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Studies on Colour Image Segmentation Method Based on Finite Left Truncated Bivariate Gaussian Mixture Model with K-Means

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Abstract - Colour Image segmentation is one of the prime requisites for computer vision and analysis. Much work has been reported in literature regarding colour image segmentation under HSI colour space and Gaussian mixture model (GMM). Since the Hue and Saturation values of the pixel in the image are non-negative. And may not be meso-kurtic, it is needed left truncate the Gaussian variate and is used to represent these two features of the colour image. The effect of truncation can not be ignored in developing the model based colour image segmentation. Hence in this paper a left truncated bivariate Gaussian mixture model is utilized to segment the colour image. The correlation between Hue and Saturation plays a predominant role in segmenting the colour images which is observed through experimental results. The expectation maximization (EM) algorithm is used for estimating model parameters. The number of image segments can be initialization of the model parameters are done with K-means algorithm. The performance of the proposed algorithm is studied by calculating the segmentation performance techniques like probabilistic rand index (PRI), global consistency error (GCE) and variation of information (VOI). The utility of the estimated joint probability density function of feature vector of the image is demonstrated through image retrievals. The image quality measures obtained for six images taken from Berkeley image dataset reveals that the proposed algorithm outperforms the existing algorithms in image segmentation and retrievals.

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Studies on Colour Image Segmentation Technique Based on Finite Left Truncated Bivariate Gaussian Mixture Model with K-Means

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Abstract - Colour Image segmentation is one of the prime requisites for computer vision and analysis. Much work has been reported in literature regarding colour image segmentation under HSI colour space and Gaussian mixture model (GMM). Since the Hue and Saturation values of the pixel in the image are non-negative. And may not be meso-kurtic, it is needed left truncate the Gaussian variate and is used to represent these two features of the colour image. The effect of truncation can not be ignored in developing the model based colour image segmentation. Hence in this paper a left truncated bivariate Gaussian mixture model is utilized to segment the colour image. The correlation between Hue and Saturation plays a predominant role in segmenting the colour images which is observed through experimental results. The expectation maximization (EM) algorithm is used for estimating model parameters. The number of image segments can be initialization of the model parameters are done with K-means algorithm. The performance of the proposed algorithm is studied by calculating the segmentation performance techniques like probabilistic rand index (PRI), global consistency error (GCE) and variation of information (VOI). The utility of the estimated joint probability density function of feature vector of the image is demonstrated through image retrievals. The image quality measures obtained for six images taken from Berkeley image dataset reveals that the proposed algorithm outperforms the existing algorithms in image segmentation and retrievals.

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I. Introduction

Image segmentation is a process of extracting useful information from the images through features and dividing the whole image into various homogeneous groups in which, the pixels within the group are more homogeneous and are heterogeneous between the groups. It is an important technology for image processing and understanding. The structural characteristics of objects and surfaces in an image can be determined by segmenting the image using image domain properties. One of the major advantages of image segmentation is denoising. Denoising is the process of removing unwanted noise from the image. Segmentation specifically attempts to separate structure from noise on a local scale. It is one of the most important steps in computer vision and analysis.

For the last three decades lot of work has been reported in literature regarding image segmentation methods (Lucchese L. et al (2001), Srinivas Y. and Srinivas Rao K. (2007), Majid Fakheri et al (2010), Siddhartha Bhattacharyya (2011)). The image segmentation methods can be divided into two categories depending upon the type of image. The images can be broadly categorized into two types namely, gray level images and colour images. A gray level image is usually characterized by pixel intensity (Farag A..A.. et al (2004), Seshashayee M. et al (2011), Srinivas Yerramalle et al (2010)). But in colour images the colour is a perceptual phenomenon related to human response to different wavelengths in the visible electro-magnetic spectrum. In colour images the features that represent the image pixel are highly influenced by three feature descriptions namely, intensity, colour and texture. Among these features colour is the most important one in segmenting the colour images since intensity and texture features also be embedded in colour features. (Fesharaki and Hellestrand (1992), Kato Z. et al (2006), Kang Feng et al (2009), Kaikuo Xu et al (2011)). A better colour space than the RGB space in representing the colours of human perception is the HSI space, in which the colour information is represented by Hue and Saturation values. Thus the human perception of image can be characterized through a bivariate random variable consisting of Hue and Saturation which can be measured using generic structure of a colour appearance model (Sangwine et al (1998)).

Ferri and Vidal (1992), Lee E. et al(2010), Dipti P. and Mridula J. (2011) and others have reviewed colour image segmentation techniques. Among these

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model based image segmentation methods are more efficient than the edge based or threshold or region based methods (Lucchese L. et al (2001)). In model based image segmentation the whole image is divided into different image regions and each image region is characterized by a suitable probability distribution. For ascribing a probability model to the feature vector of the pixels in the image region, it is needed to study the statistical characteristics of the feature vector.

In image segmentation it is customary to consider that the whole image is characterized by a finite Gaussian mixture model. That is, the feature vector of each image region follows a Gaussian distribution (Haralick and Shapiro (1985), Shital Raut et.al (2009), Kato Z. et al (2006), Mantas Paulinas and Audrius Usinskas (2007), Rahman Farnoosh et al (2008), Sujaritha M. and Annadurai S. (2010)). The image segmentation methods based on Gaussian mixture model work well only when the feature vector of the pixels are having infinite range and the distribution of the feature vector is symmetric and meso-kurtic. But in many colour images the feature vector represented by Hue and Saturation are having finite values (say nonnegative) and may not be mesokurtic and symmetric. Hence, to have an accurate image segmentation of these sorts of colour images it is needed to develop and analyze image segmentation methods based on truncated bivariate mixture distributions.

Here, it is assumed that the feature vector in different image regions follows a left truncated bivariate Gaussian distribution and the feature vector of the whole image is characterized by a finite left truncated bivariate Gaussian mixture model. This assumption is made since the Hue and Saturation values of the pixel which represents the bivariate feature vector can take nonnegative values only. Hence, the range of the Hue and Saturation values are to be left truncated at zero. The effect of the truncated nature of Hue and Saturation cannot be ignored, since the leftover probability is significantly higher than zero in the left tail end of the distribution. This left truncated nature of the bivariate feature vector can approximate the pixels of the colour image more close to the reality.

