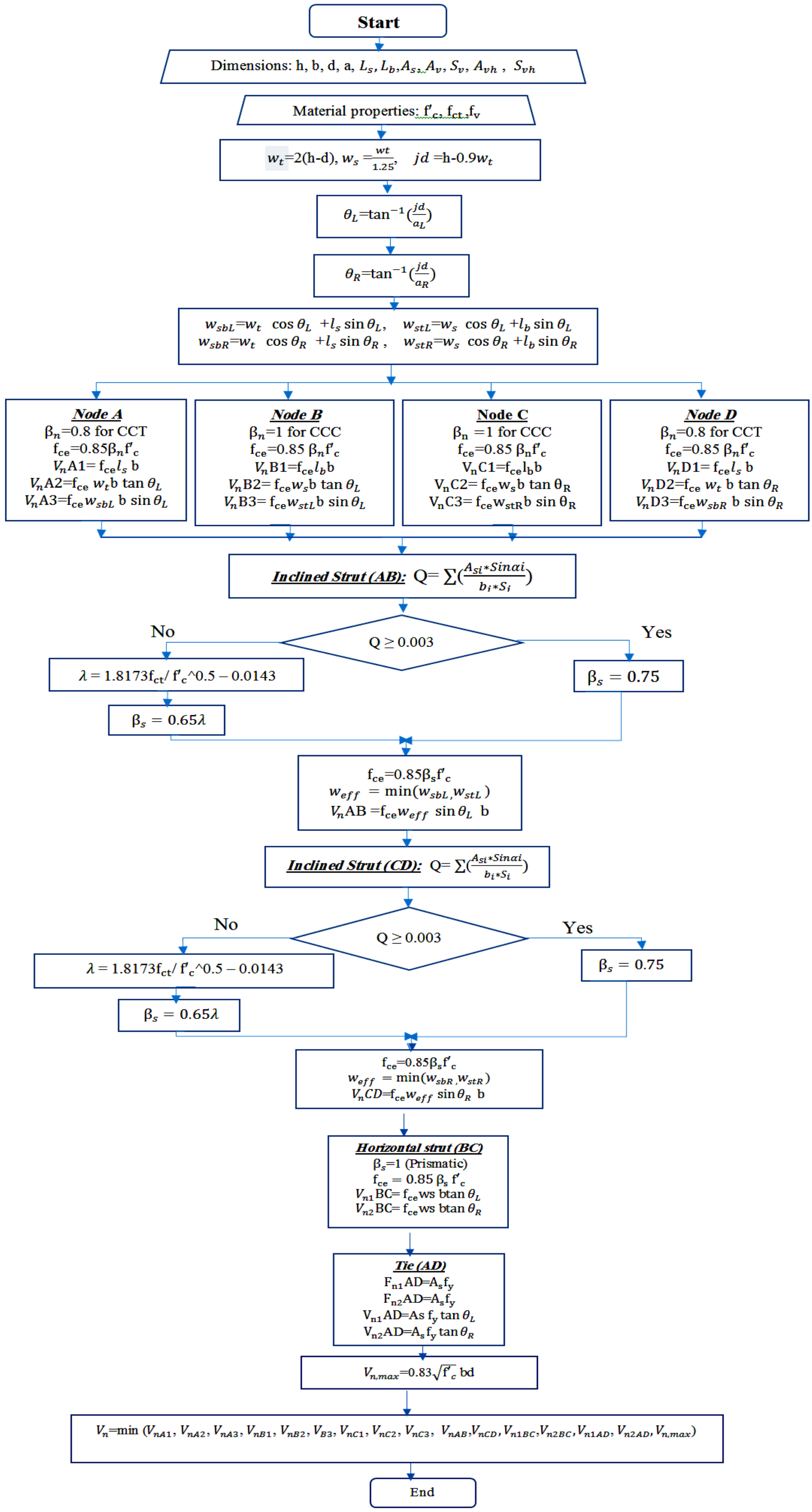


Figure 8. STM Flow chart for light weight and normal weight reinforced concrete deep beams subjected to 2-concentrated force.

