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Student learning in an electric circuit theory course: Critical aspects and task design

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Abstract

Understanding time-dependent responses, such as transients, is important in electric circuit theory and other branches of engineering. However transient response is considered difficult to learn since familiarity with advanced mathematical tools such as Laplace transforms is required. Here, we analyse and describe a novel learning environment design, the problem-solving lab, for learning transient response**.** This design merges problem-solving classes and labs, allowing students to engage in deep learning through the integrated use of tools like paper and pencil, MATLAB®, simulations, and experiments. A key element in this design is the systematic use of variation in line with variation theory. We describe critical features for learning transient response, and ways to facilitate the establishment of links between the 'worlds' of theories/models and objects/events for students. We contend that an integrated use of tools, and systematic use of variation, is crucial for learning and establishing these links.

Keywords: Engineering education research, science education research, variation theory, practical epistemologies, labwork, learning of a complex concept

Introduction

Understanding time-dependent responses, such as transients, is important in many branches of engineering. Hence, such understanding is also important in engineering education where, for example, it is not sufficient for students to learn solely about DC-circuit theory in electric circuit theory classes. However, alternating currents and transient response are considered to be relatively complex topics in electric circuit theory, as the mathematics involved is rather advanced. For instance, mathematical tools, concepts, and re-presentations based on complex functions and various transforms, such as the Laplace transform, are commonly used.

In this paper we describe and analyse a novel learning environment design, the 'problem-solving' lab, for learning transient response in electric circuits. In this design problem-solving classes and labs were replaced by extended 'problem-solving labs'. The development of this novel learning environment is part of a larger systemic framework (Bernhard 2008) of 'design-based-research' or 'design experiment' (e.g. Brown 1992, Design-Based Research Collective 2003) aiming to develop 'conceptual labs' that facilitate insightful learning. An important aim for all of the courses in this framework is for students to learn to establish links between the 'world' of theories and models and the 'world' of objects and events.

According to Lo *et al.* (2004, p. 192) the "benefits of design experiments are that we will be able to contribute to theory development, and improve practice at the same time". Ference Marton has further extended these ideas with a stronger emphasis on teacher involvement in the study and he defines a learning study as "a systematic attempt to achieve an educational objective and learn from that attempt" (Marton 2001 cited in Lo *et al.* 2004, p. 192).

Our previous studies on various response labs focussed on the course structure (Bernhard *et al.* 2007, Carstensen and Bernhard 2007) and the 'complex-concepts' theory, the latter being a method for analysing and re-presenting student discourse (Carstensen and Bernhard 2004). We have also connected our work to the threshold concept theory (Carstensen and Bernhard 2008), and an earlier paper concentrated on the learning and teaching of AC-electricity (Bernhard and Carstensen 2002). Here, we present a fine-grained analysis of the task structure in transient response labs using the 'learning of a complex-concepts' and variation theories as the main analytical tools. We are especially interested in how the new integrated learning environment, and the tools made available within it, enable students to establish links between the 'worlds' of theories/models and objects/events.

Theoretical framework and methodology for analysis and design

When designing the course tasks, we were inspired by previous successes in designing environments for learning mechanics (e.g. Bernhard 2003, 2008, Bernhard *et al.* 2007), where the design principles also were based on *variation theory* (e.g. Marton and Booth 1997, Marton and Pang 2006, Marton and Tsui 2004), a concept based in phenomenography and phenomenology, which states that learning occurs through the experience of difference rather than the recognition of similarity. In other words, variation theory states that the experience of discernment, simultaneity (synchronic and diachronic), and variation are necessary conditions for learning. Hence, critical features of the object of learning can be discerned and brought into the students' focal awareness by varying some features while keeping others invariant. Furthermore, one of the main themes of variation theory is that the pattern of variation inherent in the learning situation is fundamental to the development of certain capabilities. This theory can be used both as a theoretical design guide for potential learning environments and as a tool for analysing the task structure of an existing learning environment (See also Fraser and Linder 2009). A detailed description and analysis of a task structure is provided below.

Three concepts important to learning environments are the *intended object of learning*, the *enacted object of learning*, and the *lived object of learning*. The intended object of learning is the subject matter, along with the skills and values that the teacher or curriculum planner expects the students to learn. The enacted object of learning is the space of learning comprised within a learning environment, i.e. what is actually made *possible* for the student to learn. The lived object of learning is the way students see, understand, and make sense of the object of learning, along with the relevant capabilities the students develop as a result.

