Eur. Phys. J. B **48**, 265–269 (2005)
DOI: 10.1140/epjb/e2005-00398-5

THE EUROPEAN
PHYSICAL JOURNAL B

# Study of traffic flow at an unsignalized T-shaped intersection by cellular automata model

Q.-S. Wu[a], X.-B. Li, M.-B. Hu, and R. Jiang

School of Engineering Science, University of Science and Technology of China, Hefei, Anhui 230026, P.R. China

Received 8 July 2005
Published online 16 December 2005 – 

**Abstract.** In this paper, the traffic flow at an unsignalized T-shaped intersection in which there are three input directions of vehicles and two right-turnings and one left-turning has been investigated by using the cellular automata traffic model. The interactions between vehicles on different lanes and effects of traffic flow states of different roads on capacity of T-shaped intersection system are analyzed. The phase transition characteristics of traffic states on different lanes are studied. The research indicates that the model can be applied to the real traffic analysis and traffic forecast.

**PACS.** 89.40.-a Transportation – 45.70.Vn Granular models of complex systems; traffic flow – 05.70.Fh Phase transitions: general studies

## 1 Introduction

Unsignalized intersections exist widely in urban traffic system. They play an important role in the control of the traffic network system. The study of unsignalized intersection is the base of other intersections' study. The interaction between vehicles on different lanes, and the effects of different traffic policies on the traffic capacity are not only of scientific significance on the development of traffic theory, but also of guiding importance for establishing suitable urban traffic policies.

The studies usually concentrate on the analysis and calculation of the capacity of T-shape intersection system. In general, the methods can be classified into two types [1]: Theoretical Analysis Method and Empirical Analysis Method. The theoretical analysis method, including the gap acceptance theory and queuing theory, predicts the capacity of intersections theoretically under some supposed conditions. However, the above theories assume that all drivers are uniform. That is, different drivers will response to the same traffic condition with the same manner. Obviously, this assumption is not in agreement with the real conditions. Empirical analysis method, including delay analysis method and integrated analysis method, studies the dependence of traffic capacity on different factors with observation data. Empirical analysis methods have to gather great amount of data, and the universality of the results is limited.

Comparing with other theories and models, the cellular automata (CA) traffic model has a flexible evolution rules, and is easy to be parallelly realized on computer. Much work has been done in recent years on the use of cellular automata for modeling various aspects of city traffic [2–9]. Ruskin [10] and Wang [11] presented a CA model to simulate the traffic flow at the intersections. They follow the idea of gap acceptance theory, but incorporate more flexible rules, so that the spatial and temporal details of vehicles' interactions can be reproduced. But this CA model treats the vehicles at the intersection either stopping (velocity = 0) or moving with velocity equals one. It ignores the effect of variability of moving vehicles' velocity, so that the corresponding evolution rules are established based on the spatial gap between vehicles. This leads to the result that the vehicles cannot pass in some cases where it is possible to pass in real conditions, so that the predicted traffic capacity is lower than that of the reality.

Ceder and Eldar (2002) find that, the traffic capacity of cross intersection can be improved when split into two T-shape intersections apart from each other with distance $L$ [12]. Moreover, a majority of unsignalized intersections in urban traffic system are T-shape intersections. Therefore, the study of T-shape intersection is important. But the related theories and models are lacked.

This paper presents a new CA traffic model for an unsignalized T-shaped intersection based on our previous work [13] in which there are three input directions of vehicles and two right-turnings and one left-turning. This model considers the variation of vehicles' velocity and use temporal gap but not spatial gap to determine which vehicle is allowed to pass the intersection. The result of this new model agrees well with real traffic.

[a] e-mail: qswu@ustc.edu.cn

## 2 Model

The illustration of T-shape intersection and the traffic is shown in Figure 1a. The lanes are divided to CA cells as shown in Figure 1b. Every vehicle takes a single cell. The lengths of lane 1–5 and 2–4 are both $L = 1999$. The lengths of lane 3 and 6 are $L_3 = L_6 = 500$. The intersection locates at the middle point of lane 1–5 ($b = 1000$).

