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STUDY ON CONFORMAL FDTD FOR ELECTROMAGNETIC SCATTERING BY TARGETS WITH THIN COATING

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Abstract—In order to simulate the electromagnetic scattering of targets with thin-coating accurately, a conformal finite-difference time-domain (CFDTD) method based on effective constitutive parameters is presented in this paper. Two kinds of coating problems are considered. For a coated target with medium backing material, the CFDTD formulations on conformal cells are the same as those of the conventional FDTD, but the parameters in FDTD formulations are replaced by effective constitutive parameters to include the curved coating message of target. For a coated target with perfectly conducting (PEC) backing material, the contour-path integral is used to exclude the curved PEC part, and effective constitutive parameters are then introduced to include the coating message. The bistatic radar cross section (RCS) of coated spheres with medium backing material and with PEC backing material are computed, respectively, to validate the presented CFDTD scheme. The backscattering of a composite airfoil, which is made of radar absorbing material (RAM) and metal framework, and coated by fiberglass-reinforced plastics, is also analyzed to demonstrate the feasibility of presented scheme.

1. INTRODUCTION

One important measure for reducing radar cross section (RCS) of an aircraft is to implement coating radar absorbing material (RAM) on its strong scattering centers, such as on its front and back edge, and the lip of inlet channel, or to use composite structures to absorb electromagnetic wave [1]. There are many methods used to analyze the electromagnetic scattering of coated targets, such as asymptotic ray solution [2], surface impedance boundary conditions (SIBC) [3], finite-element boundary-integral (FE-BI) method [4], boundary integral equationimpedance boundary conditions (BIE-IBC) method [5] and finite-difference time-domain (FDTD) method [6–12]. The FDTD method [13, 14] has unique advantages in dealing with targets of complex shape or inhomogeneous medium, and can get wideband RCS of a target by once computation, so it has been widely used to analyze many electromagnetic problems. However, the conventional FDTD method may introduce significant errors in computing the electromagnetic scattering of targets with curved surface, in particular, of thin coating due to the staircasing approximation. In order to reduce the errors, two approaches have been developed. One is subgrid technique, in which a majority of the target discrete with coarse cells and some special parts discrete with fine cells. But the reflection between coarse-fine cells boundary in this approach is very difficult to be eliminated. Another valid approach is conformal FDTD (CFDTD) technique, in which the cells near boundary of target are dealt with CFDTD schemes. In recent years, CFDTD methods for targets with curved PEC surface [15–20] and curved medium surface [21, 22] have been studied by many literatures. However, few of CFDTD schemes for targets with thin coating are reported.

In this paper, a CFDTD method based on effective constitutive parameters is presented. For a coated target with medium backing material, the CFDTD formulas about E-field and H-field samples on conformal cells are the same as those of conventional FDTD, but the constitutive parameters in CFDTD formulas are replaced by effective parameters to include the curved coating message. The effective values of permittivity and electric conductivity for E-field samples on conformal cells are calculated by weighted-length of backing medium, coating and free space; the effective values of permeability and equivalent magnetic loss for H-field samples are however calculated by weighted-area of backing medium, coating and free space. For a coated target with perfectly conducting (PEC) backing material, the contour-path integral is used in advance to simulate the curved PEC boundary accurately. Effective parameters are then introduced to simulate the influence of coating. The effective value of permittivity and electric conductivity for E-field samples on conformal cells are calculated by weighted-length of coating and free space, and the effective values of permeability and equivalent magnetic loss for H-field samples are however calculated by weighted-area of the coating and free space. The bistatic RCS of coated spheres with medium backing material and with PEC backing material are computed to validate the presented CFDTD method. Finally, the backscattering of a composite airfoil, which made of RAM and metal framework, and coated by fiberglass-reinforced plastics, is analyzed using this CFDTD method.