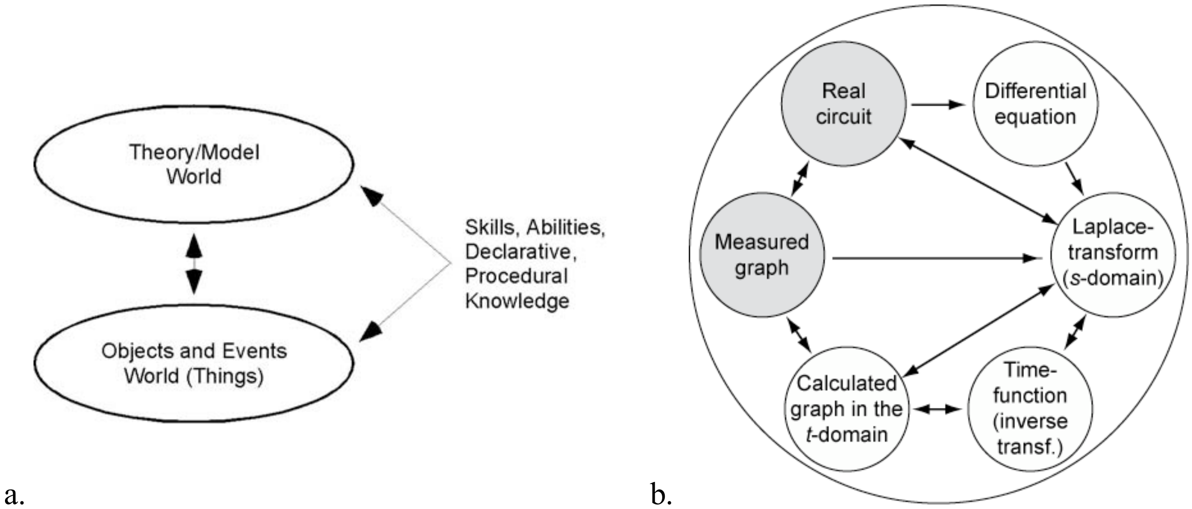

Figure 1. a) Categorisation of knowledge based on a modelling activity (Vince and Tiberghien 2002). b) The categorisation in Figure 1a has been developed further into a model of 'learning of a complex concept', in this case interpreted and applied to the example of an electric circuit theory course lab about transient response. The shaded circles represent knowledge located in the world of objects/events and the other circles represent knowledge in the world of theories/models.

An important object of learning in engineering education is that students should understand and learn to *use* theories and models and link them to objects and events. Indeed, Tiberghien (1999) proposed that the 'worlds' of theories/models and objects/events should be seen as the main analytic categories in the analysis of knowledge (see Figure 1a), and not the traditional dualistic categories - theoretical and practical knowledge. She also states that the most difficult links are the ones between the two 'worlds', although that is the main goal for labwork. The ability to make links between mathematical models and measurement data, or graphs stemming from mathematical calculations and/or derived measurement data, is also often seen as the fundamental purpose of lab work (e.g. Psillos and Niedderer 2002). However, recent research has suggested that students or novices have problems establishing such relationships, and thus explicitly linking the theory/model and object/event 'worlds' is important in education (ibid.).

Variation theory is a theory on conditions for learning. However variation theory does not provide tools that can be used to identify critical aspects. In order to identify critical aspects, we have previously developed a model for 'learning of a complex concept' (Carstensen and Bernhard 2004, 2008, Carstensen *et al.* 2005) that extends Tiberghien's (1999) model of object/event and theory/model 'worlds' (Figure 1b). In this model "single concepts" are illustrated as nodes or "islands" that may be connected by links, while the links students actually make (identified by analysing the lived object of learning), or are supposed to establish (identified by analysing of the intended object of learning), are represented by arrows. The arrows in Figure 1b show all possible links to make, and their directions. The nodes and links in our model are found by looking for “gaps” in the actions and conversations of students. A gap corresponds to a non-established link, and thus a critical aspect. When a gap is filled and the students establish a relation between two nodes this is re-presented by a link (arrow). This methodology is a further development of Wickman’s (2004) practical epistemologies, which was based on work by Wittgenstein (1953/2003)..

To study the in situ development of the students’ lived object of learning, we videotaped some lab-groups (each comprising 2-3 students) using a digital camcorder, obtaining 56 h of video from a couple of transient response lab sessions. The method allows studying how the students’ lived object of learning develops during the lab (“process study”), as well as the study of students’ lived object of learning at the end of the lab (“learning outcome”). Students’ conversations and other actions have been transcribed and analysed using our model, which enables analysis of longer sequences of video-recordings.

The idea behind our model was that knowledge is built by learning the component pieces, the islands, and by learning the whole object of learning through making explicit links. Hence, the more links that are made, the more complete the knowledge becomes (cf. for example Roth 1995, pp. 56-58). It is important to understand that the model of ‘learning of a complex concept’ is not merely describing a clustering of concepts or something complicated, it is describing *a* complex (i.e. a *whole* made up of interrelated *parts*).

Object of learning: Some aspects of transient response

As mentioned above, learning is always related to some intended object of learning. Hence, to help understand the object of learning and the relevant task structure in the transient response lab, in this section we highlight some of the theory related to transient response in electric circuits. A more complete description, including the limits and conditions for the presented models and theories, can be found in standard textbooks, such as the book used in the course (Nilsson and Riedel 2001).

It is often convenient to regard an electric circuit as a system (See Figure 2a) that transforms one or more input (source) signals $x(t)$ into one or more output (response) signals $y(t)$. In electric circuits these signals may be voltages as well as currents.

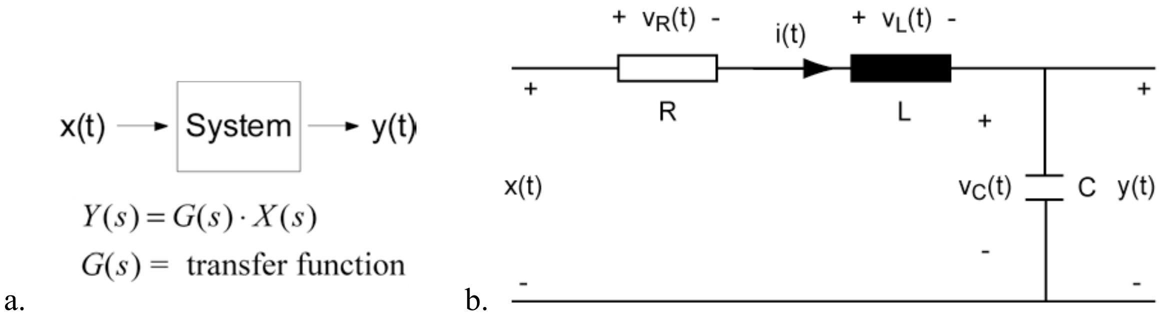

Figure 2. a) A general model for a system and b) the system, an *RLC*-circuit, studied in the transient response lab.