The vehicles on lane 1 will turn right to lane 6 with turning probability $P_{t1}$ when they get to cell $b$, and the other vehicles steer straightly forward to lane 5. The vehicles on lane 2 will turn left to lane 6 with probability $P_{t2}$, and the other vehicles steer forward to lane 4. The vehicles on lane 3 will all turn right to lane 5. Here we do not consider the left-turning of vehicles on lane 3. So, the T-shape intersection has three input directions of vehicles, five steering paths, and two traffic conflicts. As shown in Figure 1, the left-turning vehicles on lane 2 will conflict with the straight or right-turning vehicles on lane 1 at cell $b$. And at cell $c$, the right-turning vehicle of lane 3 will conflict with the straight vehicles on lane 1. This paper simulates the two conflicts and the effect of conflicts on the traffic flow at the intersection.

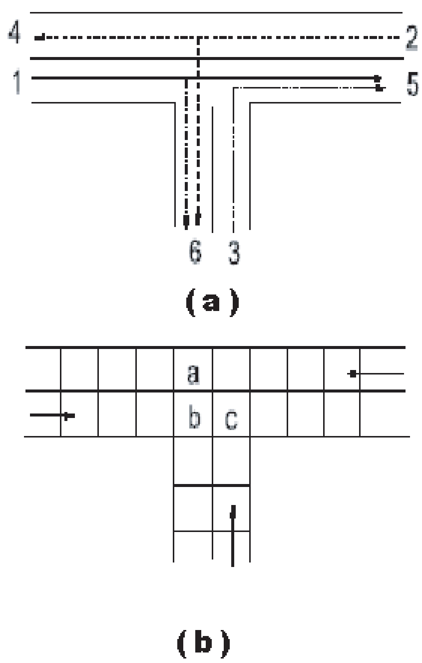


**Fig. 1.** Illustration of T-shape intersection with three input directions of vehicles.

### 2.1 The evolution rules for vehicles steering straightly at non-turning cells

For non-turning vehicles on all lanes and the vehicles tending to turn but not close to the turning cells, the velocity and position are updated according to the Nagel-Schreckenberg (NaSch) CA traffic model [2]. That is:

– velocity update: $v_n(t+1) = min(v_{max}, v_n(t)+1, d_n(t))$;
– randomization with probability $p$:

$$v_n(t+1) = \begin{cases} max(v_n(t+1) - 1, 0), & \text{if } (rand() < p); \\ v_n(t+1), & \text{if } (rand() \geq p) \end{cases}$$

– position update: $x_n(t+1) = x_n(t) + v_n(t+1)$.

Here $x_n$, $v_n$ is the position and velocity of vehicle $n$, $d_n = x_{n-1} - x_n - 1$ is the gap before vehicle $n$ (it is assumed that vehicle $n-1$ precedes vehicle $n$), $v_{max} = 5$ is the maximum velocity of the vehicles at the straight lane.

### 2.2 The turning rules at the turning points

For the turning vehicles, when they approach close to the turning cells where $T - x_n \leq 5$ ($T$ denote cell $a$, $b$ or $c$), the maximum velocity will decrease to $v_{maxt} = 2$. They arrive at the corresponding turning point with velocity zero. That is, when the vehicles are at cells $x_n$ from $T - x_n \leq 5$ to turning cells $T$, the velocity update rule change to: $v_n(t+1) \rightarrow min(v_{maxt}, v_n(t)+1, d_n(t), T - x_n(t))$. Then the conflicts at the turning point are handled with the following rules:

– **(i) The conflict of vehicles on lane 1 with the left-turning vehicles on lane 2**

The turning vehicle $i$ on lane 2 arrives at cell $a$ first, from where it turn to lane 6. If vehicle $i$ turn successfully, the velocity after turning is one and it will reach cell $b$ at the next time-step. But it may conflict with the vehicle $j$ on lane 1 reaching or passing cell $b$. Which vehicle can reach or pass cell $b$ is determined with the temporal gap difference. Obviously, the time for vehicle $i$ to reach cell $b$ is $t_i = 1$. The time for vehicle $j$ is calculated with the following formula:

$$t_j = \frac{b - x_j}{min(v_{max}, v_j + 1, d_j)}. \tag{1}$$

Here $v_{max}$ for vehicle $j$ is $v_{maxt}$ but not $v_{max}$ if it is a turning vehicle. Comparing $t_i$ with $t_j$, if $t_i < t_j$, then vehicle $i$ reach cell $b$ first and vehicle $j$ wait at the cell next to cell $b$; if $t_i > t_j$, then vehicle $j$ reach or pass cell $b$ first and vehicle $i$ wait at cell a for the next opportunity to reach cell $b$; if $t_i = t_j$, then it is determined with a preferential probability $P_{12}$ which vehicle to pass cell $b$ first.