2. CONFORMAL FDTD METHOD FOR COATED TARGETS

In Cartesian coordinate system, we discrete targets with cuboid Yee cells, the sizes of cell in x, y, z directions are Δx, Δy, Δz, respectively. We assume E-field samples on the midpoints of cell's edges, and H-field samples on the centers of cell's flanks. Fig. 1 shows the mesh truncation at the boundary of a coated target, in which the backing is material 1 and coating is material 2. The constitutive parameters of material 1 and material 2 are ε_1, σ_1, μ_1, σ_{m1}, and ε_2, σ_2, μ_2, σ_{m2}, respectively. Outside of material 2 is free-space.

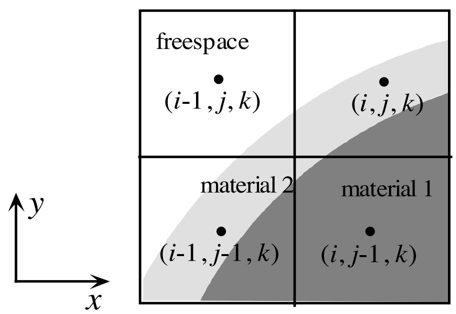


Figure 1. The mesh truncation of a coated target.

From Fig. 1 we can see that there are three types of cells near the surface of the coated target: ① Cells include material 1, material 2 and the free-space, such as $(i,\ j,\ k)$ and $(i-1,\ j-1,\ k)$. ② Cells include material 2 and the free-space, such as $(i-1,\ j,\ k)$. ③ Cells include material 1 and material 2, such as $(i,\ j-1,\ k)$. These three types of cells are all called conformal cell, and the cells total in material 1 or total in free space are called conventional cell. The E-field and H-field on conventional cells are computed using conventional FDTD. Hereinafter, we analyze the CFDTD method for coated targets whose backing material is medium and PEC, respectively, by using the x-y flank of cell $(i,\ j,\ k)$ in Fig. 1.

2.1. CFDTD for Coated Targets with Medium Backing

The E-field samples and H-field samples on the conformal cells are assumed to be as same as those on conventional FDTD cells. A and C are E_x-field samples, B and D are E_y-field samples, and F is H_z-field samples, respectively, as shown in Fig. 2.

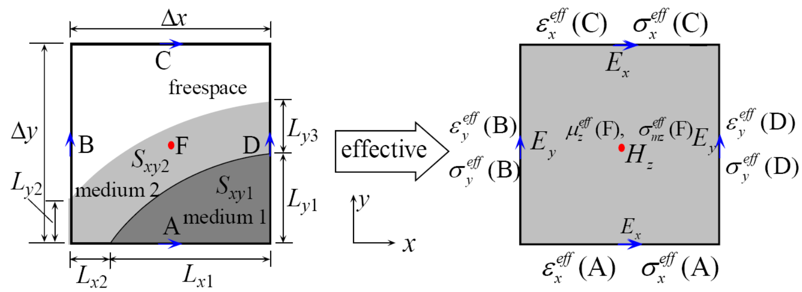


Figure 2. Effective constitutive parameters of coated target with medium backing.

The effective values of permittivity and electric conductivity for E-field samples on conformal cells are calculated by weighted-length of backing material (medium 1), coating (medium 2) and free space. The effective permittivity and electric conductivity at E-field samples A, B, C and D in Fig. 2 can be expressed, respectively, as

$$\begin{cases} \varepsilon_x^{eff}(\mathrm{A}) = [L_{x1} \cdot \varepsilon_1 + L_{x2} \cdot \varepsilon_2] / \Delta x \\ \varepsilon_y^{eff}(\mathrm{B}) = [L_{y2} \cdot \varepsilon_2 + (\Delta y - L_{y2}) \cdot \varepsilon_0] / \Delta y \\ \varepsilon_x^{eff}(\mathrm{C}) = \varepsilon_0 \\ \varepsilon_y^{eff}(\mathrm{D}) = [L_{y1} \cdot \varepsilon_1 + L_{y3} \cdot \varepsilon_2 + (\Delta y - L_{y1} - L_{y3}) \cdot \varepsilon_0] / \Delta y \end{cases} \tag{1}$$