The output $y(t)$ is dependent on both the specific input signal $x(t)$ and the system's characteristics, and this dependency can be quite difficult to work through in the time domain since differential integral equations may be involved. However, if Laplace transforms of the input $x(t)$ and output $y(t)$ are used, $X(s)$ and $Y(s)$ respectively, this dependency can be expressed as $Y(s) = G(s)\cdot X(s)$, where $G(s)$ is known as the transfer function. Notably, the transfer function $G(s)$ only depends on the system.

In most cases the transfer function can be written as a ratio of two polynomials:

$$G(s) = \frac{B(s)}{A(s)} = \frac{b_m s^m + b_{m-1}s^{m-1} + \cdots + b_1 s + b_0}{a_n s^n + b_{n-1}s^{n-1} + \cdots + a_1 s + a_0} = K\frac{(s-z_1)(s-z_2)\cdots(s-z_m)}{(s-p_1)(s-p_2)\cdots(s-p_n)}$$

Here, z_i are zeros and p_i are the poles of the system, which are important in determining the response characteristics. They are either real or exist in complex conjugate pairs, since a_i and b_i are real for all physical systems. Complex conjugate poles, $\sigma \pm j\omega_d$, will give cause to contributions (in the time-domain) in the form $k\cdot e^{\sigma t}\sin(\omega_d t+\varphi)$ and distinct real poles, p, in the form $k\cdot e^{pt}$.

The general form of the transfer function for a second order system is:

$$G(s) = \frac{b_2 s^2 + b_1 s + b_0}{a_2 s^2 + a_1 s + a_0}$$

In circuit theory, control theory, and physics, $G(s)$ is also commonly expressed in a second order system using one of the special forms: $G_1(s) = B(s)/(s^2 + 2\alpha s + \omega_n^2)$ or $G_2(s) = B(s)/(s^2 + 2\zeta\omega_n s + \omega_n^2)$. However, the parameters in these (or similar) forms of modelling are only applicable to a second-order system. In addition, the damping ratio ζ in the second type of expression is not an independent parameter, but coupled to the natural frequency of the undamped system ω_n, since $\zeta = a_1/(2\omega_n)$. Conversely, poles and zeros can be used to determine the responses and stability of systems of any order, from first to higher order systems[1], and are therefore more generally applicable. Hence, neither $G_1(s)$ nor $G_2(s)$ were part of our object of learning.

The above theory is general and applicable to many types of systems, not only those used in electrical engineering. For instance, both a simple *RLC*-circuit and a spring with a viscous damping are examples of second order systems (n=2). The circuit used in the studied lab is shown in Figure 2b, and the relationship between the input voltage $v_{in}(t)$ and the current $i(t)$ through the circuit can be written as:

$$x(t) = v_{in}(t) = v_R(t) + v_L(t) + v_C(t) = R \cdot i(t) + L\frac{d}{dt}i(t) + \frac{1}{C}\int_{-\infty}^{t} i(t)dt$$

One of the tasks in the lab was to experimentally and mathematically determine $i(t)$ and the voltage across the capacitor $v_C(t)$, when $v_{in}(t)$ was a voltage step. However, many students find it difficult to obtain the solutions by solving the differential equations, and in some cases it is not even possible without Laplace transforms. But, when using standard procedures for Laplace transforms, the differential equation can be written as:

$$X(s) = V_{in}(s) = V_R(s) + V_L(s) + V_C(s) = R \cdot I(s) + sL \cdot I(s) + \frac{1}{sC} \cdot I(s) =$$
$$= \left(R + sL + \frac{1}{sC} \right) \cdot I(s) = Z(s) \cdot I(s)$$

Thus the differential equation is transformed into an algebraic expression in terms of the complex frequency s. The solution (in terms of s) can then be transformed to the time-domain by using the inverse Laplace transform.

If $v_{in}(t)$ is a voltage step, the Laplace transform of the input signal will be $V_{in}(s) = E/s$**.**

In our lab sessions two responses were studied: the current $i(t)$ through the circuit and the voltage $v_C(t)$ across the capacitor. The corresponding transfer functions and step responses in the s-domain are shown in Table 1 below:

[1] The order of a system is equivalent to the order of the polynomial function in the denominator of the transfer function, i.e. the highest order of derivatives in the differential equation. For a circuit this corresponds to the number of *independent* capacitances and inductances.