– **(ii) The conflict of straightly steering vehicle on lane 1 with the right-turning vehicles on lane 3**

The turning vehicle m on lane 3 must reach cell $c$ first and then drive to lane 5 accelerating from velocity zero. It may conflict with the straightly steering vehicle n reaching or passing cell $c$. We calculate the time for vehicle m and n to reach cell $c$:

$$t_m = \frac{c - x_m}{min(v_{maxt}, v_m + 1, d_m)} \tag{2}$$

$$t_n = \frac{c - x_n}{min(v_{max}, v_n + 1, d_n)}. \tag{3}$$

Note that the $v_{max}$ in formula (2) is $v_{maxt} = 2$, but in formula (3) $v_{max} = 5$ which is the maximum velocity of straightly steering vehicle. Comparing $t_m$ with $t_n$, the vehicle with less time will reach or pass cell $c$ first, whereas the other vehicle should wait. If $t_m = t_n$, it is determined with a preferential probability $P_{13}$ which vehicle reach cell $c$ first.

In the first conflict, the vehicle $i$ and $j$ compete to take cell $b$. Here velocity of vehicle $i$ is one, whereas the velocity of vehicle $j$ is a certain speed in its driving. That is, vehicle $i$ and $j$ have a kind of inequality in competition. In the second conflict, vehicle m and n compete to take cell $c$. Vehicle m turns from lane 3, so that its maximum speed is limited to $v_{max} = 2$, while the straightly steering vehicle n has the maximum speed of $v_{max} = 5$. That is, the competition has also a kind of inequality for lane 1 and lane 3.

### 2.3 Deceleration

For all the straightly steering or turning vehicles, the speed is randomly decelerated with probability $p = 0.3$, in order to simulate the drivers' over-decelerating reaction.

Open boundary condition is used to simulate the input and output of vehicles on lane 1, lane 2 and lane 3. We assume that the first cells at the entrances of lane 1, 2 and 3 correspond to $x = 1$, and the entrance regions of lanes include $v_{max}$ cells, i.e., the vehicles can enter each lane from the cells $1, 2, ..., v_{max}$. In one time step, when the update of the vehicles on the lane is completed, we check the positions of the last vehicles on each lane, which are denoted as $x_{last}$. If $x_{last} > v_{max}$, a vehicle with velocity $v_{max}$ is injected with probability $P_{in1}$, $P_{in2}$, $P_{in3}$ at the cell $min(v_{max}, x_{last} - v_{max})$. Near the exits of the lanes, the leading vehicle is removed if $x_{first} > L$ ($L$ denotes the position of the last cell on the lane) and the following vehicle becomes the new leading vehicle.

## 3 Simulation result

As a preliminary work, we simulate the effect of turning probability $P_t$ on the traffic flow when changing the input probability $P_{in}$ on a single lane without considering other lanes' influence. As shown in Figure 2, the flow of single lane increases with the input probability until a saturated flux is reached. The turning vehicle acts as a bottleneck for the lane because it slows down near the turning point. One can see that this bottleneck effect is more obvious when $P_t$ is bigger because the maximum flow and the critical point of $P_{in}$ are smaller. When $P_t = 1$, the saturate flow is about 0.31, and the critical point of $P_{in}$ is about 0.31 also. In general, the flow will saturate when the input probability is 0.5. The simulations for lane 1, 2 and 3 have the same result. The turning probability for lane 3 is always one because all the vehicles on it will turn right.

Then we show the simulation results when vehicles enter from three lanes. Figure 3 shows the effects of lane 3

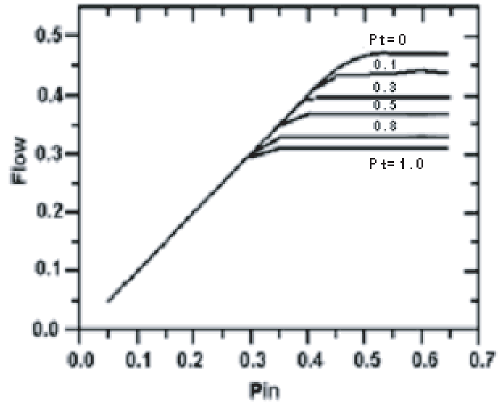


**Fig. 2.** The effect of turning probability $P_t$ on the flow with the changing of input probability $P_{in}$ without the conflicts of vehicle flows.

