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Study on Synchronization of Two Eccentric Rotors Driven by Hydraulic Motors in One Vibrating System

In this article the synchronization of two eccentric rotors driven by hydraulic motors (TERDHM) in one vibrating system is studied. The differential equations of TERDHM motion are derived, and the synchronous and stable rotating conditions of TERDHM are established. It is found from the analysis and simulation that the angular velocities of TERDHM act on each other and both vary in a small range approaching their mean value. The synchronous rotation speed is equal to the average angular velocity of TERDHM, and the phase difference between them depends on the difference of the flow rates of the two hydraulic motors.

INTRODUCTION

A new kind of vibrating system is introduced in this article. In this system two eccentric rotors that excite vibration are separately driven by two hydraulic motors instead of two electric motors. It is known that a hydraulic motor has many advantages, such as it is very easy to change and control its rotating speed and it is smaller and lighter than an electric motor. These advantages are very applicable to vibrating machines.

Although the synchronous motion theory of two eccentric rotors driven by two electric motors has been studied in great detail (Blekhman, 1981, 1988; Wen, 1982), the many theoretical problems of a vibrating system excited by TERDHM have not been analyzed thoroughly. These include the motion law of TERDHM, the rotating speed characteristics, the phase difference between two rotors, and, particularly, the possibility of maintaining synchronous rotation. Based on the analytic method of nonlinear theory, the above problems are analyzed in detail with the condition that the hydraulic motors are fixed displacement motors.

The results of the simulation and calculation are consistent with theoretical analysis.

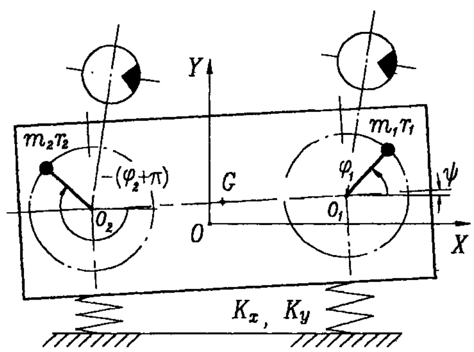

FIGURE 1 The mechanical model of the vibrating system.

Received 17 January 1995; Revised 20 February 1997.
Shock and Vibration, Vol. 4, No. 5,6, pp. 305–310 (1997)
ISSN 1070-9622/97/$8.00

MECHANICAL MODEL AND DIFFERENTIAL EQUATION OF MOTION OF VIBRATING SYSTEM

The simplified mechanical model of the system is shown in Fig. 1, where xoy is a fixed coordinate axis and $G(x, y)$ is the mass center of the vibrating body. When the system is static, G overlaps with origin O. In Fig. 1 two eccentric rotors turn in opposite directions, and the displacements of the eccentric masses $m_1(x_1, y_1)$, $m_2(x_2, y_2)$ can be written as

$$\begin{aligned} x_1 &= x + l\cos\psi + r_1\sin\phi_1, \\ x_2 &= x - l\cos\psi - r_2\sin\phi_2, \\ y_1 &= y + l\sin\psi + r_1\cos\phi_1, \\ y_2 &= y - l\sin\psi + r_2\cos\phi_2, \end{aligned} \tag{1}$$

where ϕ_i, r_i are the rotating angle and eccentricity of the eccentric rotor, $i = 1, 2$; ψ is the angular displacement of the vibrating body about the mass center G; l is the length between G and the center O_i of the rotating shaft, here $l_i = l$; and x, y are the displacements of the mass center G in the x, y directions.

The kinetic energy T, potential energy V, and dissipation function D of the system can be expressed as follows:

$$\begin{aligned} T = {} & \frac{1}{2}M(\dot{x}^2 + \dot{y}^2) + \frac{1}{2}I\dot{\psi}^2 + \frac{1}{2}J_1\dot{\phi}_1^2 + \frac{1}{2}J_2\dot{\phi}_2^2 \\ & + m_1r_1\dot{\phi}_1(\dot{y}\cos\phi_1 - \dot{x}\sin\phi_1) \\ & - m_2r_2\dot{\phi}_2(\dot{y}\cos\phi_2 + \dot{x}\sin\phi_2) \\ & + (m_1 - m_2)l\dot{\psi}(\dot{y}\cos\psi - \dot{x}\sin\psi) \\ & + m_1lr_1\dot{\psi}\dot{\phi}\cos(\phi_1 - \psi) \\ & - m_2lr_2\dot{\psi}\dot{\phi}_2(\phi_2 + \psi), \end{aligned} \tag{2}$$