$$\begin{cases} \sigma_x^{eff}(\mathrm{A}) = [L_{x1} \cdot \sigma_1 + L_{x2} \cdot \sigma_2] / \Delta x \\ \sigma_y^{eff}(\mathrm{B}) = L_{y2} \cdot \sigma_2 / \Delta y \\ \sigma_x^{eff}(\mathrm{C}) = 0 \\ \sigma_y^{eff}(\mathrm{D}) = [L_{y1} \cdot \sigma_1 + L_{y3} \cdot \sigma_2] / \Delta y \end{cases} \tag{2}$$

where the superscript "*eff*" denotes the effective values of parameters. L_{x1} and L_{x2} denote the length of medium 1 and medium 2, respectively, on the lower edge. L_{y2} denotes the length of medium 2 on the left edge. L_{y1} and L_{y3} are the length of medium 1 and medium 2, respectively, on the right edge.

The effective values of magnetic permeability and equivalent magnetic loss for H-field samples are calculated by weighted-area of medium 1, medium 2 and free space. Then, the effective magnetic permeability and equivalent magnetic loss at the H-field sample F in Fig. 2 can be expressed as

$$\mu_z^{eff}(\mathrm{F}) = \left[S_{xy1}\cdot\mu_1 + S_{xy2}\cdot\mu_2 + (\Delta x\Delta y - S_{xy1} - S_{xy2})\cdot\mu_0\right]/\Delta x\Delta y \quad (3)$$

$$\sigma_{mz}^{eff}(\mathrm{F}) = \left[S_{xy1}\cdot\sigma_{m1} + S_{xy2}\cdot\sigma_{m2}\right]/\Delta x\Delta y \quad (4)$$

where S_{xy1} and S_{xy2} denote the areas of medium 1 and medium 2 on the x-y flank, respectively.

(1) Modification of E-field formulas

The time stepping expressions of $E_x(i+1/2,\, j,\, k)$ and $E_y(i,\, j+1/2,\, k)$ on the conformal cells can be expressed as

$$\begin{aligned} &E_x^{n+1}(i+1/2,\, j,\, k) \\ =&\ \mathrm{CA}_x \cdot E_x^n(i+1/2,\, j,\, k) \\ &+\mathrm{CB}_x\cdot\left\{\left[H_z^{n+1/2}(i+1/2, j+1/2, k) - H_z^{n+1/2}(i+1/2, j-1/2, k)\right]\Big/\Delta y\right. \\ &\left.-\left[H_y^{n+1/2}(i+1/2, j, k+1/2) - H_y^{n+1/2}(i+1/2, j, k-1/2)\right]\Big/\Delta z\right\} \end{aligned} \quad (5)$$

$$\begin{aligned} &E_y^{n+1}(i,\, j+1/2,\, k) \\ =&\ \mathrm{CA}_y \cdot E_y^n(i,\, j+1/2,\, k) \\ &+\mathrm{CB}_y\cdot\left\{\left[H_x^{n+1/2}(i, j+1/2, k+1/2) - H_x^{n+1/2}(i, j+1/2, k-1/2)\right]\Big/\Delta z\right. \\ &\left.-\left[H_z^{n+1/2}(i+1/2, j+1/2, k) - H_z^{n+1/2}(i-1/2, j+1/2, k)\right]\Big/\Delta x\right\} \end{aligned} \quad (6)$$

where the coefficients CA_x, CB_x and CA_y, CB_y are, respectively

$$\begin{cases} \mathrm{CA}_x = \left[1 - \dfrac{\sigma_x^{eff}(i+1/2, j, k)\Delta t}{2\varepsilon_x^{eff}(i+1/2, j, k)}\right]\Big/\left[1 + \dfrac{\sigma_x^{eff}(i+1/2, j, k)\Delta t}{2\varepsilon_x^{eff}(i+1/2, j, k)}\right] \\ \mathrm{CB}_x = \left[\dfrac{\Delta t}{\varepsilon_x^{eff}(i+1/2, j, k)}\right]\Big/\left[1 + \dfrac{\sigma_x^{eff}(i+1/2, j, k)\Delta t}{2\varepsilon_x^{eff}(i+1/2, j, k)}\right] \end{cases} \quad (7)$$