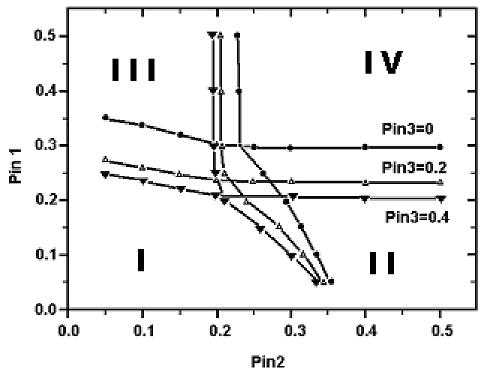


**Fig. 3.** The phase diagram in the $(P_{in1}, P_{in2})$ space for different input probability $P_{in3}$.

on the traffic of lane 1, 2 and on the whole intersection. Here turning probability $P_{t1}$ and $P_{t2}$ are 0.5 and preferential probability $P_{12}$ and $P_{13}$ are 0.5. The phase diagram in $(P_{in1}, P_{in2})$ for $P_{in3} = 0, 0.2, 0.4$ is classified into four regions. Region I corresponds to free flow on both lane 1, 2. Region II (III) corresponds to free flow on lane 1 (2) and congestion flow on lane 2 (1). Region IV corresponds to congestion flow on both lane 1 and 2.

When $P_{in3} = 0$, vehicles enter the system only from lane 1 and 2. One can see that the free flow region of lane 1 is larger for $P_{in1}$ than that for lane 2. When $P_{in2}$ is large enough, the critical changing point from free flow to congestion flow for lane 1 is about $P_{in1} = 0.3$. When $P_{in1} < 0.3$, the traffic on lane 1 will remain as free flow no matter what the traffic condition on lane 2 is. For lane 2, when $P_{in1}$ is large enough, the critical changing point from free flow to congestion flow is about $P_{in2} = 0.23$. When $P_{in2} > 0.23$, lane 2 will be congestion flow because of the heavy traffic on lane 1. Simulation reveals that the saturated flow on lane 1 is 0.296, whereas that of lane 2 is 0.23. The differences above arise from the rule inequality competing for cell $b$. The turning vehicles on lane 2 are more likely to wait at cell $a$ because it starts from static state to compete with the moving vehicle on lane 1 to

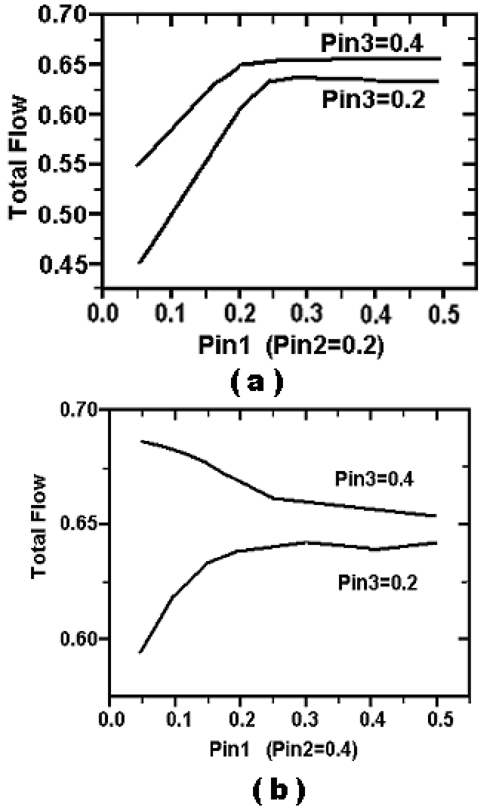


**Fig. 4.** The variation of intersection capacity with input probability.

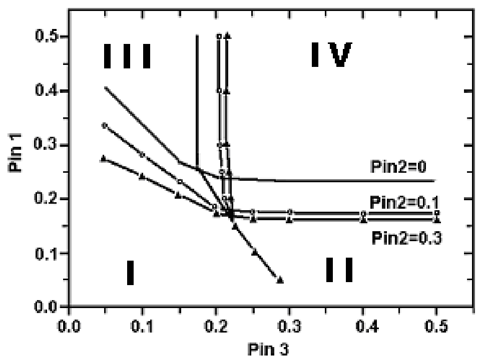


**Fig. 5.** Phase diagram in the $(P_{in1}, P_{in3})$ space with different input probability $P_{in2}$.

take cell $b$. So the free flow region on lane 2 in Figure 3 is smaller than that of lane 1.

