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STUDIES IN RADIO SOURCE STRUCTURE

II. The Relaxation of Relativistic Electron Spectra in Self-absorbed Radio Sources

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Summary

A calculation is presented of the synchrotron spectrum of self-absorbed radio sources. It is well known that electron spectra $N(E)\propto E^{-\gamma}$, when γ is a constant $>\frac{1}{3}$, give rise to radiation spectra whose logarithmic slope is 2·5. It is shown that other forms of $N(E)$ can give rise to radiation spectra either steeper or flatter than this. Such spectra might arise when an initial power law spectrum relaxes. The nature of this relaxation is studied and is shown to depend crucially on whether $\gamma \gtrless 3$. The relevance of these results to observed radio sources is discussed.

1. *Introduction.* Self-absorption of synchrotron radiation by relativistic electrons has been discussed in papers by Twiss (1958) and Le Roux (1961). The process is believed to be responsible for the low frequency cut-off observed in the spectra of some radio sources, especially those with small angular diameter and high surface brightness (Slish 1963, Williams 1963, Hornby & Williams 1966). The flux from a homogeneous source at frequencies ν for which the optical depth is large would be proportional to $\nu^{2\cdot5}$ if the electrons had a power law energy distribution of the kind which often occurs in radio sources. In Section 2 we consider the spectrum of the emitted radiation when the electron distribution has other forms.

The situation within sources of the former kind clearly does not correspond to thermodynamic equilibrium, and there will be a tendency for an initial power law electron distribution to relax towards a Maxwellian distribution. In Sections 3 and 4 we discuss the nature of this relaxation in some simple cases, and the consequent changes in the spectrum of the emitted radiation, showing how they depend on the slope of the original power law spectrum.

The evolution of an actual source is complicated by other effects, besides synchrotron emission and absorption, which can influence the energy spectrum of the electrons. For example, the electrons will undergo collisions with the other particles in the source, and they will interact with radiation through the Compton effect. In Sections 2, 3 and 4 we consider idealized models in which these effects are neglected. The condition that this should be a valid approximation is discussed in Section 5, where the possible relevance of this work to actual radio sources is also considered.

2. *The radiation spectrum.* We consider the spectrum of the radiation from simple model radio source in which synchrotron self absorption occurs. W

suppose that the magnetic field is uniform throughout the source (and of strength H), and that a uniform density of relativistic electrons is present, with an isotropic distribution of velocities and a differential energy spectrum denoted by $N(E)$. We assume that not enough ionized gas is present to cause appreciable absorption, and that the Razin effect is negligible; the Faraday rotation however may be large.

The intensity of the radiation emitted from unit volume, per steradian, in a direction making an angle θ with the magnetic field, is

$$I(\nu, \theta) = \frac{1}{4\pi} \int N(E) p(\nu, E, \theta) dE \tag{1}$$

where $p(\nu, E, \theta)$ is the power radiated per unit frequency interval at frequency ν by an electron whose velocity makes an angle θ with the direction of H. This formula makes use of the fact that each electron radiates only in a narrow cone in its direction of motion, and therefore only those electrons with pitch angle $\sim \theta$ need be considered.

Now $p(\nu, E, \theta)$ is given by

$$p(\nu, E, \theta) = \sqrt{3} \frac{e^3 (H \sin\theta)}{mc^2} \frac{\nu}{\nu_c} \int_{\nu/\nu_c}^{\infty} K_{5/3}(x) dx \tag{2}$$

where

$$\nu_c(E) = \frac{3e(H \sin\theta)}{4\pi mc} \left(\frac{E}{mc^2}\right)^2 \tag{3}$$

The function $p(\nu, E, \theta) \propto \nu^{1/3}$ for $\nu \ll \nu_c$, and cuts off exponentially for $\nu \gg \nu_c$.

The absorption coefficient per unit length is given by the formula

$$\mu(\nu, \theta) = -\frac{c^2}{8\pi\nu^2} \int_0^{\infty} E^2 \frac{d}{dE} \left(\frac{N(E)}{E^2}\right) p(\nu, E, \theta) d\theta \tag{4}$$

(Le Roux 1961)

At high frequencies for which absorption is unimportant, the radiation intensity within the source is proportional to $I(\nu, \theta)$ the constant of proportionality depending on the dimensions of the source. At low frequencies where the optical depth is $\gg 1$ the intensity at a point within the source is

$$F(\nu, \theta) = \frac{I(\nu, \theta)}{\mu(\nu, \theta)} \tag{5}$$

The surface brightness of the source at frequencies for which it is opaque will of course be proportional to $F(\nu, \theta)$.

We now consider the form of $F(\nu, \theta)$ for various forms of the electron energy distribution $N(E)$.

