D.W. And you think your films, or others like them that might be produced, would be of value to students who are not going to be geometers, who are not going to be mathematicians or scientists?

C G Yes, sure. The fact that these things are beautiful is indicating that there is another dimension to geometry than proof and definition and axiomatics What there is as well is a realm of sensitivity You are giving general education to people when they sense that ballet is a universe where geometry has a place, and where they see geometry where ballet has a place. Well, you can see that these entities which are capable of being generated one from the other, relating one to another, are giving you nourishment for your imagination, and you can spill out into any field because you have been prepared as a proper geometer of this kind. Whereas if you have been put into a straitjacket and only used as a machine, then a machine can do it, you don't have to do it.[4]

Notes

[1] The scenario for "The Foundations of Geometry" is based on unpublished notes written by Gustave Choquet and incorporated in a curriculum proposal submitted by the Academy of Sciences to the Ministry of Education in France.

[2] "Animated Geometry" are 1-3 minute, silent, black-and-white animated films devised and produced in the 1950's by J.L. Nicolet A number of these films have been revised, computer-animated, and reissued in colour by Educational Solutions Inc , 80 Fifth Avenue, New York, NY 10011, U S A

[3] The label "Thales theorem" is attached to different geometrical results in different countries. In France it is applied to: "A line parallel to a side of a triangle divides the other two sides in the same proportion".

[4] The film "Foundation of Geometry" is silent, in colour, and has a duration of 16¾ minutes. The computer animation was carried out by Serge Chicoine and André Fourrier of the University of Montreal Enquiries about the availability of the film should be addressed to Educational Solutions Inc.

Students' Errors in the Mathematical Learning Process: a Survey*

HENDRIK RADATZ

Student's errors in mathematics education are not simply a result of ignorance and situational accidents. Most student mistakes are not due to unsureness, carelessness, or unique situational conditions, as was assumed at the beginnings of the behaviouristic theory of education. Rather, student errors are the result or the product of previous experience in the mathematics classroom According to the present state of error research, student errors

- are causally determined, and very often systematic;
- are persistent and will last for several school years, unless the teacher intervenes pedagogically;
- can be analyzed and described as error techniques;
- can be derived, as to their causes, from certain difficulties experienced by students while receiving and processing information in the mathematical learning process, or from effects of the interaction of variables acting on mathematics education (teacher, curriculum, student, academic environment, etc.).

Student errors "illustrate" individual difficulties; they show that the student has failed to understand or grasp certain concepts, techniques, problems, etc., in a "scientific" or "adult" manner. Analyzing student errors may reveal the faulty problem-solving process and provide information on the understanding of and the attitudes toward mathematical problems.

From this perspective, error analysis gains importance in two respects First, with regard to the requirements of academic practice, as an opportunity to diagnose learning difficulties, as a method of developing criteria for differentiating mathematical education, and as a means to create more awareness and support for the performance and understanding of individual students. Second, error analysis seems to be a remarkable starting point for research on the mathematical teaching-learning process. Error analysis must be considered a promising research strategy for clarifying some fundamental questions of mathematics learning.

The following is an attempt to give a brief historical survey. Of course this enumeration will not be exhaustive, nor will evaluation be free of a certain subjectivity (See Table on next page)

Error analysis in mathematical education has a long history, and is characterized by very different approaches and interests, which were shaped both by the contemporary mainstream of the sciences concerned, pedagogy and psychology, and by the objectives and forms of organization determined by the respective educational policies

Three focal points and tendencies can be discerned:

1 Arithmetic constitutes the dominating subject matter area for the majority of the studies It is only after the recent incisive reform concerning the content of mathematical curricula that research interest has extended to other topics of mathematical education.

2. Error theory was developed in Germany sporadically, with great intervals in between, and with little continuity, as opposed to its development in the U.S., for instance

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3 There is a pluralism of theoretical approaches to and explanatory attempts about the causes of student errors in the mathematical learning process, while error techniques in certain subject matter areas (e.g. written calculation methods within the set **N**) can be very succinctly described.