In this method of segmentation, the number of image regions is obtained by *K*-means algorithm for which the initial value of the number of components is identified from the number of peaks in the image histogram. The model parameters are estimated by using Expectation Maximization (EM) algorithm. The EM-algorithm is one of the most preferred method of estimating the model parameters in mixture distributions (Mclanchlan G. and Krishnan T. (1997)). The EM-algorithm requires the updated equations of the model parameters which are derived for the left truncated bivariate Gaussian mixture model. The initialization of the model parameters for carrying the EM-algorithm is done through feature vector of the pixel intensities of the image regions obtained through *K*-means clustering and moment method of estimation. An image segmentation algorithm with component likelihood maximization under Bayesian frame work is also developed and analyzed.

The efficiency of the developed image segmentation algorithm is studied by conducting experimentation with six images namely, OSTRICH, POT, TOWER, BEARS, DEER and BIRD which are taken randomly from Berkeley image data set. The segmentation performance measures namely, probabilistic rand index (PRI), global consistency error (GCE) and variation of information (VOI) are computed for the six images and presented. A comparative study of these measures with those obtained from the finite Gaussian mixture model reveals that this algorithm performs better than the Gaussian mixture model with *K*-means and having clear boundaries.

Using the estimated joint probability density functions of the feature vector of pixels of each image, the images are retrieved. The efficiency of the developed algorithm in image retrieval is also studied by computing the image quality metrics like maximum distance, image fidelity, mean square error, signal to noise ratio and image quality index and the results are presented. A comparative study of these quality measures with those obtained from the Gaussian mixture model with *K*-means revealed that this algorithm performs better.

II. Finite Left Truncated Bivariate Gaussian Mixture Model

The effect of truncation in bivariate Gaussian distribution has been discussed by several researchers (Norman L.Johnson, Samuel Kotz and Balakrishnan (1994)). The probability density function of the left truncated Gaussian distribution (truncated at zero) is,

$$g(x,y;\theta)=\frac{f(x,y)}{\int_0^\infty\int_0^\infty f(x,y)\,dxdy}\ ,\ 0<x<\infty\,;\,0<y<\infty \qquad (1)$$

Where, zero is the truncation point for both the Hue and saturation, $f(x,y)$ is the probability density function of the bivariate Normal distribution is

$$f(x,y)=\frac{1}{2\pi\sqrt{1-\rho^2}\sigma_1\sigma_2}\exp\left\{\frac{-1}{2(1-\rho^2)}\left[\left(\frac{x-\mu_1}{\sigma_1}\right)^2-2\rho\left(\frac{x-\mu_1}{\sigma_1}\right)\left(\frac{y-\mu_2}{\sigma_2}\right)+\left(\frac{y-\mu_2}{\sigma_2}\right)^2\right]\right\}$$

$$-\infty<x<+\infty\,;\,-\infty<y<+\infty,$$
$$\sigma_1>0\,;\,\sigma_2>0\,;\,-1<\rho<1,$$
$$-\infty<\mu_1<+\infty\,;\,-\infty<\mu_2<+\infty \qquad (2)$$

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The value of $\left[1-\int_0^\infty\int_0^\infty f(x,y)\,dxdy\right]$ is significant based on the values of the parameters. This distribution includes the skewed, asymmetric bivariate distributions as particular cases for limiting and specific values of the parameters. The various shapes of the frequency curves of the left truncated bivariate Gaussian distribution are shown in Figure1.

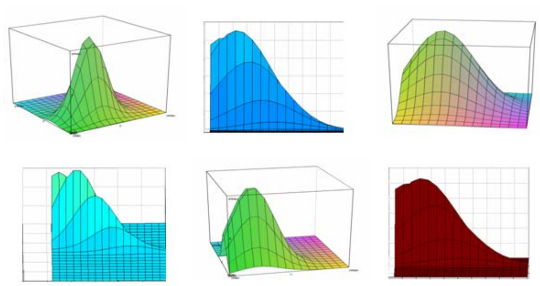

Fig1 : Shapes of left truncated bivariate Gaussian frequency surfaces

Following the heuristic arguments given by Bengt Muthen (1990), the mean value of 'X'(hue) is obtained as

$$\mathrm{E(X)} = \mu_1 + \sigma_1 \mathrm{A} \quad (3)$$

Where,

$$\mathrm{A} = \phi\left(\frac{-\mu_1}{\sigma_1}\right)\left[1-\Phi\left[\left(\left(\frac{-\mu_1}{\sigma_1}\right)-\rho\left(\frac{-\mu_1}{\sigma_1}\right)\right)c\right]\right]+\rho\phi\left(\frac{-\mu_2}{\sigma_2}\right)\left[1-\Phi\left[\left(\left(\frac{-\mu_1}{\sigma_1}\right)-\rho\left(\frac{-\mu_2}{\sigma_2}\right)\right)c\right]\right]$$

and $c = \left(1-\rho^2\right)^{-1/2}$, ϕ, Φ are the ordinate and area of standard Normal distribution. Similarly the mean value 'Y'(saturation) is

$$\mathrm{E(Y)} = \mu_2 + \sigma_2 \mathrm{B} \quad (4)$$

Where,

$$\mathrm{B} = \phi\left(\frac{-\mu_2}{\sigma_2}\right)\left[1-\Phi\left[\left(\left(\frac{-\mu_2}{\sigma_2}\right)-\rho\left(\frac{-\mu_2}{\sigma_2}\right)\right)c\right]\right]+\rho\phi\left(\frac{-\mu_1}{\sigma_1}\right)\left[1-\Phi\left[\left(\left(\frac{-\mu_2}{\sigma_2}\right)-\rho\left(\frac{-\mu_1}{\sigma_1}\right)\right)c\right]\right]$$

and c is as given in equation (3)

