Students' performance in reasoning and proof in Taiwan and Germany: Results, paradoxes and open questions

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Abstract: In different international studies on mathematical achievement East Asian students outperformed the students from Western countries. A deeper analysis shows that this is not restricted to routine tasks but also affects students' performance for complex mathematical problem solving and proof tasks. This fact seems to be surprising since the mathematics instruction in most of the East Asian countries is described as examination driven and based on memorising rules and facts. In contrast, the mathematics classroom in western countries aims at a meaningful and individualised learning. In this article we discuss this "paradox" in detail for Taiwan and Germany as two typical countries from East Asia and Western Europe.

Kurzreferat: Internationale Vergleichsstudien haben gezeigt, dass Schülerinnen und Schüler ostasiatischer Länder bessere Ergebnisse im Fach Mathematik erzielen als Lernende aus westlichen Ländern. Dies betrifft dabei nicht nur Routineaufgaben, sondern auch komplexe Problemlöse- und Beweisaufgaben. Die Ergebnisse scheinen zu überraschen, da der Mathematikunterricht in ostasiatischen Ländern oft als prüfungsorientiert und auf Auswendiglernen ausgerichtet beschrieben wird. Im Gegensatz gibt es in westlichen Ländern das Ziel, ein verständnisvolles und individualisiertes Lernen zu ermöglichen. In diesem Beitrag diskutieren wir dieses „Paradox" für die Länder Taiwan und Deutschland als zwei typische Beispiele aus Ostasien und Westeuropa.

ZDM-Classification: D43, E53

1. Introduction

During the last decade international comparative studies revealed repeatedly differences between the mathematical achievement in East Asian countries and countries from Western Europe and North America. Although one cannot say that the mathematics education in East Asia (resp. Western countries) is homogenous, there exist several similarities which go back to a common cultural basis (Leung, 2001). However, the findings of video studies like TIMSS 1995 and TIMSS-R 1999 (Stigler, Gonzales, Kawanaka, Knoll & Serrano, 1999; Hiebert et al., 2003) give evidence that mathematics instruction in East Asian countries can be different with respect to various aspects as a comparison between mathematics lessons from Hong Kong and Japan indicates. Similar results were shown for the mathematics instruction in Germany, Switzerland and the Netherlands. Against this background a starting point of the discussion and investigation of possible reasons for the differences in the mathematical achievement between East Asian and the Western countries should preferably start with the comparison of two countries.

In the following we will present different aspects of the mathematical achievement, mathematics instruction and the school system in Taiwan and Germany. On the one hand, it is shown how different these countries are. On the other hand, there remain open questions regarding the higher achievement of Taiwanese junior high school students in mathematics.

2. Students' performance in mathematics achievement studies

2.1 International studies

In the last years different studies on mathematical achievement have revealed major differences between the lower secondary students' performance from Germany and Taiwan. Although there does not exist a direct comparative study so far, one can conclude from the results of international studies that Taiwanese junior high school students belong to the group of high performing students whereas their German counterparts show only moderate achievements. In the following we present briefly some selected results to illustrate these differences.

2.1.1 Mathematics achievement – Taiwan

Taiwan participated as one of 20 populations in the International Assessment of Educational Progress study (IAEP) in 1992 (Lapointe, Mead & Askew, 1992). The IAEP study collected different types of data from students of age 9 and age 13, in particular, there was a mathematical achievement test which covered different areas of mathematics like arithmetic, algebra, geometry etc. In the following we will report about the results for the Taiwanese sample of age 13 (N= 1780).

As presented in Table 1 Taiwan belongs to the high achieving countries. The Taiwanese students solved on average about 73% of the test items correctly whereas the international average in this study was about 58%.

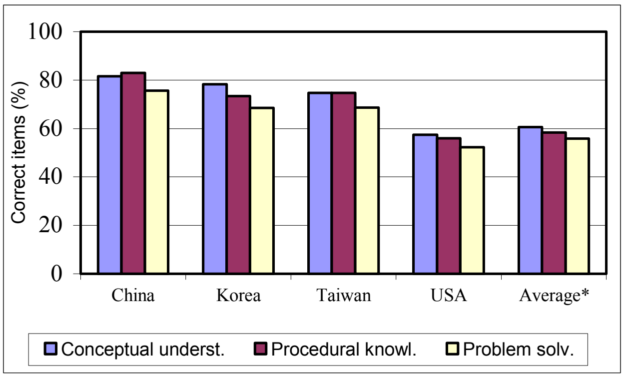
Table 1: Mathematical achievement of students of age 13 in different countries.

Country	*Correct items*	*Rank*
P. R. China*	80.2 %	1
Korea	73.4 %	2
Taiwan	72.7 %	3
USA	55.3 %	16
IAEP Average**	58.3 %	

* Sample restricted to 20 provinces and cities
** Average of the country scores

If we distinguish the results into the different areas of mathematics we get a similar result: Taiwanese students belong to the high achieving students. The same holds, if the test items are arranged into groups which address different cognitive levels (conceptual understanding, procedural knowledge, problem solving). For all three levels the Taiwanese sample achieves top scores (see Figure 1). This means, particularly, that the 13 years old Taiwanese show a high problem solving performance.

Figure 1: Results for items addressing different cognitive levels



In addition to the 1992 IAEP study Taiwan participated in the TIMS study 1999 (Mullis et al. 2000). Overall the Taiwanese sample comprised 5772 grade 8 students (M = 14.2 years) from 150 schools. In the mathematics test the Taiwanese students achieved a score of 585 which means rank 3 of all 39 participating countries (see Table 2).

Table 2: Mathematical achievement (TIMSS 1999) in different countries.