It is remarkable that studies seldom extend beyond the national horizon. None of the American publications quotes a German study, and the converse is almost equally true.

The great German interest in error analysis during the twenties must be understood, on the one hand, as due to the increasing importance of empirical-scientific pedagogy, which made use of the insights of experimental psychology, and which mostly ousted philosophical pedagogy from its former predominant role after World War I. On the other hand, a reform program began to take effect at that time which advocated fostering individuality, adapting all educa-

Table: Overview of accessible publications on errors in mathematical education

	GERMAN PUBLICATIONS	**ANGLO-AMERICAN PUBLICATIONS**
	Ranschburg 1904, 1916 Hylla 1916	Phelps 1913 Gist 1917, Uhl 1917, Counts 1917 Thorndike 1917
1920	Weimer 22, 23 Kieβling 25, Korn 26 Weimer 25, 26; Rose 28; Seemann 29; Kieβling 28, 29; Weimer 29; Fettweis 29	Osburn 24, Myers 24 Buswell 25, 26 Brueckner 30
1930	Seemann 31; Weimer 31; Kieβling 32 Kieβling 34 Weimer 39	Burge 32 Brueckner 35 Schonell 37, Williams 37 Grossnickel 39
1940	Seemann 49	Holland 42
1950	Schaffrath 59 Monsheimer 60; Kieβling 60	Brueckner 55 Schonell 57
1960	Rose 61 Schaffrath 64 Krueger 65 Schlaak 68	Harvey 65 Schacht 67 Smith 68; Roberts 68
1970	Glück 71 Baumgarten 72; Jäkel 72 Pippig 75 Pippig 77; Radatz 77, Andelfinger 77; Birkhan 78; Wiegard 78 Radatz 79	West 71; Carman 71; Ashlock 72; Reisman 72; Lankford 72; Ellis 72; Erlwanger 73, Baxter 73; Cox 73; Kane 74, Cox 75; Dodd 75; Engelhardt 75; Crawford 75; Hucherson 75; Blankenship 76; Ashlock 76; Burrows 76; Lietke 76; Robitaille 76; Brown 77; Ginsburg 77; Grabner 77, Engelhard 77; Davis 78; Kent 78; Hopkins 78; Laursen 78

tional work to the child and his/her characteristics, and to the child's respective stage of development. On the part of psychology, the influence of three schools of thought can be recognized: psycho-analysis, Gestalt theory, and the so-called psychology of thinking (Denkpsychologie). These headings, however, subsume very complex and internally inconsistent theories. Freud, in particular, stressed error analysis as an epistemological method in his early lectures.

In Germany, Weimer is considered the founder of didactically oriented error research. His objective was to establish an error pattern in order to explain individual mistakes for all educational subjects and all age groups. In doing so, he distinguished, under the overall concept of "wrong" ("das Falsche"), between mistake (Irrtum), falsification (Fälschung), and errors (Fehler). Weimer grouped errors in five categories (familiarity errors, perseverance errors, similarity errors, mixed errors, and errors due to emotion or volition), making allowance for psycho-analytical insights in doing so.

Following Weimer, Seemann was less concerned with the diagnosis of poor individual performance and with developing suitable aids than with providing a psychological foundation for a didactical method for teaching mathematics. For him, mathematics errors were not the result of change, but phenomena arising from laws which were being created by a wrong combination of determining tendencies, the so-called "principle of least effort" having a dominating role in this. For arithmetic, Seemann distinguished three groups of errors: mechanical errors, associative errors, and functional errors.

Kießling was predominantly concerned with so-called error-proneness (Fehlsamkeit) as a personal disposition for mistakes, and with theories of error assessment and error prevention. While Fettweis pointed out errors in algebra, Rose attempted to establish a classification of the causes of error in mathematical education (inattention, ignorance of rules, confusion of concepts, and inability to recognize the characteristic features of a mathematical problem).