Current	Capacitor voltage
Transfer function	
$G_I(s)=\frac{I(s)}{V_{in}(s)}=\frac{1}{L}\frac{s}{s^2+\frac{R}{L}s+\frac{1}{LC}}$	$G_{V_C}(s)=\frac{V_C(s)}{V_{in}(s)}=\frac{1}{LC}\frac{1}{s^2+\frac{R}{L}s+\frac{1}{LC}}$
Derivation of G(s) and step responses in the s-domain	
$I(s)=V_{in}(s)\frac{1}{Z(s)}=\frac{E}{s}\frac{1}{sL+R+\frac{1}{sC}}=$ $=\frac{E}{s}\frac{\frac{1}{L}s}{s^2+\frac{R}{L}s+\frac{1}{LC}}=\frac{E}{L}\frac{1}{s^2+\frac{R}{L}s+\frac{1}{LC}}$	$V_C(s)=V_{in}(s)\frac{Z_C(s)}{Z_L(s)+Z_R(s)+Z_C(s)}=$ $=\frac{E}{s}\frac{\frac{1}{sC}}{sL+R+\frac{1}{sC}}=\frac{E}{LC}\frac{1}{s}\frac{1}{s^2+\frac{R}{L}s+\frac{1}{LC}}$
Step response in the time domain	
For examples of i(t) see Figure 4a	For examples of $v_C(t)$ see Figure 4b

Table 1. Step responses and transfer functions for the current and the capacitor voltage.

In terms of variation, we can already see that two different types of transfer function are obtained: one with a first order (G_I) and one with a zero order (G_{V_C}) numerator. In addition, the roots of the denominator $s^2 + sR/L + 1/(LC)$ will be qualitatively different depending on the values of R, L, and C.

Task structure

For students to learn a specific object of learning they must become focally aware of its critical features, i.e. students have to discern these features. Such features, along with the pattern of variation in the task structure, constitute the enacted object of learning, i.e. what is possible for the students to learn. Therefore, we provide a quite detailed analysis of the task structure in terms of what is varied and what remains invariant in the different tasks. A brief summary of the task structure in the new and old versions of the lab is presented in Table 2. The first four tasks (1a, 1b, 2 and 3) in the new structure were not part of the original design but, as discussed below, inclusion of these new tasks has proved to be essential for students' learning.

Brief description of task		New course	Old course
Scheduled time	Lecture Class (problem-solving) Problem-solving lab/Lab	4 h - 2×4 h	4 h 6 h 4 h
1a.	Simulate the step response for six systematically varied transfer functions	•	-
1b.	Obtain "by hand" the mathematical function in the time-domain for the six-step responses in task 1a.	•	-
2.	Obtain the expression for the transfer function of an *RLC*-circuit with R= 100 Ω, L= 100 mH and C= 100 μF.	•	-
3.	Calculate the step response $y(t)$ for some values of R, L and C that correspond to the values of the real circuit (used in coming tasks).	•	-
4a.	Measure the step responses $i(t)$ and $V_C(t)$ for a real *RLC*-circuit. R is varied while L and C are kept constant.	•	•
4b.	Measure the step responses $i(t)$ and $V_C(t)$ for the *RLC*-circuit in task 4a. L is varied while R and C are kept constant.	•	•
4c.	Measure the step responses $i(t)$ and $V_C(t)$ for the *RLC*-circuit in task 4a. C is varied while R and L are kept constant.	•	•
5a.	Fit a mathematical function to the four different experimental curves for $i(t)$ obtained in task 4a.	•	•
5b.	Use the fits obtained in task 5a to calculate the values of R, L and C.	•	•

Table 2. An overview of the task structure and organisation in the transient response lab according to the new and old designs.

Task 1a-b: Simulate and calculate the step response for six systematically varied transfer functions

In this first task (or, more strictly, set of several related tasks), students studied six systematically varied transfer-functions:

$$G_a(s) = \frac{3s+5}{s^2+2s+5} \quad G_b(s) = \frac{3s+5}{s^2+2s+1} \quad G_c(s) = \frac{3s+5}{s^2+2s+0.75}$$
$$G_d(s) = \frac{5}{s^2+2s+5} \quad G_e(s) = \frac{5}{s^2+2s+1} \quad G_f(s) = \frac{5}{s^2+2s+0.75}$$

Two separate dimensions of variation were used in this task. One was variation in the s^0-term a_0 of the denominator polynomial, while a_1, a_2, and the numerator remained invariant, and the other was in the s^1-term b_1 of the numerator polynomial, while b_0 and the denominator remained invariant. Varying a_0 in the denominator of $G(s)$ results in different types of poles, as shown in Table 3.

a_2	a_1	a_0	Roots of $a_2s^2+a_1s+a_0$ (poles)	
1	2	5	$-1+\sqrt{1-5}=-1+2j$	$-1-\sqrt{1-5}=-1-2j$
1	2	1	-1	-1
1	2	0.75	$-1+\sqrt{1-0.75}=-0.5$	$-1-\sqrt{1-0.75}=-1.5$

Table 3. Roots of the different denominator polynomials of $G_a(s)$ – $G_f(s)$.

Initially, the students were asked to calculate the step response function in the time domain for the transfer function G_a “by hand”, thereby obtaining the inverse transform of $1/s \cdot G(s)$. They were also instructed to use MATLAB® and Simulink® to simulate the step response of G_a. The students were then instructed to do the same for the transfer functions $G_b(s)$-$G_f(s)$, before comparing the resultant time-domain step responses in an attempt to relate the observed changes in the graphs to changes in the coefficients. In particular, the students were asked to notice the final values of the obtained curves and their initial behaviour, while trying to relate them to the transfer function's parameters.

The step responses for the six different $G(s)$ are compiled together in graphical form in Figure 3, where several important characteristics can be observed:

1. The different types of solutions (complex conjugate, double or two distinct real roots) of the denominator polynomial (the poles of the transfer function), result in three qualitatively different ways of approaching the steady-state,

2. The steady-state value of the step response can be seen to depend on the transfer function's limit-value when s approaches zero, i.e. the ratio b_0/a_0.

