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Study of a Fuzzy Production Inventory Model with Deterioration Under Marxian Principle

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Abstract The basic concern of the Capitalistic approach of manufacturing industry is to minimize costs and to maximize profit. Through uncontrolled rate of exploitation, the manufacturer creates large quanta of social surplus. This ultimately results in the economic injustice and inequality among social economic classes. This current article makes an initiation to develop an economic production quantity model with deterioration under Marxian approach of socio-political economy. Here, two Marxian economic production quantity models are developed aiming at the minimization of exploitation rate and reduction of social surplus. The notion of marginal profit is considered here to reduce the social surplus (Net profit) incurred in the production plant itself. Fuzzy system has also been studied to get the variability of the exploitation in the model. Sensitivity analysis, graphical illustrations are made to validate the model.

Keywords EPQ · Deterioration · Exploitation · Marxian economy · Labor cost · Fuzzy system · Optimization

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1 Introduction

The inventory control problems are much celebrated optimization problems in the domain of operation research. In this context, economic order quantity (EOQ) model and economic production quantity (EPQ) model are two popular lot-sizing approaches among the inventory management technologies. Here, optimal order/production quantity (lot size) is the matter of concern, aiming at the minimization of cost or maximization of the profit. Initially, the literature was grown with the simple assumptions of constant demand and production rate etc. Gradually, the literature of the lot-sizing modelling was improved and matured incorporating more reliable assumptions serving different purposes [1, 2]. In reality, the demand and pricing are a very crucial factor in any retailing business model. The demand rate may vary with respect to time [3, 4], stock level [5], selling price [6], various dealing policy and so many other components in different supply situations. The deterioration or decay of items in inventory is also a very practical concern closely associated with the modelling which was considered by Ghare and Schrader [7] for the first time. Again, the deterioration rate may not be constant always in reality. The deterioration rate in terms of the two parameter Weibull distribution was considered by Emmons [8]. Till now, several investigations [9–19] were engaged to develop inventory models with deterioration of items. Very recently, the constant rates of deterioration are considered to manifest the memory-based investigations of the EPQ models by Rahaman et al. [20, 21].

In reality, the parameters and decision variables carry uncertainty in their value which create uncertain environment of decision-making. Fuzzy logic is one of the finest tools to define uncertainty involved in the process of decision-making. The concept of fuzzy uncertainty was given by Zadeh [22]. Subsequently, Bellman and Zadeh [23] considered the fuzzy logic for decision-making problems. Till date, many investigations [24–32] have explored the intuitive meanings of fuzziness and the potential applications of fuzzy sense in the fields of technology and management. The concept of learning experience-based uncertainty decision-making problem was introduced through the establishment of triangular dense fuzzy sets (TDFS) by De and Beg [33]. One of the extensions of TDFS through considering continuous time variable, namely cloudy fuzzy set was developed by De and Mahata [34]. De [35] also extended the concept of triangular dense fuzzy set to the triangular dense fuzzy lock set (TDFLS). The optimization of inventory control problem is one kind of decision-making problems where data regarding various parameters are ambiguous in nature. This motivates the study of inventory control problem in an uncertainty phenomenon described by fuzzy sets. In this context, some studies [36–39] on the inventory control problems have been carried out utilizing the concepts of TDFS, TDFLS and cloudy fuzzy set.

Again, the Marxian economics is a revolutionary approach against classical political economics. The radical idea of this socio-political economy containing the analysis on the crisis of capitalism, the surplus product and surplus value was innovated by Marx and Engels [40, 41]. The social surplus is beehive of the Capitalistic economy. It is defined to be a measure of wealth value and product over and above the necessity of a social system to its productivity. The social surplus in the form of net profit is expected to be exponential in a Capitalistic society in the favor of the Bourgeoisie (Capitalist). The social surpluses are gathered in the hand of Bourgeoisie through the exploitation to the proletariat (worker) which ultimately creates discrimination and injustice in society. In reality, the existence of social surplus and exploitation has ancient tradition in the history of mankind. From that moment when the human civilization emerged out of hunting and gathering stage of social surplus was the source of net wealth. The royal family life style of many ancient civilizations proves the existence of exploitation and social surplus in those days' society. However, the Capitalistic idea of productivity is much organized trap to make exponential growth of the surplus value with huge exploitation. Against the Capitalism, Marx's advocacy of revolutionary socialism was experimented in many countries like Soviet Union, China, Vietnam Cuba etc. Many studies and investigations on Marxian economics [42–54] enriched the literature as well. However, the Marxian economy faced a huge challenge from the Capitalistic world and seems to be dominated in modern era.

After brief reviewing of literature the following shortcomings in the existing literature can be pointed out and those are tried to be overcome in the current article:

(i) The existing approaches of describing the inventory control problems worried only on the cost minimization and profit maximization. The traditional strategy is not interested to bother about the exploitation and discrimination in society.

(ii) The Marxian approach of socio-political economy made great enthusiasm initially. However, it seems to be dominated by the growing establishment of Capitalistic economy.

(iii) This current article makes an initiation to develop an economic production quantity (EPQ) model inspired by the Marxian view of socio-political economics. Here, the minimization of exploitation rate is included as an objective along with cost minimization and profit maximization. Also, total profit is restricted to be little part of the total cost. The EPQ model with marginal profit and exploitation is termed as the Marxian economic order quantity (MEPQ) model in this paper.

The rest of the article is organized as the following: The Sect. 2 gives a very short description on the fuzzy numbers used in this article and fundamental Marxian Theorem. The notations and assumptions to describe the proposed models are presented in the Sect. 3. The Sect. 4 formulates the mathematical models based on the assumptions. The fuzzification of one of the models described in the Sect. 4 is made in the Sect. 5. The Sect. 6 represents a comparison numerically among various models. The sensitivity analysis of the fuzzy model is done in the Sect. 7. The graphical analysis is presented in the Sect. 8. Finally, the conclusion over the current study is made in Sect. 9.

2 Preliminaries

2.1 Fundamental Marxian Theorem [41]

The main notion of this theory is to establish the theory of socialism and communism that focuses critiques of capitalism. One of the essential key points of this theory is 'value of labour power'. According to Marx, the actual wages paid in labour-time or socially necessary human labour time for the production of a commodity with the given highest level of technological set up available in society is called 'value of labour power'. The 'surplus value' means the extra money earned (drawn) from a production system at its end after selling the whole commodities explicitly. This includes the unpaid labour power to produce the commodities. Let, s_p be the selling value of one unit of a commodity, c_p be the production cost of one unit, v be the value of labour power to produce one unit and s be the surplus value of one unit of that commodity then, $s_p = c_p + v + s$. Here c_p is called c_p-constant capital and v is called v-variable capital. Also, the rate of exploitation r is given by $r = \frac{s_p - (c_p + v)}{c_p + v}$ is called the fundamental Marxian theorem.

2.2 L and R Triangular Fuzzy Number and it's α-Cuts

Definition 1 A fuzzy set $\tilde{A}$ on a crisp set A is an ordered pair given by $\tilde{A} = \{(x, \mu_{\tilde{A}}(x))\}$, where x is the element of A and $\mu_{\tilde{A}}(x)$ is the corresponding member function and $\mu_{\tilde{A}}(x) \in [0, 1]$ for all $x \in A$.

Definition 2 The α-cut of the fuzzy set $\tilde{A}$ of X is given by $A^{\alpha} = \{x : \mu_{\tilde{A}}(x) \geq \alpha, x \in X, \alpha \in [0, 1]\}$. By definition the α-cut is a crisp set. This is also called the interval of confidence, α-level set etc.