This development of error analysis was abruptly curtailed in 1933 by the Nazis seizing power, as they were opposed to the individualistic foundation of the prevailing educational system, and considered mathematics less important as compared to other subjects, which seemed better suited to promote the Nazi educational aims.

During the last decades, it has been difficult to discern systematic research on error theory in mathematical education. Accessible publications are either reprints of dated work [Seemann, Rose, Monsheimer-Türpitz], short surveys in handbooks and pedagogical encyclopedias [Knabe, Kießling, Kirchhof], or produced as by-products from other efforts [Krüger, Schlaak, Glück, Grissemann, Schaffrath].

Upon analyzing performance tests in solving text problems, Schlaak has observed certain focal points of error: inadequate understanding of texts, incorrect number determination, etc. Grissmann, in his study on legasthenia and calculation performance, has pointed out some causal factors in calculation problems, as they can be derived from the casuistic-phenomenological research literature. The studies of Schaffrath, Krüger, and Glück are essentially preparatory work for diagnostic calculation tests, Glück treating so-called calculation errors at length. Upon analyzing performance in calculation toward the end of the second form, he was able to distinguish five error types: changes of operation, additive or multiplicative proximity, part results, first digit only correct, and copying errors.

Two important studies continuing this discussion are those of Pippig. In his publication on calculation deficiencies and how to overcome them from a psychological perspective, Pippig considers those errors having their cause primarily in the psychological mechanisms of the imitative process. Pippig considers efforts directed at studying the calculation errors of students especially important as the ability to solve formal problems is a necessary, but not sufficient, condition for higher achievement in mathematical education. In his contribution to overcoming thinking errors, Pippig treats the errors and difficulties encountered when working at text problems. He describes causes of error in the different stages of the solution process.

Birkhan has used the analysis of mathematics students' errors as a tool of cognitive psychology, with the conceptual system of information processing theory as a backdrop. Radatz is concerned with pointing out the logical and regular character of most mistakes in mathematical education, and with the attempt to regroup error causes across individual mathematics content areas according to certain aspects of receiving and processing information by students.

In the USSR, a number of questions and problems were raised by the fundamental change of school structure and the related curricula reform at the beginning of the sixties. These problems led to manifold pedagogical-didactical research efforts. Among these, analyzing student errors or individual academic learning difficulties are the favoured methods to win new knowledge pertaining to mathematics-specific abilities, and to aspects of the mathematics teaching-learning process. The theoretical background is furnished by the statements of Marxist psychology. Unfortunately, the major part of these Soviet studies was made accessible in the shape of translations into English only within recent years. As examples for this, we should like to mention the studies by Kuzmitskaya and Menchinskaya. Kuzmitskaya has determined four error causes in the study of difficulties encountered by slightly retarded children in the attempt to solve mathematical problems: insufficient short-term memory, insufficient understanding of the problem conditions, errors due to lack of command of oral calculation rules, and incorrect use of the four basic operations. Menchinskaya stresses the regular character of student errors in mathematics education, and emphasizes the complexity of the processes of potential causes of error. She names four causal areas, which cannot be distinctly separated from one another:

- errors due to incorrect implementation of an operation;
- errors due to insufficient quality of conceptual understanding;
- mechanical errors due to lack of interest or to diversion;
- errors due to application of inappropriate rules or algorithms.