(a) *Monoenergetic spectrum.* If the electrons all have the same energy E_0, then

$$I(\nu, \theta) = \frac{N_0}{4\pi} p(\nu, E_0, \theta) \tag{6}$$

and the absorption coefficient can be written in the form

$$\mu(\nu, \theta) = \frac{c^2}{8\pi\nu^2} \frac{N_0}{E_0^2} \left[\frac{d}{dE} (E^2 p(\nu, E, \theta))\right]_{E=E_0} \tag{7}$$

When the optical depth is large the value of $F(\nu, \theta)$ is

$$\frac{8}{3}\frac{E_0\nu^2}{c^2}.$$

The spectrum of the radiation from a source containing a monoenergetic distribution of electrons is therefore of the form shown in Fig. 1. If $\mu(\nu_c, \theta) \ll 1$, there will be a frequency range over which the intensity $\propto \nu^{1/3}$, but at lower frequencies the spectrum will be proportional to ν^2. If, however, N_0 is so high that $\mu(\nu_c, \theta) \gg 1$, there will be no frequency range over which the spectrum has a slope $+\frac{1}{3}$. Since $p(\nu, E, \theta)$ cuts off rapidly for $\nu \gg \nu_c$, a further increase in the value of N_0 would not cause the frequency at which the radiation spectrum peaks to greatly exceed ν_c.

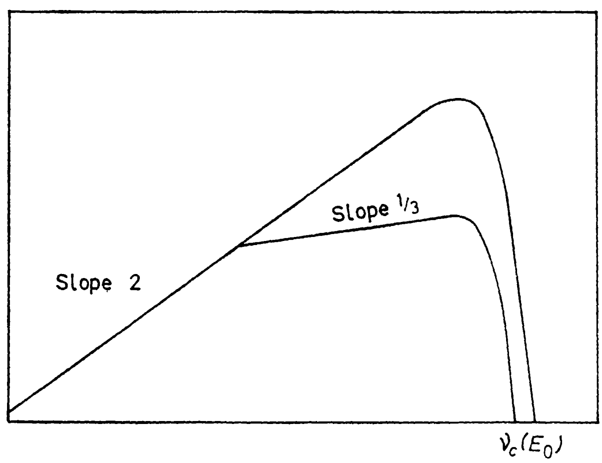

FIG. 1. *The spectrum from a source containing a monoenergetic distribution of electrons. The slope of the spectrum is 2 at frequencies for which the optical depth is large, and $\frac{1}{3}$ when the source is transparent.*

This spectrum is, incidentally, the same as would arise from a Maxwellian distribution of relativistic electrons with temperature T such that $kT = \frac{3}{4}E_0$.

(b) *Power law spectra.* Suppose now that the differential energy spectrum of the relativistic electrons is given by

$$N(E)dE = KE^{-\gamma}dE \tag{8}$$

for $E_1 < E < E_2$, where K and γ are constants.

If $\gamma > \frac{1}{3}$, the radiation at frequencies ν such that $\nu_c(E_1) \ll \nu \ll \nu_c(E_2)$ will almost all be due to electrons with energy E such that $\nu \sim \nu_c(E)$. The limits of integration in (1) can then be replaced by 0 and ∞, so that we obtain

$$I(\nu, \theta) = \frac{\sqrt{3}}{\gamma+1}\Gamma\left(\frac{3\gamma-1}{12}\right)\Gamma\left(\frac{3\gamma+19}{12}\right)\frac{e^3}{4\pi mc^2} \times \left(\frac{3e}{2\pi m^3c^5}\right)^{(\gamma-1)/2} K(H\sin\theta)^{(\gamma+1)/2}\,\nu^{-((\gamma-1)/2)} \quad (9$$

If, on the other hand, $\gamma < \frac{1}{3}$, the main contribution to $I(\nu, \theta)$ at all frequencie comes from the highest energy electrons present. The condition that the mai contribution to the absorption coefficient $\mu(\nu, \theta)$ should come from electrons fc which $\nu_c(E) \sim \nu$ is that $\gamma > -\frac{2}{3}$. If this condition is satisfied, the limits of integration in (4) can be taken as 0 and ∞, and we obtain

$$\mu(\nu, \theta) = \frac{\sqrt{3}}{4} \Gamma\left(\frac{3\gamma+2}{12}\right) \Gamma\left(\frac{3\gamma+22}{12}\right) \frac{e^3}{2\pi m} \left(\frac{3e}{2\pi m^3 c^5}\right)^{\gamma/2} K(H \sin\theta)^{(\gamma+2)/2} \nu^{-((\gamma+4)/2)} \tag{10}$$

When $\gamma < -\frac{2}{3}$ the main contribution to (4) comes from the electrons with $E \sim E_2$.

The synchrotron radiation spectrum from a source with electron energy distribution (8), for different values of γ, will therefore have the following form.

$\gamma < -\frac{2}{3}$

This case is similar to the source with a monoenergetic spectrum already discussed, since the main contribution to both the emission and the absorption at all frequencies comes from the highest energy electrons present.