Table 3. Numerical Example No. 2, (Two Non- Central Concentrated Force)

<table>
<tr><td colspan="12">Input data</td></tr>
<tr><td>h=400 mm</td><td>L=1000 mm</td><td colspan="2">d=361 mm</td><td>a_L =270 mm</td><td colspan="2">a_R=470 mm</td><td>b_w =150 mm</td><td colspan="2">f_{ct}=3.9 MPa</td><td>f'c=38 MPa</td><td>Bearing Plates=(60 * 150) mm</td></tr>
<tr><td>f_y=460 MPa</td><td>Vertical web reinf.= ϕ6mm@70mm c/c</td><td colspan="2">f_{yv} =442 MPa</td><td>Horizontal web reinf.= ϕ6mm@70mm c/c</td><td colspan="2">f_{yh} =442 MPa</td><td>Bottom and upper covers=25 mm</td><td colspan="2">Side cover= 15 mm</td><td colspan="2">Main longitudinal reinf.= 3ϕ16 mm</td></tr>
<tr><td colspan="12">Output data</td></tr>
<tr><td colspan="3"><u>Node A,</u> Figure 11-a., β_n=0.8 for CCT [ACI 318M-14, Table 23.9.2] [1], f_{ce}=25.84MPa, V_nA1 =232.56kN, V_nA2 =369.4kN, V_nA3 =287.3kN</td><td colspan="3"><u>Node B,</u> Figure 11-b., β_n=1(CCC) [ACI 318M-14, Table 23.9.2] [1], f_{ce}=32.3 MPa, V_nB1 = 290.7kN, V_nB2 = 369.4 kN, V_nB3 = 322 kN</td><td colspan="3"><u>Node C,</u> Figure 11-c., β_n=1 (CCC) [ACI 318M-14, Table 23.9.2] [1], f_{ce}= 32.3 MPa, V_nC1 = 290.7 kN, V_nC2 = 211.7 kN, V_nC3 = 237.6 kN</td><td colspan="3"><u>Node D,</u> Figure 11-d., β_n=0.8 for CCT [ACI 318M-14, Table 23.9.2] [1], f_{ce}=25.84 MPa, V_nD1 =232.56kN, V_nD2 =211.7kN, V_nD3 =218.5 kN</td></tr>
<tr><td colspan="3"><u>Inclined Strut (AB)</u> $\theta_L = \alpha_2$=50.7°, α_1 =90°-50.7°=39.3° , Q=7.57*10^{-3}>0.003, β_s = 0.75 [ACI 318M-14, Table 23.4.3] [1], f_{ce}=24.225 MPa , w_{eff} = 5.9mm, $V_{n1}AB$= 241.5 kN</td><td colspan="3"><u>Inclined Strut (CD)</u> $\theta_R = \alpha_2$=35°, α_1 =90°- 35°=55°, Q= 7.49*10^{-3}>0.003 , β_s = 0.75 [ACI 318M-14, Table 23.4.3] [1], f_{ce}=24.225 MPa, w_{eff} = 85.5mm, $V_{n2}AB$ =178.2 kN →min</td><td colspan="3"><u>Horizontal strut (BC)</u> β_s=1 (Prismatic) [ACI 318M-14, Table 23.4.3] [1], f_{ce} = 32.3 MPa, V_{n1}BC = 369.4 kN, V_{n2}BC = 211.7 kN</td><td colspan="3"><u>Tie (AD)</u>
As=603.18mm^2
V_{n1}AD= 339 kN, V_{n2}AD= 194.3 kN</td></tr>
</table>

Therefore, maximum nominal shear, [ACI 318-14(9), section 9.9.2.1], [1], $V_{n,max}$=277 kN, Minimum value of Vn =178.2 kN, Ultimate design V_u= $\phi V n_{min}$=133.65 kN, (ϕ =0.75), Ultimate capacity load of deep beam is Pu = 2*133.65 = 267.3 kN