$$\begin{cases} \mathrm{CA}_y = \left[1 - \dfrac{\sigma_y^{eff}(i, j+1/2, k)\Delta t}{2\varepsilon_y^{eff}(i, j+1/2, k)}\right]\Big/\left[1 + \dfrac{\sigma_y^{eff}(i, j+1/2, k)\Delta t}{2\varepsilon_y^{eff}(i, j+1/2, k)}\right] \\ \mathrm{CB}_y = \left[\dfrac{\Delta t}{\varepsilon_y^{eff}(i, j+1/2, k)}\right]\Big/\left[1 + \dfrac{\sigma_y^{eff}(i, j+1/2, k)\Delta t}{2\varepsilon_y^{eff}(i, j+1/2, k)}\right] \end{cases} \quad (8)$$

(2) Modification of H-field formulas

The time stepping expression of $H_z(i+1/2, j+1/2, k)$ on the conformal cells can be expressed as

$$\begin{aligned} & H_z^{n+1/2}(i+1/2, j+1/2, k) \\ = \; & \mathrm{CP}_z \cdot H_z^{n-1/2}(i+1/2, j+1/2, k) \\ & -\mathrm{CQ}_z \cdot \left\{ \left[E_y^n(i+1, j+1/2, k) - E_y^n(i, j+1/2, k) \right] / \Delta x \right. \\ & \left. - \left[E_x^n(i+1/2, j+1, k) - E_x^n(i+1/2, j, k) \right] / \Delta y \right\} \end{aligned} \quad (9)$$

where the coefficients CP_z and CQ_z are, respectively

$$\begin{cases} \mathrm{CP}_z = \dfrac{1 - \dfrac{\sigma_{mz}^{eff}(i+1/2, j+1/2, k)\Delta t}{2\mu_z^{eff}(i+1/2, j+1/2, k)}}{1 + \dfrac{\sigma_{mz}^{eff}(i+1/2, j+1/2, k)\Delta t}{2\mu_z^{eff}(i+1/2, j+1/2, k)}} \\ \mathrm{CQ}_z = \dfrac{\dfrac{\Delta t}{\mu_z^{eff}(i+1/2, j+1/2, k)}}{1 + \dfrac{\sigma_{mz}^{eff}(i+1/2, j+1/2, k)\Delta t}{2\mu_z^{eff}(i+1/2, j+1/2, k)}} \end{cases} \quad (10)$$

The other E- and H-field samples on the conformal cells can be computed with the above-mentioned manner.

2.2. CFDTD for Coated Targets with PEC Backing

When the backing material of a coated target is PEC, the constitutive parameters of material 1 in Fig. 1 are $\varepsilon_1 = \varepsilon_0$, $\sigma_1 \to \infty$, $\mu_1 = \mu_0$, $\sigma_{m1} = 0$. In this paper, we use the contour-path integral on conformal cells to fit the curved PEC boundary, and introduce effective parameters at the E-field and H-field samples to simulate the influence of thin coating on the scattering character of targets. There are differences between PEC backing and medium backing when effective values of constitutive parameters being calculated. If the edge or the flank of a conformal cell is partly filled with PEC, we need to calculate the effective parameters by weighted length or area of the parts except PEC. i.e., the effective values of electrical permittivity and electric conductivity for E-field samples are calculated by weighted-length of coating and free space; the effective values of magnetic permeability and equivalent magnetic loss for H-field samples are however calculated by weighted-area of coating and free space.

2.2.1. Effective Parameters

Now, we analyze the technique of effective parameters using a Yee cell's flank, including PEC backing material, coating and free space. As Fig. 3 shows, A, B, C and D are E-field samples, and F is H-field sample. Because the lower edge is filled with PEC and coating, effective parameters of A are equal to the corresponding parameters of coating. The right edge is filled with PEC, coating and free space, effective parameters of D are equal to the corresponding parameters by weighted-length of coating and free space. The left and upper edges are not filled with PEC, so effective parameters of B and C are calculated the same as the case of medium backing.