$-\frac{2}{3} < \gamma < \frac{1}{3}$

The emission $I(\nu, \theta) \propto \nu^{1/3}$ (the main contribution coming from electrons with $E \sim E_2$), but the absorption coefficient is given by (10), and $\mu(\nu, \theta) \propto \nu^{-((\gamma+4)/2)}$. The radiation from the source would thus be proportional to $\nu^{1/3}$ at frequencies for which absorption was not significant, but at low frequencies would be proportional to $\nu^{(3\gamma+14)/6}$. The slope would therefore be between 2 and 2·5, depending on the value of γ.

$\gamma > \frac{1}{3}$

In this case the radiation spectrum has a slope $-((\gamma-1)/2)$ at frequencies for which the optical depth $\ll 1$. However if $\nu \ll \nu_m$ (where ν_m, which will be a function of direction, is the frequency at which the optical depth ~ 1), the slope of the spectrum is 2·5. We then have

$$F(\nu, \theta) = \left(\frac{8\pi m^3 c^7}{3e}\right)^{1/2} (H \sin\theta)^{-1/2} f(\gamma) \nu^{2\cdot 5}, \tag{11}$$

where

$$f(\gamma) = \frac{\Gamma\left(\frac{3\gamma-1}{12}\right) \Gamma\left(\frac{3\gamma+19}{12}\right)}{(\gamma+1)\,\Gamma\left(\frac{3\gamma+2}{12}\right) \Gamma\left(\frac{3\gamma+22}{12}\right)} \tag{12}$$

The function $f(\gamma)$ is a decreasing function of γ, reflecting the fact that the mean energy (and therefore the 'equivalent temperature') of the electrons responsible for emitting and absorbing radiation of a given frequency is slightly higher for a flat spectrum than for a steeper one.

These results are of course only applicable for emission at frequencies between $\nu_c(E_1)$ and $\nu_c(E_2)$.

(c) *More general spectra.* The radiation spectrum can obviously be calculated from (1), (4) and (5) for any form of the function $N(E)$. We shall merely make a few remarks which will be relevant to the discussion of the subsequent sections.

Contrary to the conclusion of Twiss (1958) and Slish (1965), there is no chance of getting negative absorption (i.e. amplification), for any energy distribution o ultrarelativistic electrons emitting synchrotron radiation (Wild, Smerd & Weiss 1963). This may be possible for mildly relativistic electrons which radiate at distinct harmonics of their gyrofrequency (Bekefi, Hirshfield & Brown 1961), but cannot occur if the electrons are highly relativistic and may be assumed to emit radiation with a continuous spectrum. However McCray (1966) has pointed out that synchrotron maser action may occur if a sufficient quantity of ionized gas is present in the source.

The results of (b) show that the spectrum of an opaque source is not sensitive to changes in γ, provided that γ remains $>\frac{1}{3}$. Therefore the slope of $2 \cdot 5$ would not be significantly altered if the electron spectrum were not an exact power law,

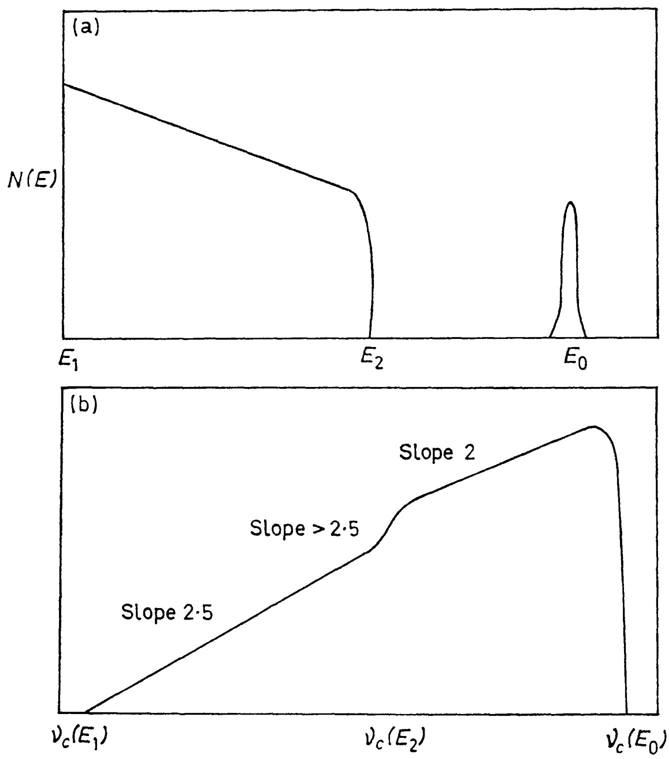

FIG. 2. (b) *shows the spectrum of the radiation from a source in which the electron distribution has the form illustrated in* (a). *The electron distribution is a power law* (*with index* $\gamma > \frac{1}{3}$) *between energies* E_1 *and* E_2, *together with a monoenergetic spectrum at a higher energy* E_0. *If the optical depth of the source is large at all relevant frequencies, the spectrum will have a slope* $> 2 \cdot 5$ *at frequencies* $\sim \nu_c(E_2)$.