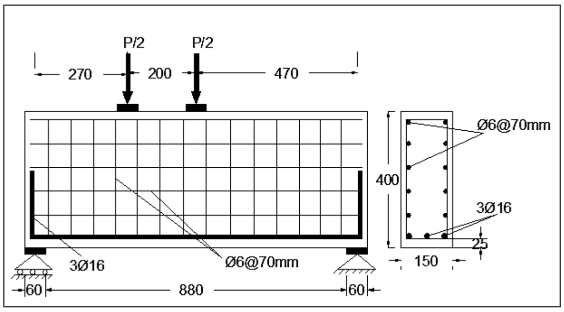
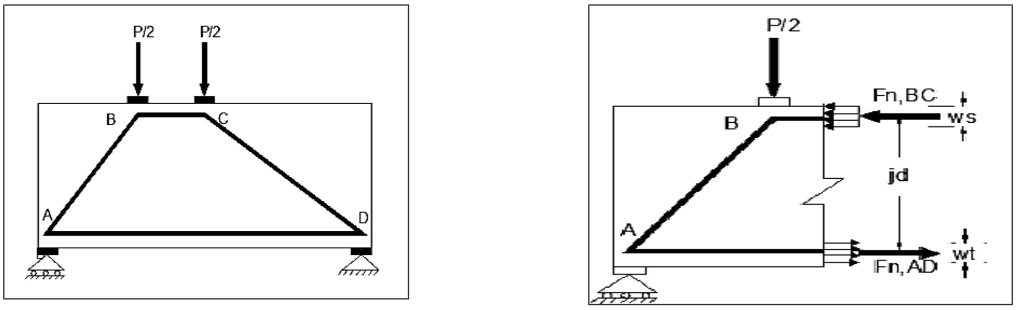


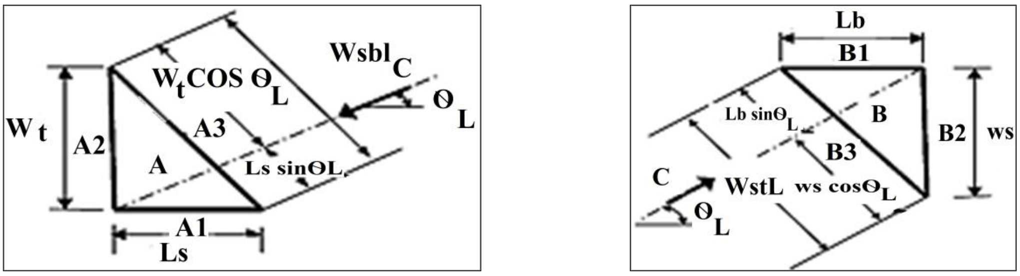
Figure 9. Details of (B.2F), all dimensions in (mm)



a): STM for (B.2F)

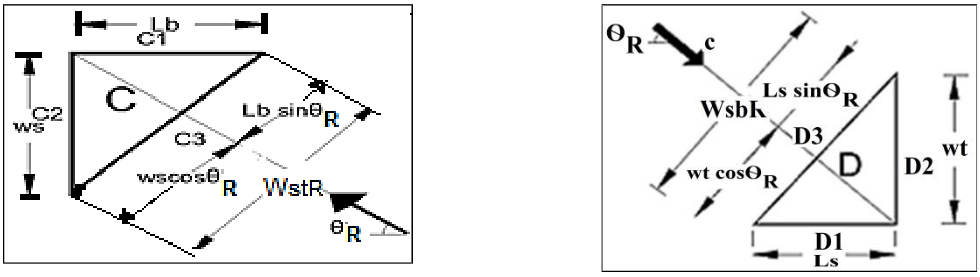
b):Free body diagram for the left half of (B.2F)