The Variance of X is

$$\mathrm{V(X)} = \sigma_1^2\,\mathrm{R} - 2\mathrm{A}\,\sigma_1\,\mathrm{A} + \mathrm{A}^2$$

$$= \sigma_1^2\,\mathrm{R} - \mathrm{A}^2(2\sigma_1 - 1) \quad (5)$$

Where,

$$\mathrm{R} = \left[\pi+\left(\frac{-\mu_1}{\sigma_1}\right)\phi\left(\frac{-\mu_1}{\sigma_1}\right)\left[1-\Phi\left[\left(\left(\frac{-\mu_1}{\sigma_1}\right)-\rho\left(\frac{-\mu_1}{\sigma_1}\right)\right)c\right]\right]+\rho^2\left(\frac{-\mu_1}{\sigma_1}\right)\phi\left(\frac{-\mu_1}{\sigma_1}\right)\left[1-\Phi\left[\left(\left(\frac{-\mu_1}{\sigma_1}\right)-\rho\left(\frac{-\mu_1}{\sigma_1}\right)\right)c\right]\right] - c^{-1}\rho\phi\left(\frac{-\mu_1}{\sigma_1}\right)\left[-\phi\left[\left(\left(\frac{-\mu_1}{\sigma_1}\right)-\rho\left(\frac{-\mu_1}{\sigma_1}\right)\right)c\right]\right]\right],$$

and c and A is given in equation (3). The Variance of Y is

$$\mathrm{V(Y)} = \sigma_2^2\,\mathrm{T} - 2\mathrm{B}\,\sigma_2\,\mathrm{B} + \mathrm{B}^2$$

$$= \sigma_2^2\,\mathrm{T} - \mathrm{B}^2(2\sigma_2 - 1) \quad (6)$$

Where,

$$\mathrm{T} = \left[\pi+\left(\frac{-\mu_2}{\sigma_2}\right)\phi\left(\frac{-\mu_2}{\sigma_2}\right)\left[1-\Phi\left[\left(\left(\frac{-\mu_2}{\sigma_2}\right)-\rho\left(\frac{-\mu_2}{\sigma_2}\right)\right)c\right]\right]+\rho^2\left(\frac{-\mu_2}{\sigma_2}\right)\phi\left(\frac{-\mu_2}{\sigma_2}\right)\left[1-\Phi\left[\left(\left(\frac{-\mu_2}{\sigma_2}\right)-\rho\left(\frac{-\mu_2}{\sigma_2}\right)\right)c\right]\right] - c^{-1}\rho\phi\left(\frac{-\mu_2}{\sigma_2}\right)\left[-\phi\left[\left(\left(\frac{-\mu_2}{\sigma_2}\right)-\rho\left(\frac{-\mu_2}{\sigma_2}\right)\right)c\right]\right]\right],$$

c and B are as given in equations (3) and (4) respectively. The Covariance of (X, Y) is

$$\mathrm{COV\,(X, Y)} = \sigma_1\sigma_2\mathrm{U} - \mathrm{AB}\,(\sigma_1 + \sigma_2 - 1) \quad (7)$$

where,

$$\mathrm{U} = \left[\rho\pi+\rho\left(\frac{-\mu_1}{\sigma_1}\right)\phi\left(\frac{-\mu_1}{\sigma_1}\right)\left[1-\Phi\left[\left(\left(\frac{-\mu_2}{\sigma_2}\right)-\rho\left(\frac{-\mu_1}{\sigma_1}\right)\right)c\right]\right] - c^{-1}\phi\left(\frac{-\mu_1}{\sigma_1}\right)\left[-\phi\left[\left(\left(\frac{-\mu_2}{\sigma_2}\right)-\rho\left(\frac{-\mu_1}{\sigma_1}\right)\right)c\right]\right] + \rho\left(\frac{-\mu_2}{\sigma_2}\right)\phi\left(\frac{-\mu_2}{\sigma_2}\right)\left[1-\Phi\left[\left(\left(\frac{-\mu_1}{\sigma_1}\right)-\rho\left(\frac{-\mu_2}{\sigma_2}\right)\right)c\right]\right]\right],$$

c, A and B are as given in equations (3) and (4) respectively.

Since the entire image is a collection of regions, which are characterized by left truncated bivariate normal distribution, it can be characterized through a *K*-Component finite left truncated bivariate Gaussian distribution and its probability density function is of the form

$$h(x,y) = \sum_{i=1}^{K}\alpha_i g_i(x_i, y_i; \theta) \quad (8)$$

Where, *K* is the number of regions, $\alpha_i > 0$ are weights such that $\sum_{i=1}^{K}\alpha_i = 1$ and $\theta = \left\{\mu_{1i}, \mu_{2i}, \sigma_{1i}^2, \sigma_{2i}^2, \rho_i\right\}$ is the set of parameters. $g_i(x_i, y_i / \theta_i)$ given in equation (1) represent the probability density function of the i^{th} image region. α_i is the probability of occurrence of the i^{th} component of the finite left truncated bivariate Gaussian mixture model (FLTBGMM) i.e., the probability that the feature belongs to the i^{th} image region.

The mean vector representing the entire image is

$$E(\mathrm{W}^{\mathrm{T}}) = \begin{bmatrix} \sum_{i=1}^{K}\alpha_i E_i(X) \\ \sum_{i=1}^{K}\alpha_i E_i(Y) \end{bmatrix} \quad (9)$$

Where, $E(X_i)$ and $E(Y_i)$ are given in equations (3) and (4) for the i^{th} image region.

III. Estimation of the Model Parameters by EM-Algorithm

To obtain the estimation of the model parameters, we utilized the EM-algorithm by maximizing the expected likelihood function for carrying out the EM-algorithm. The likelihood function of bivariate observations $(x_1, y_1), (x_2, y_2), (x_3, y_3), \ldots, (x_N, y_N)$ drawn from an image with probability density function

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$$L(\theta) = \prod_{s=1}^{N} h(x_s, y_s; \theta)$$

$$= \prod_{s=1}^{N} \left(\sum_{i=1}^{K} \alpha_i g_i(x_s, y_s; \theta) \right) \quad (10)$$

$$= \prod_{s=1}^{N} \left(\sum_{i=1}^{K} \alpha_i \frac{\exp\left\{ \frac{-1}{2(1-\rho_i^2)} \left[\left(\frac{x_s - \mu_{1i}}{\sigma_{1i}} \right)^2 - 2\rho_i \left(\frac{x_s - \mu_{1i}}{\sigma_{1i}} \right) \left(\frac{y_s - \mu_{2i}}{\sigma_{2i}} \right) + \left(\frac{y_s - \mu_{2i}}{\sigma_{2i}} \right)^2 \right] \right\}}{2\pi \sqrt{1-\rho_i^2}\, \sigma_{1i} \sigma_{2i} \int_0^\infty \int_0^\infty f_i(x, y; \theta)\, dx\, dy} \right)$$