When $P_{in3}$ increases, one can see that region I shrinks. The vehicles on lane 1–5 cannot drive freely after passing cell $b$ because they have to compete with the turning vehicles from lane 3. So that the critical changing point from free flow to congestion flow decrease and the free flow region shrinks. Meanwhile, the traffic on lane 2 is affected by lane 1, so that the critical changing point for lane 2 also moves left and the free flow region shrinks.

We note that when $P_{in1}$ is nearly zero, the critical changing point of lane 2 converges to $P_{in2} = 0.35$ for different $P_{in3}$. This is because the traffic on lane 2 will not be affected by lane 3 when there is no input vehicle on lane 1. The traffic on lane 2 can be described by the single lane simulation with turning probability $P_{t2} = 0.5$. So we conclude that the traffic on lane 3 affects lane 2 through lane 1.

Figure 4 shows the variation of intersection capacity with input probability $P_{in1}$, $P_{in2}$ and $P_{in3}$. In Figure 4a, the capacity of T-shape intersection increases until it is saturated when $P_{in1}$ increases. Here $P_{in2} = 0.2$, and $P_{in3}$ is 0.2 and 0.4 respectively. Simulation reveals that for all the cases with $P_{in2} < 0.25$, the tendency is the same. This is because that the traffic on lane 2 remains as free flow when $P_{in2} < 0.25$, and qualitatively it can not affect the flows on lane 1 and lane 3.

As shown in Figure 4b, when $P_{in2} = 0.4$, lane 2 is in congestion flow, and the intersection capacity changes differently. When $P_{in3} = 0.2$, lane 3 is in free flow and the flux remains unchanged. With the increase of $P_{in1}$, the flux on lane 1 increases whereas the flux on lane 2 decreases. But the decrement of lane 2 flux is lesser than the increment of lane 1 flux. So the intersection capacity increases with $P_{in1}$ until it is saturated. But when $P_{in3} = 0.4$, the flux increment of lane 1 cannot compensate the decrement of flux on lane 2 and 3, so that the intersection capacity first decreases obviously and then decreases slowly.

Next we investigate the dependence of lane 1, 3 flux and the intersection capacity on the input probability $P_{in2}$. Let the turning probability $P_{t1} = 0$ and $P_{t2} = 1$, that is, all vehicles on lane 1 will steer straightly and all vehicles on lane 2 will turn left. The preferential probability $P_{12}$ and $P_{13}$ are both 0.5. The phase diagram in the $(P_{in1}, P_{in3})$ space for different $P_{in2}$ is shown in Figure 5.

When $P_{in2} = 0$, the traffics on lane 1, 3 interact with each other. From Figure 5, one can see that the free flow region of lane 1 is larger than that of lane 3. The critical changing point of lane 1 is about 0.25, and the critical changing point of lane 3 is about 0.17. In the congestion flow region, the saturated flux on lane 1 is 0.23, whereas the saturated flux on lane 3 is 0.17. These differences arise from the rule inequality of competing for cell $c$ because the maximum speed for lane 1 is $v_{max} = 5$, while the maximum speed for the turning vehicles on lane 3 is $v_{maxt} = 2$.

Moreover, we also get the simulation result for the case of turning probability $P_{t1} = 0.5$. Now the saturated flux of lane 1 and 3 is 0.26 and 0.23 respectively. That is, when the number of straightly steering vehicles on lane 1 decreases, the conflict between lane 1 and 3 is lightened, so that the maximum fluxes of both lanes increase.

When $P_{in2} = 0.1$, the congestion flow region of lane 1 increases. But the congestion flow region of lane 3 decreases a little. This is because the competition of lane 2 and lane 1 decreases the disturbance of lane 1 on lane 3 traffic. So the effect of lane 2 on lane 3 is positive.