but were 'curved' so that different values of γ were appropriate in different ranges of E. However if $\gamma < \frac{1}{3}$ in some energy range, there is a possibility, which does not appear to have been discussed in the literature, that the radiation spectrum may have a positive slope steeper than $2 \cdot 5$ at some frequencies. We can best illustrate how such a spectrum might arise by considering the radiation from a source in which a power law distribution of electrons is present with $N(E) \propto E^{-\gamma}$ between E_1 and E_2 (as in case (b)) and $\gamma > \frac{1}{3}$, but there is also a monoenergetic electron distribution present with a higher energy E_0 (as in case (a)). This electron distribution is illustrated in Fig. 2, together with the radiation spectrum which would be expected if the optical depth were >1 at frequencies up to $\sim \nu_c(E_0)$. The spectrum at high frequencies (between $\nu_c(E_2)$ and $\nu_c(E_0)$ will have a slope 2, whereas below $\nu_c(E_2)$ it will have a slope $2\cdot5$. There will be an intermediate range of frequencies in the neighbourhood of $\nu_c(E_2)$ where the slope of the spectrum will be $>2\cdot5$. This spectrum can be interpreted in the following way. At high frequencies the brightness temperature of the radiation corresponds to the energy E_0, whereas at slightly lower frequencies at which absorption due to the electrons with the power law energy distribution becomes important, the brightness temperature corresponds to the energy E_2.

It is not necessary that there should be a complete break in the electron energy spectrum in order to obtain a radiation spectrum whose slope is steeper than $2\cdot5$. A sufficient condition is that the electron spectrum should approximate to a power law with $\gamma<\frac{1}{3}$ in some energy range, and to a power law with a larger value of γ at lower energies.

We note that in sources with a uniform magnetic field, which we have been considering in this section, $F(\nu, \theta)\propto(\sin\theta)^{1/2}$. However ν_m will be lower for small values of θ.

3. *Energy balance of electrons emitting and absorbing synchrotron radiation.* In Section 4 we shall consider how the energy distribution of the relativistic electrons within a radio source evolves as the electrons emit and absorb synchrotron radiation. In this section we consider the energy balance of an individual electron within a source containing a uniform magnetic field. We assume that the electron emits and absorbs radiation predominantly at frequencies for which the source is opaque, so that it is in the presence of radiation of intensity $F(\nu, \theta)$. We suppose that the electrons have a spectrum $N(E)\propto E^{-\gamma}$ $(\gamma>\frac{1}{3})$.

Each electron spontaneously emits synchrotron radiation at a rate depending on its energy E and on $(H\sin\theta)$. However the net rate at which energy is absorbed from the radiation field (i.e. absorption minus stimulated emission) depends not only on these parameters but also on the energy spectrum of the other electrons in the source, since it is proportional to $F(\nu, \theta)$ and so to $f(\gamma)$. Indeed it is only through $f(\gamma)$ that individual electrons are aware of the energy distribution of the other electrons in the source (or at least those with the same pitch angle). We might therefore expect to find that the electrons tend to gain energy for small values of γ, for which $f(\gamma)$ is large, and to lose energy for large values of γ, for which $f(\gamma)$ is small. Accordingly there should be a critical value γ_{crit} of γ, for which the overall gains and losses are in balance.

We now determine γ_{crit}. To do this we evaluate the energy which each electron absorbs from the radiation field. If the electron emits and absorbs at frequencies $\ll\nu_m$, the intensity of the radiation is given by (11). By integrating (4) by parts it becomes clear that the contribution of a single electron of energy E and pitch angle θ to the absorption coefficient at a frequency ν is

$$\frac{c^2}{8\pi\nu^2}\frac{1}{E^2}\frac{d}{dE}[E^2p(\nu, E, \theta)] \tag{13}$$

(The electron only absorbs radiation travelling at an angle $\sim\theta$ to the magnetic field direction.) The rate at which energy is absorbed from the radiation field by such an electron is therefore

$$\frac{c^2}{2E^2}\int_0^\infty\frac{F(\nu, \theta)}{\nu^2}\frac{d}{dE}[E^2p(\nu, E, \theta)]d\nu. \tag{14}$$

When the electrons in the source have a power law energy spectrum, so that $F(\nu, \theta)$ is given by (11), this is

$$\frac{8}{5\sqrt{3}}\frac{e^2m^3c^7}{E^2}(H\sin\theta)^{1/2}f(\gamma)\frac{d}{dE}E^2\nu_c^{3/2}\int_0^\infty K_{5/3}\left(\frac{\nu}{\nu_c}\right)\left(\frac{\nu}{\nu_c}\right)^{5/2}d\left(\frac{\nu}{\nu_c}\right) \qquad (15)$$