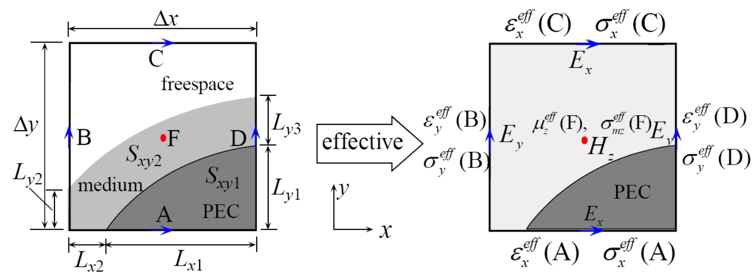


Figure 3. Effective constitutive parameters of coated target with PEC backing.

The effective electrical permittivity and conductivity for E-samples A, B, C, and D can be expressed as

$$\begin{cases} \varepsilon_x^{eff}(\mathrm{A}) = \varepsilon_2 \\ \varepsilon_y^{eff}(\mathrm{B}) = \left[L_{y2} \cdot \varepsilon_2 + (\Delta y - L_{y2}) \cdot \varepsilon_0\right] / \Delta y \\ \varepsilon_x^{eff}(\mathrm{C}) = \varepsilon_0 \\ \varepsilon_y^{eff}(\mathrm{D}) = \left[L_{y3} \cdot \varepsilon_2 + (\Delta y - L_{y1} - L_{y3}) \cdot \varepsilon_0\right] / (\Delta y - L_{y1}) \end{cases} \tag{11}$$

$$\begin{cases} \sigma_x^{eff}(\mathrm{A}) = \sigma_2 \\ \sigma_y^{eff}(\mathrm{B}) = (L_{y2} \cdot \sigma_2) / \Delta y \\ \sigma_x^{eff}(\mathrm{C}) = 0 \\ \sigma_y^{eff}(\mathrm{D}) = (L_{y3} \cdot \sigma_2) / (\Delta y - L_{y1}) \end{cases} \tag{12}$$

where L_{x1} and L_{x2} are the length of PEC and coating, respectively, on the lower edge, L_{y2} is the length of coating on the left edge, L_{y1} and L_{y3} are the lengths of PEC and coating, respectively, on the right edge.

The flank in Fig. 3 is filled with PEC, coating and free space, effective parameters of F are equal to the corresponding parameters by weighted-area of coating and free space. So effective magnetic permeability and equivalent magnetic loss for H-field sample F can be written as

$$\mu_z^{eff}(\mathrm{F}) = [(\Delta x\Delta y - S_{xy1} - S_{xy2})\cdot\mu_0 + S_{xy2}\cdot\mu_2] \,/\, (\Delta x\Delta y - S_{xy1}) \quad (13)$$

$$\sigma_{mz}^{eff}(\mathrm{F}) = (S_{xy2}\cdot\sigma_{m2}) \,/\, (\Delta x\Delta y - S_{xy1}) \quad (14)$$

where S_{xy1} and S_{xy2} are the areas of PEC and coating, respectively, on the cell's flank.

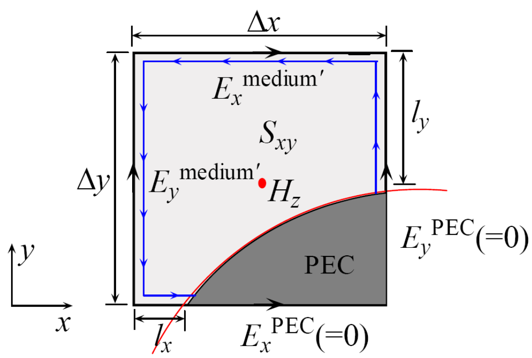


Figure 4. Contour-path integral for a conformal cell with PEC.

2.2.2. Modification of E- and H-field Time Stepping Expressions

When the conformal cells include perfectly electric conductor, we use the contour-path integral to fit the curved PEC surface. From Faraday's law

$$\oint \boldsymbol{E}\cdot d\boldsymbol{l} = -\mu\frac{\partial}{\partial t}\iint \boldsymbol{H}\cdot d\boldsymbol{s} - \sigma_m\iint \boldsymbol{H}\cdot d\boldsymbol{s} \quad (15)$$