3. It is apparent that the initial behaviour of the response function depends on the numerator polynomial, and not on the variation of a_0 in the denominator polynomial.

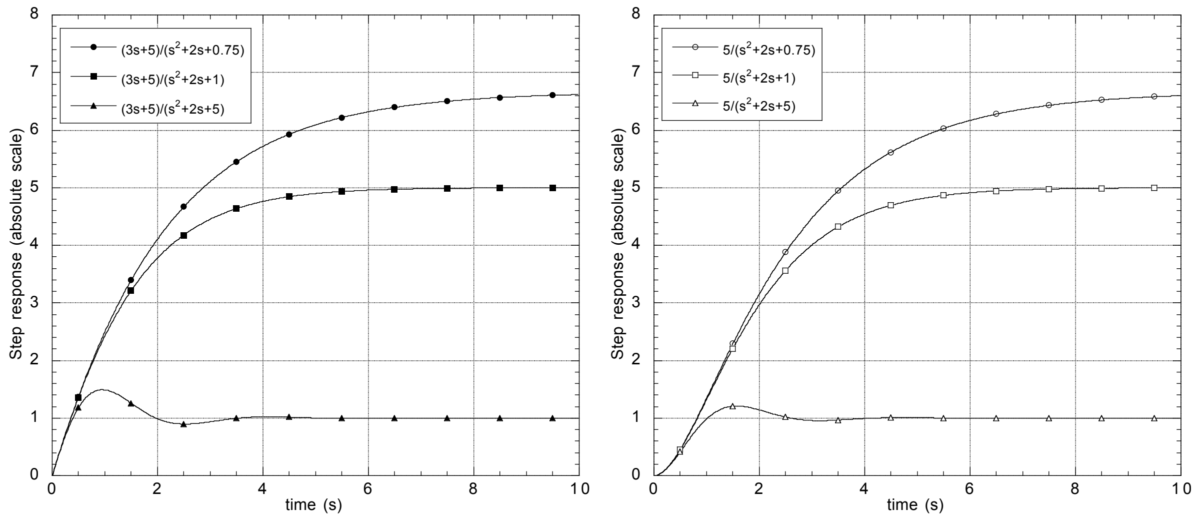

Figure 3. Step responses for transfer functions with different denominators, a) with $3s+5$ in the numerator [$G_a(s)$–$G_c(s)$], and b) with 5 in the numerator [$G_d(s)$–$G_f(s)$].

Instructing the students to obtain the response through simulations and "hand" calculations 'opened-up' for awareness of the connections between the mathematical parameters of the transfer function and the resultant step response. Furthermore, the values of the numerical coefficients a_0, a_1, and a_2 were chosen to allow simple "hand" calculations, ensuring that the physical meaning of the obtained parameters and functions, and not the mathematical manipulation, were in the students' focal awareness.

Tasks 2-3

The students' next task was to derive $G(s)=Y(s)/X(s)$ for the circuit in Figure 2b, where R= 100 Ω, L= 100 mH, and C= 100 μF. Students were then asked, based on physical as well as mathematical reasoning, how (and why) $y(t)$ would be affected if C was changed from 100 μF to 10 μF. This assignment was built upon in task 3, in which students were asked to calculate the response $y(t)$ for different values of R, L, and C used in the lab when $x(t)$ was a step, i.e. $x(t) = 0$ V, $t < 0$; 1 V, $t \geq 0$. Students were then asked to identify *the* coefficients in the time function in relation to R, L, and C, with a hint that a comparison with task 1 would be helpful. In essence, these tasks involved the derivation of $G_{V_c}(s)$ and $V_C(s)$, and the calculation of $v_C(t)$ through the inverse transformation of $V_C(s)$.

Task 4

This was the first experimental task in the new lab design and the first actual task in the old version of the lab. The current $i(t)$ through and the voltage $v_C(t)$ over the capacitor in an *RLC*-circuit was measured by sensors connected to a computer-based system, which collected, processed and visually presented the experimental data. The voltage step $v_{in}(t)$ was generated by using a low-frequency positive square wave. In the first experimental task, R_{res} was varied while L and C were kept invariant.

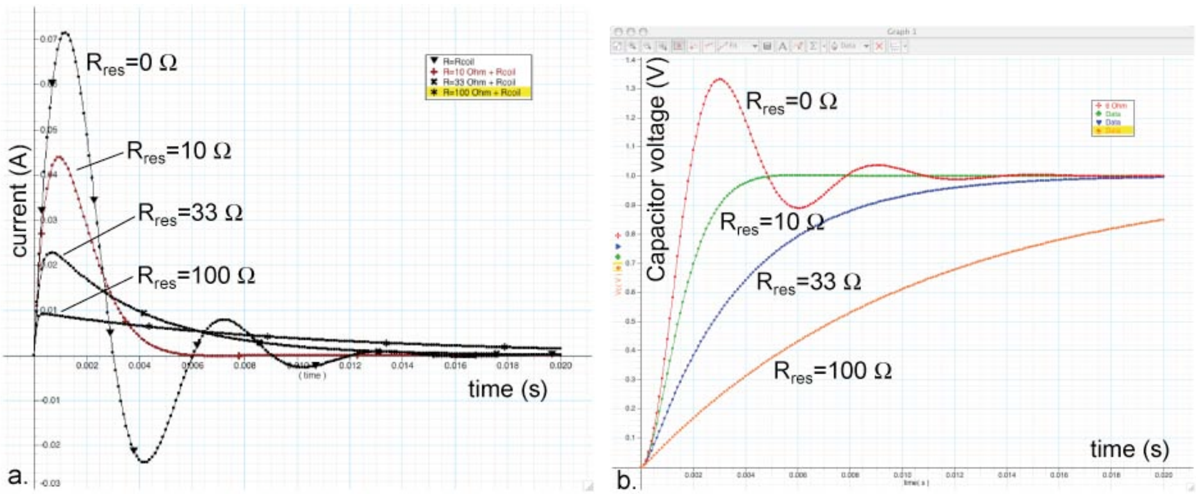

Figure 4. Experimental curves for the current (a) and the capacitor voltage (b) for different values of R_{res} (L=8.2 mH and C=100 μF).