Evaluating the integral and simplifying, we obtain for the rate of absorption of energy

$$\frac{2}{3}e^4\frac{(H\sin\theta)^2}{m^4c^7}E^2g(\gamma) \qquad (16)$$

where

$$g(\gamma)=\frac{9\sqrt{3}}{8\pi}\Gamma\left(\frac{31}{12}\right)\Gamma\left(\frac{11}{12}\right)f(\gamma) \qquad (17)$$

Expression (16) can be compared with the total rate of energy loss suffered by an electron of energy E emitting synchrotron radiation, which is

$$\frac{2}{3}e^4\frac{(H\sin\theta)^2}{m^4c^7}E^2 \qquad (18)$$

$g(\gamma)$ is plotted in Fig. 3. It is of course proportional to the radiation intensity, and $g(\gamma)=1$ for $\gamma=\gamma_{\text{crit}}=3$.

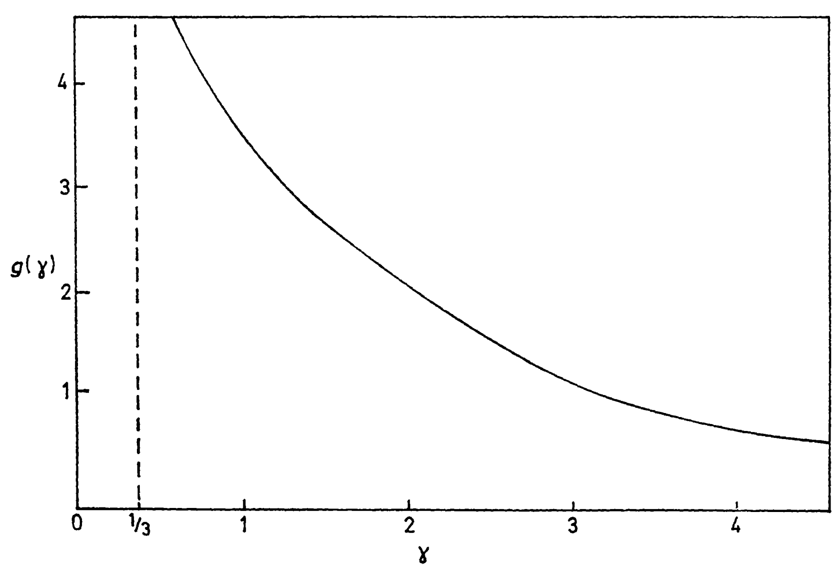

FIG. 3. *The function* $g(\gamma)$.

If a power law electron spectrum has an exponent $\gamma<3$, the equilibrium intensity of radiation is high enough for the energy absorbed by individual electrons to exceed the energy radiated. On the other hand if $\gamma>3$ the electrons do not absorb enough radiation to compensate for their synchrotron energy losses. We shall discuss the consequences of this in the next section.

The energy gains or losses of the electrons are at the expense of the radiation field, and a discussion of the energy exchange between the electrons and the photons will perhaps give some further insight into the distinction between the cases $\gamma>3$ and $\gamma<3$. When the radiation intensity within the source is $F(\nu, \theta)$

(equation (11)) the low energy electrons in a given volume, by definition, emit and absorb equal numbers of photons of frequency ν. This does *not* imply (as the preceding work makes clear) that each individual electron absorbs energy from the radiation field at a rate which exactly balances its emission rate.

Each electron of energy E and pitch angle θ will absorb photons from the radiation field whose mean energy depends on the *shape* of the radiation spectrum ($\propto \nu^{2\cdot 5}$ in this case) but is independent of its intensity, and thus of γ (assuming $\gamma > \frac{1}{3}$). The mean energy of the electrons which emitted these photons does, however, depend on γ, being lower for larger values of γ. There will be one particular value of γ for which the photons are, on average, absorbed by electrons with the same energy as those which emitted them. When γ has this value, but not otherwise, the electrons will absorb and emit energy at equal rates (i.e. $\gamma = \gamma_{\mathrm{crit}}$). This argument would thus provide us with an alternative, but essentially equivalent, method of calculating γ_{crit}.

If $\gamma > \gamma_{\mathrm{crit}}$ each photon in the radiation field is likely to be reabsorbed by an electron with higher energy than the one which emitted it. An individual electron therefore loses energy, and the energy which it radiates will be absorbed by another electron and re-emitted as radiation at (on average) higher frequencies. Most of this energy will escape freely from the source in the form of radiation at frequency $\gtrsim \nu_m$. Thus even the low energy electrons can lose energy, though their synchrotron radiation cannot escape directly. If $\gamma < \gamma_{\mathrm{crit}}$ each electron absorbs energy from the radiation field and re-emits some of it at lower frequencies. The low energy electrons therefore gain energy at the expense of those which radiate at frequencies $\sim \nu_m$.