Figure 10. Strut – Tie model for 2-concentrated forces loaded beam (B.2F)



a) Faces of support nodal zone A for (B.2F)

b) Faces of load nodal zone B for (B.2F)



c) Faces of load nodal zone C for (B.2F) **d) Faces of support nodal zone D for (B.2F)**

Figure 11. Nodes in 2-concentrated forces loaded beam (B.2F)

2.3 Uniformly Distributed Load

Many researchers went to the conclusion that when deep beam is subjected to uniformly distributed load, it could be considered as deep beam under two concentrated forces that should equal to the uniformly distributed load in value [15-17]. Figure 12 shows the principal stress paths in the simply supported deep beam subjected to a non-central uniformly distributed load. It is worth to mention that the uniformly distributed load can be substituted by equivalent two equal forces or equivalent two unequal forces. This Substitution is allowed only if the equality of the maximum moments the most fundamental value in the Strut-Tie model application of the both systems is guaranteed. Figures 13-a and 13-b show how the bending moment for the two equivalent equal concentrated forces are closer to the bending moment of uniformly distributed load than the bending moment when the two forces are unequal, Figure 13-c. That is why in this investigation the two equivalent equal two forces were taken into consideration as a substitution of uniformly distributed load.

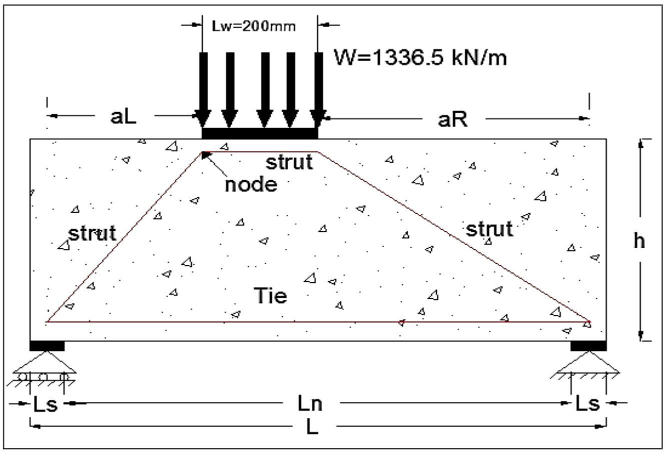


Figure 12. Strut – Tie model for uniformly distributed loaded beam (B.U)

Based on that, the prediction of strength capacity for the reinforced concrete deep beam subjected to non-central uniformly distributed loading (B.U) shown in Figure 13-b and 9 can be obtained by the same procedure shown in Figure 8. It was considered that the equivalent two concentrated forces are equal, so the strength capacity can be calculated by the following:

Maximum nominal shear [ACI 318-14(9), section 9.9.2.1] [1]

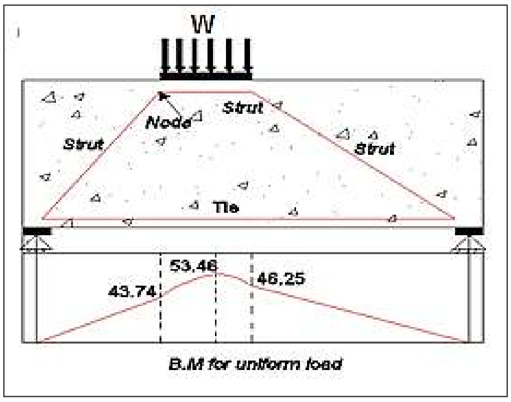
$V_{n,max}$=277 kN

Minimum value of V_n =178.2 kN

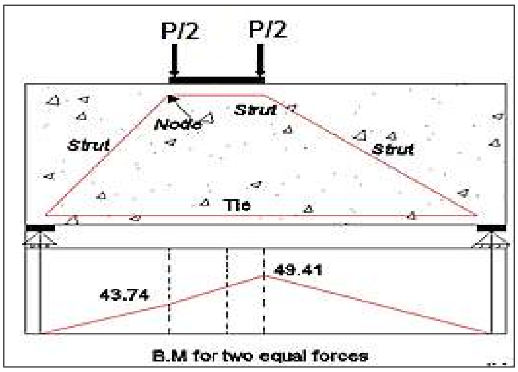
Ultimate design $V_u = \phi Vn_{min}$=133.65 kN, (ϕ =0.75)