Definition 3 The fuzzy number is a fuzzy set given by $F : \mathcal{R} \rightarrow [0, 1]$ which satisfies the following properties:

(i) F is upper semi- continuous.
(ii) $F(x) = 0$ for $x < \gamma$ and $x > \delta$ for some γ, δ.
(iii) There exist two real numbers α, β such that $\gamma \leq \alpha \leq \beta \leq \delta$ such that
 (a) $F(x)$ is monotonic increasing on $[\gamma, \alpha]$
 (b) $F(x)$ is monotonic decreasing on $[\beta, \delta]$
 (c) $F(x) = 1$ for $\alpha \leq x \leq \beta$.

Definition 4 The L- Triangular fuzzy number given $\tilde{A}$ is given by its membership functions.

$$\mu(\tilde{A}) = \begin{cases} 0 \, if \, x \leq a(1-\rho) \\ \dfrac{x - a(1-\rho)}{a\rho} \, if \, a(1-\rho) \leq x \leq a \\ 1 \, if \, x \geq a \end{cases}$$

Here, ρ stands for the measurement of deviation of fuzziness for the R-triangular fuzzy number.

The left α-cut of $\tilde{A}$ given by $A(\alpha) = a(1-\rho) + \alpha a\rho$

Definition 5 The R- Triangular fuzzy number given $\tilde{B}$ is given by its membership functions.

$$\mu(\tilde{B}) = \begin{cases} 0 \, if \, x \leq b \\ \dfrac{b(1+\sigma) - x}{b\sigma} \, if \, b \leq x \leq b(1+\sigma) \\ 1 \, if \, x \geq b(1+\sigma) \end{cases}.$$

Here, σ stands for the measure of deviation of fuzziness for the R-triangular fuzzy number.

The right α-cut of $\tilde{B}$ given by

$B(\alpha) = b(1+\sigma) + \alpha b\sigma$

3 Notations and Assumptions

Here, we have studied Marxian Economic order quantity (MEPQ) model under two different approaches considering with and without the marginal profit. The following notations and assumptions are used in the current paper to describe the proposed models.

Symbol	Meaning
Notations (both normal and Marxian profit model)	
K	Production rate per cycle (week)
D	Demand rate per cycle (week)
ϕ	Rate of deterioration (a fraction)
s_c	Set up cost per cycle ($)
p_c	Production cost per unit produced items per unit time ($)
h_c	Holding cost per unit item per unit time ($)
d_c	Deterioration cost per unit item per unit time ($)
v	Labor cost to produce per unit item ($)
s_p	Earned selling price (revenue) per unit item per unit time ($)
$I(t)$	Inventory level at time t
Decision variables	
T	Total time cycle (week)
T_1	Total production duration (week)
Q	Economic lot size
R	Rate of exploitation
s_p	Unit selling price per unit item ($)
Objective functions	
Y	Total earned revenue over whole cycle (To be maximized from the producer's side) ($)
R	Rate of exploitation (To be minimized from the producer's side for Marxian system) (%)
Z	Total cost (To be minimized from the producer's side)
W	Compound objective function

3.1 Assumptions

The following common assumptions have been considered to develop the proposed inventory models:

3.1.1 EPQ Model (Capitalistic Approach)

(i) The production rate K and demand rate D are assumed to be uniform and constant. The rate of the product is assumed to be greater than the rate of the demand i.e., $K > D$
(ii) There is constant rate of deterioration ϕ of the items in the inventory throughout the total cycle time
(iii) Shortages are not allowed
(iv) Replenishment rate is instantaneous
(v) The time horizon is infinite
(vi) In non-Marxian system(traditional), the producers try to deprive (exploit) the labors at the rate R by keeping no bounds.

3.1.2 Marxian EPQ Model with Normal Profit (Model 1)

The sixth assumption in the general EPQ model of Capitalistic approach is replaced by the additional objective of minimization of exploitation rate R to develop the Model 1. The minimization of exploitation aims at the reduction of social discrimination.

3.1.3 Marxian EPQ Model with Marginal Profit (Model 2)

Along with the assumptions for EPQ model with exploitation minimization objective, here an approach to reduce the social surplus is directed through the assumption that the sales revenue being considered as a decision variable under the constraints of restricted profit. Here, the total profit is assumed to be a small component of the total cost and accordingly the sales revenue is to be restricted.

4 Formulation of Mathematical Models

Initially, the inventory level is assumed to be the zero. The inventory cycle is started at $t = 0$ with the finite production rate K. In the time interval $0 \le t \le T_1$, the production phase is going on meeting up the demand rate $D(<K)$ and facing deterioration of the rate ϕ. The production rate is sufficiently bigger over the demand and deterioration rate to make the inventory level increasing in this time interval. At $t = T_1$, the production phase is ended with maximum inventory level in the whole cycle. In the non-productive phase $(T_1 \le t \le T)$, the inventory level gradually decreases due to meeting up the demand and facing the deterioration of the items. Finally, the whole economic cycle is closed at $t = T$ with a zero level of the inventory. Fig. 1 depicts the situation graphically.

In the productive phase $(0 \le t \le T_1)$, the dynamics of the inventory level can be depicted mathematically by following differential equation

$$\frac{dI(t)}{dt} = K - D - \phi I(t) \tag{1}$$

Also, the dynamics of the inventory level in the non-productive phase $(T_1 \le t \le T)$ can be described as

$$\frac{dI(t)}{dt} = -D - \phi I(t) \tag{2}$$

The boundary conditions are given as

$$I(0) = 0 = I(T) \tag{3}$$

Solving Eq. (1) and Eq. (2) and using the boundary conditions given by Eq. (3), the following derivations regarding the inventory level are obtained.

For productive phase $(0 \le t \le T_1)$:

$$I(t) = \frac{K - D}{\phi}\left(1 - e^{-\phi t}\right) \tag{4}$$

For non-productive phase $(T_1 \le t \le T)$:

$$I(t) = \frac{D}{\phi}\left\{e^{\phi(T-t)} - 1\right\} \tag{5}$$

Also, the continuity of the function $I(t)$ at $t = T_1$ gives the following relation

$$\frac{K - D}{\phi}\left(1 - e^{-\phi T_1}\right) = \frac{D}{\phi}\left\{e^{\phi(T-T_1)} - 1\right\}$$

i.e.,

$$T = T_1 + \frac{1}{\phi} log\left|\frac{K - D}{D}\left(1 - e^{-\phi T_1}\right) + 1\right| \tag{6}$$

The economic lot size will be

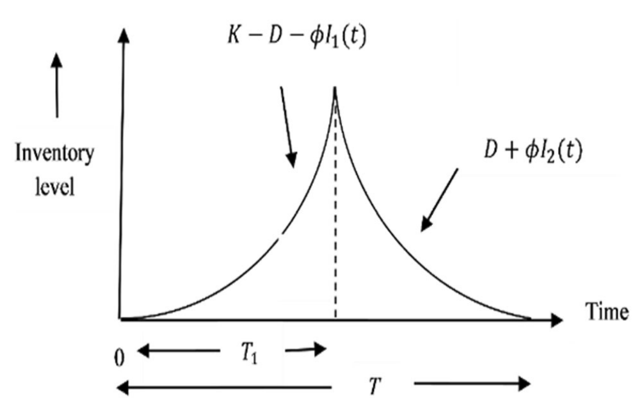

Fig. 1 EPQ model with deterioration

$$Q = DT \tag{7}$$

Costs and revenue

The set up cost per inventory cycle is

$$SC = s_c \tag{8}$$

The total production cost is

$$PC = p_c K T_1 \tag{9}$$

The total holding cost

$$HC = h_c \int_0^T I(t)dt = h_c \left\{ \int_0^{T_1} I(t)dt + \int_{T_1}^T I(t)dt \right\}$$