Country	*Scores*	*SD*	*Rank*
Singapore	604	79	1
Korea	587	79	2
Taiwan	585	104	3
Hong Kong	582	73	4
USA	502	88	19
Int. Average*	500	100	

* Average of the individual scores

In the TIMS studies benchmarks (top 10%, upper quarter, median, lower quarter) were used to get a more detailed description of the distribution of high achieving students to the different countries. For example, the Top 10% Benchmark, located at score 616, is determined as follows: an item was included if at least 65 % of students scoring at the scale point 616 answered the item correctly and less than 50 % of students scoring at the Upper Quarter Benchmark answered it correctly. An investigation of the items of the Top 10% Benchmark yields the following description:

> Students can organize information, make generalizations, and explain solution strategies in nonroutine problem solving situations. They can organize information and make generalizations to solve problems; apply knowledge of numeric, geometric, and algebraic relationships to solve problems (e.g., among fractions, decimals, and percents; geometric properties; and algebraic rules); and find the equivalent forms of algebraic expressions (Mullis et al., 2000, p. 42).

The other benchmarks are determined and described in an analogous way (see Mullis et al., 2000, p. 42 for more information). The distribution of the TIMSS 1999 students to the different benchmarks is given in Figure 2 (for some countries).

Figure 2: Students per country scoring at TIMSS 1999 benchmarks.

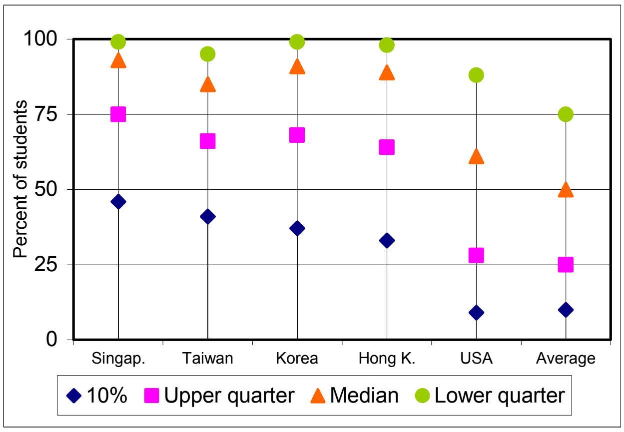


Figure 2 points up that there are huge differences between the East Asian countries and Western countries. About 41% of the Taiwanese students achieve the top 10% and already 85% belong to the upper half of the TIMSS 1999 sample whereas the students from the USA are distributed similar to the average values.

2.1.2 Mathematics achievement –Germany

As already mentioned before till now there is no study in which Taiwan and Germany took part. German students participated in TIMSS 1995 (Beaton et al. 1996) and PISA 2000 (Deutsches PISA-Konsortium, 2001). In both studies the achievement of the German sample was moderate. In TIMSS 1995 German sample comprised N=2870 grade 8 students (M = 14.8 years) and 2893 grade 7 students (M = 13.8 years) from 134 resp. 132 schools. Table 3 gives some results.

Table 3: Mathematical achievement (TIMSS 1995) in different countries.

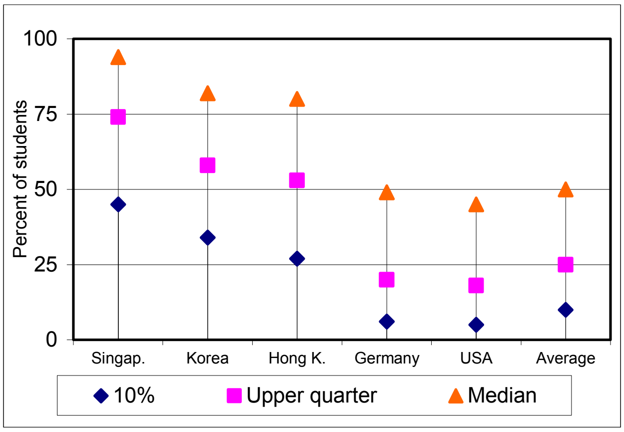
Country	*Score grade 7*	*SD*	*Score grade 8*	*SD*
Singapore	601	93	643	88
Korea	577	105	607	109
Hong Kong	564	99	588	101
Germany	484	85	509	90
USA	476	89	500	91
Int. Average*	500	100	500	100

* Average of the individual scores

Like in TIMSS 1999 some benchmarks were defined. However, in the 1995 study there are only three: the Top 10% Benchmark, the Upper Quarter and the Median. Results for the grade 8 sample are given in Figure 3.

One can see that the samples of Singapore, Korea and Hong Kong distribute in a similar way like in TIMSS 1999. The German sample is comparable with the US sample. However, one has to be careful in interpreting these results since the benchmarks are based on the average results and the sample of TIMSS 1995 and 1999 are not equal.

Figure 3: Students per country scoring at TIMSS 1995 benchmarks.



In PISA 2000 the German sample comprised N= 5073 15 year old students from 219 schools (for the extended German sample about 48000 students in 1466 schools). In contrast to the IAEP and the TIMS study the basis for the items in PISA was not the curriculum. The aim of the mathematical part of PISA was to survey the mathematical literacy of the 15 year old students in the participating countries (OECD, 1999). Nevertheless, the items were curriculum valid for the German sample.

Again we give a short overview of the results (Table 4).

Table 4: Mathematical achievement (PISA 2000) in different countries.

Country	*Score*	*SD*
Hong Kong*	560	?
Japan	557	87
Korea	547	84
USA	493	98
Germany	490	103
Int. Average**	500	100

* Hong Kong joined the 1st cycle of PISA in 2002

** Average of the individual scores

Like in TIMSS 1995 the German sample scores similar to the US sample. A deeper analysis of the German results for TIMSS 1995 and PISA revealed that German students perform well with routine items or items which require one solution step. Problems occur, if a solution is based on a combination of arguments, like proofs or complex modelling tasks (Blum & Neubrand, 1998, Deutsches PISA-Konsortium, 2001).