The qualitatively different ways steady-state was approached are shown in Figure 4, which in this case depended on the value of the resistance R (with L and C constant). The different responses for $i(t)$ and $v_C(t)$, due to differences in the order of the numerator polynomial in their respective transforms (see Table 1), are also shown in Figure 4. The values of R_{res}, L, and C used in the task are presented in Table 4, together with the resultant mathematical expression of the current $i(t)$ (obtaining the latter was part of task 5). In addition, the total resistance was higher than the nominal resistor value R_{res} since the coil resistance R_{coil} was ≈ 6 Ω.

R_{res} (Ω)	R_{tot} (Ω)	L (mH)	C (μF)	Roots of $s^2 + \frac{R_{tot}}{L}s + \frac{1}{LC}$		$i(t)$ (A)
0	6	8.2	100	–366+1042j	–366–1042j	$0.1170e^{-366t}\sin(1042t)$
10	16	8.2	100	–976+517j	–976–517j	$0.2357e^{-976t}\sin(517t)$
33	39	8.2	100	–272	–4484	$0.0290(e^{-272t}-e^{-4484t})$
100	106	8.2	100	–95	–12832	$0.0096(e^{-95t}-e^{-12832t})$

Table 4. Variations in terms of R_{res}, with L, C, and E constant. Note that the frequency, ω_d, of the damped system changes with R and is not equal to ω_n.

The second sub-task (task 4b) involved obtaining experimental curves when L was varied (while R and C were kept constant), and the third sub-task (task 4c) involved varying C (while R and L were kept constant). The values for the variations used in task 4c, along with the results in mathematical form, are presented in Table 5.

R_{res} (Ω)	R_{tot} (Ω)	L (mH)	C (μF)	Roots of $s^2 + \frac{R_{tot}}{L}s + \frac{1}{LC}$		$i(t)$ (A)
0	6	8.2	100	–366+1042j	–366–1042j	$0.1170e^{-366t}\sin(1042t)$
0	6	8.2	330	–366+485j	–366–485j	$0.2514e^{-366t}\sin(485t)$
10	16	8.2	100	–976+517j	–976–517j	$0.2357e^{-976t}\sin(517t)$
10	16	8.2	330	213	1739	$0.0799(e^{-213t}-e^{-1739t})$

Table 5. Variations in terms of C, where R_{res}= 0 and 10 Ω, while L and E are constant.

Task 5

In the final task students were asked to fit polynomial functions to each of the four measured curves for $i(t)$, using tools for manual user-defined fitting incorporated in provided software. As shown in Figure 4a, there are two qualitatively different responses, type $i(t) = ae^{bt}\sin(ct+d)$ and $i(t) = ae^{bt} + ce^{dt}$, and only the former type was provided in the lab-instructions. The software also allowed the students to display both the measured and calculated graphs in the same diagram. Hence, the students had to decide the type of function to fit to as well as appropriate values for the constants. They were also required to calculate the corresponding R, L, and C values from their fitted curves. To do this, a fit to measured data in the form,

$$i(t) = ae^{-bt}\sin\omega t \;\Rightarrow\; I(s) = a\frac{\omega}{(s+b)^2+\omega^2} \text{ or } i(t) = ae^{-bt} + ce^{-dt} \;\Rightarrow\; I(s) = \frac{a}{s-b} + \frac{c}{s-d}$$

could be compared to the form for $I(s)$ (see Table 1) for calculating R, L, and C. Alternatively, the nominal values for R, L, and C could be used to calculate a first approximation of $i(t)$.

In addition, the nominal and fitted values of R disagreed due to the resistance of the coil, while the nominal and fitted values of L and C were usually in good agreement. The qualitatively different functions obtained for $i(t)$ can be found in Table 4.

Results and discussion

Our study has many similarities with a 'learning study' (Lo *et al.* 2004). Therefore the design of the tasks and analysis of the task structure in terms of variation theory are the most important aspects of the study (For a more detailed analysis of the learning process and the learning outcomes see Carstensen and Bernhard (2004) and Carstensen (forthcoming)). For each task we systematically varied one of the primary parameters that are open to variation and kept other primary parameters invariant. However, it is also important to note that there was variation between the tasks. For example, in the first task students encountered transfer functions in the forms $G(s) = (b_1 s + b_0)/(a_2 s^2 + a_1 s + a_0)$ and $G(s) = b_0/(a_2 s^2 + a_1 s + a_0)$. However, results corresponding to $G(s) = b_1 s/(a_2 s^2 + a_1 s + a_0)$ were encountered during the experimental measurement of $i(t)$ in task four, and during the fitting of an appropriate mathematical expression for $i(t)$ to experimental data in task five. On the other hand, the experimental $v_C(t)$ corresponded to a transfer function in the form $G(s) = b_0/(a_2 s^2 + a_1 s + a_0)$.