$$= h_c \left[\int_0^{T_1} \frac{K-D}{\phi} \left(1 - e^{-\phi t}\right) dt + \int_{T_1}^T \frac{D}{\phi} \left\{ e^{\phi(T-t)} - 1 \right\} dt \right]$$

$$= h_c \left[\frac{K-D}{\phi} \left\{ T_1 + \frac{1}{\phi} \left(e^{-\phi T_1} - 1 \right) \right\} + \frac{D}{\phi^2} \left\{ e^{\phi(T-T_1)} - 1 - \phi(T - T_1) \right\} \right]$$

$$= h_c \left\{ \frac{KT_1 - DT}{\phi} \right\}$$

i.e.,

$$HC = h_c \left\{ \frac{KT_1 - DT}{\phi} \right\} \tag{10}$$

The total deterioration cost is

$$DC = d_c (KT_1 - DT) \tag{11}$$

The total labour cost to produce is

$$LC = vQ = vDT \tag{12}$$

So, the total cost can be obtained as

$$Z = SC + PC + HC + DC + LC$$

$$= s_c + p_c K T_1 + h_c \left(\frac{KT_1 - DT}{\phi} \right) + d_c (KT_1 - DT) + vDT$$

i.e.,

$$Z = s_c + p_c K T_1 + h_c \left(\frac{KT_1 - DT}{\phi} \right) + d_c (KT_1 - DT) + vDT \tag{13}$$

The total sales revenue is

$$Y = s_p DT \tag{14}$$

Therefore, the rate of exploitation will be

$$R = \frac{s_p DT}{Z} - 1 \tag{15}$$

4.1 EPQ Model with Normal Profit (Model 1)

In the traditional EPQ modelling approach, the total average profit is obtained as

$$TAP = \left(\frac{Y - Z}{T} \right) \tag{16}$$

And so, the optimization problem will be

$$\begin{cases} Maximize \left(\frac{Y-Z}{T} \right) \\ \text{Subject to } Y = s_p DT \\ Z = s_c + p_c K T_1 + h_c \left(\frac{KT_1 - DT}{\phi} \right) + d_c (KT_1 - DT) + vDT \\ T = T_1 + \frac{1}{\phi} log \left| \frac{K-D}{D} \left(1 - e^{-\phi T_1}\right) + 1 \right|, \text{ Q} = \text{DT} \end{cases} \tag{17}$$

4.2 Marxian EPQ Model with Normal Profit (Model 2)

The multi objectives optimization problem from the manufacturer's side in the Marxian approach with normal profit is

$$\begin{cases} Maximize\ Y = s_p DT \\ \text{Minimize } z = s_c + p_c K T_1 + h_c \left(\frac{KT_1 - DT}{\phi} \right) + d_c (KT_1 - DT) + vDT \\ \text{Minimize } R = \frac{s_p DT}{z} - 1 \\ \text{Subject to } T = T_1 + \frac{1}{\phi} log \left| \frac{K-D}{D} \left(1 - e^{-\phi T_1}\right) + 1 \right|, \text{Q} = \text{DT} \end{cases} . \tag{18}$$

The equivalent single objective optimization problem corresponding to the Eq. (18) is given by

$$\begin{cases} Maximize\ W = \frac{Y-Z}{R'} = \frac{1}{R'} \left[s_p DT - \left\{ s_c + p_c K T_1 + h_c \left(\frac{KT_1 - DT}{\phi} \right) + d_c (KT_1 - DT) + vDT \right\} \right] \\ Subject\ to\ R' = \frac{s_p DT}{z}, \text{ Q} = \text{DT}, R = R' - 1 \\ T = T_1 + \frac{1}{\phi} log \left| \frac{K-D}{D} \left(1 - e^{-\phi T_1}\right) + 1 \right| \end{cases} \tag{19}$$

4.3 Marxian EPQ Model with Marginal Profit (Model 3)

The Model 3 restricts the Model 2 with barrier to the uncontrolled profit maximization. The concept of marginal profit is utilized here considering the total profit to be small component of the total cost. For this Marxian economic production quantity (EPQ) model with marginal unit selling price s_p is considered to be the decision of the variable and is obtained as

$$s_p = \frac{z}{DT} + \frac{\partial}{\partial T}\left(\frac{z}{DT}\right)$$
$$= \frac{s_c}{DT} + \left(p_c + \frac{h_c}{\phi} + d_c\right)\frac{KT_1}{DT} + \left(v - \frac{h_c}{\phi} - d_c\right) - \frac{s_c}{DT^2} + \left(p_c + \frac{h_c}{\phi} + d_c\right)\frac{K}{D}\left(\frac{T\frac{dT_1}{dT} - T_1}{T^2}\right)$$
$$= \frac{s_c}{DT}\left(1 - \frac{1}{T}\right) + \left(p_c + \frac{h_c}{\phi} + d_c\right)\left\{\frac{KT_1}{DT}\left(1 - \frac{1}{T}\right) + \frac{e^{\phi(T-T_1)}}{T}\right\} + \left(v - \frac{h_c}{\phi} - d_c\right)$$

i.e.,

$$s_p = \frac{s_c}{DT}\left(1 - \frac{1}{T}\right) + \left(p_c + \frac{h_c}{\phi} + d_c\right)\left\{\frac{KT_1}{DT}\left(1 - \frac{1}{T}\right) + \frac{e^{\phi(T-T_1)}}{T}\right\} + \left(v - \frac{h_c}{\phi} - d_c\right) \quad (20)$$

Therefore, the optimization problem for marginal profit is given by

$$\begin{cases} Maximize\ W = \dfrac{Y - Z}{R'} \\ = \dfrac{1}{R'}\left[s_pDT - \left\{s_c + p_cKT_1 + h_c\left(\dfrac{KT_1 - DT}{\phi}\right) + d_c(KT_1 - DT) + vDT\right\}\right] \\ Subject\ to\ R' = \dfrac{s_pDT}{z}, \mathrm{Q} = \mathrm{DT}, R = R' - 1, \\ = T_1 + \dfrac{1}{\phi}log\left|\dfrac{K-D}{D}\left(1 - e^{-\phi T_1}\right) + 1\right| \\ s_p = \dfrac{s_c}{DT}\left(1 - \dfrac{1}{T}\right) + \left(p_c + \dfrac{h_c}{\phi} + d_c\right)\left\{\dfrac{KT_1}{DT}\left(1 - \dfrac{1}{T}\right) + \dfrac{e^{\phi(T-T_1)}}{T}\right\} + \left(v - \dfrac{h_c}{\phi} - d_c\right) \end{cases} \quad (21)$$

5 Formulation of Fuzzy Mathematical Model

In real domain of production-manufacturing and marketing, there are often seen several parameters which are used to represent different costs and revenue as well as in the measurement of production rate and demand pattern carrying uncertainty. Fuzzy mathematical modeling proves its greatness to tackle this uncertain situation. In this section, the MEPQ model with normal profit (Model 2) is reconsidered in the fuzzy environment considering the parameters to be of the following types:

(i) The unit selling price of the product is assumed to be R- Triangular fuzzy number $\tilde{s}_p$

(ii) The unit cost prices (setup cost, production cost, holding cost, deterioration cost, labour cost), the production rate and demand are assumed to be L Triangular fuzzy numbers, namely $\tilde{s}_c$, $\tilde{p}_c$, $\tilde{h}_c$, $\tilde{d}_c$, $\tilde{v}$, $\tilde{K}$, $\tilde{D}$, respectively.

In the fuzzy version of the Model 2, only the deterioration rate is taken as the crisp input.