The time stepping expression of $H_z(i+1/2, j+1/2, k)$ can be written as

$$\begin{aligned} & H_z^{n+1/2}(i+1/2, j+1/2, k) \\ = \; & \mathrm{CP}_z \cdot H_z^{n-1/2}(i+1/2, j+1/2, k) \\ & -(\mathrm{CQ}_z/S_{xy}) \cdot \left[E_y^n(i+1, j+1/2, k) \cdot l_y - E_y^n(i, j+1/2, k) \cdot \Delta y \right. \\ & \left. -E_x^n(i+1/2, j+1, k) \cdot \Delta x + E_x^n(i+1/2, j, k) \cdot l_x\right] \end{aligned} \quad (16)$$

where l_x and l_y denote the lengths of the lower and right edges except PEC, respectively. S_{xy} denotes the area of the cell's flank except PEC. Corresponding to Fig. 3, $l_x = L_{x2}$, $l_y = \Delta y - L_{y1}$, $S_{xy} = \Delta x \Delta y - S_{xy1}$. The form of coefficients CP_z and CQ_z are similar to equation (10), but the effective permeability and equivalent magnetic loss are replaced by those in equations (13) and (14). When $l_x = \Delta x$ and $l_y = \Delta y$, the area $S_{xy} = \Delta x \Delta y$, equation (16) degenerate to the case of conformal cells without PEC.

The time-step of FDTD is determined by the shape and area of cells. So the small contour path area on conformal cells requires a small time-step in FDTD computation according to Courant stability condition. The time-step used in CFDTD is however usually determined by conventional cell size that may cause instability. In order to eliminate instability arising from the very small distorted cell, we use a so called 1/6 criterion as implemented in [15]. That means when the ratio of the distorted cell area filled by coating and free space to the regular cell area $S_{xy}/(\Delta x \Delta y) < 1/6$, we take the approximation of $S_{xy} = 1/6 \Delta x \Delta y$.

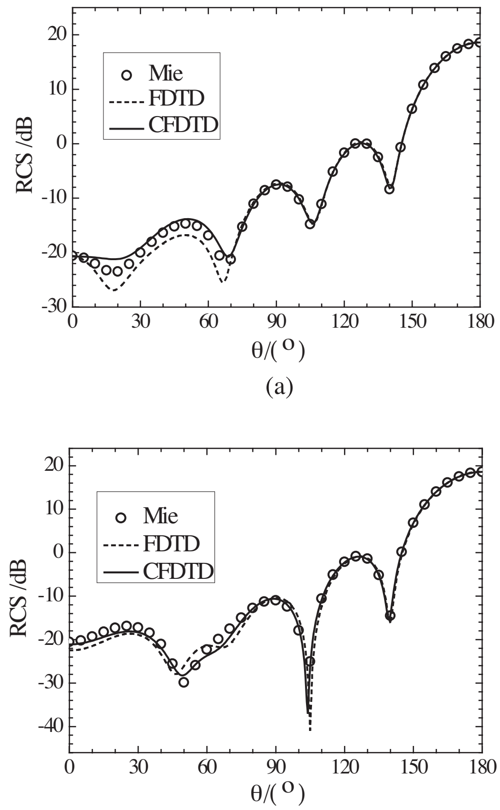
The CFDTD formulas for Ex- and Ey-field samples on the conformal cells are similar to equations (5) and (6), but the effective constitutive parameters in coefficients of CA_x, CB_x, CA_y, CB_y are replaced by those in equations (11)–(14).

3. NUMERICAL RESULTS

In order to validate the CFDTD method presented in this paper, bistatic RCS of coated spheres with medium backing and with PEC backing are computed, respectively.