2.1.3 Mathematics achievement – summary

If we sum up the presented results of the international studies, then we can say, that East Asian students, in particular the Taiwanese students, belong to the top achieving students in international comparisons. The German students and the US students achieve only moderate results. In TIMSS 1999 about 40% of the Taiwanese students scored on the top 10% level, in contrast in TIMSS 1995 only 6% of the German students reached this level. Though we have no direct comparison between Taiwanese and German students one can assume that the differences in the mathematical achievement are huge. This affects particularly items which require more than only a basic knowledge. However, we have to take into account that these studies are descriptive and give no insight about the reasons of these results.

2.2 Studies on reasoning and proof

In this section we will present some results from Taiwan and Germany regarding the lower (junior) secondary students' competencies in reasoning and proof.

2.2.1 Results from Germany

For Germany we present results of a study with 524 students from 27 grade 8 classes of the German Gymnasium (high attaining students) (cf. Reiss, Hellmich & Reiss, 2002, Heinze & Reiss, in press). The study focussed on the question of students' abilities to perform geometrical proofs and is mainly based on two tests on reasoning and proof, the first was administered at the end of grade 7 and the second in the mid of grade 8 after a teaching unit on reasoning and proof (congruence geometry)[1]. The test items are based on a competency model with three levels of competency: (I) basic competency (applying facts and rules, e.g. for calculations), (II) argumentative competency (one-step-argumentation) and (III) argumentative competency (combining several arguments). The competency model was confirmed by the empirical data (see Table 5).

Table 5: Percentage of correct items for the three levels of competency and upper, middle and lower third of the students.

Percentage	Levels of Competency					
	Test 1			Test 2		
	L I	L II	L III	L I	L II	L III
Lower third	46	25	6	30	23	4
Middle third	64	62	19	57	36	8
Upper third	79	88	50	74	55	26
Average	64	60	26	54	38	13
N = 524	Test 1: M = 51 (s = 19)			Test 2: M = 37 (s = 16)		

The data show that low-achieving students were hardly able to solve any items on level III whereas high-achieving students performed well on level I and level II items and satisfactorily on level III tasks (Reiss, Hellmich & Reiss, 2002, Heinze & Reiss, in press).

A deeper analysis of students' responses to the test items together with additional interviews with ten students of the sample indicate that even high-achieving students have difficulties generating a proof idea and combining arguments to a proof (Heinze & Reiss, in press). However, the low-achieving students are hardly able to deal with proofs, in particular, they have substantial deficits in their declarative and methodological knowledge.

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[1] In Germany geometry is taught in each grade on the lower secondary level. The geometry curriculum in grade 7 contains e.g. relationships between angles and properties of triangles. In grade 8 the congruence theorems are used for teaching the mathematical proof.

These results for students' performance on reasoning and proof items supports the findings of TIMSS 1995 and the PISA study: German students perform well in task which requires the application of simple rules and one step argumentations, but most of them fail, if they have to combine different arguments or facts to a chain of arguments. It seems that they do not have the ability to generate an idea or a strategy for a solution of non routine tasks.

Furthermore, the results indicate significant differences with respect to the achievement of distinct classrooms (range of mean scores in grade 7 between 22 and 68% of the number of possible points respectively between 12 and 59% in grade 8). A multilevel analysis for the achievement test (grade 8) revealed that the mean pre-test scores on classroom level have more influence on the individual post-test achievement than the individual pre-test scores. It is reasonable to assume that an essential portion of this influence is based on instruction, since there are hardly any differences in respect to other factors like the number of students per class, their social background etc.

Regarding the presented results on German students' competencies in reasoning and proof we have to stress that in contrast to TIMSS and PISA only high attaining students were considered (from the German school type Gymnasium, see next section for information about the school system).

2.2.2 Results from Taiwan

In Taiwan, the ongoing *Development of Adolescents' Competence on Mathematical Argumentation* (DACMA) project investigates Taiwanese junior high students' competence and conception of mathematical argumentation in a nation-wide questionnaire survey (cf. Lin et al., 2003). Part of the project focus on the students' performance on constructing geometric proof. The sample involves 1144 of grade 7, 1088 of grade 8, and 1083 of grade 9 students. The test items were adapted from the English survey of Healy & Hoyles (1998) and partly modified according to local responses from Taiwanese students found in a pre-pilot study (Lin & Chen, 1999). Moreover, the project develops some new items collected from local mathematics classroom teaching which are meaningful to Taiwanese teachers and students. The survey was processed in January 2003 while grade 9 students had just learnt the formal geometric proof and grade 7 and 8 students had not yet learnt the junior high geometry.

The items of this study can also be classified into the three level competency model (see Section 2.2.1). Items of level I are calculating questions (with three steps). Although the properties used in level I questions are learnt in elementary school, Taiwanese grade 7 and 8 students have no experiences of applying these properties to solve geometric questions with more than one calculation step. The level II and III items for grade 7 and 8 students are in the form of 'judging and then explaining'. These items ask students to judge whether the given properties are true, and then ask students to explain why. This form is quite unfamiliar to students because school tests always propose an assumed true setting and ask for a numerical answer or explanation. Some results of the tests are given in Table 6.

Table 6: Percentage of correct* and incomplete** solutions for the three levels of competency in different grades .

grade	Percentage	Levels of Competency		
		L I	L II	L III
7	Correct*	37.0	18.6	11.4
	Incomplete**	10.8	18.1	37.9
8	Correct*	47.1	26.1	13.2
	Incomplete**	0.8	20.3	35.3
9	correct		46.9	36.4
	incomplete		5.5	23.2

* Correct solution means that the students judge correctly and then construct a solution which would get the best score in school tests

** Incomplete solution means that the students judge correctly and understand the crucial elements for a solution, but lack of some process or detailed explanation

The data shows that Taiwanese junior high students' competence is increasing slightly from grade 7 to grade 8 but significantly from grade 8 to grade 9. It is reasonable because Taiwanese student learn geometry from the second semester of grade 8 to the first semester of grade 9 and the test was processed after the first semester. Nearly 47% of grade 9 students can construct a correct proof for level II items and more than 36% can do it for level III items.