This implies

$$\log L(\theta) = \sum_{s=1}^{N} \log \left(\sum_{i=1}^{K} \alpha_i g_i(x_s, y_s; \theta) \right) \quad (11)$$

The updated equations of EM-algorithm for estimating the model parameters are

$$\alpha_k^{(l+1)} = \frac{1}{N} \sum_{s=1}^{N} \left[t_k(x_s, y_s; \theta^{(l)}) \right]$$

$$= \frac{1}{N} \sum_{s=1}^{N} \left(\frac{\alpha_k^{(l)} g_k(x_s, y_s; \theta^{(l)})}{\sum_{i=1}^{K} \alpha_k^{(l)} g_i(x_s, y_s; \theta^{(l)})} \right) \quad (12)$$

Where, $g_k(x_s, y_s; \theta^{(l)})$ is as given in equation (1).

For updating μ_{1k} we have,

$$\mu_{1k}^{(l+1)} \sum_{s=1}^{N} t_k(x_s, y_s; \theta^{(l)}) - \sum_{s=1}^{N} t_k(x_s, y_s; \theta^{(l)}) x_s + \sum_{s=1}^{N} t_k(x_s, y_s; \theta^{(l)}) \sigma_{1k}^{(l)} \left[\frac{\rho_k^{(l)}(y_s - \mu_{2k}^{(l)})}{\sigma_{2k}^{(l)}} + \left[A - \rho_k^{(l)} B \right] \right] = 0 \quad (13)$$

Similarly for updating μ_{2k} , we have ,

$$\mu_{2k}^{(l+1)} \sum_{s=1}^{N} t_k(x_s, y_s; \theta^{(l)}) - \sum_{s=1}^{N} t_k(x_s, y_s; \theta^{(l)}) y_s + \sum_{s=1}^{N} t_k(x_s, y_s; \theta^{(l)}) \sigma_{2k}^{(l)} \left[\frac{\rho_k^{(l)}(x_s - \mu_{1k}^{(l)})}{\sigma_{1k}^{(l)}} + \left[B - \rho_k^{(l)} A \right] \right] = 0 \quad (14)$$

Where,

$$t_k(x_s, y_s; \theta^{(l)}) = \frac{\alpha_k^{(l)} g_k(x_s, y_s; \theta^{(l)})}{h(x_s, y_s; \theta^{(l)})} = \frac{\alpha_k^{(l)} g_k(x_s, y_s; \theta^{(l)})}{\sum_{i=1}^{K} \alpha_i^{(l)} g_i(x_s, y_s; \theta^{(l)})} ,$$

$$g_k(x_s, y_s; \theta^{(l)}) = \frac{\exp\left\{ \frac{-1}{2(1-\rho_k^2)} \left[\left(\frac{x_s - \mu_{1k}}{\sigma_{1k}} \right)^2 - 2\rho_k \left(\frac{x_s - \mu_{1k}}{\sigma_{1k}} \right) \left(\frac{y_s - \mu_{2k}}{\sigma_{2k}} \right) + \left(\frac{y_s - \mu_{2k}}{\sigma_{2k}} \right)^2 \right] \right\}}{2\pi \sigma_{1k} \sigma_{2k} \sqrt{1-\rho_k^2} \int_0^\infty \int_0^\infty f_k(x, y; \theta)\, dx\, dy}$$

A and B are as given in equations (3) and (4) respectively. The updated equations for σ_{1k}^2 at $(l+1)^{th}$ iteration is,

$$\sum_{s=1}^{N} t_k(x_s, y_s; \theta^{(l)}) \left[\left[\left(\frac{x_s - \mu_{1k}^{(l)}}{\sigma_{1k}^{(l+1)}} \right)^2 - \frac{\rho_k (x_s - \mu_{1k}^{(l)})(y_s - \mu_{2k}^{(l)})}{\sigma_{1k}^{(l+1)} \sigma_{2k}^{(l)}} \right] - D + \rho_k^{(l)} E \right] = 0 \qquad (15)$$

Where, $t_k(x_s, y_s; \theta^{(l)})$ is given in equation (14),

$$D = \pi \sigma_{1k} \sigma_{2k} + \sigma_{1k} \sigma_{2k} c^{-1} \rho_k \left[\phi\left(\frac{-\mu_{1k}}{\sigma_{1k}} \right) \left[\phi \left[\left((\frac{-\mu_{1k}}{\sigma_{1k}}) - \rho_k (\frac{-\mu_{1k}}{\sigma_{1k}}) \right) c \right] \right] \right]$$

$$+ \sigma_{2k} \left(\rho_k^2 + 1 \right) (-\mu_{1k}) \phi\left(\frac{-\mu_{1k}}{\sigma_{1k}} \right) \left[1 - \Phi \left[\left((\frac{-\mu_{1k}}{\sigma_{1k}}) - \rho_k (\frac{-\mu_{1k}}{\sigma_{1k}}) \right) c \right] \right],$$

And

$$E = \rho_k \pi \sigma_{1k} \sigma_{2k} + \sigma_{1k} \sigma_{2k} c^{-1} \left[-\phi\left(\frac{-\mu_{1k}}{\sigma_{1k}} \right) \left[-\phi \left[\left((\frac{-\mu_{2k}}{\sigma_{2k}}) - \rho_k (\frac{-\mu_{1k}}{\sigma_{1k}}) \right) c \right] \right] \right]$$

$$+ \rho_k \sigma_{1k} \left[(-\mu_{2k}) \phi\left(\frac{-\mu_{2k}}{\sigma_{2k}} \right) \left[1 - \Phi \left[\left((\frac{-\mu_{1k}}{\sigma_{1k}}) - \rho_k (\frac{-\mu_{2k}}{\sigma_{2k}}) \right) c \right] \right] \right]$$

$$+ \rho_k \sigma_{2k} \left[(-\mu_{1k}) \phi\left(\frac{-\mu_{1k}}{\sigma_{1k}} \right) \left[1 - \Phi \left[\left((\frac{-\mu_{2k}}{\sigma_{2k}}) - \rho_k (\frac{-\mu_{1k}}{\sigma_{1k}}) \right) c \right] \right] \right]$$