The very long tradition of error analysis in mathematics education research in the U.S. is evident in Buswell's [1925] monograph quoting 31 studies already existing at that time which explicitly deal with arithmetical errors Among these Uhl's study [1917] is especially important, as the latter, for the first time, supplemented the analysis of written exercises by classroom observations and diagnostic interviews in order to analyze error techniques and error causes. By means of this multi-level research method, Buswell was able to identify a multitude of error types in the four basic types of calculation. Continuing these studies, Brueckner and Burge attempted to typify error techniques. As to content, the American studies of that time were almost exclusively concerned with errors and difficulties encountered in the four written calculation methods within the quantity of natural numbers (the "four rules"). Research interest focused on five objectives:

1. Listing all potential error techniques;
2. Determining the frequency distribution of these error techniques across age groups;
3. Analyzing special difficulties, particularly encountered when doing written division, and when operating with zero;
4. Determining the persistence of individual error techniques;
5. Attempting to classify and group errors [see Grossnickel, Holland, Buswell, Schonell, Brueckner et al]

These approaches were continued, during recent years, by the studies of Roberts, Lankford, Engelhard, and particularly Cox, which also served to formulate their statements more precisely

A further group of American contributions to this topic is essentially concerned with the opportunities for error analysis offered to the mathematics teacher, with curricular and methodological activities adapted to these, and with tests of the efficiency of special training programs aimed at reducing error frequency Related to these are efforts to sensitize teachers for so-called "diagnostic teaching of mathematics", in which analysis of student errors plays an important role [Ashlock, Reisman, Robitaille], or to inform them of this method. A third research orientation may be seen in Erlwanger's or Ginsburg's attempts to use error analysis as a method of investigating basic structures in the mathematical teaching-learning process In this, they are strongly influenced by Piaget's methods, as their studies are predominantly based on diagnostic talks and interviews, and on analyzing individual cases For Erlwanger, Ginsburg, and others, most student errors are not of an accidental character, but are due to very individual problem-solving strategies and rules which go back to earlier experience and understanding of mathematics. In this, the most recent American developments in research on error analysis are converging, in their methods, and in their fundamental insights as well, with the Soviet Union research approaches on one hand, and with the efforts made in the French IREMs on the other [Brousseau, Jaulin-Mannoni, Salin].

To conclude, we should like to sketch some research deficits and desirable research fields which have been suggested to us by an analysis of the accessible literature. Some research levels which have as yet hardly been worked at, are the following:

- errors in the non-arithmetical content areas of mathematical education;
- conscious and subconscious processes in incorrect attempts to solve mathematical problems;
- detailed analyses of error causes derivable from interaction effects between the variables intervening in the mathematical teaching-learning process (teacher, curriculum, student, etc);
- development of special didactical aids for treating particular learning difficulties and errors

Beyond that, the problems of describing causes and of separating causal areas, of investigating the process character of the individual solution path, or of the methods of analysis in the field of research and teaching practice, will be topics for current and future efforts at research and didactical development.

We conclude this brief survey with a quote from one of Freud's lectures in 1908:

> *It must be stressed again that we do not assert that every single mistake (Fehlleistung) is logical (sinnreich), despite the fact that I consider this likely. For us, it will suffice to demonstrate such a logic frequently in different forms of mistakes.*

Bibliography

Andelfinger, B , Diagnostische und therapeutische Verfahren im Mathematikunterricht der Sekundarstufe I In: *Beiträge zum Mathematikunterricht*, Hannover: Schroedel 1977, S. 20 - 23

Ashlock, R B , *Underachievers in Elementary Mathematics. Diagnosing and Correcting Difficulties in Computation.* University of Maryland 1973

Ashlock, R B., *Error Patterns in Computation.* Columbus, Ohio 1976

Baumgarten, A., Fehleranalyse der Schüler im Mathematikunterricht — ein Mittel, Selbständigkeit und Denkvermögen zu entwickeln In: *Mathematik in der Schule* 10 (1972), S. 566 - 577

Baxter, H H., Prediction of Errors Type in Computation of Sixth Grade Mathematics Students Pennsylvania State University, Dissertation, 1973 DAI 35 A: 251

Birkhan, G., Analyse von Denkfehlern in der Mathematikdidaktik als kognitionspsychologisches Untersuchungsinstrument. In: *Komplexe menschliche Informationsverarbeitung*, Bern: Huber 1979, S. 495-502.