The grade 7 and 8 students have not learnt any geometric content in junior high school. However, more than 18% of grade 7 students can judge and proof correctly for level II items and more than 11% for level III items; the grade 8 students perform better. Moreover, many students judge correctly and construct an incomplete proof. If we sum up correct and incomplete proofs together, 36.7% and 46.4% of grade 7 and grade 8 students respectively can judge correctly and at least understand the crucial elements for proving level II items, and 49.3% and 48.5% for level III items. These results show that Taiwanese students have some potential competence in geometric proof before they learn it, and which does not come from learn geometric proof directly. There are maybe some implicit attributes which are beneficial to learn geometric proof in the learning and teaching context.

Furthermore, although grade 7 and 8 students have no experience of solving items with three calculation steps (level I), 37% resp. 47% of them can organize those properties needed into a chain to solve such an item. But those who can judge and prove correctly a simple one-step argumentation item (level II) are significantly less. Thus, students perform well in challenging calculation items which are in a familiar setting. However, simple one-step argumentations in an unfamiliar setting seem to be difficult for them. This indicates that Taiwanese students' performance maybe influenced by the familiarity of the item setting.

3. Mathematics instruction in Taiwan and Germany

In this section, firstly, we will give some basic facts about the school system and the organisation of the mathematics classroom in Taiwan and Germany. Then in Section 3.2 and 3.3 we give empirical results about the teaching style in Taiwan (resp. the Taiwanese cultural area) and in Germany.

3.1 Mathematics classroom in Taiwan and Germany – some basic facts

There are several differences between Taiwan and Germany with regard to the school system and the conditions for the mathematics classroom.

In Taiwan the compulsory education starts with the age of six. The students visit the elementary school in the first six years and then for three years the junior high school (grades 7–9). After that there is the possibility to go to the senior high school (grades 10–12) and, finally at the age of 18, to apply for a university or college. During the time of compulsory education, e.g. in grades 1–9, all students are taught together - there is no separation of the student population into different school types or different classes based on the individual proficiency level. Though there is a national curriculum, some junior high schools have a better reputation than others. This is mainly based on the fact how many of their students are able to pass the entrance test of a senior high school with good reputation. The acceptance of the different senior high schools depends on the percentage of students which pass the entrance examinations of the best universities.

Like in Taiwan in Germany the compulsory education starts in the age of six. The students visit the elementary school for four or six years depending on the state where they live[2]. After the elementary school in grade 5 resp. grade 7 the student population is separated with respect to their proficiency level into three tracks: the Hauptschule (until grade 9), the Realschule (until grade 10) and the Gymnasium (until grade 12 or 13). In general, the distribution of the students to these tracks is based on reports written by the elementary school teachers, however students can change the tracks after every school year but it is very difficult. Further education at a university requires a final examination of a Gymnasium or an equivalent certification. Besides the three school tracks there exist some so-called Gesamtschulen (comprehensive schools) in which a separation based on the students' performance is made internally and only in some subjects.

Regarding the conditions for the mathematics classroom in Taiwan and Germany some data collected in the above cited international studies is given in Table 7.

The results in Table 7 show that, on the one hand, for the Taiwanese students there is more time to learn mathematics in school. On the other hand, the German students are taught in much more smaller classes, such that a more intensive individual support by the teacher is possible.

Table 7: Conditions for mathematics classroom.

<table>
<tr><th></th><th colspan="6">Taiwan*</th><th colspan="6">Germany**</th></tr>
<tr><td>Class size</td><td colspan="6">M = 39</td><td colspan="6">M = 24.1***</td></tr>
<tr><td rowspan="2">Percentage of classes with specific size</td><td colspan="2">1-20</td><td colspan="2">21-35</td><td colspan="2">36 -</td><td colspan="2">1-20</td><td colspan="2">21-30</td><td colspan="2">30-</td></tr>
<tr><td colspan="2">0 %</td><td colspan="2">14%</td><td colspan="2">86%</td><td colspan="2">25%</td><td colspan="2">72%</td><td colspan="2">3%</td></tr>
<tr><td>Mathematics instruction per year (hours)</td><td colspan="6">126 h</td><td colspan="6">112 h***</td></tr>
<tr><td rowspan="2">Percentage of students with specific amount of math instruction per week</td><td colspan="3">2 - 3.5 h</td><td colspan="3">3.5 – 5 h</td><td colspan="3">2 - 3.5 h</td><td colspan="3">3.5 – 5 h</td></tr>
<tr><td colspan="3">51%</td><td colspan="3">48%</td><td colspan="3">85%</td><td colspan="3">12%</td></tr>
</table>

*TIMSS 1999 ** TIMSS 1995 *** PISA 2000

Unfortunately, we have no detailed data about the number of students visiting cram schools after a regular school day. PISA 2000 revealed huge differences between Western and East Asian countries. About 17% of the German sample responded that they visit or visited a cram school or got extra lessons (not specifically in mathematics!) for a certain amount of time. In contrast, in countries like Japan and Korea more than 60% of the students reported of these kind of out-school-studying. There are still a lot of open questions about the influence of these private institutions on the learning of mathematics and their way of teaching. Lin & Tsao (1999) described the teaching style of specific Taiwanese cram schools for the preparation of entrance examinations as "one-task-one-rule".

3.2 Taiwanese teaching style

The results of the TIMSS video studies 1995 and 1999 revealed the existence of a typical culture of mathematics classroom resp. a typical teaching style in different cultural areas (Stigler el al., 1999; Hiebert et al., 2003). In the following we describe differences between the Taiwanese and the German teaching style.