The preceding discussion has assumed that the magnetic field within the source is uniform, but in fact it still applies if there are irregularities with length scales longer than the mean free path of the radiation under consideration. Our treatment must be modified if there are small scale irregularities, so that radiation emitted at one point in the source is reabsorbed at places where the direction or strength of the magnetic field may be different.

If the magnetic field has strength $\sim H$ but is randomly oriented, the absorption coefficient $\mu(\nu)$ and the emission per unit volume $I(\nu)$ will be independent of direction, and can be calculated by averaging (9) and (10) over all directions. The equilibrium intensity $F(\nu)$ when the optical depth is large is then

$$F(\nu) = \left(\frac{8\pi m^3 c^7}{3e}\right)^{1/2} H^{-1/2} f(\gamma) \left\{ \frac{\Gamma\left(\frac{\gamma+7}{4}\right)\Gamma\left(\frac{\gamma+10}{4}\right)}{\Gamma\left(\frac{\gamma+8}{4}\right)\Gamma\left(\frac{\gamma+9}{4}\right)} \right\} \tag{19}$$

where $f(\gamma)$ is defined in (12).

Since the coefficient in braces is greater than unity, the energy absorbed by an electron moving at right angles to H will, for a given γ, be greater relative to its losses than in the case when the magnetic field is uniform. However the electrons with small pitch angles do not, as in the case when the magnetic field is uniform, see a greater intensity of radiation. Therefore the ratio of the energy gains and losses suffered by individual electrons depends on whether their pitch angles are conserved as they traverse the irregularities in the magnetic field.

The condition that the electrons moving perpendicular to the magnetic field should gain energy is $\gamma < \sim 3\cdot 4$. Therefore, if this condition is satisfied, some electrons will gain energy while others with small pitch angles will suffer a net loss of energy. If $\gamma > 3 \cdot 4$ all the electrons lose energy.

The other situation which is amenable to an exact calculation is that in which the pitch angles of the electrons are frequently randomized. It can be shown that the critical value of γ for which the energy gains and losses of the electrons are in balance is then $\sim 2 \cdot 8$. This result also applies to the case when the magnetic field is uniform and the electrons conserve their pitch angle, if there is some scattering process which isotropizes the radiation field.

Finally we note that if the magnetic field strength is non-uniform the electrons in regions of strong field tend to gain energy at the expense of those in regions where the magnetic field is weak.

4. *Development of the spectrum.* The preceding calculation shows that, in a source with a uniform magnetic field, the electrons which radiate mainly at frequencies $\ll \nu_m(\theta)$ will gain or lose energy according as the index of the initial power law spectrum $\gtrless \gamma_{\text{crit}} = 3$. The rate of change of the energy of each electron is initially proportional to E^2, and so the form of the electron energy spectrum will alter, and will not remain a power law (assuming no further injection of electrons). This situation is in contrast to true thermodynamic equilibrium, in which the average gains and losses of all the electrons (which would then have a Maxwellian distribution) are equal: the electrons of above average energy will tend to lose more energy than they gain, and vice versa, but the relative numbers in different energy intervals is such as to leave the overall distribution unchanged. In the case of a power law spectrum, no temperature is defined, so that, provided the radiation has its equilibrium intensity (i.e. that the source has a high optical depth) all the electrons behave in the same way and the spectrum becomes distorted. The object of this section is to discuss the evolution of the electron spectrum and the associated changes in the radiation emitted from the source.

We suppose that at $t = 0$ the velocities of the electrons are isotropically distributed with energy spectrum $N(E) = K.E^{-\gamma}$. The rate of loss (or gain) of energy is proportional to $(H \sin \theta)^2$, and so if we assume that the electrons conserve their pitch angles their energy distribution will not remain isotropic. However the electrons only interact with radiation travelling almost in their direction of motion. Provided, therefore, that other emission and absorption processes can be neglected, and provided also that the radio photons are not scattered through appreciable angles, we may consider separately the electrons with different pitch angles θ.

The behaviour of the spectrum of the electrons which radiate predominantly at frequencies $\gg \nu_m(\theta)$ is straightforward and very well known. Even if there were no high energy cut-off in the electron spectrum at $t = 0$, at later times $t > 0$ no electrons would survive with energy greater than

$$E_{\text{max}} \sim \frac{m^4 c^7}{e^4 (H \sin \theta)^2 t} \tag{20}$$

The radiation spectrum, which at $t = 0$ obeys a power law of slope $-((\gamma - 1)/2)$ will cut off exponentially above a frequency $\sim \nu_c(E_{\text{max}})$. If $\gamma < 1$ a peak develops in the electron spectrum at $E \sim E_{\text{max}}$ because of the 'piling up' of electrons which initially had high energies.

The situation is more complicated for electrons which radiate at frequencies $\lesssim \nu_m(\theta)$, since the synchrotron losses can then be compensated by absorption.