Suppose, the fuzzy unit selling price is expected to at least reach the value s_{p0} and most desired value is expected to be greater than $s_{p0}(1+\sigma_0)$, where σ_0 is the measurement of fuzzy deviation. Then, the R Triangular fuzzy unit selling price $\tilde{s}_p$ can be represented through its membership functions as the following

$$\mu(\tilde{s}_p) = \begin{cases} 0 & if\ s_p \le s_{p_0} \\ \dfrac{s_{p_0}(1+\sigma_0) - s_p}{s_{p_0}\sigma_0} & if\ s_{p_0} \le s_p \le s_{p_0}(1+\sigma_0) \\ 1 & if\ s_p \ge s_{p_0}(1+\sigma_0) \end{cases}. \quad (22)$$

Then, the right α-cut of $\tilde{s}_p$ is

$$s_p(\alpha) = s_{p_0}(1+\sigma_0) - \alpha s_{p_0}\sigma_0 \quad (23)$$

Similarly, the fuzzy unit costs $\tilde{A}_k$ $(k = 1,2,3,4,5$ and where $\tilde{A}_1 = \tilde{s}_c, \tilde{A}_2 = \tilde{p}_c, \tilde{A}_3 = \tilde{h}_c, \tilde{A}_4 = \tilde{d}_c, \tilde{A}_5 = \tilde{v})$, production rate $\tilde{A}_6 (= \tilde{K})$ and demand rate $\tilde{A}_7 (= \tilde{D})$ are expected to be at most reach the values A_{k0} and most desired values are expected to be lesser than $A_{k0}(1+\rho_k)$, where ρ_k are the measure of fuzzy deviation. Then, the L Triangular fuzzy parameters $\tilde{A}_k$ $(k = 1,2,3,4,5,6,7)$ can be represented through its membership functions as the following

$$\mu(\tilde{A}_k) = \begin{cases} 0 & if\ A_k \le AA_{k_0}(1-\rho_k) \\ \dfrac{A_k - A_{k_0}(1-\rho_k)}{A_{k_0}\rho_k} & if\ A_{k_0}(1-\rho_k) \le A_k \le A_{k_0} k = 1,2,3,4,5,6\ \text{and}\ 7. \\ 1 & if\ A_k \ge A_{k_0} \end{cases} \quad (24)$$

Then, the left α-cut of $\tilde{A}_k$ $(k = 1,2,3,4,5,6,7)$ is

$$A_k(\alpha) = A_{k_0}(1-\rho_k) + \alpha A_{k_0}\rho_k \quad (25)$$

The above fuzzy replication of Model 2 contributes to the fuzzy sense on the function describing total profit, total cost, exploitation rate, time and lot size in the system given by the Eq. (19). Suppose, $\tilde{Y}$, $\tilde{Z}$, $\tilde{Q}$, $\tilde{R}$, $\tilde{T}$ are representing the fuzzy analogue ofthe total profit, total cost, exploitation rate, lot size and time, respectively. Then, the fuzzy replica of the Eq. (19) can be obtained as

$$\begin{cases} Maximize\ \widetilde{W} \cong \dfrac{\widetilde{Y}-\widetilde{Z}}{\widetilde{R}'} \\ \widetilde{Z} \cong \widetilde{s}_c + \widetilde{p}_c\tilde{K}T_1 + \widetilde{h}_c\left(\dfrac{\tilde{K}T_1 - \tilde{D}\widetilde{T}}{\phi}\right) + \widetilde{d}_c\left(\tilde{K}T_1 - \tilde{D}\widetilde{T}\right) + \widetilde{v}\tilde{D}\widetilde{T} \\ \widetilde{Y} = \widetilde{s}_p\tilde{D}\widetilde{T}, \widetilde{R}' \cong \dfrac{\widetilde{Y}}{\widetilde{Z}}, \widetilde{Q} \cong \tilde{D}\widetilde{T}, \widetilde{R} = \widetilde{R}' - 1 \\ \widetilde{T} \cong T_1 + \dfrac{1}{\phi}\log\left|\dfrac{\tilde{K}-\tilde{D}}{\tilde{D}}\left(1-e^{-\phi T_1}\right)+1\right| \end{cases} \tag{26}$$

Using the α-cuts of the fuzzy variables described by the Eq. (23) and Eq. (25), the α-cuts of different fuzzy valued functions mentioned in the Eq. (26) can be obtained as

$$\begin{cases} W(\alpha) \cong \dfrac{Y(\alpha)-Z(\alpha)}{R'(\alpha)} \\ Y(\alpha) = s_p(\alpha)A_7(\alpha)T(\alpha) \\ Z(\alpha) \cong A_1(\alpha) + A_2(\alpha)A_6(\alpha)T_1 + A_3(\alpha)\left(\dfrac{A_6(\alpha)T_1 - A_7(\alpha)T(\alpha)}{\phi}\right) \\ \quad + A_4(\alpha)(A_6(\alpha)T_1 - A_7(\alpha)T(\alpha)) + A_5(\alpha)A_7(\alpha)T(\alpha) \\ R'(\alpha) \cong \dfrac{Y(\alpha)}{Z(\alpha)}, Q(\alpha) \cong A_7(\alpha)T(\alpha), \\ T(\alpha) \cong T_1 + \dfrac{1}{\phi}\log\left|\dfrac{A_6(\alpha)-A_7(\alpha)}{A_7(\alpha)}\left(1-e^{-\phi T_1}\right)+1\right| \\ R(\alpha) = R'(\alpha) - 1 \end{cases} \tag{27}$$

For the defuzzification of the system given by the Eq. (27), the defuzzified value W is chosen as the value of $W(\alpha)$ corresponding to possible highest level of aspiration. So, the defuzzified non-linear optimization problem equivalent to (27) is

$$\begin{cases} Maximize\ \alpha \\ W \geq W(\alpha) \\ W(\alpha) \cong \dfrac{Y(\alpha)-Z(\alpha)}{R'(\alpha)}, Y(\alpha) = s_p(\alpha)A_7(\alpha)T(\alpha), \\ Z(\alpha) \cong A_1(\alpha) + A_2(\alpha)A_6(\alpha)T_1 + A_3(\alpha)\left(\dfrac{A_6(\alpha)T_1 - A_7(\alpha)T(\alpha)}{\phi}\right) \\ \quad + A_4(\alpha)(A_6(\alpha)T_1 - A_7(\alpha)T(\alpha)) + A_5(\alpha)A_7(\alpha)T(\alpha) \\ T(\alpha) \cong T_1 + \dfrac{1}{\phi}\log\left|\dfrac{A_6(\alpha)-A_7(\alpha)}{A_7(\alpha)}\left(1-e^{-\phi T_1}\right)+1\right| \\ R'(\alpha) \cong \dfrac{Y(\alpha)}{Z(\alpha)}, Q(\alpha) \cong A_7(\alpha)T(\alpha) \\ T(\alpha) \cong T_1 + \dfrac{1}{\phi}\log\left|\dfrac{A_6(\alpha)-A_7(\alpha)}{A_7(\alpha)}\left(1-e^{-\phi T_1}\right)+1\right| \\ R(\alpha) = R'(\alpha) - 1 \end{cases} \tag{28}$$

6 Numerical Simulation

In this section, four different lot size models (namely, Model 1, Model 2, Model 3 and the fuzzy version of the Model 2) are taken for the numerical optimization. Following the below mentioned algorithm, the numerical optimizations of the EPQ model, MEPQ model with normal profit, MEPQ model with marginal profit and fuzzy MEPQ model with normal profit are carried out using the LINGO 18.0 software.

7 Solution Algorithm of the Proposed Models

Step 1 Solve the crisp EPQ model (given by the Eq. 17) traditionally.

Step 2 Convert the crisp EPQ model into MEPQ model introducing another optimization function, namely, the rate of exploitation.

Step 3 Construct the multi-objective (profit maximization and exploitation minimization) optimization problem for the crisp MEPQ model with normal profit.