Ultimate capacity load of deep beam is $P_u = 2\times 133.65 = 267.3\ kN$, $P_u = W_u \times (L_W = 0.2m)$,

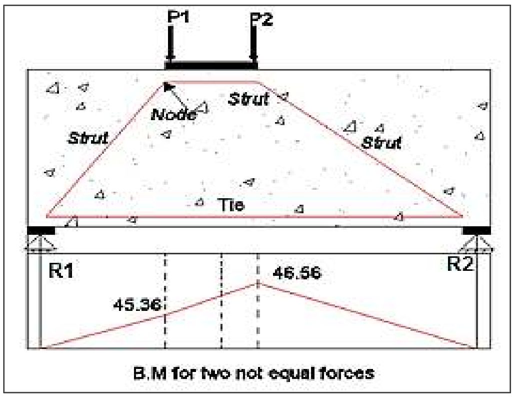
$\therefore$ W_u =1336.5 kN/m, this is similar to the numerical example No.2.



a) Uniformly distributed load when W= 1336.5 kN/m



b) Equivalent two equal concentrated forces when $W*L_w$=1336.5*0.2=2(P/2)= 2(133.65) kN



c) Equivalent two unequal concentrated forces when P1=162 kN, P2=105.2 kN, R1=168 kN, and R2=99.1 kN

Figure 13. Moment diagrams for (B.U)

3. Load non-Centrality Effect

The deep beam specimens B.1F, B.2F and B.U were reanalyzed with different load positions in order to study the effect of load non-centrality. For every beam, load was moved from the center of the beam span towards the left support (to the position after which the beam became not deep). Generally, as shown in Table 4 it was observed that moving load from span center towards the left support leads to noticeable decrease in the deep beam ultimate capacity. In case of B.1F, the decrease was 14.7%, 26.8% and 30.2% when left shear span to effective depth ratio (a_L/d) was decreased by 21%, 43% and 50%, respectively as shown in Figures 14 and 17. In case of B.2F and B.U, when the left shear span to effective depth ratio (a_L/d) decreased by 27%, 53% and 63%, the ultimate load capacity of the concrete deep beams decreased by 14.7%, 26.9% and 30.5%, respectively as shown in Figures 15 and 18 in addition to Figures 16 and 19. Also, Figure 17, 18 and 19 show the effect of a_L/d ratio on the ultimate capacity of specimens in all group.

That decrease in the ultimate capacity of beams took place because the load non-symmetry which led to different shear span to effective depth ratios (a/d) on both sides of the deep beams. Only a single strut was used between the applied load and the left support, which is the shorter left strut that made a big angle (θ_L). Therefore, in this case, the longer right portion of the beam required longer strut with a shallow angle (θ_R) which would not be safe or practical. That is why the ACI 318M-14 [1] requires a minimum angle of 25 degrees between struts and ties in order to insure the effectiveness of the strut-tie concepts.