The updated equations for σ_{2k}^2 at $(l+1)^{\text{th}}$ iteration is

$$\sum_{s=1}^{N} t_k(x_s, y_s; \theta^{(l)}) \left[\left[\left(\frac{y_s - \mu_{1k}^{(l)}}{\sigma_{2k}^{(l+1)}} \right)^2 - \frac{\rho_k (x_s - \mu_{1k}^{(l)})(y_s - \mu_{2k}^{(l)})}{\sigma_{1k}^{(l)} \sigma_{2k}^{(l+1)}} \right] - G + \rho_k^{(l)} E \right] = 0 \qquad (16)$$

where,

$$G = \pi \sigma_{1k} \sigma_{2k} + \sigma_{1k} \sigma_{2k} c^{-1} \rho_k \left[-\phi\left(\frac{-\mu_{2k}}{\sigma_{2k}} \right) \left[-\phi \left[\left((\frac{-\mu_{2k}}{\sigma_{2k}}) - \rho_k (\frac{-\mu_{2k}}{\sigma_{2k}}) \right) c \right] \right] \right]$$

$$+ \sigma_{1k} \left(\rho_k^2 + 1 \right) (-\mu_{2k}) \phi\left(\frac{-\mu_{2k}}{\sigma_{2k}} \right) \left[1 - \Phi \left[\left((\frac{-\mu_{2k}}{\sigma_{2k}}) - \rho_k (\frac{-\mu_{2k}}{\sigma_{2k}}) \right) c \right] \right],$$

$t_k(x_s, y_s; \theta^{(l)})$ and E are as given in equations (14) and (15) respectively and

Therefore the updated equation for estimating ρ_k is

$$\sum_{s=1}^{N} t_k(x_s, y_s; \theta^{(l)}) \left[-\frac{\rho_k}{(1-\rho_k^2)^2} \left[\left(\frac{x_s - \mu_{1k}}{\sigma_{1k}} \right)^2 + \left(\frac{y_s - \mu_{2k}}{\sigma_{2k}} \right)^2 \right] \right. \qquad (17)$$

$$\left. -\frac{1+\rho_k^2}{(1-\rho_k^2)^2} \left[\left(\frac{x_s - \mu_{1k}}{\sigma_{1k}} \right) \left(\frac{y_s - \mu_{2k}}{\sigma_{2k}} \right) \right] + \frac{\rho_k (D+F)}{(1-\rho_k^2)^2} + \frac{(1+\rho_k^2) E}{(1-\rho_k^2)^2} \right] = 0$$

Where, $t_k(x_s, y_s; \theta^{(l)})$, D, E and G are as given in equations (14) ,(15) and (16) respectively and

$$F = \pi\sigma_{1k}\sigma_{2k} - \sigma_{1k}\left[(\mu_{2k})\ \phi\left(\frac{-\mu_{2k}}{\sigma_{2k}}\right)\left[1-\Phi\left[\left(\left(\frac{-\mu_{2k}}{\sigma_{2k}}\right) - \rho_k\left(\frac{-\mu_{2k}}{\sigma_{2k}}\right)\right)c\right]\right]\right]$$

$$+\sigma_{1k}\rho_k^{\,2}\left[(-\mu_{2k})\phi\left(\frac{-\mu_{2k}}{\sigma_{2k}}\right)\left[1-\Phi\left[\left(\left(\frac{-\mu_{2k}}{\sigma_{2k}}\right) - \rho_k\left(\frac{-\mu_{2k}}{\sigma_{2k}}\right)\right)c\right]\right]\right]$$

$$+\sigma_{1k}\sigma_{2k}\rho_k c^{-1}\left[-\phi\left(\frac{-\mu_{2k}}{\sigma_{2k}}\right)\left[-\phi\left[\left(\left(\frac{-\mu_{2k}}{\sigma_{2k}}\right) - \rho_k\left(\frac{-\mu_{2k}}{\sigma_{2k}}\right)\right)c\right]\right]\right]$$

Solving equations (12), (13), (14), (15), (16) and (17) iteratively we get the revised estimates α_k μ_{1k}, μ_{2k}, σ_{1k}^2, σ_{2k}^2 and ρ_k.

IV. Initialization of Parameters by K-Means

The efficiency of the EM-algorithm in estimating the parameters is heavily dependent on the number of regions in the image. The number of mixture components taken for *K*–means algorithm is obtained, by plotting the histogram of the pixel intensities of the whole image. The mixing parameter α_k and the region parameters μ_{1k}, μ_{2k}, σ_{1k}^2, σ_{2k}^2, ρ_k are unknown as prior. A commonly used method in initializing parameters is by drawing a random sample in the entire image data (Mclanchan G. and Peel D. (2000)). This method performs well, if the sample size is large, and the computation time is heavily increased. When the sample size is small, some small regions may not be sampled. To overcome this problem, we use *K*-means algorithm to divide the whole image into various homogeneous regions. In *K*-means algorithm the centroids of the clusters are recomputed as soon as pixel joins the cluster.

The initial values of α_i can be taken as $\alpha_i = \frac{1}{K}$, where, *K* is the number of image regions obtained from the *K*-means algorithm (Rose H. Turi (2001)). *K*-means algorithm uses an iterative procedure that minimizes the sum of distances from each object to its cluster centroid, over all clusters. This procedure consists of the following steps.

1) Randomly choose *K* data points from the whole dataset as initial clusters. These data points represent initial cluster centroids.
2) Calculate Euclidean distance of each data point from each cluster centre and assign the data points to its nearest cluster centre.
3) Calculate new cluster centre so that squared error distance of each cluster should be minimum.
4) Repeat step 2 and 3 until clustering centers do not change.
5) Stop the process.