Step 4 Convert the multi-objective optimization problem into a single objective optimization problem.

Step 5 Solve the single objective optimization problem (given by the Eq. 19).

Step 6 Convert the MEPQ model with normal profit into the MEPQ model with marginal profit restricting the value of the unit selling price by the Eq. (20)

Step 7 Construct the multi-objective optimization problem for the MEPQ model with marginal profit and convert it to an equivalent single objective optimization problem.

Step 8 Solve the single objective optimization problem (given by the Eq. 21).

Step 9 Convert the crisp MEPQ model with normal profit (given by the Eq. 19) into its fuzzy analogue based on the consideration of R Triangular fuzzy unit selling price and L-Triangular fuzzy unit costs, demand rate and production rate.

Step 10 Rewrite the fuzzy single objective optimization problem (given by Eq. 26) in terms of α-cuts of the fuzzy variables and parameters.

Step 11 Defuzzify the problem given by the Eq. (27)

Step 12 Solve the problem given by the Eq. (28)

Step 13 Compare the results of the crisp EPQ model, crisp MEPQ model with normal profit, crisp MEPQ model with marginal profit, fuzzy MEPQ model with normal profit.

Step 14 End.

The above-mentioned algorithm is used to obtain the optimum values of the objective functions and decision variables under the consideration of the following numerical inputs:

(a) Crisp EPQ model with normal profit: The values of associated crisp parameters are taken as $s_p = 45$, $s_c = 500$, $p_c = 28$,$h_c = 2.5$, $d_c = 1.5$, $v = 0.5$, $D = 1000$, $K = 1400$ and $\phi = 0.005$.
(b) Crisp MEPQ model with normal profit: The values of associated crisp parameters are taken as $s_p = 45$, $s_c = 500$, $p_c = 28$,$h_c = 2.5$, $d_c = 1.5$, $v = 0.5$, $D = 1000$, $K = 1400$ and $\phi = 0.005$.
(c) Crisp MEPQ model with marginal profit: Here, all the values of the crisp parameters remain same as the crisp MEPQ model with normal profit with an exception of s_p. Instead of being an input parameter, here, s_p is regarded as the decision variable.
(d) Fuzzy MEPQ model with normal profit: The fuzzy version of the MEPQ model with normal profit is illustrated numerically considering the following values of the centers of the fuzzy parameters:

$s_{p_0} = 45$, $s_{c_0} = 500$, $p_{c_0} = 28$, $h_{c_0} = 2.5$, $d_{c_0} = 1.5$,$v_0 = 0.5$,$D_0 = 1000$, $K_0 = 1400$.

Also, the deviation (range) of the fuzzy variables and parameters are taken as$\sigma_0 = 0.3$ and $\rho_k = 0.2$ (for $k = 1, 2, \ldots, 7$), where σ_0 and ρ_k(for $k = 1, 2, \ldots, 7$) are referred by the Eq. (22) and Eq. (24). The value of the only crisp parameter ϕ is set to be 0.005.

Then, the optimum values of the objective functions and the decision variables involved in the above four models in the mentioned range of the inputs are presented by Table 1.

In Table 1, the manifestation of the EPQ models is carried out in different scenarios. From the comparison in the table, it is perceived that only the addition of exploitation rate as an objective function to be minimized, does not contribute to any significant change of the situation of costs, profit and deprivation if the remaining setup is maintained just like the traditional EOQ model. Because, the Crisp EPQ and MEPQ models with normal profit show the same numerical outputs in Table 1. In the next step of the numerical simulation, we are looking for an alternative business policy under the marginal profit. The selling price is restricted to be bound, aiming at a very marginal component of cost as the net profit. And it is seen in Table 1 that the MEPQ model with marginal profit can successfully handle the objective of minimization of social surplus as well as the exploitation. On the other hand, the fuzzy MEPQ model with normal profit gives a huge quantum of profit through even more exploitation rate than the crisp MEPQ model with normal profit. This shows that uncertainties involved in the parameters make more and more gathering of social surplus. In the sense of profit maximization objective of manufacturer's side, the fuzzy modeling is the best among the four modeling. However, in the Marxian view on socio-political economy MEPQ model with marginal profit gives the best perspective.

8 Sensitivity Analysis

The sensitivity of the fuzzy model with respect to the fuzzy deviation parameters.

σ_0 (deviation of the fuzzy unit selling price) and ρ_k(where $k = 1, 2, \ldots, 7$ and representing the deviation of the fuzzy costs, production rate and demand rate) as well as the non-fuzzy parameters ϕ (rate of deterioration) has been carried out in this section and they are noted in Table 2.Varying the values of σ_0, ρ_k (where $k = 1, 2, \ldots, 7$) and ϕ corresponding to the optimal solution from -50% to $+50\%$, the effect of the individual changes on the optimal solution is checked here. The percentage of changes of the total profit with respect to the percentage of changes in the individual inputs is considered as the measurement of the sensitivity of the optimal solution against the corresponding input parameters.

The following observations and explanations are made from Table 2:

(i) The changes in the value of deviation of fuzziness of the variable s_p do not contribute directly in the changes of exploitation and net profit as per expectation. It is seen that by increasing σ_0 for 50% of its value, the values of R^* and P^* change with negative impact. Again, decreasing σ_0 for 50% of its value, though the exploitation rate can be decreased but the net profit is increased with a

Table 1 Optimal solutions of the EPQ models ($P^* = Total\,profit$)

Model	s_p(\$)	α^*	T_1^*(Weeks)	T^*(Weeks)	Q^*(Units)	R^*(%)	Y^*(\$)	Z^*(\$)	$P_* = Y^* - Z^*$
Crisp EPQ with normal profit	–	–	1.431	2.000	2000	–	90,000	59,010.59	30,989.41
Crisp MEPQ with normal profit	–	–	1.431	2.000	2000	52.51	90,000	59,010.59	30,989.41
Crisp MEPQ with marginal profit	29.76	–	1.431	2.000	2000	0.85	59,514.95	59,010.69	504.26
Fuzzy MEPQ with normal profit	48.37	0.75	1.431	2.000	1900	72.51	91,912.50	53,280.90	38,631.60

Table 2 Sensitivity of the optimal solution with respect to fuzzy and non-fuzzy parameters