Table 4. Effect of load non-centrality on ultimate capacity

Specimen	**Group**	**Load Type**	**Shear Span (mm)**		**Angle (degrees)**		**Ultimate capacity**		a_L/d	θ_L/θ_R	**% Decrease in P_u**
			a_L	a_R	θ_L	θ_R	P_u(kN)	**Wu (kN/m)**			
1	A	Single Concentrated Force	470	470	34.4	34.4	250.35	-	1.3	1	-
2			370	570	41.0	29.5	213.37	-	1.02	1.39	14.7
3			270	670	50	25.6	183.2	-	0.74	1.95	26.8
4			235	705	53.8	24.54	174.7	-	0.65	2.19	30.2
1	B	Two Concentrated forces	370	370	41.7	41.7	313.5	-	1.02	1	-
2			270	470	50.7	35	267.3	-	0.74	1.44	14.7
3			170	570	62.7	30	228.9	-	0.47	2.09	26.9
4			135	605	67.7	28.6	217.86	-	0.37	2.36	30.5
1	C	Uniformly Distributed Load	370	370	41.7	41.7	-	1567.5	1.02	1	-
2			270	470	50.7	35	-	1336.5	0.74	1.44	14.7
3			170	570	62.7	30	-	1144.5	0.47	2.09	26.9
4			135	605	67.7	28.6	-	1089.3	0.37	2.36	30.5

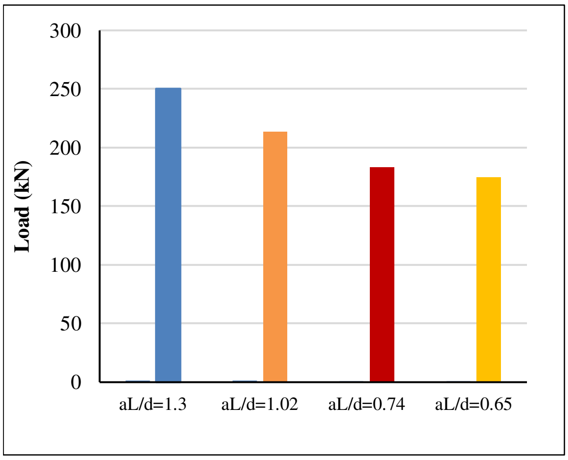


Figure 14. Effect of a_L/d ratio on the ultimate capacity of specimens in group A

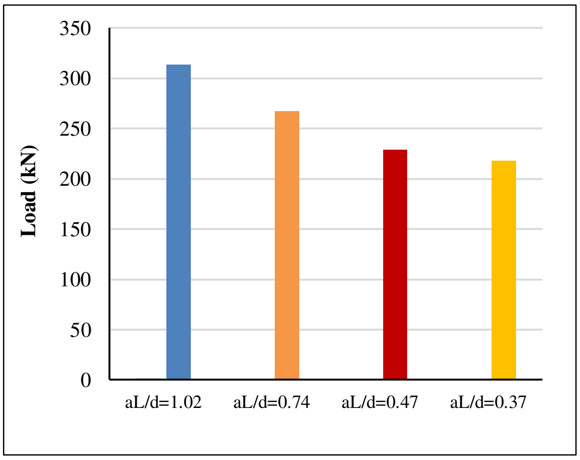


Figure 15. Effect of a_L/d ratio on the ultimate capacity of specimens in group B

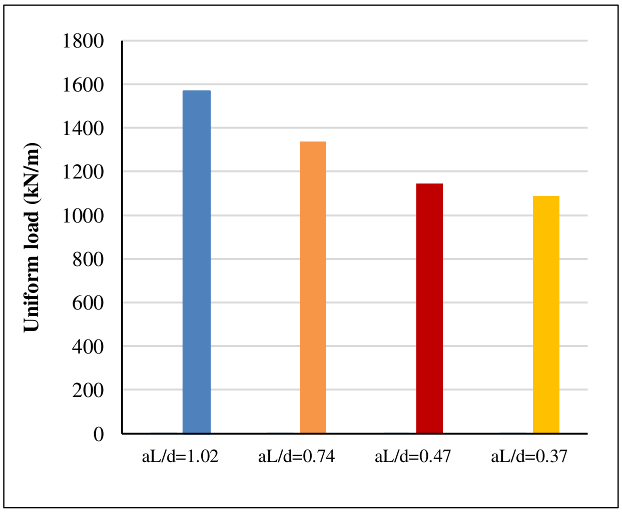
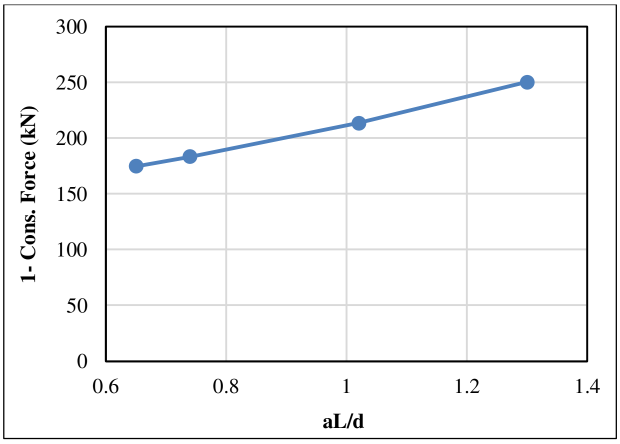