A first insight into the instructional pattern of Taiwanese mathematics teachers is given by the small video study of Fwu & Wang (2002). In this study three to four mathematics lessons of three teachers (two young and one experienced teacher) were videotaped and analyzed. One of the results was an instructional pattern common to the three teachers. This pattern consists of the six steps

(1) review of previous materials,
(2) presentation of the topic for the day,
(3) presentation of definitions of terms and rules,
(4) demonstration with examples,
(5) practice,
(6) assignment of homework

and is described as follows:

> At the beginning of a class, the teacher usually starts with a check of the homework assignment or gives a quiz to review material taught in the previous period. He/she usually calls on students to write up the procedures and solutions on the blackboard (…). When the teacher moves on to the new topic, the students "automatically" take out the math textbook and turn

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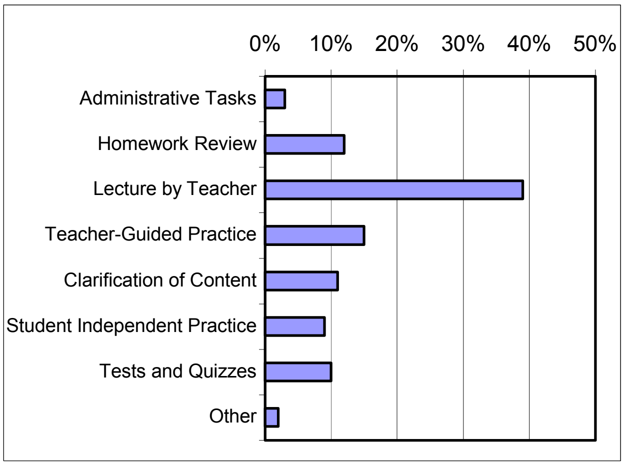
[2] In Germany there are 16 states (Bundesländer) which are responsible for the education.

to the exact page from which the new topic begins. The teacher then presents the new terms and rules by either contrasting them with the previously established ones which can not apply to the new situation or by highlighting the "knack" of deriving correct answers to math problems in the new section being studied. At this stage, the teacher usually asks some closed questions to check if students get the point. (…) He/she then demonstrates with two to four problems with different degrees of difficulty to show how to apply the rules to get the answer. (…) To check if the students have learnt the rules and skills, the teacher calls on some students to practice problems from the textbook on the blackboard while other students do the same problems at their seats. (…) With the correct procedures and answers listed on the board, the teacher will then ask the whole class to check their own answers against the "standard" ones. This cycle of teacher demonstration and student practice at the board and in seats is usually repeated several times, occupying the major block of time in an instruction period. At the end of the class, the teacher usually gives homework either from the textbooks or from self-produced worksheets. He/she may also announce a quiz to be held in the next period on the topic just taught (Fwu & Wang, 2002, Section 3).

In the described Taiwanese mathematics lessons the teachers focus on demonstrating procedures, on the one hand, and on asking the students to practice procedural skills on the other hand. Thus, there is not something like a teacher – student discourse which focus on discovering new rules or exploring the meaning of concepts in different contexts. These results of Fwu & Wang (2002) are consistent with the findings of a teacher questionnaire which was administered in TIMSS 1999. The mean values of the teacher responds to the question how much time they spend on different activities in a mathematics lessons are given in Figure 4 (cf. Mullis et al., 2000, p. 205).

Figure 4: How much time spend teacher on various activities in mathematics lessons.



About 39% of the lesson time is used for a lecture-style teacher presentation, another 11% for clarification of student problems, which are, in general, again in a lecture-style presentation. In addition to these 50% teacher presentations we have teacher guided practice or homework review which is also teacher guided.

Regarding the question, whether the teachers emphasize procedures or reasoning and problem-solving activities the data of TIMSS 1999 also confirms the findings of Fwu & Wang (2002). In TIMSS 1999 the responds of four teacher questions were combined to a index of teacher emphasis on mathematics reasoning and problem-solving. The questions were about how of the teachers asks the students to (1) explain the reasoning behind an idea, (2) represent and analyse relationships using tables, charts or graphs, (3) work on problems for which there is no immediately obvious method of solution and (4) write equations to represent relationships. The respond categories were 1 = never or almost never, 2 = in some lessons, 3 = in most lessons, 4 = every lesson. Table 8 gives the mean values for different countries.

Table 8: Percentage of students which are taught by teachers with specific emphasis on mathematics reasoning and problem-solving.

Country	*1 - 2.25*	*2.25 - 3*	*3 - 4*
Japan	7%	45%	49%
Korea	13%	66%	21%
USA	24%	57%	18%
Taiwan	29%	58%	13%
Hong Kong	38%	56%	6%
Int. Average	25%	61%	15%

Table 8 shows that the index of emphasis on reasoning and problem-solving is for teachers from Taiwan and Hong Kong below the international average (in contrast to Japan). It seems that, in general, Taiwanese teachers are satisfied if students are able to apply procedures; they rarely require reasoning activities.

These findings for the Taiwanese mathematics classroom are comparable with that for Hong Kong. In the TIMSS 1999 video study Hong Kong was the country where teacher was talking most, the students' verbal contributions were limited to short utterances and on average 84% of the problems per lesson were posed by the teacher with an apparent indent to using procedures - the highest percentage in TIMSS 1999 video (Hiebert et al., 2003, pp. 131-132).

3.3 German teaching style

Regarding the German mathematics classroom the most important study is the TIMSS 1995 video study (N=100 German mathematics lessons in grade 8) comparing classrooms in Germany, Japan, and the United States (Stiegler et al., 1999; Klieme, Schümer & Knoll, 2001). The results characterized the typical German teaching style as guiding students through the development of a procedure or a concept by asking them to orally fill in relevant information (so-called "fragend-entwickelnde" (=questioning-developing) teaching style):