Blankenship, C S , Reduction of Systematic Inversion Errors in Subtraction Washington, University, Dissertation 1976.

Brown, J.S. and Burton, R.B , Diagnostic Models for Procedure bugs in Basic Mathematics Skills Bolt, Beranek and Newmann Inc Cambridge, Mass. Report 3669; 1977

Brueckner, L.J., *Diagnosis and Remedial Teaching in Arithmetic* Boston: Houghton Mifflin, 1930

Brueckner, L J , Diagnosis in Arithmetic. In: *34th NSSE Yearbrook*, Bloomington 1935, S. 269 - 302.

Brueckner, L J and Bond, G L , *The Diagnosis and Treatment of Learning Difficulties.* New York: Appleton-Century-Croft, 1955.

Burge, L., Types of Error and Questionable Habits of Work in Multiplication In: *The Elementary School Journal*, 32 (1932), S. 185 - 194

Burrows, J.K , A Review of the Literature on Computational Errors with Whole Numbers University of British Columbia, Vancouver, MEDIC Report, 7, 1976

Buswell, G.T. and Judd, C.H , *Summary of Educational Investigation Relating to Arithmetic.* Chicago: University of Chicago, 1925

Buswell, G T. and John, L., *Diagnostic Studies in Arithmetic* Chicago: University of Chicago, 1926.

Carman, R A , Mathematical Mistakes. In: *Mathematics Teacher* 64 (1971) 2, S. 109 - 115.