In the above algorithm, the cluster centers are only updated once all points have been allocated to their closed cluster centre. The advantage of *K*-means are that it is a very simple method, and it is based on intuition about the nature of a cluster, which is that the within cluster error should be as small as possible. The disadvantage of this method is that the number of clusters must be supplied as a parameter, leading to the user having to decide what the best number of clusters for the image is (Rose H.Turi, (2001)) .Success of *K*-means algorithm depends on the parameter *K*, number of clusters in image. After determining the final values of *K* (number of regions), we obtain the initial estimates of the parameters μ_{1k}, μ_{2k}, σ_{1k}^2, σ_{2k}^2, ρ_k and α_k for each image region and with the method of moments given by Bengt Muthen (1990) for Truncated Bivariate Normal Distribution with initial parameters as $\alpha_i = 1/K$ for i=1,2,…,*K*

$\mu_{1k} = \overline{x}_{1k}$ is the k^th^ region sample mean of the Hue angle.

$\mu_{2k} = \overline{y}_{1k}$ is the k^th^ region sample mean of the Saturation.

$\sigma_{1k} = s_{1k}$ (Sample Standard Deviation of the k^th^ segment of Hue - angle)

$\sigma_{2k} = s_{2k}$ (Sample Standard Deviation of the k^th^ segment of - Saturation)

ρ_k is the correlation coefficient between Hue and Saturation of the k^{th} image region.

Substituting these values as the initial estimates, we obtain the refined estimates of the parameters by using the EM-algorithm.

V. Segmentation Algorithm

After refining the parameters the prime step is image segmentation, by allocating the pixels to the segments. This operation is performed by segmentation algorithm. The image segmentation algorithm consists of four steps

Step 1) Plot the histogram of the whole image.
Step2) Obtain the initial estimates of the model parameters using *K*-Means algorithm and moment estimators as discussed in section IV.
Step3) Obtain the refined estimates of the model parameters μ_{1k}, μ_{2k}, σ_{1k}^2, σ_{2k}^2, ρ_k and α_k for i= 1,2,…,*K* using the EM-algorithm with the updated equations given in section III.

Step4) Assign each pixel into the corresponding j[th] region (segment) according to the maximum likelihood of the j[th] component L_j.

That is

$$L_j = \max_{j \in k} \left\{ \frac{\exp\left\{ \frac{-1}{2(1-\rho_k^2)} \left[\left(\frac{x_s - \mu_{1k}}{\sigma_{1k}} \right)^2 - 2\rho_k \left(\frac{x_s - \mu_{1k}}{\sigma_{1k}} \right) \left(\frac{y_s - \mu_{2k}}{\sigma_{2k}} \right) + \left(\frac{y_s - \mu_{2k}}{\sigma_{2k}} \right)^2 \right] \right\}}{2\pi\sigma_{1k}\sigma_{2k}\sqrt{1-\rho_k^2} \int_0^\infty \int_0^\infty f_k(x, y, \theta)\, dx dy} \right\}$$

VI. EXPERIMENTAL RESULTS

To demonstrate the utility of the image segmentation algorithm developed in this paper, an experiment is conducted with six images taken from Berkeley images dataset (http://www.eecs.berkeley.edu/Research/Projects/CS/vision/bsds/BSDS300/html/dataset/images.html.). The images namely, OSTRICH, POT, TOWER, BEARS, DEER and BIRD are considered for image segmentation. The feature vectors of the whole image is taken as input for image segmentation. The feature vector of the image are assumed to follow a mixture of left truncated bivariate Gaussian distribution. That is, the image contains K regions and the feature vector of the each image region follow a left truncated bivariate Gaussian distribution with different parameters. The number of segments in each of the six images considered for experimentation is determined by the histogram of pixel intensities. The histograms of the six images are shown in Figure 2.

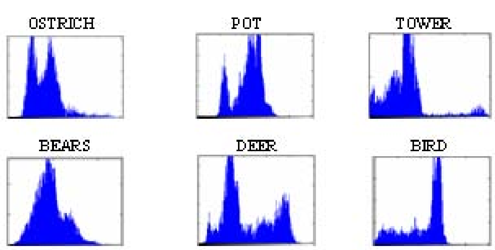

Figure 2 : Histograms Of The Images

The initial estimates of the number of regions K in each image are obtained and given in Table 1.

Table1 : Initial Estimates of K

IMAGE	OSTRICH	POT	TOWER	BEARS	DEER	BIRD
Estimate of K	2	3	4	2	4	3

After assigning these initial values of K to each image data set, the K-means algorithm is performed. The initial values of the model parameters $\mu_{1i}, \mu_{2i}, \sigma_{1i}^2, \sigma_{2i}^2, \rho_i$ and α_i for i=1,2,…,K for each image region of the images are computed by using the method given in section IV. Using these initial estimates, the refined estimates of the model parameters for each image region are obtained by using EM-algorithm given in section III. The computed values of the initial estimates and the final estimates of the model parameters K, $\mu_{1i}, \mu_{2i}, \sigma_{1i}^2, \sigma_{2i}^2, \rho_i$ and α_i for i=1,2…,K for each image are shown in Tables **-2a, 2b, 2c, 2d, 2e and 2f** for different images.

Table-2a
Estimated Values of The Parameters For OSTRICH Image
Number of image regions (K=2)

Parameters	Estimation of Initial Parameters by K-means		Estimation of Final Parameters by EM-Algorithm	
	Regions(i)		Regions(i)	
	1	2	1	2
α_i	1/2	1/2	0.2627	0.7373
μ_{1i}	0.1781	0.1940	0.2054	0.2798
μ_{2i}	0.3321	0.7613	0.2505	0.7775
σ_{1i}^2	0.0016	0.0004	0.0287	0.0772
σ_{2i}^2	0.0126	0.0207	0.0747	0.0768
ρ_i	-0.4310	0.6996	-0.6163	0.2840

Table-2b
Estimated Values of The Parameters For POT Image
Number of image regions (K=3)

Parameters	Estimation of Initial Parameters by K-means			Estimation of Final Parameters by EM-Algorithm		
	Regions(i)			Regions(i)		
	1	2	3	1	2	3
α_i	1/3	1/3	1/3	0.4888	0.2019	0.3093
μ_{1i}	0.5532	0.4946	0.1517	0.5505	0.5223	0.3089
μ_{2i}	0.2168	0.1125	0.1219	0.1958	0.0810	0.1106
σ_{1i}^2	0.0004	0.0027	0.0029	0.0248	0.0269	0.6202
σ_{2i}^2	0.0008	0.0018	0.0035	0.0358	0.0328	0.0469
ρ_i	0.1666	0.3570	-0.7230	-0.4604	0.9867	0.1373