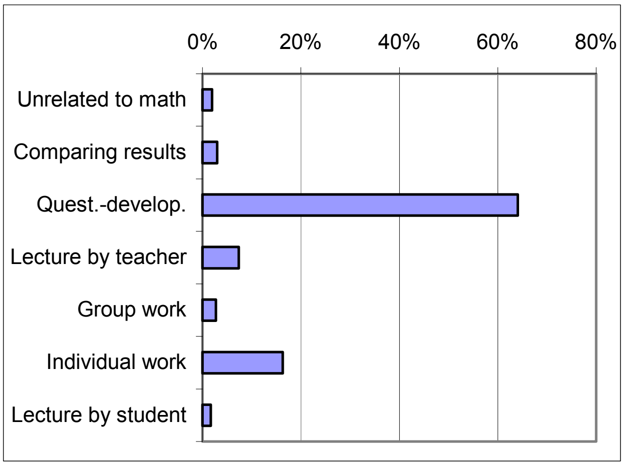
> The teacher organizes the lesson so that most of the mathematical work during the lessons is done as a whole class. The teacher does not lecture much to the students; instead, she guides students through the development of the procedure by asking students to orally fill in the relevant information. (…) If the problem is a relatively new one, the teacher generally works the problem at the board, eliciting ideas and procedures from the class as work on the problem progresses. If the problem is one they have already been introduced to, a student might be called to the chalkboard to work the problem. The problem might be slightly different than problems students have worked before but the method to solve the problem has been introduced previously and applied in related situations. The class is

> expected to monitor the student's work, to catch errors that are made, and to help the student when he or she gets stuck.
> The teacher keeps the student and class moving forward by asking questions about next steps and about why such steps are appropriate. After two or three similar problems have been worked in this way, the teacher summarizes the activity by pointing to the principle or property that guides the deployment of the procedure in these new situations. For the remaining minutes of the class period, she assigns several problems in which students practice the procedure in similar situations (Stiegler et al., 1999, pp. 133-134).

Though there is a teacher – student discourse in the German mathematics classroom, it is not ensured that the required students' activities and responses are on a high cognitive level. In contrast, as already Voigt (1984) in an in-depth case study showed, within the German teaching style students generally do not have to solve complex problems, because the problems are solved step by step in a classroom discourse which is strictly led by the teacher. The required students answers are mostly only on an elementary level and the complex problem is transformed into a series of closed simple question (cf. Klieme, Schümer & Knoll, 2001).

The findings of the Augsburg video study with 22 lessons dealing with geometrical reasoning and proof in grade 8 fits to the above mentioned teaching style. As shown in Figure 5 about 64% of the lesson time was used for the typical German questioning-developing teaching style (cf. Heinze & Kraft, in press).

Figure 5: How much time spend teacher on various activities in mathematics lessons.



About 16% of the time was used for individual work, however, this student work mainly consists of making geometrical drawings.

Regarding the teaching of proof in these lessons an analysis of the proving processes showed that the negative effects of the German teaching style are dominating here, too. In particular, the stages in the proving process, in which the problem situation is explored, additional information is collected and a proof idea is generated, were underemphasized by the teachers (cf. Heinze & Reiss, 2004). In general, the teacher developed the proof step by step at the blackboard by asking questions to the students.

Though the German teaching style seems to be much more student oriented than the Taiwanese teaching style, we have to state for the Augsburg video study that in a mathematics lesson about 40% of the German students do not participate in the teacher – student discourse at all (Heinze & Kraft, in press).

4. Paradoxes, conjectures and open questions

In this section we will discuss the previous presented facts for Germany and Taiwan and close with some remarks and open questions.

4.1 The situation in Germany - discussion

As described in Section 2 one of the main difficulties of German lower secondary students is to use their mathematical knowledge for solving problems which require more than only one argument. In particular, the ability to investigate a problem situation, to generate a strategy, to identify appropriate arguments and to combine arguments to a solution is not acquired in the German mathematics classroom even in the German Gymnasium. A study with upper secondary students showed that these difficulties are still existing in grade 13 (Reiss & Thomas, 2000).

Since Heinze & Reiss (in press) showed in their study with 27 classes of grade 8 that there are strong class effects, one can assume that it is, in fact, the mathematics instruction that plays an important role in developing this students' ability. If we consider the results concerning the German teaching style, then we can hypothesise that this questioning-developing teaching style indeed prevents the students from solving complex mathematical problems on their own. Generally, they do not have to explore problem situations on their own, they do not have to generate strategies for solutions on their own etc. In contrast, during the mathematics lessons the students mostly remain on the level of answering one-step problems ask by their teachers.

4.2 The paradox of the Taiwanese learner

The situation for Taiwan is much more difficult to explain. There exist only a few empirical results concerning the mathematics instruction in this East Asian country. However, these findings fits into the general descriptions of the Taiwanese/Chinese teaching style which was already mentioned in other publications (e.g., Lin & Tsao, 1999; Leung, 2001): The mathematics classroom is teacher dominated, student involvement is minimal, there are large class sizes, the instruction is content-oriented and examination driven, the students' activities consist mainly of practicing and memorising mathematical concepts and procedures.

From an educational point of view memorising concepts and practicing procedural skills establishes mainly an instrumental understanding of mathematics (in the sense of Skemp, 1978). In the Taiwanese mathematics classroom there seems to be hardly any starting point for the students to build up mathematical knowledge or a mathematical understanding individually. Cooperative learning settings, learning material which can be chosen individually, phases of exploration of mathematical concepts or statements, reasoning or argumentation activities, i.e. elements of the mathematics classroom which are considered as learning opportunities fostering a relational understanding of mathematics do hardly occur in Taiwanese mathematics lessons.

However, the outcome of the mathematics education in Taiwan seems to be different as the results of the studies cited in Section 2 indicate. Taiwanese students are more successful than their counterparts from the e.g., USA even for complex problem-solving items. A similar result we have for the sample of Hong Kong which achieved the best results of all countries in PISA 2000, a study that emphasize the aspect of mathematical modelling.

The phenomenon that East Asian students[3] belong to the high achieving samples in comparative studies though their education consists mainly of memorising and practicing procedures went down in the literature as "The paradox of the Chinese learner" (cf. Watkins & Biggs, 1996).