The calculations of Section 3 tell us whether these electrons initially gain or lose energy. The net rate of loss (or gain) of energy is proportional to E^2, and therefore the slope of the electron spectrum will gradually alter. The intensity of radiation within the source will then no longer be given exactly by (11), and this in turn affects the subsequent rate of change of the energy of the electrons.

A quantitative calculation of the development of the electron spectrum in a source would involve a detailed treatment of the transfer of radiation in a specific model. We shall merely indicate the qualitative behaviour to be expected.

The nature of the evolution obviously depends on whether $\gamma \gtrless \gamma_{\text{crit}} = 3$, and so we discuss the two cases separately.

Case I: $\gamma > \gamma_{\text{crit}}$

This is the simpler of the two cases. Even the low energy electrons for which synchrotron absorption is important will lose energy—the energy which they can absorb from the radiation field is insufficient to balance the spontaneous synchrotron emission. Therefore, because the rate of energy loss $\propto E^2$, the electron spectrum will tend to steepen. A decrease in the equilibrium radiation intensity will be associated with this steepening, and the energy absorbed will then become even less important relative to the emitted energy. The time-scale for appreciable changes to manifest themselves is comparable with the synchrotron lifetime, and so the high frequency cut-off will have moved downwards to $\sim \nu_m$ before the low frequency spectrum is appreciably distorted. The frequency ν_m then decreases, the spectrum at frequencies $\ll \nu_m$ maintaining a slope very close to 2·5, since this slope is not sensitive to the slight distortion of the energy spectrum of the low energy electrons. The lifetime of the electrons in the source does not greatly exceed their synchrotron lifetime in the absence of absorption.

Figs. 4(a) and 4(b) show qualitatively the development of the electron spectrum and the associated changes in the radiation from the source.

Case II: $\gamma < \gamma_{\text{crit}}$

As in the previous case, the high frequency cut-off will fall to $\sim \nu_m$ before any change becomes evident in the spectrum at lower frequencies. The low energy electrons radiating at frequencies $\ll \nu_m$ initially gain energy at a rate proportional to E^2. The consequent flattening of the electron spectrum will result in an increase of the radiation intensity in optically thick regions, so increasing the disparity between the energy gain and loss rates for the electrons. A peak may develop in the electron spectrum, since the low energy electrons gain energy while the high energy electrons emitting at frequencies above ν_m lose energy rapidly. If this peak in the electron distribution becomes sufficiently well marked, the situation will resemble that illustrated in Fig. 2. The spectrum of the radiation at frequencies below ν_m would then not have a slope of 2·5, but there would be a frequency range in which the slope was even steeper. The lifetime of the electrons could be very much greater than their synchrotron lifetime, since those which radiate at frequencies $\sim \nu_m$ could attain an energy distribution in which they absorb energy at a rate which nearly balances their emission.

We note that a power law spectrum with γ exactly equal to γ_{crit} is unstable—if γ is slightly below γ_{crit} the spectrum flattens, whereas if γ exceeds γ_{crit} it steepens. In sources where electrons with $N(E) \propto E^{-\gamma}$ are being injected at a steady rate, we can distinguish two cases: if $\gamma > 2$ the electrons radiating both above and below ν_m will have a power law spectrum with index $-(\gamma+1)$, whereas if $\gamma < 2$ the spectrum of the low energy electrons is distorted from a power law and a peak develops at energies such that $\nu_c(E) \sim \nu_m$.