In addition, in the first task a_0 was varied, while the variation of R in the later tasks corresponded to a variation of a_1. Thus, the students experienced qualitatively different ways of approaching steady-states (under-damped, critically damped and over-damped), as well as qualitatively different initial responses and steady-states.

Although only a single *primary parameter* was varied at a time, it is extremely important, especially for engineers, to understand that changing one parameter in a system can cause several changes in the system's *response*. Hence, education must challenge and counter the commonly reported tendency for "local reasoning" (e.g. Duit and von Rhöneck 1997). This is one of the intentions behind the design of task 1 (see Figures 3a and 3b), where the students experience changes in the s^0-term a_0 in the denominator, which leads to changes in the steady-state magnitude and to the system reaching a steady-state in qualitatively different ways. Hence, variation theory should not be misconceived as requiring changes in one parameter that only effect one change in the response, since that is physically impossible in most real world cases. For instance, a change in the resistance R will change all the parameters a, b, c in both the under-damped response $a \cdot e^{bt}\sin(ct)$ and the over-damped response $a(e^{bt}-e^{ct})$, as shown in Table 4 and Figure 4.

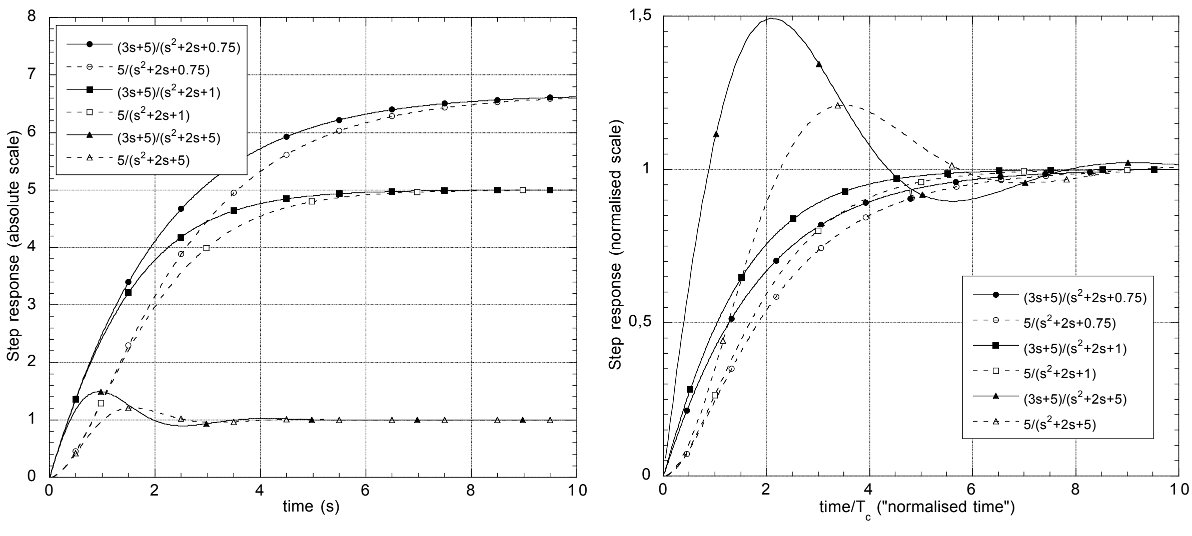

Figure 5. a) Step responses for the different transfer functions $G_a(s)$–$G_b(s)$ and b) those plotted on a 'normalised' scale (T_c=1/ω_n).

Different re-presentations, along with other symbolic tools, bring different aspects of a situation to the foreground (while sending others to the background). Therefore, the choice of re-presentations leads to different enacted objects of learning. For instance, it has been suggested that "[t]he only way a student would be able to compare the outputs of [$G_a(s)$ – $G_f(s)$] effectively would be to plot them on a normalized basis" (Fraser and Linder 2007). However, comparing the normalised plot in Figure 5b with the curves in Figures 3 and 5a it is clear that many of the critical *global* features of the response cannot be discerned from a normalised plot. Further, students have often only been previously exposed to responses with a magnitude of 1 or normalised plots, which could result in unintended learning outcomes. For example, some students in our study believed that something was wrong when the final values, as seen in Figures 3 and 5a, were not 1, prompting them to enquire about the perceived discrepancy. However, in most practical cases (e.g. in amplifier circuits) the magnitude is likely to be different from 1, and it is important for students to be aware of this. In the studied transient response lab an important part of the intended object of learning was to reduce the tendency for "local reasoning" by highlighting the global features of responses, which are not clear in normalised plots. Hence, we strongly disagree with the claim that a normalised plot is "the only way" to compare the outputs effectively, and instead we contend that the "effectiveness" of a learning mechanism depends on the intended object of learning.