4.3 Remarks and open questions

Within the last decade in the mathematics education and the psychology literature the paradox of the Taiwanese/Chinese learner was discussed and ideas for the investigation of this phenomenon were planned (e.g., ICMI 2001; Leung, 2001; Watkins & Biggs, 1996). In particular, the ICMI study group for a comparative study on mathematics education in East Asia and the West discussed an extensive program including different research perspectives to collect empirical data on different levels (e.g., administration, classroom, individual) for different variables (e.g., mathematics achievement, beliefs, attitudes). The main aims of this study are to gain a deeper understanding of various aspects of mathematics learning and teaching and to develop a process of self-reflection on our traditional ways that we often take for granted (ICMI, 2001, p.116).

From our point of view, interesting research questions are, whether the different teaching cultures in Germany and Taiwan really lead to a different level of mathematical understanding (e.g., instrumental versus relational understanding) and if there are differences in Taiwan and Germany in the types or strategies of learning mathematics.

Regarding the first question Lin & Tsao (1996) hypothesise that Taiwanese students indeed acquire mainly an instrumental understanding of mathematics. Based on the examination culture in Taiwanese schools which is enforced by the teaching and learning culture "one-task-one-rule" in the cram schools, the students are not learning mathematics but exam maths:

> From the point of view of the students interviewed, mathematics means computation, problem solving becomes choosing the right problem routine and learning mathematics is memorizing. (…) Exam maths simply comprises sets of rules (Lin & Tsao, 1996, p. 233).

According to Lin & Tsao (1996) the excellent results of the Taiwanese students in the IAEP 1992 study go back to fact, that

> the items in the IAEP written mathematics test can be regarded as just another kind of exam maths, so it would be expected that Taiwanese students would perform well" (Lin & Tsao, 1996, p. 234).

The results of the DACMA project (Section 2.2.2) seem to support this conclusion. For grade 7 and 8 Taiwanese students, they have not learnt geometry in junior high school, can organize their knowledge learnt in elementary school to solve a difficult and inexperienced question, but cannot retrieve a simple principle to judge and explain why a property is true. These two kinds of question are different in posing form, the harder is familiar as school test and the easier is not. Nevertheless, Taiwanese students perform better in a familiar and difficult question than in an unfamiliar easy one. This shows that a difficult school-test-like question is easier to solve it than a simple unfamiliar one.

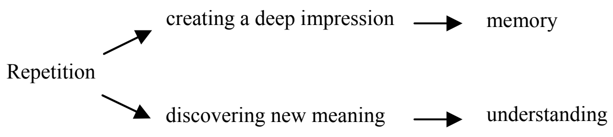
To what extent can similar conclusion be drawn for the TIMSS items with high difficulty index or the PISA items of the competency level 4 and 5? Are these items also more or less exam maths? Further research is needed to find out which level of mathematical understanding Taiwanese students achieve? Moreover, based on the previous hypotheses a comparison to the German students seems to be interesting. Since the German teaching style is less emphasizing the procedural aspects it is the question whether the German students have a deeper mathematical understanding (though they have problems to use their mathematics knowledge).

Another approach to explain the paradox of the Chinese learner refers to the way of learning mathematics. It is the question, whether Taiwanese students (or more general: Chinese students) practice a different type of repetition or memorising than the German students. According to a model of Dahlin & Watkins (2000) repetition plays two different roles (cf. Figure 6):

> On the one hand, repetition can be associated with creating a "deep impression" on the mind, and therefore with memorisation. On the other hand, repetition can be used to deepen and develop understanding by discovering new meaning (Dahlin & Watkins, 2000, p. 80).

The "deep impression" can be created in different ways: by an intense emotional experience, by repeated recitation or by understanding.

Figure 6: The role of repetition (Dahlin & Watkins, 2000, p. 81).



Another similar model is proposed by Marton, Dall'Alba & Tse (1996). They distinguish between mechanical rote learning and memorisation with understanding. The latter separates into "memorisation what is understood" and "understanding through memorisation". Understanding through memorisation is not equal to mechanical repetition. It means that each time a different aspect is focussed or a different point of view is taken.

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[3] Notice that Japan is an exceptional case (see Table 7 or TIMSS 1999 video study: Hiebert et al, 2003).

Regarding the differences in the mathematical achievement between East Asian and Western countries it was argued that the differences in the role of memorisation are one of the key points for the higher performance of the East Asian students (Stevenson & Stigler, 1992; Biggs & Watkins, 1996). In an interview study Dahlin & Watkins (2000) showed that there are indeed differences between German and Chinese (Hong Kong) secondary students. Only in interviews with German students statements occur which were categorized as "Repetition helps understanding by checking it". This means that the function of understanding is mainly to check the understanding one has already achieved. Moreover, the conception "Repetition plus 'attentive afford' can lead to new meaning" was significantly more common among Hong Kong students than among German students. "Attentive afford" means that one thinks of the meaning (e.g., of a sentence) several times to develop the understanding.

However, till now there is not very much empirical evidence for this hypothesis, since there are only a few studies with comparatively small samples. Nevertheless, it might be one piece of the puzzle.