Parameters	% Change in input parameters	T_1^*(Weeks)	T^*(Weeks)	Q^*(Units)	R^*(%)	Y^*($)	Z^*($)	α^*	$P^* = Y^* - Z^*$	$\frac{P^*-P_*}{P_*} \times 100\%$
σ_0(Deviation of the fuzzy selling price)	+ 50	0.715	1.000	998.00	54.17	45,112.10	29,261.55	0.99	15,850.55	− 58.97
	+ 25	1.431	2.000	1920.00	70.72	92,880.00	54,403.45	0.8	38,476.55	− 0.40
	− 25	1.431	2.000	1920.00	65.96	90,288.00	54,403.45	0.8	35,884.55	− 7.11
	− 50	3.357	4.685	4403.45	64.67	207,072.10	125,747.40	0.7	81,324.70	110.51
ρ_1(Deviation of the fuzzy set up cost)	+ 50	1.431	2.000	1880.00	79.81	92,214.00	52,155.05	0.7	40,058.95	3.69
	+ 25	1.431	2.000	1900.00	72.53	91,912.50	53,274.65	0.75	38,637.85	0.02
	− 25	1.431	2.000	1900.00	72.49	91,912.50	53,287.15	0.75	38,625.35	− 0.02
	− 50	2.179	3.045	2983.77	58.61	138,297.80	87,191.58	0.9	51,106.22	32.29
ρ_2(Deviation of the fuzzy production cost)	+ 50	3.650	5.092	4989.87	56.44	231,280.30	147,837.40	0.9	83,442.90	116.00
	+ 25	1.431	2.000	1860.00	84.39	92,488.50	50,158.20	0.65	42,330.30	9.57
	− 25	1.431	2.000	1900.00	70.38	91,912.50	53,946.85	0.75	37,965.65	− 1.72
	− 50	3.376	4.711	4334.27	73.29	218,447.30	126,056.4	0.6	92,390.90	139.16
ρ_3(Deviation of the fuzzy holding cost)	+ 50	1.431	2.000	1880.00	76.89	92,214.00	52,129.82	0.7	40,084.18	3.76
	+ 25	1.431	2.000	1840.00	85.63	92,736.00	49,957.20	0.6	42,778.80	10.74
	− 25	1.431	2.000	1840.00	85.44	92,736.00	50,009.70	0.6	42,726.30	10.60
	− 50	1.431	2.000	1840.00	85.34	92,736.00	50,035.95	0.6	42,700.05	10.53
ρ_4(Deviation of the fuzzy deterioration cost)	+ 50	1.431	2.000	1880.00	76.76	92,214.00	52,169.93	0.7	40,044.07	3.66
	+ 25	1.431	2.000	1900.00	72.51	91,912.50	53,280.85	0.75	38,631.65	0.00
	− 25	1.431	2.000	1840.00	85.53	92,736.00	49,983.53	0.6	42,752.47	10.67
	− 50	1.431	2.000	1840.00	85.53	92,736.00	49,983.61	0.6	42,752.39	10.67
ρ_5(Deviation of the fuzzy labour cost)	+ 50	1.431	2.000	1900.00	72.58	91,912.50	53,257.15	0.75	38,655.35	0.06
	+ 25	1.431	2.000	1880.00	76.80	92,214.00	52,155.95	0.7	40,058.05	3.69
	− 25	1.431	2.000	1880.00	76.71	92,214.00	52,184.15	0.7	40,029.85	3.62
	− 50	7.208	10.020	9318.75	65.36	463,375.00	280,228.8	0.65	183,146.20	374.08
ρ_6(Deviation of the fuzzy production rate)	+ 50	1.377	2.000	1790.00	80.60	89,007.75	49,284.93	0.65	39,722.82	2.82
	+ 25	1.515	2.151	1990.11	76.31	97,615.07	55,365.71	0.7	42,249.36	9.36
	− 25	1.453	2.000	1910.00	76.96	93,685.50	52,940.85	0.7	40,744.65	5.47
	− 50	1.484	2.000	1930.00	81.60	95,969.28	52,847.54	0.65	43,121.74	11.62
ρ_7(Deviation of the fuzzy demand rate)	+ 50	1.495	2.000	1840.00	86.07	92,736.01	49,838.04	0.6	42,897.97	11.04
	+ 25	1.462	2.000	1840.00	85.80	92,735.97	49,912.32	0.6	42,823.65	10.33
	− 25	1.400	2.000	1840.00	85.28	92,736.00	50,051.60	0.6	42,684.40	10.49
	− 50	1.371	2.000	1840.00	85.04	92,736.00	50,116.93	0.6	42,619.07	10.32
ϕ(Crisp rate of deterioration)	+ 50	1.432	2.000	1996.00	93.17	90,089.46	58,816.69	0.99	31,272.77	− 19.05
	+ 25	1.431	2.000	1920.00	68.28	91,584.00	54,422.34	0.8	37,161.66	− 3.80
	− 25	3.587	3.587	3299.93	82.68	166,316.50	91,037.59	0.6	75,278.91	94.86
	− 50	1.430	2.000	1840.00	85.66	92,736.00	49,948.72	0.6	42,787.28	10.76

huge amount. The explanation is that whenever the values of σ_0 change, it makes a new adjustment in the values of production time and total cycle time. Thus, the rate of exploitation and the net profit changes accordingly.

(ii) Decreasing the values of the deviation ρ_1 of fuzziness of the parameter s_c i.e., actually increasing the value of s_c, the exploitation can be decreased as expected. The net profit also follows the rule accordingly with an exception -50% value of ρ_1. In this case, the value of the net profit is increased due to the bigger values of the production time and the total cycle time.

(iii) The changes in the value of deviation of fuzziness of the variable p_c give the similar results as s_p.That is, here also the values of the production time and total cycle time change according to the values of ρ_2 and thus the changes in the values of exploitation and net profit do not follow the expectation.

(iv) The changes in the value of deviation of fuzziness of the variable h_c and v give almost the similar results as s_c.

(v) The changes in the value of deviation of fuzziness of the variable d_c and D give different results for different level of aspiration α.

(vi) The net profit and the exploitation rate are uniformly decreased as the value of deviation of fuzziness of the variable K is increased.

9 Graphical Illustrations

Here, we draw several graphs based on the numerical data obtained in Tables 1 and 2. In Fig. 2, the crisp EPQ and MEPQ models with normal profit represent the same total profit. The fuzzy MEPQ models with normal profit assure for the best result for the Capitalistic point of view, while the MEPQ model with marginal profit gives the best result for the Marxian point of view.

Fig. 3 represents a two-dimensional plotting of total profit against the exploitation rate. The curve in Fig. 3 has several ups and downs in its progression. The highest peak of the total profit corresponds for the exploitation rate of 65.36%. The non-uniform progress of the curve shows the dependency of the total profit on the other parameters and decision variables.

The graphical version of the sensitivity analysis presented by Table 2 is given by Figs. 4, 5, 6, 7, 8, 9, 10, 11 and 12.

Figure 4 shows the variation of profit value with respect to the variation of fuzzy deviation parameter of unit selling price. It indicates that the increase of fuzzy deviation parameter causes the decrease of profit value but profit value may assume zero around the zero change of that parameter exclusively. Figure 5 indicates the profit variation with respect to the changes of the setup cost. For low set up cost, profit is maximum but for higher set up cost, the profit value kept a bound.

Figure 6 describes the variation of profit value with respect to the % changes of unit production cost. It shows that, within the change -25% to $+25\%$, the profit is minimum, but beyond that it began to increase. Figure 7 reveals the profit variation due to the changes of unit holding cost. It is seen that, from -50% to -25% change, the profit function is kept to be constant value near \$43000, but at no change, the profit began to decrease. Then, it began to increase up to $+25\%$ change and becomes lower value for further changes.

Figure 8 describes "Z" type curve with respect to the changes of the unit deterioration cost showing the profit range $\$(38500 - 43000)$. Figure 9 indicates a "L" type curve due to the changes of the total labor cost giving the profit range $\$(30000 - 180000)$.

Figure 10 discusses the profit variation of the Marxian EPQ model due to the change of fuzzy deviation parameter of demand rate, describing a monotonic decreasing at 0% change then increasing at $+ 25\%$ change and finally decreasing for further changes keeping profit range $\$(38000 - 43500)$. Figure 11 explores the profit variation due to production rate and it gives a "V" shaped curve around no change performed.

Figure 12 depicts the profit variation of the Marxian EPQ model due to the changes of the deterioration rate. The curve is increasing from -50% to -25%. Then it begins to decrease for further changes keeping the profit range $\$(28000 - 73000)$. Fig. 13 gives a three-dimensional surface on the interdependency among the total profit, lot size and cycle time of the MEPQ model. On the other hand, Fig. 14 represents the surface response for the total profit function with respect to the variation of rate of exploitation and unit selling price exclusively.