When $P_{in2} = 0.3$, the free flow region of lane 1 decreases more and the free flow region of lane 3 grows bigger. We note that when $P_{in1} < 0.15$ and $P_{in3} > 0.22$, the critical changing point of lane 3 for different $P_{in2}$ are the

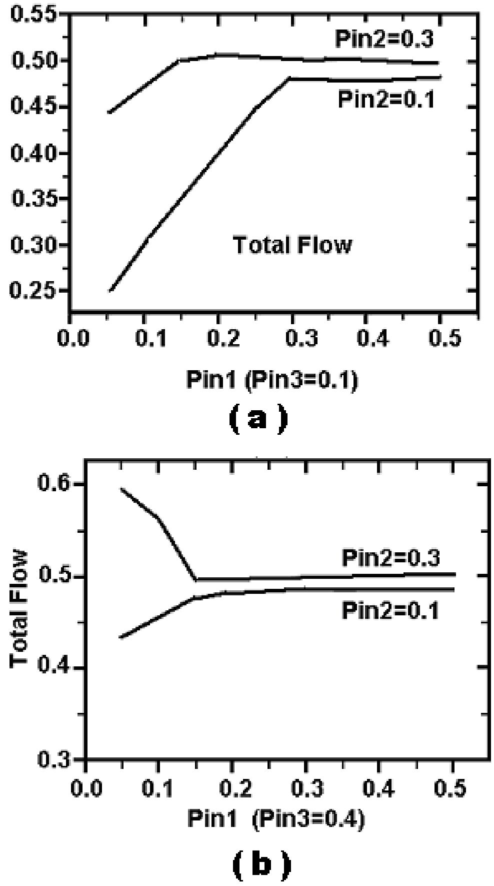


**Fig. 6.** The variation of intersection capacity with input probability.

same. That is, when $P_{in1}$ is small enough, the traffic flow on lane 1 remains free flow and it will not be affected by lane 2, so that the traffic on lane 3 will not be affected by lane 2.

Figure 6 shows the dependence of intersection capacity on the input probability $P_{in1}$ with different $P_{in2}$ and $P_{in3}$. The characteristics of these curves are like those in Figure 4.

The simulation results reported above are qualitatively and almost quantitatively in agreement with the measurement results of Chodur [14]. We believe that the rules and conflict judgment criterions in this model are compatible with the real traffic conditions and the model can reflect the basic characteristics of T-shape intersection.

## 4 Conclusions

We simulate the traffic flow at an unsignalized T-shape intersection with three input direction by using a new cellular automata model. The phase diagram and the variation of intersection capacity with vehicle input probability, and the interaction between vehicles on different lanes are discussed. When the fluxes have not reached saturation, the increase of input flux on lane 3 will decrease the flux on lane 1 and lane 2. The increase of input flux on lane 2 will decrease the flux on lane 1 but increase the flux of lane 3. The variation of capacity is determined by the net increment or decrement of flux on all three lanes. This new model can be applied to the practical traffic analysis and traffic forecast.

This work is financially supported by the National Natural Science Foundation of China (Grant No. 10272101, 10404025, 10532060) and the Open Foundation Project of the Key Laboratory of the State Ministry of Education on Transportation Intelligence Technology and System at the Beijing Jiaotong University.

## References

1. N.H. Gartner, C.J. Messer, A.K. Rathi, Federal Highway Administration of US Department of Transportation, 8-1–8-41 (1996)
2. K. Nagel, M. Schreckenberg, J. Phys. I (France) **2**, 2221 (1992)
3. B.S. Kerner et al., J. Phys. A **35**, 9971 (2002)
4. W. Knospe et al., J. Phys. A **33**, L477 (2000)
5. P. Berg, A. Woods, Phys. Rev. E **64**, 035602 (2001)
6. H.K. Lee et al., Phys. Rev. Lett. **92**, 238702 (2004)
7. R. Jiang, Q.S. Wu, J. Phys. A **36**, 381 (2003)
8. D. Chowdhury, L. Santen, A. Schadschneider, Physics Reports **329**, 199 (2000)
9. D. Helbing, Rev. Mod. Phys. **73**, 1067 (2001)
10. H.J. Ruskin, R.L. Wang, Lecture Notes in Computer Science **2329**, 381 (2002)
11. R.L. Wang, H.J. Ruskin, Lecture Notes of Computer Science **2667**, 577 (2003)
12. A. Ceder, K. Eldar, Transportation Research Part A **36**, 699 (2002)
13. X.B. Li, R. Jiang, Q.S. Wu, Int. J. Modern Phys. B **18**, 2703 (2004)
14. J. Chodur, *Transportation Research Circular EC108*, 4th International Symposium on Highway Capacity, (2000), p. 357