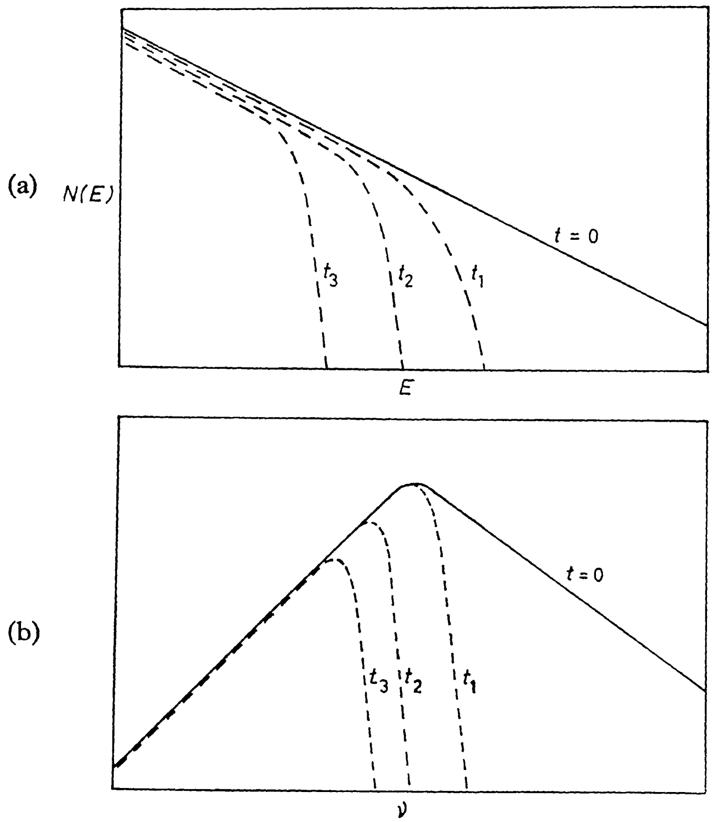

FIG. 4. (a) *Illustration of the evolution of the electron energy distribution in a source with a uniform magnetic field. At* $t=0$ *the electrons have an energy spectrum* $N(E) \propto E^{-\gamma}$ *where* $\gamma > 3$ *(case I). The curves show the form of the distribution at* $t=0$ *and at three later times* t_1, t_1, t_2, *and* t_3. (b) *The radiation spectrum emitted from the source at times* $t=0$, t_2 *and* t_3.

5. *Relation to observations.* This work has shown that the emission and absorption of radio photons in a source opaque to synchrotron radiation at the relevant frequencies will distort a power law electron distribution $N(E) \propto E^{-\gamma}$ in a manner which depends crucially on the value of γ. Furthermore this distortion can cause the slope of the low frequency radio spectrum to depart from the usual value of 2·5. To decide whether these results are applicable to actual radio sources, we must consider whether other losses suffered by the electrons, or deviations of actual sources from the idealized models discussed here, are serious enough to drastically modify our conclusions.

The relativistic electrons in radio sources will lose energy by the inverse Compton effect, whose importance depends on the relative energy density of the radiation and the magnetic field. A necessary condition for inverse Compton losses to be unimportant is that

$$H \gg 2 \times 10^{-7} \nu_m^{7/5} \qquad (21)$$

where H is in gauss, and ν_m is measured in Mc/s. This criterion only takes accoun of radiation at frequencies $\lesssim \nu_m$, and the Compton effect could be more important if a significant intensity of higher frequency radiation were present in the source. If $\gamma < \gamma_{\mathrm{crit}}$, and the electrons are able to set up an energy distribution for which their synchrotron gains and losses balance, the Compton losses could be significant even if (21) is satisfied, since the electrons are then able to survive for much longer than their synchrotron lifetime.

The spectra of some radio sources exhibit a maximum at 10^2–10^3 Mc/s, below which the flux falls off steeply. This is probably due to the occurrence of synchrotron self-absorption at lower frequencies, although there are other absorption effects which might also operate (Hornby & Williams 1966, Williams 1963). The magnetic field strength in these sources, assuming equipartition between the magnetic and particle energy densities, can be as high as 10^{-3}–10^{-2} gauss (though there is evidence in one or two cases that the magnetic field must be weaker than its equipartition value). In a field of this strength the lifetime of the relevant electrons would be short ($< 10^4$ years) compared to the probable lifetime of the source. One might therefore expect to observe signs of deviations from an original power law spectrum in such sources. Neither Compton losses, nor collisional losses, would be likely to be significant in a source with these parameters. The fact that the radio emission from these sources is generally polarized at high radio frequencies indicates that the magnetic field has large-scale regularity, though we would not expect complete uniformity. The model discussed in the previous section may therefore be relevant. The fact that radio spectra usually correspond to a value of $\gamma < 3$ suggests that the sources may resemble our case II.

Observations of these sources are not sensitive enough to distinguish between spectra of slope 2, 2·5, or $> 2 \cdot 5$. Moreover, even if precise observations were available, they would be open to a variety of interpretations. If the electrons had a power law distribution, the existence of inhomogeneities within the source would cause the spectrum to have a slope $< 2 \cdot 5$. The most conclusive evidence for the presence of a non-power law electron energy distribution would be a spectrum steeper than 2·5, though even this could alternatively be due to absorption by ionized gas in the source.

The source 1934–63 (Kellermann 1966) has a peculiar spectrum which peaks at ~ 1000 Mc/s and falls off rapidly both above and below this frequency. It has small linear dimensions, and probably a high magnetic field strength and it may be an example of a source in which the electron distribution has relaxed from a power law.

Quasi-stellar sources may contain components which are opaque up to very high radio frequencies ($\gtrsim 10^5$ Mc/s), and where the magnetic field strength is 1–1000 gauss. The relativistic electrons would almost certainly be continually injected into these regions. However, if they were injected with a power law energy spectrum with $\gamma < 2$, they would quickly attain an approximate Maxwellian distribution in which their synchrotron gains and losses could be in close balance. The main energy losses of the electrons would then be those due to the Compton effect, which are not exactly compensated by absorption. It is possible that the optical continuum from quasars could be emitted in this way (Rees & Sciama 1965, 1966).

The details of the radiation spectrum which would be produced under these conditions, and the associated flux of X-rays and γ-rays which would result from 'higher order' Compton scattering will be discussed in the next paper of this series.

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