10 Conclusion

In this current article, an economic production quantity model of deteriorated items has been experimented in different economic policies and phenomena. Traditional EPQ model has been viewed in the light of the Marxian socio-political economics. Initially, the crisp EPQ model is modified through introducing an additional objective of exploitation reduction under normal profit scenario. It is seen that, the MEPQ model with normal profit does not bring any changes in the whole economic situation as the outcomes coincides with that of the traditional EPQ model. Later, the MEPQ model with normal profit is further modified in two different ways. Firstly, the normal profit scheme is replaced by the notion of marginal profit and the model is renamed as the MEPQ model with marginal profit. Here, a small share of the total cost is expected to be ultimate profit. And it is established from the numerical simulation, the MEPQ model with marginal profit can fulfill the purpose of reducing the social surplus accordingly the Marxian point of view. The MEPQ model with normal profit is also viewed in the uncertain environment by fuzzy decision-making setup. The fuzzy consideration is seemed to be favored for the goal of the Bourgeoisie. The manufactures can manipulate the uncertainty in their interest to earn more social surplus even larger quanta

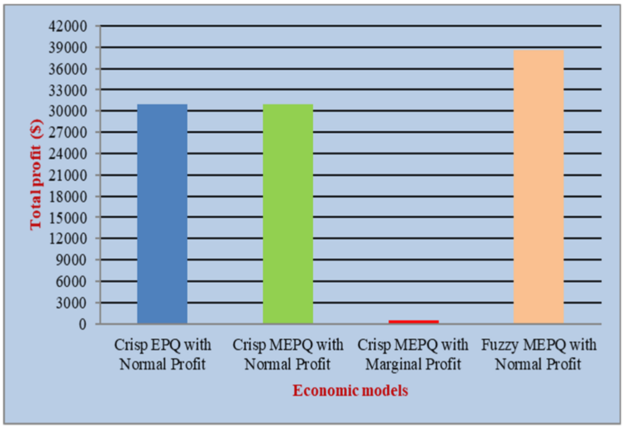

Fig. 2 Total profit in different economic model

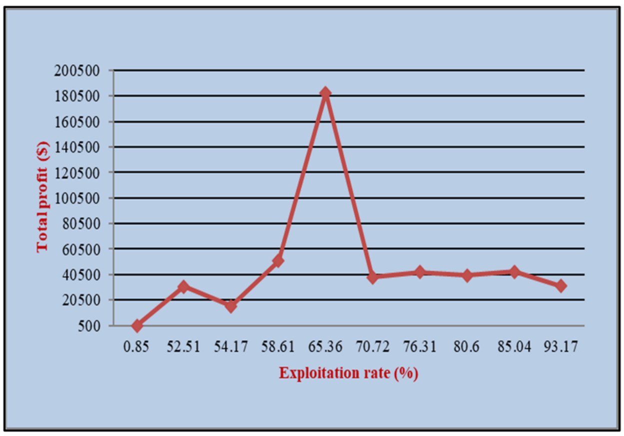

Fig. 3 Total profit versus exploitation rate

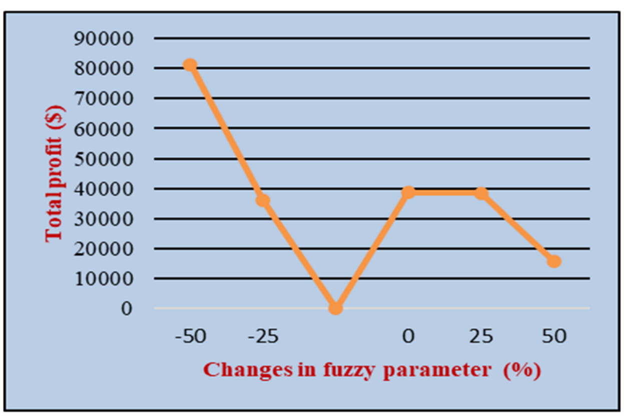

Fig. 4 Total profit versus change of fuzzy deviation of unit selling price

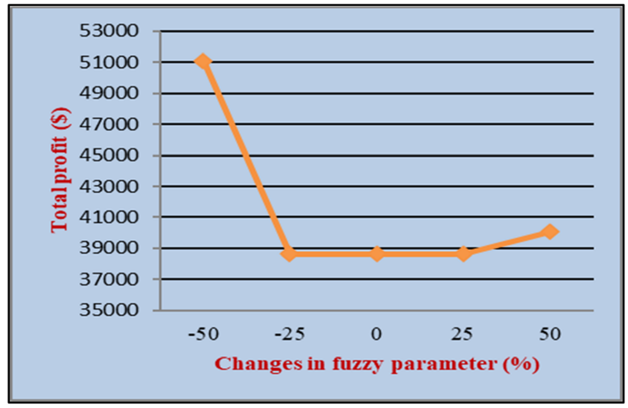

Fig. 5 Total profit versus change of fuzzy deviation of set up cost

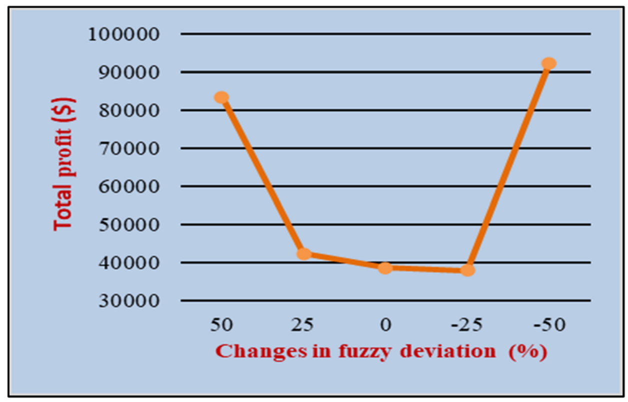

Fig. 6 Total profit versus change of fuzzy deviation of unit production cost

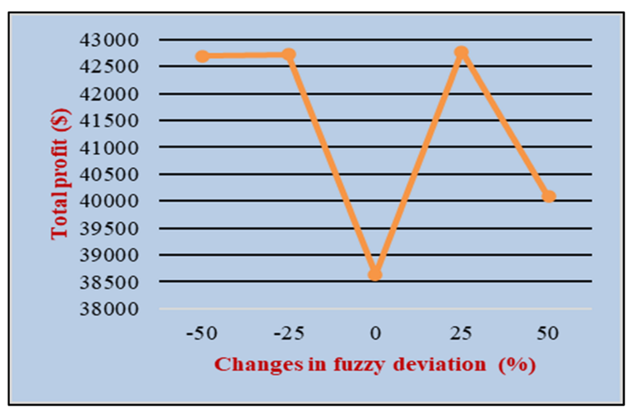

Fig. 7 Total profit versus change of fuzzy deviation of unit holding cost

compared to that of the crisp case. So, fuzzy MEPQ model with normal profit stands just opposite to the Marxian view. Traditionally, the cost minimization and profit maximization are the celebrated objectives to the manufacturer. So, a decision maker has to fix his/her goal on increasing profit. But, the purely Capitalistic approach in profit maximization is resulting the high exploitation and deprivation of the peoples of working class as well as middle class people. Though this policy contributes a huge surplus in the manufactures' hand but it may create negative reaction