Table-2c
Estimated Values of The Parameters For TOWER Image
Number of Image Regions (K =4)

Parameters	Estimation of Initial Parameters by K-means				Estimation of Final Parameters by EM-Algorithm			
	Regions(i)				Regions(i)			
	1	2	3	4	1	2	3	4
α_i	1/4	1/4	1/4	1/4	0.1999	0.1523	0.1872	0.4606
μ_{1i}	0.1519	0.5699	0.1505	0.5738	0.4011	0.6276	0.1616	0.5743
μ_{2i}	0.1937	0.2789	0.6176	0.7724	0.2408	0.3519	0.5047	0.7721
σ_{1i}^2	0.0033	0.0073	0.0011	0.0006	0.1268	0.1045	0.0043	0.0213
σ_{2i}^2	0.0148	0.0148	0.0291	0.0059	0.1055	0.1696	0.6936	0.0539
ρ_i	0.1561	-0.0259	0.0386	-0.1086	0.9061	0.7744	-0.1902	0.1944

Table-2d
Estimated Values of The Parameters For BEARS Image
Number of Image Regions (K =2)

Parameters	Estimation of Initial Parameters by K-means		Estimation of Final Parameters by EM-Algorithm	
	Regions(i)		Regions(i)	
	1	2	1	2
α_i	1/2	1/2	0.4531	0.5469
μ_{1i}	0.4787	0.2364	0.4867	0.4067
μ_{2i}	0.4532	0.2600	0.4171	0.3375
σ_{1i}^2	0.0027	0.0154	0.0560	0.0667
σ_{2i}^2	0.0129	0.0170	0.0786	0.1439
ρ_i	0.2044	-0.6378	0.1263	0.6274

Table-2e
Estimated Values of The Parameters For DEER Image
Number of Image Regions (K =4)

Parameters	Estimation of Initial Parameters by K-means				Estimation of Final Parameters by EM-Algorithm			
	Regions(i)				Regions(i)			
	1	2	3	4	1	2	3	4
α_i	1/4	1/4	1/4	1/4	0.0703	0.4769	0.2775	0.1753
μ_{1i}	0.1299	0.1144	0.2324	0.3016	0.1388	0.1969	0.2349	0.3185
μ_{2i}	0.6989	0.4560	0.2354	0.1262	0.6847	0.4538	0.2045	0.1039
σ_{1i}^2	0.0004	0.0001	0.0015	0.0026	0.1153	0.1372	0.0366	0.0650
σ_{2i}^2	0.0104	0.0018	0.0019	0.0011	0.1435	0.0125	0.0525	0.1987
ρ_i	-0.1355	-0.0338	-0.0833	-0.0591	-0.0868	-0.0264	-0.2712	-0.2248

Table-2f Estimated Values of The Parameters For BIRD Image Number of Image Regions (K=3)						
Parameters	Estimation of Initial Parameters by K-means			Estimation of Final Parameters by EM-Algorithm		
	Regions(i)			Regions(i)		
	1	2	3	1	2	3
α_i	1/3	1/3	1/3	0.1029	0.6941	0.2031
μ_{1i}	0.1290	0.5948	0.1425	0.1677	0.6833	0.1946
μ_{2i}	0.6899	0.1143	0.2136	0.6031	0.0965	0.0722
σ_{1i}^2	0.0047	0.0029	0.0048	0.0242	0.2669	0.0237
σ_{2i}^2	0.0330	0.0015	0.0150	0.1369	0.0135	0.5790
ρ_i	0.0834	-0.0504	-0.1409	-0.4398	0.1672	-0.0101

Substituting the final estimates of the model parameters, the probability density function of the feature vector of each image are estimated. Using the estimated probability density functions and the image segmentation algorithm given in section V, the image segmentation is done for each of the six images under consideration. The original and segmented images are shown in Figure 3.

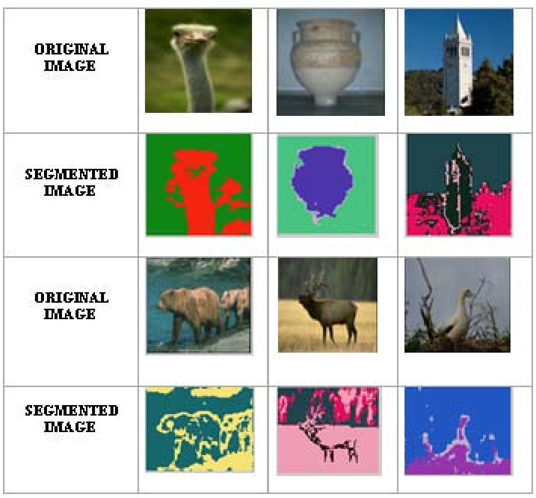

Figure 3 : Original and Segmented Images

VII. Performance Evaluation

After conducting the experiment with the image segmentation algorithm developed in this paper, its performance is studied. The comparison is based on three performance measures namely, Probabilistic Rand Index (PRI) given by Unnikrishnan R. and et.al (2007), the Variation of Information (VOI) given by Meila M. (2005), and Global Consistency error (GCE) given by Martin D. and et al (2001). The objective of the segmentation methods are based on regional similarity measures in relations to their local neighborhood.

The performance of developed algorithm using finite left truncated bivariate Gaussian mixture model (FLTBGMM) is studied by computing the segmentation performance measures namely, PRI, GCE and VOI for the six images under study. The computed values of the performance measures for the developed algorithm and the earlier existing finite Gaussian mixture model (GMM) with K-means algorithm are presented in Table 3 for a comparative study.