Counts, G.S , Arithmetic Tests and Studies in the Psychology of Arithmetic In: *Supplementary Educational Monographs*, I, No 4, University of Chicago Press, 1917.
Cox, L.S., Systematic Errors in the Addition (Subtraction, Multiplication, Division) Algorithm in Normal and Handicapped Population Kansas City: University of Kansas Medical Center, 1973
Cox, L.S., Diagnosing and Remediating Systematic Error in Addition and Subtraction Computations In: *The Arithmetic Teacher* 22 (1975), S. 151 - 157
Cox, L.S , Systematic Errors in the Four Vertical Algorithms in Normal and Handicapped Population. In: *Journal Res Math Ed* , 6 (1975) 4, S 202 - 220
Davis, E J. and Cooney, I J , Identifying Errors in Solving Linear Equations. In: *Mathematics Association of Two-Year Colleges Journal*, 3 (1978) 2 S. 170 - 178.
Dodd, C A., Jones, G A. and Lamb, C E., Diagnosis and Remediation of Pupil's Error: Exploratory Study In: *School Science and Mathematics* (1975), S 270 - 276
Ellis, L.C , A Diagnostic Study of Whole Number Computations of Certain Elementary Students Dissertation, 1972. DAI 33 A, 1972., 2234
Engelhardt, J M , A Proposal for Generating a Mathematics Learning Model from the Analysis of Students' Errors. Athens: Paper, Workshop, University of Georgia, 1975.
* Engelhard, J.M , Analysis of Children s Computation Erros: A Qualitative Approach. In: *British Journal Educ Psych* , 47 (1977)., S. 149 - 154
Erlwanger, S H., Benny's Conception of Rules and Answers in IPI Mathematics. In: *Journal of Children's Mathematical Behavior*, 1 (1973) 2, A 7 - 26
Fettweis, E., Versuch einer psychologischen Erklärung von Schülerfehlern in der Algebra. In: *Mathematisch naturwissenschaftlicher Unterricht* 60 (1929), S. 214 - 219
Ginsburg, H , *Children s Arithmetic The Learning Process* New York: V Nostrand, 1977.
Ginsburg, H., The Psychology of Arithmetic Thinking. *The Journal of Children's Mathematical Behavior*, 1 (1977) 4, S. 1 - 89.
Gist, A., Errors in the Fundamentals of Arithmetic In: *School and Soviet*, 6 (1917), S 175 - 177
Glück, G., *Rechenleistung und Rechenfehler* Tübingen, Universität. Dissertation, 1971.
Grossnickle, F.E., Constancy of Error in Learning Division with a Two-Figure Divisor In: *Journal of Educational Research*, 33 (1939), S 189 - 196.
Harvey, L F and Kyte, C.C., Zero Difficulties in Multiplication In: *The Arithmetic Teacher*, 12 (1965), S 45 - 50.
Hopkins, M.H., *Teaching Arithmetic A Diagnostic Approach*. Brandenton, Fl., 1977.
Hutcherson, L R., Errors in Problem Solving in Sixth-grade Mathematics. Texas, University, Dissertation, 1975, DAI 36 A, 6459 - 6460
Hylla, E , Analyse und Rechenfehler In: *Zeitschrift für Pädagogische Psychologie*, 17 (1916), S. 319f.
Jäkel, E., Didaktische Differenzierung auf der Grundlage langfristiger Fehleranalysen im Mathematikunterricht der Klasse 8 In: *Mathematik in der Schule*, 10 (1972), S 645 - 656
Kane, R B , Byrne, M A and Hater, H A., *Helping Children Read Mathematics*. New York, 1974.
Kent, D., The Dynamic of Puts In: *Mathematics Teaching*, 82 (1978), S. 32 - 36
Kent, D , Three Words Used in Mathematics. In: *Cambridge Journal of Education*, 8 (1978), S 45 - 51
Kießling, A., *Die Bedingungen der Fehlsamkeit*. Leipzig, 1925
Kießling, A., Der gegenwärtige Stand der Fehlerforschung In: *Die Deutsche Schule*, 32 (1928), S 158 - 162
Kießling, A., Alltagsfehler und Lernfehler. In: *Zeitschrift für Pädagogische Psychologie*, 30 (1929), S. 517 - 521
Kießling, A., Fortschritte und Fehlerforschung. In: *Zeitschrift für Pädagogische Psychologie*, 33 (1932), S 184 - 188
Kießling, A., Über Grundlagen und Bedeutung einer differentiellen Fehlerkunde. In: *Zeitschrift für Psychologie*, 31 (1934), S 388 - 393.
Kießling, A. and Rombach, J , Fehlerkunde. In: Rombach, J (Hrsg): *Lexikon der Pädagogik*, 1, Freiburg, 1960, S 1175 - 1178
Korn, G , Über Rechenleistung und Rechenfehler Eine pädagogisch-psychologische Untersuchung In: *Zeitschrift für angewandte Psychologie*, 25 (1926), S 145f