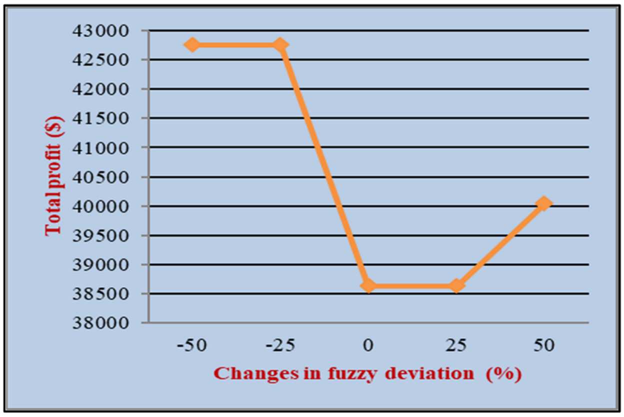

Fig. 8 Total profit versus change of fuzzy deviation of deterioration cost

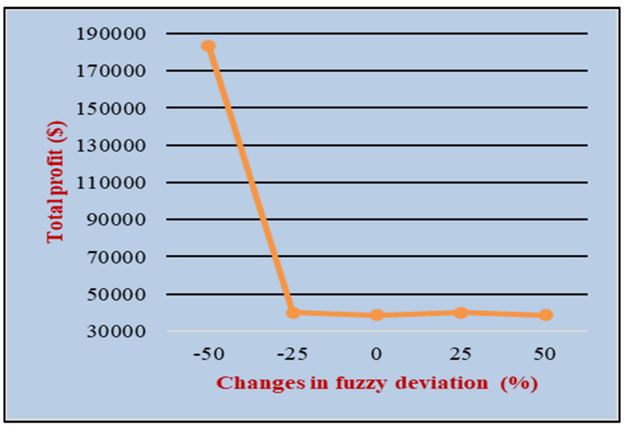

Fig. 9 Total profit versus change of fuzzy deviation of labour cost

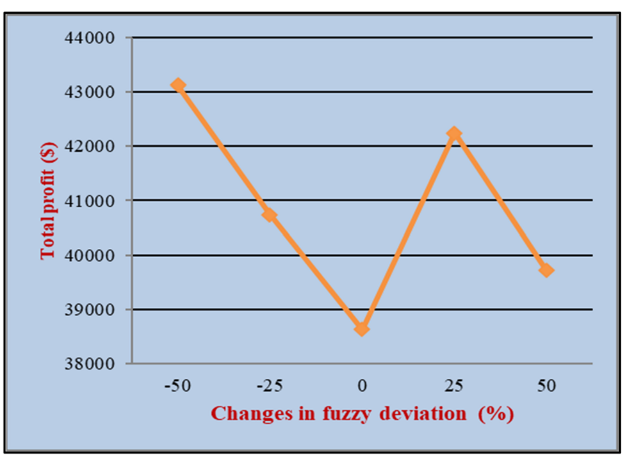

Fig. 10 Total profit versus change of fuzzy deviation of demand rate

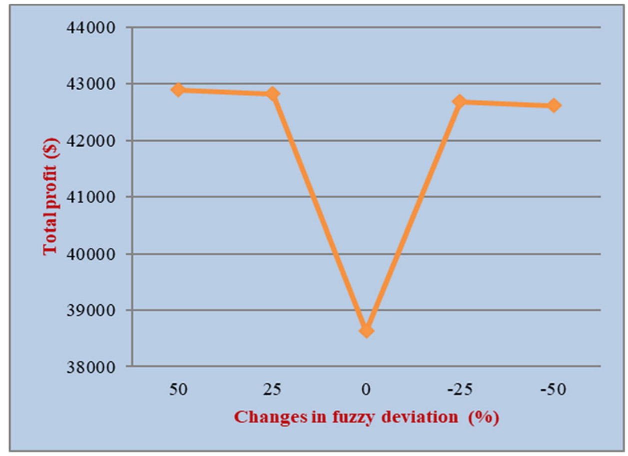

Fig. 11 Total profit versus change of fuzzy deviation of production

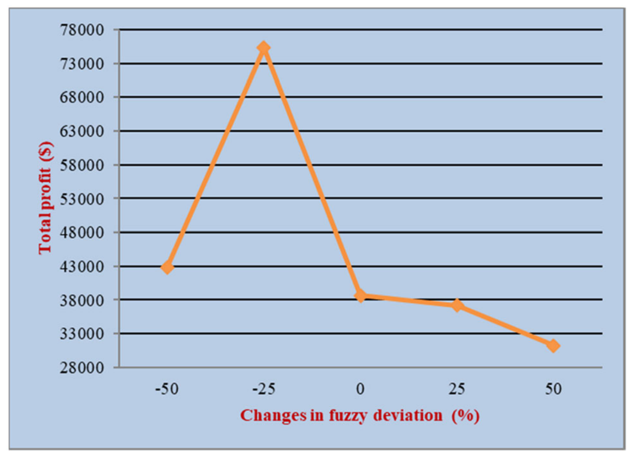

Fig. 12 Total profit versus change of fuzzy deviation of deterioration rate

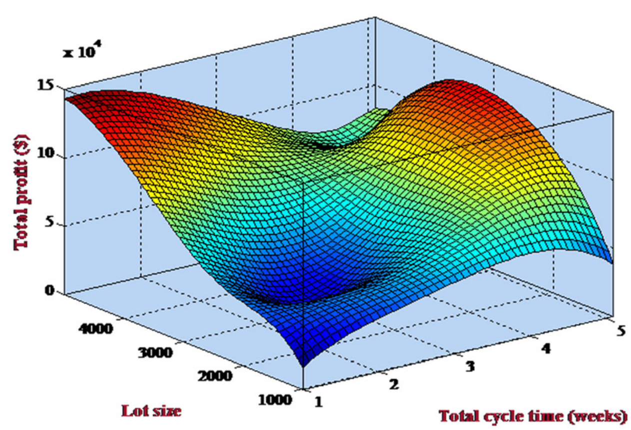

Fig. 13 Interdependency of total profit, lot size and cycle time

from exploited section. There may be caused unrest, struggle for revolution which ultimately hampers the productivity. Therefore, the MEPQ model with marginal profit can be an effective model in this regard. However, the Capitalistic approach of the economic system dominates the current era of the economic activity throughout the whole world. So, we cannot neglect the objectivity of the Bourgeoisie society. In this article, we have shown the both

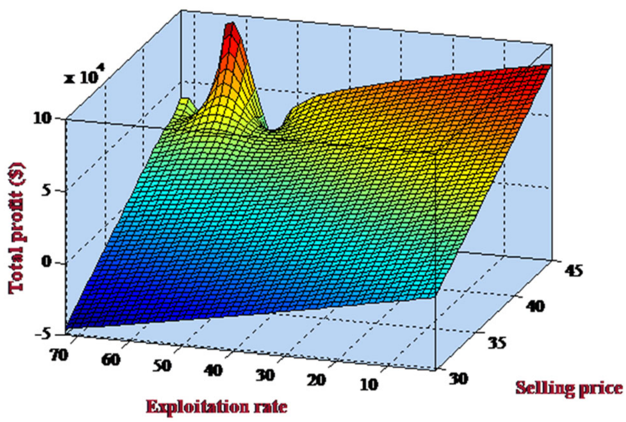

Fig. 14 Interdependency of total profit, exploitation rate and selling price

sides of the coin manifesting two different models in favor of both the Marxian and Capitalistic aspirations.

The present study can claim its novelties in the below mentioned grounds: Firstly, up to the authors' knowing, Marxian philosophy is experimented in the discussion of a lot-sizing problem for first time in literature. Secondly, the mathematical foundation of the manufacturing inventory model having multi-valued objective function has been studied here within in a single objective function. Thirdly, a model is developed on manpower exploitation in production farm keeping a new direction of modern research.

However, the authors admit their limitations for developing a fully theoretical and hypothetical studies. So, the notion of alternative policy of lot-sizing management introduced here has to be modified with real data and business problem in future. Again, the whole world is going through a pandemic due to COVID-19 and global economy is going through very difficult time. At least for this hard time, the manufacturers can reduce the exploitation rate and earning of social surplus maintaining the principal of marginal profit. Then, the consumers and the working class with very low income can survive and the production-supply consumption cycle run as well. In this context, some modifications on the MEPQ model with marginal profit introduced here can be a fruitful mathematical aid.

Declarations

Conflict of interest The authors declare that there is no conflict of interest regarding publication of this article.

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