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Study of limit cycles of an autonomous system of differential equations

R.F. Matlak

The thesis concerns the system

$$\text{(S)} \qquad \frac{dx}{dt} = a + bx + cy + dx^2 + exy + fy^2 \ , \quad \frac{dy}{dt} = xy \ ,$$

studied earlier by Čerkas [1] and Il'in [4].

In Part One of the thesis critical points of (S), both in the finite (x, y)-plane and at infinity, are analysed, which leads to a characterisation of these points and to criteria for the existence of two saddles (anti-saddles) on either of the coordinate axes. As an application of these, all possible configurations of singularities are found for the system.

The cases of a strong (weak) focus or a centre are then examined in greater detail. It is shown, with the aid of results of Coppel [2], that (S) cannot have two weak foci nor a weak focus and a centre. Further, using the process of Poincaré and Liapunov, conditions are obtained for the stability of the weak focus and for the generation of a limit cycle by small perturbations which change a weak focus into a strong one.

The main result of the thesis is the following criterion for the stability of a limit cycle of (S):

If $\gamma \equiv (\phi(t), \psi(t))$ is a limit cycle of (S) about the strong (weak) focus $F \equiv (0, \eta)$, then γ is stable (unstable) when

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$$\int_\gamma [b+e(\mathrm{sign}\eta)\exp\psi(t)]dt$$

is negative (positive).

Part Two concerns two systems related to (S).

The first is a numerical example of a system derived by employing the method of Poincaré and Liapunov. It is shown that the system possesses a unique limit cycle. The calculations have been carried out with the aid of the Runge-Kutta method.

The second system, studied by Kukles and Šahova [5], is of the form

$$\text{(T)} \qquad \frac{dx}{dt} = -y + dx^2 + exy + fy^2 , \quad \frac{dy}{dt} = x .$$

Using the method of characteristic exponents and an appropriate topographic system, a new proof is given of the known result that (T) has no limit cycles.

Two results which arose from the work for this thesis appeared in [3] and [6].

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