In both the old and new lab designs (see Table 2 for a summary of similarities and differences in task structure) the intended object of learning was for students to establish all the links displayed in Figure 1b. However, when we used our method for analysing the learning of complex concepts to examine the students' actions in the old lab design we noticed several 'lingering gaps' (Wickman 2004). No students from any group noticed that they should use two qualitatively different functions when trying to fit polynomial curves to the experimental curves for $i(t)$. Instead, they tried to fit a damped sinusoidal function, even to the curves where R_{res}=33 and 100 Ω. An example of this can be seen in the following excerpt, which took place a few minutes after the students had begun the measurements and fitting for R_{res} = 33 Ω:

```
2002_Group_1 _Tape_3 5:19
1   Anne       This is the hard thing, one doesn't understand anything.
2   Betty      No, exactly.
3   Anne       Do you find this to be a damped sine?
4              Both start to laugh
5   Betty      Yes!
6   Anne       No, it can't be.
```

They then compared the new curve with the previous ones, and noticed the "sharp peak" at the top of the curve. However, they still tried to fit a damped sine to the experimental graph and hence could not obtain a good fit. After about a quarter of an hour they asked the teacher about their problems:

```
2002_Group_1 _Tape_3 17:19
7   Betty       It looks so strange!
8   Anne        Yes, it does.
9   Teacher1    Which one are you doing?
10  Anne        33 ohm.
11  Teacher1    Is it obvious that it is a damped sine?
12  Anne        No, but we didn't have any other guess.
13  Teacher1    What alternatives are there?
14  Anne        We don't know.
```

There was a lot of surface discussion in all of the groups regarding the appearance of the curve and what it should look like for the report, rather than trying to understand what the curve meant or relating it to theories presented in the lectures and/or things supposedly learnt in earlier experiments. Although the relevant topics had been ‘covered’ in the lectures and problem-solving classes, the students were still not aware of the possible alternatives, as seen in turn 14 above. Despite our intentions, our analysis showed that the students found making the intended links between the theory/model and object/event ‘worlds’ very difficult. In other words, the students’ lived object of learning after instruction did not correspond to the intended object of learning. Hence, not all of the critical aspects were brought into students’ focal awareness in the initial lab design.

Three new tasks (see Table 2) were introduced prior to the old, tasks in the new lab design, while the lab duration was also extended from 4 h to 2×4 h. However, since the extended time for the lab was used to replace classroom-based problem solving sessions, the amount of time that students were “taught” Laplace transforms as applied to circuits was not extended (See Carstensen and Bernhard 2007, for more details).

Although tasks 4-5 were identical in the old and new lab designs, the students’ courses of actions, and thus the lived object of learning, were quite different. The students initially had problems making links between theory/model and object/event ‘worlds’ in both the old and new lab designs. Indeed, in turns 15-19 from the new lab design, Tess and Benny also struggle with what type of curve should fit the curve for $R_{res} = 33\ \Omega$:

```
2003_Group_1 _Tape_1_Session2
15  Tess        Well, I don't want to interrupt, but I don't think that's the
                right formula.
16  Benny       You never know.
                (continues writing on the keyboard to see what happens, about
                one minute later Tess tries to interrupt again)
17  Tess        See, this here, this is for the damped sine-wave (points at
                the instructions). It looks like this
18  Benny       Yea.
19  Tess        And that's not our curve!
```

However, after Tess points out that it couldn't be a damped sine-wave in turn 19, Benny's and Tess' course of action is different from that of the students in the first version of the lab. After noticing the 'gap' in their knowledge they turn to different resources, such as the textbook and lecture notes, to fill the 'gap' and after a few minutes Tess suggests that an exponential function should be involved:

```
20  Tess        And I think this is for the capacitor. Yes this is the one
                for a capacitor (.) But try something with an exponential
                function.
```

Benny's and Tess' next actions were to use different resources to fill the gaps. By the end of the lab session this had resulted in them not only making all of the fits, but they also established all of the links in Figure 1b. Indeed, in the new lab design all of the studied groups worked differently to those in the old design, and the previously observed trial-and-error behaviour disappeared. Discussions within and between the groups were focused on relevant subject matter, rather than topics such as, "Is this curve good enough for the report?" Instead of focusing on what to report, the students focussed on what should be learnt, i.e. they made links between all of the components in our model shown in Figure 1b.

Conclusions and implications

Our results indicate that the theory of variation can be useful when designing new learning environments and improving existing ones. Variation theory states that the object of learning's critical features should be brought into the students' focal awareness, and that such critical features can be discerned through the use of systematic variation. However, the theory does not establish which features are critical and which features should be varied, and this is a question for empirical research that must be sensitive to the actual object of learning. The methodology for analysing the 'learning of a complex concept' is one possible methodology for finding critical features. In this study we have shown that variation in itself is not sufficient, and through a fine-grained analysis we found variations, and the appropriate use of tools, that *open up for* learning of our intended object of learning. It was found to be critical to introduce tasks 1-3 in order to open up for learning of the whole complex concept (Shown in Figure 1b).

Several tools were made available to students in the integrated environment of the new lab design, including paper and pencils, mathematical tools such as MATLAB®, toolboxes such as Simulink®, circuit simulation software such as PSpice, and tools for computer-based measurements on real circuits. Furthermore, the new labs were designed so that students were prompted to use several 'tools' to understand and handle the subject matter. Although many of these tools were available to the students in the old lab design, they were seldom used in an integrated fashion. In addition, students need repeated practice in interpreting different aspects of circuit theory formalism and relating it to the real world, and vice versa. This must be explicitly addressed in instruction, since such links between theories/models and objects/events are difficult for students to establish by themselves (cf. e.g. McDermott 1997). We contend that the integrated use of tools in the problem-solving labs, as well as the systematic use of variation, is crucial when students are establishing the links between the 'world' of theories/models and the 'world' of objects/events, and when developing a holistic integration of different tools.

Problem-solving labs are powerful learning environments, and we propose that the principles behind our design could be applied to the development of learning environments for other objects of learning in engineering and science.

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