Figure 16. Effect of a_L/d ratio on the ultimate capacity of specimens in group C



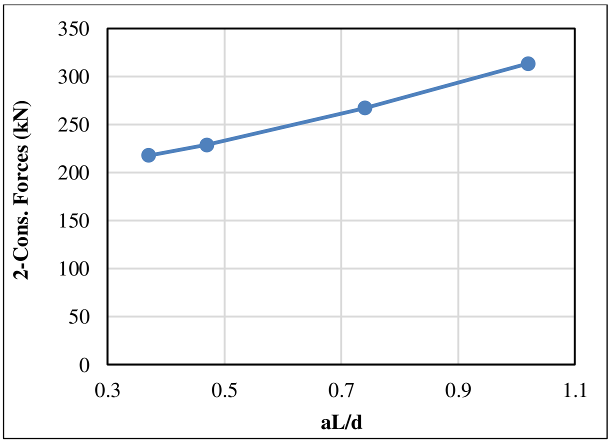


Figure 17. Effect of a_L/d ratio on the ultimate capacity of specimens in group A

Figure 18. Effect of a_L/d ratio on the ultimate capacity of specimens in group B

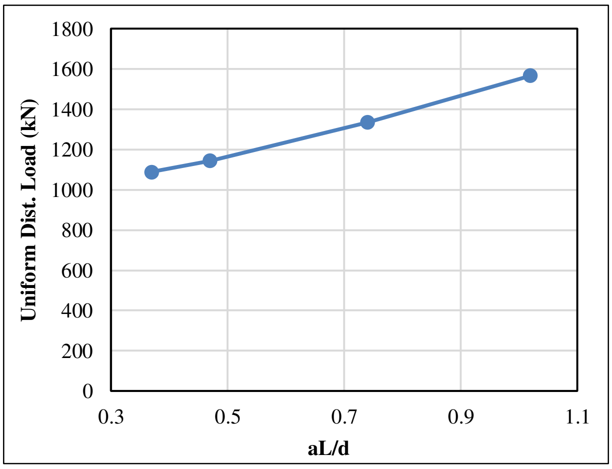


Figure 19. Effect of a_L/d ratio on the ultimate capacity of specimens in group C

4. Conclusion

Detailed prediction procedures for RC deep beams that subjected to different non-central loadings are presented here. In addition to that, the effect of load non-centrality is investigated in this study. It was found that moving load from the span center towards one of the supports leads to a noteworthy decrease in the beam ultimate capacity. It is true that moving the load towards the support makes the near strut shorter with bigger strut-tie angle which makes the load transferring to the near support faster, but at the same time, the other strut which lies near the other support becomes longer with smaller strut-tie angle. This longer strut with smaller strut-tie angle makes stresses transferring to the other support slower and goes through the far bigger portion of the deep beam. Accordingly, the far bigger portion of the beam becomes weaker and thus it will be taken into confederation as a governing ultimate capacity value.

Based on that, in case of one concentrated force, it was observed that the ultimate load capacity decreased by 30.2% when the left shear span to the effective depth ratio (a_L/d) decreased by 50%. While in case of two concentrated forces and uniformly distributed load, it was observed that changing (a_L/d) ratio by 63% led to decrease in the deep beam ultimate capacity by 30.5%.