Literature

Beaton, A. E.; Mullis, I. V. S.; Martin, M. O.; Gonzalez, E. J.; Kelly, D. L.; Smith, T. A. (1996): Mathematics Achievement in the Middle School Years: IEA's Third International Mathematics and Science Study (TIMSS). IEA. Boston: Center for the Study of Testing, Evaluation, and Educational Policy

Blum, W.; Neubrand, M. (Hrsg.) (1998): TIMSS und der Mathematikunterricht. Informationen, Analysen und Konsequenzen. Hannover: Schroedel

Dahlin, B.; Watkins, D. (2000): The role of repetition in the process of memorising and understanding: A comparison of the views of German and Chinese secondary school students in Hong Kong. British Journal of Educational Psychology 70, pp. 65 – 84

Deutsches PISA-Konsortium (Ed.) (2001): PISA 2000. Basiskompetenzen von Schülerinnen und Schülern im internationalen Vergleich. Opladen: Leske + Budrich

Fwu, B. J.; Wang, H. H. (2002): Practice makes perfect on the blackboard: A cultural analysis of mathematics instructional patterns in Taiwan. In I. Vakalis; D. Hughes Hallett; D. Quinney; C. Kourouniotis (Eds.), 2nd International Conference on the Teaching of Mathematics (ICTM 2). Iraklion (Greece), New York: Wiley, no page numbers

Healy, L.; Hoyles, C. (1998): Justifying and proving in school mathematics. Summary of the results form a survey of the proof conceptions of students in the UK. Research Report Mathematical Sciences, Institute of Education, University of London

Heinze, A.; Kraft, E. (in press): Schülerbeteiligung im Mathematikunterricht – eine Auswertung videografierter Unterrichtstunden. In A. Heinze & S. Kuntze (Eds.), Beiträge zum Mathematikunterricht 2004. Hildesheim: Franzbecker

Heinze, A. & Reiss, K. (2004): The teaching of proof at lower secondary level – a video study. Zentralblatt für Didaktik der Mathematik (ZDM), 36(3), pp. 98 – 104

Heinze, A.; Reiss, K. (in press): Mathematikleistung und Mathematikinteresse in differentieller Perspektive. To appear in J. Doll; M. Prenzel (Eds.), Studien zur Verbesserung der Bildungsqualität von Schule: Lehrerprofessionalisierung, Unterrichtsentwicklung und Schülerförderung. Münster: Waxmann

Hiebert, J.; Gallimore, R.; Garnier, H.; Givvin, K. B.; Hollingsworth, H.; Jacobs, J.; Chui, A. M.-Y.; Wearne, D.; Smith, M.; Kersting, N.; Manaster, A.; Tseng, E.; Etterbeek, W.; Manaster, C.; Gonzales, P.; Stigler, J. (2003): Teaching Mathematics in Seven Countries: Results from the TIMSS 1999 Video Study. U.S. Department of Education. National Center for Education Statistics, Washington, DC: Government Printing Office

International Commission on Mathematical Instruction (ICMI) (2001): ICMI Comparative Study: Mathematics education in different cultural traditions: A comparative study of East Asia and the West. Discussion Document. Zentralblatt Didaktik der Mathematik 33(4), pp. 113 -122

Klieme, E.; Schümer, G.; Knoll, S. (2001): Mathematikunterricht in der Sekundarstufe I: „Aufgabenkultur“ und Unterrichtsgestaltung. - In: Bundesministerium für Bildung und Forschung (BMBF) (Ed.), TIMSS – Impulse für Schule und Unterricht. Bonn: BMBF, pp. 43 – 57

Lapointe, A. E.; Mead, N. A.; Askew, J. M. (1992): Learning Mathematics. IAEP. Princeton: Educational Testing Service.

Leung, F. K. S. (2001): In search of an East Asian identity in mathematics education. Educational Studies in Mathematics 47, 35 – 51

Lin, F. L.; Chen, Y. J. (1999): A pilot study on proving and justifying of Taiwanese junior high school students. Dept. of Mathematics, National Taiwan Normal University, Taipei, Taiwan (in Chinese)

Lin, F. L. et al. (2003): Development of Adolescents' Competence on Mathematical Argumentation. Technical Report of the NSC support project. Dep. of Mathematics, National Taiwan Normal University, Taipei, Taiwan

Lin, F.-L.; Tsao, L.-C. (1999): Exam Maths Re-examined. In: C. Hoyles; C. Morgan; G. Woodhouse (Eds.), Rethinking the mathematics curriculum. London: Falmer, pp. 228 – 239

Marton, F.; Dall'Alba, D.; Tse, K.-T. (1996): Memorising and understanding: The keys to the paradox? In D. A. Watkins; J. B. Biggs (Eds.), The Chinese Learner: Cultural, psychological, and contextual influences. Hong Kong: Comparative Education Research Centre, pp. 69 - 83

Mullis, I. V. S.; M. O. Martin; E. J. Gonzalez; K. D. Gregory; R. A. Garden; K. M. O'Connor; S. J. Chrostowski; T. A. Smith (2000): TIMSS 1999 International Mathematics Report. IEA. Boston: International Study Center Lynch School of Education

OECD (Ed.) (1999): Measuring student knowledge and skills. A new framework for assessment. Paris: OECD

Reiss, K.; Hellmich, F.; Reiss, M. (2002): Reasoning and proof in geometry: Prerequisites of knowledge acquisition in secondary school students. In A. D. Cockburn; E. Nardi (Eds.), Proceedings of the 26th Conference of the International Group for the Psychology of Mathematics Education. Volume IV. Norwich (Great Britain). University, pp. 113-120

Reiss, K.; Thomas, J. (2000): Wissenschaftliches Denken beim Beweisen in der Geometrie. Ergebnisse einer Studie mit Schülerinnen und Schülern der gymnasialen Oberstufe. mathematica didactica 23, pp. 96–112

Skemp, R. R. (1978): Relational understanding and instrumental understanding. Arithmetic Teacher 26 (3), pp. 9-15

Stevenson, H.W.; Stigler, J. W. (1992): The learning gap: Why our schools are failing and what we can learn from Japanese and Chinese education. New York: Summit Books

Stigler, J.; Gonzales, P.; Kawanaka, T.; Knoll, S.; Serrano, A. (1999): The TIMSS videotape classroom study. U.S. Department of Education. National Center for Education Statistics, Washington, DC: Government Printing Office

Voigt, J. (1984): Interaktionsmuster und Routinen im Mathematikunterricht. Theoretische Grundlagen und mikroethnographische Falluntersuchungen. Weinheim; Basel: Beltz

Watkins, D. A.; Biggs, J. B. (Eds.) (1996): The Chinese Learner: Cultural, psychological, and contextual influences. Hong Kong: Comparative Education Research Centre

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