3.1. Bistatic RCS of a Coated Sphere with Medium Backing

As the first example, we consider the bistatic scattering characteristics of a coated sphere with medium backing. The frequency of incident wave is 3 GHz. The inner radius of the sphere is 7.5 cm, and the constitutive parameters of backing medium are $\varepsilon_r = (1.5, -0.5)$, $\mu_r = (2.0, -0.3)$. The thickness of coating is 0.1 cm, and the constitutive



(a)

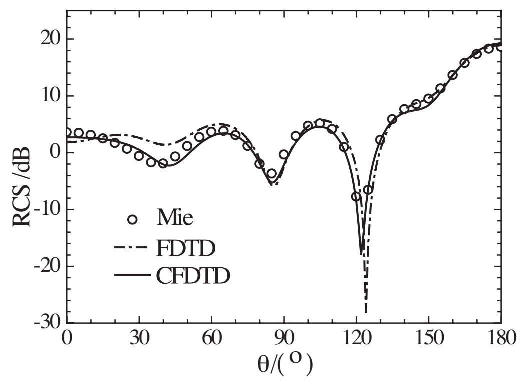
(b)

Figure 5. Bistatic RCS of a coated sphere with medium backing. (a) VV polarization, (b) HH polarization.

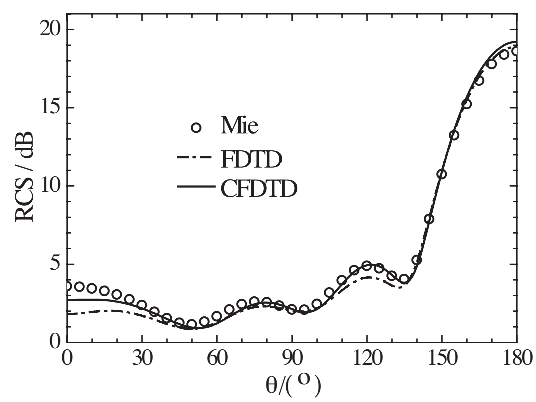
parameters are $\varepsilon_r = (2.0, -0.6)$, $\mu_r = (1.5, -0.5)$. The lattice space increments are $\delta = \Delta x = \Delta y = \Delta z = 0.5\,\text{cm}$, i.e., the thickness of coating is 0.2δ. The memory requirements and time costs of CFDTD and FDTD on computer P4 2.66 for 800 time-steps are about 9.3 Mb, 69.235 s, and 8.2 Mb, 53.64 s, respectively. Fig. 5 shows the bistatic RCS of the coated sphere, in which (a) is VV polarization and (b) HH polarization, solid line, dotted line and ◯ denote the results of CFDTD, FDTD and Mie's solution, respectively. From Fig. 5 we can see that a rather large error exist in the results of FDTD at the range of $0° \sim 70°$ (VV polarization) and $30° \sim 80°$ (HH polarization), respectively, comparing with Mies solutions. However, the results of CFDTD are in good agreement with Mies solutions.

3.2. Bistatic RCS of a Coated Sphere with PEC Backing

Next, we consider a coated PEC sphere with inner radius 0.75 m, thickness of coating is 0.01 m the frequency is 0.3 GHz. The constitutive parameters of coating are $\varepsilon_r = (4., -1.)$, $\mu_r = (3., -1.)$. The lattice space increments are $\delta = \Delta x = \Delta y = \Delta z = 0.05$ m, i.e., the thickness of coating is 0.2δ. Fig. 6 shows the bistatic RCS of the coated PEC sphere, in which (a) is VV polarization and (b) HH polarization, solid line, dotted line and ◯ denote the results of CFDTD, FDTD and Mie's solution, respectively. From Fig. 6 we can see that the results of CFDTD are coincident with Mie's solutions, but a rather large error exists in the results of FDTD.



(a)



(b)

Figure 6. Bistatic RCS of coated sphere with PEC backing. (a) VV polarization, (b) HH polarization.