5. References

[1] ACI Committee, American Concrete Institute. “Building Code Requirements for Structural Concrete (ACI 318-14) and Commentary” (2014). American Concrete Institute. ISBN: 9780870319303

[2] Rogowsky, D. M., MacGregor, J. G., and Ong, S. Y. “Tests of Reinforced Concrete Deep Beams.” In Journal Proceedings 83(4), (1986, July): 614-623.

[3] AASHTO LRFD. “Bridge Design Specification” (2008), 612.

[4] Brown, M. D., and Bayrak, O. “Minimum Transverse Reinforcement for Bottle-Shaped Struts.” ACI Structural Journal, 103(6) (2006): 813.

[5] Abdul-Razzaq, K. S., “Effect of Heating on Simply Supported Reinforced Concrete Deep Beams.” (2013).

[6] Abdul-Razzaq, K. S., Abed, A. H., & Ali, H. I. “Parameters Affecting Load Capacity of Reinforced Self-Compacted Concrete Deep Beams.” International Journal of Engineering, 5(05), (2016). DOI: https://doi.org/10.1051/matecconf/201712001004

[7] Abdul-Razzaq, K. S., & Jebur, S. F. “Experimental Verification of Strut and Tie Method for Reinforced Concrete Deep Beams under Various Types of Loadings.” Journal of Engineering and Sustainable Development, 21(06), (2017).

[8] Abdul-Razzaq, K. S. and Jebur, S. F. “Suggesting Alternatives for Reinforced Concrete Deep Beams by Reinforcing Struts and Ties.” International Conference on Advances in Sustainable Construction Materials & Civil Engineering Systems (ASCMCES-17), 120(13), (09 August, 2017): 01004. DOI: https://doi.org/10.1051/matecconf/201712001004.

[9] Abdul-Razzaq, K. S., Jebur, S. F. & Mohammed, A. H. “Concrete and Steel Strengths Effect on Deep Beams with Reinforced Struts.” International Journal of Applied Engineering Research, 13(1), (2018): 66-73.

[10] Abdul-Razzaq, K. S., Jalil, A. M. “Behavior of Reinforced Concrete Continuous Deep Beams - Literature Review.” The Second Conference of Post Graduate Researches (CPGR'2017) College of Engineering, Al-Nahrain Univ., Baghdad, Iraq - 4th Oct. 2017.

[11] Nielsen, M.P., Hoang, L.C. “Limit Analysis and Concrete Plasticity, Third Edition.” CRC Press, (2010). ISBN 9781439803967 - CAT# K10220.

[12] Martin, B. T., Sanders, D. H., Wassef, W., Cole, T. A. and Bahem, N. “Verification and Implementation of Strut-and-Tie Model in LRFD Bridge Design Specifications.” NCHRP Project, (2007).

[13] Wang, G. L., & Meng, S. P. “Modified Strut-and-Tie Model for Prestressed Concrete Deep Beams.” Engineering Structures, 30(12), (2008): 3489-3496. DOI: https://doi.org/10.1016/j.engstruct.2008.05.020.

[14] Zhang, N., & Tan, K. H. “Direct strut-and-tie model for single span and continuous deep beams.” Engineering Structures, 29(11), (2007):2987-3001. DOI: https://doi.org/10.1016/j.engstruct.2007.02.004.

[15] Nagarajan, P., and Pillai, M. “Analysis and Design of Simply Supported Deep Beams Using Strut and Tie Method.” Advances in Structural Engineering, 11(5), (2008):.419-499. DOI: 10.1260/136943308786412050.

[16] Sam-Young, N., Chang-Yong, L., and Kyeong-Min, L. “Deep Beam Design Using Strut-Tie Model.”

[17] Brown, M. D., & Bayrak, O. “Investigation of Deep Beams with Various Load Configurations.” ACI Structural Journal, 104(5), (2007): 611.