Table 3 : Segmentation performance measures

IMAGE	METHOD	PERFORMANCE MEASURES		
		PRI	GCE	VOI
OSTRICH	GMM	0.9234	0.4317	2.2761
	FLTBGMM-K	0.9782	0.4037	1.7611
POT	GMM	0.9456	0.4281	2.5973
	FLTBGMM-K	0.9796	0.4131	1.9263
TOWER	GMM	0.9615	0.4469	3.7121
	FLTBGMM-K	0.9816	0.4302	2.8194
BEARS	GMM	0.9121	0.4418	3.2693
	FLTBGMM-K	0.9831	0.4337	2.6421
DEER	GMM	0.9774	0.4829	2.2863
	FLTBGMM-K	0.9847	0.4030	1.3947
BIRD	GMM	0.9673	0.4671	2.7197
	FLTBGMM-K	0.9705	0.4226	2.3244

From the above Table 3, It is observed that the PRI values of the proposed algorithm for the six images considered for experimentation are more than that of the values from the segmentation algorithm based on finite Gaussian mixture model with K-means. Similarly GCE and VOI values of the proposed algorithm are less than that of finite Gaussian mixture model. This reveals that the proposed algorithm outperforms the existing algorithm based on the finite Gaussian mixture model.

After developing the image segmentation method it is needed to verify the utility of segmentation in model building of the image for image retrieval. Using the estimated probability density function of the images under consideration the retrieved images are obtained and are shown in Figure 4.

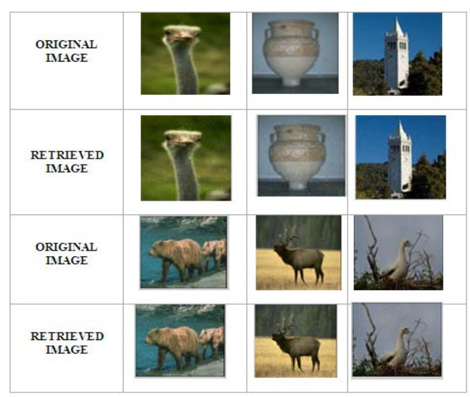

Figure 4 : Original and Retrieved Images

The Performance Evaluation of the retrieved image is done by Subjective Image Quality testing or by Objective Image Quality testing. The Objective Image Quality testing methods are often used since the numerical results of an objective measure are readily computed and allow a consistence comparison of different algorithms. There are several Image Quality measures available for Performance Evaluation of the Image Segmentation method. An extensive survey of Quality Measures is given by Eskicioglu A.M. and Fisher P.S. (1995). For the Performance Evaluation of the developed Segmentation algorithm, we consider the Image Quality Measures namely (a) Maximum Distance, (b) Image Fidelity, (c) Mean Square Error, (d) Signal to Noise Ratio and (e) Image Quality Index are computed for all the Six images with respect to the developed method and earlier methods and presented in Table- 4.

Table 4 : Comparative study of Image Quality Metrics

IMAGE	Quality Metrics	GMM	FLTBGMM-*K*	Optimal Criteria
OSTRICH	Maximum Distance	0.5013	0.5067	Close to 1
	Image Fidelity	0.7910	0.8076	Close to 1
	Mean Square Error	0.0932	0.0793	Close to 0
	Signal to Noise Ratio	13.3781	13.9959	As big as possible
	Image Quality Index	0.8102	0.8492	Close to 1
POT	Maximum Distance	0.3290	0.3957	Close to 1
	Image Fidelity	0.6729	0.6786	Close to 1
	Mean Square Error	0.0738	0.0467	Close to 0
	Signal to Noise Ratio	11.7401	13.0240	As big as possible
	Image Quality Index	0.6075	0.6174	Close to 1
TOWER	Maximum Distance	0.8481	0.8757	Close to 1
	Image Fidelity	0.5217	0.3884	Close to 1
	Mean Square Error	0.2101	0.1792	Close to 0
	Signal to Noise Ratio	8.8724	8.8488	As big as possible
	Image Quality Index	0.6271	0.5173	Close to 1
BEARS	Maximum Distance	0.5387	0.8765	Close to 1
	Image Fidelity	0.4277	0.6586	Close to 1
	Mean Square Error	0.0872	0.0484	Close to 0
	Signal to Noise Ratio	9.1217	10.7550	As big as possible
	Image Quality Index	0.5951	0.5906	Close to 1
DEER	Maximum Distance	0.6217	0.6474	Close to 1
	Image Fidelity	0.3982	0.4470	Close to 1
	Mean Square Error	0.0828	0.0547	Close to 0
	Signal to Noise Ratio	10.0629	11.8918	As big as possible
	Image Quality Index	0.3763	0.3840	Close to 1
BIRD	Maximum Distance	0.8429	0.9129	Close to 1
	Image Fidelity	0.1920	0.2349	Close to 1
	Mean Square Error	0.2013	0.0900	Close to 0
	Signal to Noise Ratio	8.9231	9.3864	As big as possible
	Image Quality Index	0.3481	0.4160	Close to 1

From the Table 4, it is observed that all the image quality metrics for the six images are meeting the standard criteria. This implies that using the proposed algorithm the images are retrieved accurately. A comparative study of proposed algorithm with that of algorithm based on finite Gaussian mixture model (GMM) and Finite left truncated bivariate Gaussian mixture model with *K*-means reveals that the mean square error of the proposed model is less than that of the finite GMM and FLTBGMM. Based on all other quality metrics also it is observed that the performance of the proposed model in retrieving the images is better than the finite Gaussian mixture model.

VIII. Conclusion

In this paper we introduce a novel and new colour image segmentation method based on left truncated bivariate Gaussian mixture model. Here it is assumed that the colour image is characterized by HSI colour space, in which the Hue and Saturation values are non negative. they are characterized by left truncated Bivariate Gaussian mixture model. The left truncated bivariate Gaussian distribution includes the Bivariate Gaussian distribution is a limiting case when the truncation points tends to infinite. It also includes several platy, meso, lefty and skewed distributions as particular cases for different values of the parameters. The model parameters are estimated by using EM-algorithm. The initialization and the number of image segments are determined through K-means algorithm and moment method of estimation. The segmentation algorithm is developed with component maximum likelihood. The experimentation with six colour images reveals that this algorithm outperforms the existing algorithms in both image segmentation and image retrievals. The image quality metrics also supported the utility of the proposed algorithm. It is possible to develop image segmentation algorithm with finite mixture of doubly truncated multivariate Gaussian distribution with more image features which require further investigations.

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