Lankford, F G , Some Computational Strategies of Seventh Grade Pupils. University of Virginia, 1972 (US Dept. of Health, Education and Welfare. Best.-Nr 2-D-013)
Laursen, K.W., Errors in First-year Algebra. In: *Mathematics Teacher*, 71 (1978), S 194 - 195
Liedtke, W , Learning Difficulties in Mathematics In: *Vector*, 18 (1976) 1, S 11 - 13
Monsheimer, O. and Türpitz, H., *Rechenbegabung-Rechenleistung-Rechenfehler*. Weinheim: Beltz 1960
Myers, G C., Persistence of Errors in Arithmetic In: *Journal of Educational Research*, 10 (1924) 2., S. 19 - 28
Osburn, W., *Corrective Arithmetic* Boston: Haughton Mifflin 1924.
Phelps, C L , A Study of Errors in Testing of Adding Ability In: *Elementary School Teacher* (1913) 14, S. 29 - 39
Pippig, G , Rechenschwächen und ihre Überwindung in psychologischer Sicht In: *Mathematik in der Schule*, 13 (1975) 11, S 623 - 628.
Pippig, G., Psychologische Überlegungen zur Überwindung von Denkfehlern. In: *Mathematik in der Schule*, 15 (1977) 1, S 26 - 41.
Radatz, H , Fehlerdiagnose und innere Differenzierung In: *Die Grundschule*, 9 (1977) 2, S 68 - 74
Radatz, H , Schülerfehler und ihre möglichen Ursachen im Mathematikunterricht In: *Westermanns Pädagogische Beiträge*, 29 (1977) 9, S 366 - 369.
Ranschburg, P , *Die Leseschwäche (Legasthenie) und Rechenschwäche (Arithmasthenie) der Schulkinder* Berlin: 1916
Reisman, F K , *A guide to the Diagnostic Teaching of Arithmetic* Columbus: Merrill, 1972
Roberts, G H., The Failure Strategies of Third Grade Arithmetic Pupils In: *The Arithmetic Teacher*, 15 (1968), S 442 - 446
Robitaille, D F , Learning Difficulties in Mathematics In: *Vector*, 17 (1976) 3, S 62 - 66
Rose, G , Fehlerquellen im Mathematikunterricht. In: *Lebendige Schule*, 16 (1961) 9
Schacht, E J , A study of the Mathematical Error of Low Achievers in Elementary School Mathematics Dissertation, DAI 28 A, 1967, 920 - 921.
Schaffrath, J.G., Gedanken zur Psychologie der Rechenfehler In: *Schweizer Erziehungsrundschau*, 32 (1959), S 41 - 48, 66 - 70
Schlaak, G., *Fehler im Rechenunterricht*. Hannover: Schroedel 1968.
Schonell, F E., *Diagnosis of Individual Difficulties in Arithmetic* Oliver and Boyd 1937
Schonell, F E. and Schonell, F.J., *Diagnosis and Remedial Teaching in Arithmetic* Oliver and Boyd 1937.
Seemann, J., *Untersuchungen über die Psychologie der Rechenfehler* Leipzig 1929.
Seemann, J , *Die Rechenfehler*. Langensalza 1931
Seemann, J , *Psychologie der Fehler und ihre Bekämpfung im Unterricht* Crailsheim 1949.
Smith, C A , A Study of Constant Errors in Subtraction and in the Application of Selected Principles of the Decimal Numeration System Made by Third and Fourth Grade Students. Dissertation, DAI 30 A, 1968, 1084.
Thorndike, E.L , *The Psychology of Arithmetic*. New York 1917
Uhl, W L., The Use of Standardized Materials in Arithmetic for Diagnosing Pupils' Methods of Work In: *Elementary School Journal*, 18 (1917), S 215 - 218
Weimer, H., Wesen und Arten der Fehler In: *Zeitschrift für Pädagogik und Psychologie*, 23 (1922), S. 17 - 25 and 24 (1923), S. 84 - 98, 267 - 282, 353 - 372.
Weimer, H., *Psychologie der Fehler*. Leipzig 1925
Weimer, H , Fehlerkunde (Leistungsfehler). In: Schwartz, H (Hrsg): *Pädagogisches Lexikon*, 2, Bielefeld 1929
Weimer, H , Fehler und Irrtum. In: *Zeitschrift für Pädagogik und Psychologie*, 32 (1931), S 48 - 52
Weimer, H., *Fehlerverhütung und Fehlervermeidung*. Düsseldorf 1939
West, T A., Diagnosing Pupils' Errors Looking for Patterns In: *Arithmetic Teacher*, 18 (1971), S. 467 - 469
Wiegard, A F , Eselsbrücken, Fehlerhäufigkeit und typische Fehler bei der schriftlichen Subtraktion — Eine Untersuchung zu den Rechenfertigkeiten im Bereich der schriftlichen Subtraktion. In: *Sachunterricht und Mathematik in der Primarstufe*, 6 (1978), S. 263 - 267.
Williams, C.L. and Whitaker, R.L., Diagnosis of Arithmetic Difficulties In: *Elementary School Journal*, 37 (1937), S 592 - 600.