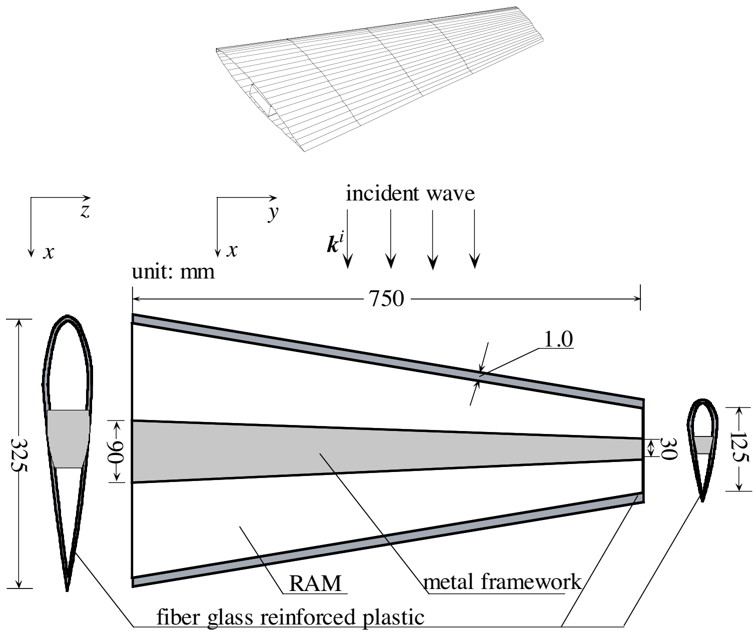


Figure 7. Section planes of a composite airfoil.

3.3. Backscattering of a Composite Airfoil Model

Finally, we consider a composite airfoil [1]. Fig. 7 shows its section planes and outlines data. The constitutive parameters of coating and RAM are $\varepsilon_r = 3.1$, $\sigma = 5.338 \times 10^{-2}$ S/m, $\mu_r = 1.0$, $\sigma_m = 8.527\,\Omega$/m and $\varepsilon_r = 4.8$, $\sigma = 1.335 \times 10^{-2}$ S/m, $\mu_r = 1.40$, $\sigma_m = 2.842 \times 10^3\,\Omega$/m. The FDTD lattice space increments are $\delta = \Delta x = \Delta y = \Delta z = 2$ mm, respectively and the thickness of the coating is 0.5δ. The time increment is $\Delta t = \delta/2c = 3.333 \times 10^{-12}$ s. The FDTD computational region is $236\Delta x \times 72\Delta y \times 424\Delta z$, which requires about 490 Mb memories. The expression of incident Gaussian pulse is

$$E^i(t) = \exp\left[-4\pi\left(t - t_0\right)^2/\tau^2\right] \tag{17}$$

where $\tau = 60\Delta t$, $t_0 = 0.8\tau$.

The direction of incident wave parallel to x-axis, and E-field parallels to y-axis. Fig. 8 shows far field backscattering of the composite airfoil and a PEC one, in which (a) is time domain response and (b) frequency domain response. From Fig. 8 we can see that the composite structure can reduce RCS of airfoil effectively.

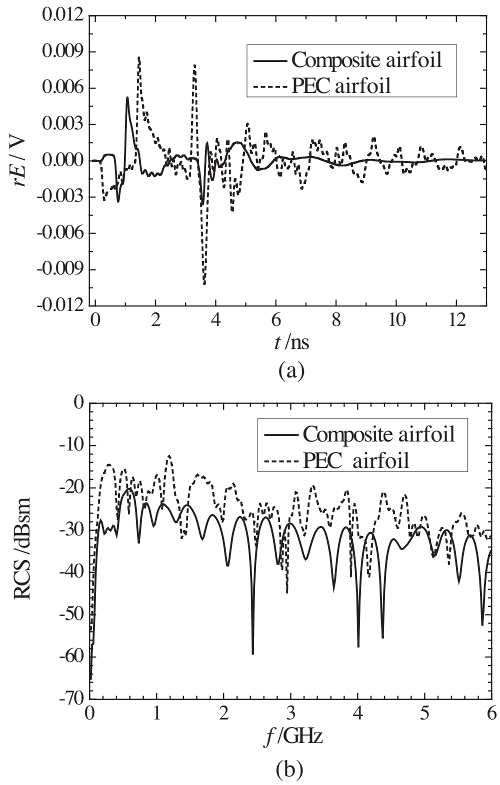


Figure 8. Far-field backscattering of airfoil. (a) time domain response, (b) bfrequency domain response.

4. CONCLUSION

The CFDTD method was presented to analyze the electromagnetic scattering of coated targets with medium backing and PEC backing. The numerical results of coated spheres with medium backing and PEC backing validated this method, respectively. Finally, it was used to analyze the backscattering of a composite airfoil. This CFDTD method also can be used to analyze scattering of multilayer coated targets.

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