$$\begin{aligned} V = {} & \frac{1}{2}k_xx^2 + \frac{1}{2}k_yy^2 + \frac{1}{2}k_\psi\psi^2 \\ & + m_1gr_1(1 + \sin\phi_1) \\ & + m_2gr_2(1 + \sin\phi_2), \end{aligned} \tag{3}$$

$$\begin{aligned} D = {} & \frac{1}{2}f_x\dot{x}^2 + \frac{1}{2}f_y\dot{y}^2 + \frac{1}{2}f_\psi\dot{\psi}^2 \\ & + \frac{1}{2}f_1(\dot{\phi}_1 - \dot{\psi})^2 \\ & + \frac{1}{2}f_2(\dot{\phi}_2 + \dot{\psi})^2, \end{aligned} \tag{4}$$

where

$$M = M_0 + m_1 + m_2,$$
$$I = I_0 + m_1l^2 + m_2l^2,$$
$$J_1 = m_1r_1^2, \qquad J_2 = m_2r_2^2,$$

and m_i is the eccentric mass of the eccentric rotor, $i = 1, 2$; M_0, I_0 are the mass of the vibrating body and moment of inertia about the mass center G without m_i; f_i is the rotating damping coefficient of the rotating shaft; f_x, f_y, f_ψ are the damping coefficients of the vibrating system in the x, y, ψ directions, respectively; k_x, k_y, k_ψ are the spring constants of the vibrating system in the x, y, ψ directions, respectively; and g is the weight acceleration. Substituting T, V, and D into the Lagrange equation,

$$\frac{d\partial T}{dt\partial \dot{q}_i} - \frac{\partial T}{\partial q_i} + \frac{\partial V}{\partial q_i} + \frac{\partial D}{\partial \dot{q}_i} = Q_i. \tag{5}$$

We can obtain

$$\begin{aligned} & M\ddot{x} + f_x\dot{x} + k_xx \\ & \quad = m_1r_1\dot{\phi}_1^2\cos\phi_1 - m_2r_2\dot{\phi}_2^2\cos\phi_2 \\ & \quad\quad + m_1r_1\ddot{\phi}_1\sin\phi_1 - m_2r_2\ddot{\phi}_2\sin\phi_2 \\ & \quad\quad + l(m_1 - m_2)(\ddot{\psi}\sin\psi - \dot{\psi}^2\cos\psi), \\ & M\ddot{y} + f_y\dot{y} + k_yy \\ & \quad = m_1r_1\dot{\phi}_1^2\sin\phi_1 + m_2r_2\dot{\phi}_2^2\sin\phi_2 \\ & \quad\quad - m_1r_1\ddot{\phi}_1\cos\phi_1 - m_2r_2\ddot{\phi}_2\cos\phi_2 \\ & \quad\quad - l(m_1 - m_2)(\ddot{\psi}\cos\psi - \dot{\psi}^2\sin\psi), \\ & I\ddot{\psi} + f\dot{\psi} + k_\psi\psi \\ & \quad = -(f_1\dot{\phi}_1 - f_2\dot{\phi}_2) \\ & \quad\quad + l(m_1 - m_2)(\ddot{x}\sin\psi - \ddot{y}\cos\psi) \\ & \quad\quad - m_1r_1l[\ddot{\phi}_1\sin(\phi_1 - \psi) \\ & \quad\quad - \dot{\phi}_1^2\cos(\phi_1 - \psi)] \\ & \quad\quad + m_2r_2l[\ddot{\phi}_2\sin(\phi_2 + \psi) \\ & \quad\quad - \dot{\phi}_2^2\cos(\phi_2 + \psi)], \end{aligned} \tag{6}$$

$$\begin{aligned} & J_1\ddot{\phi}_1 + f_1\dot{\phi}_1 \\ & \quad - f_1\dot{\psi} + m_1r_1[\ddot{y}\cos\phi_1 - \ddot{x}\sin\phi_1] \\ & \quad + m_1r_1g\cos\phi_1 \\ & \quad + m_1r_1l[\ddot{\psi}\cos(\phi_1 - \psi) \\ & \quad - \dot{\psi}^2\sin(\phi_1 - \psi)] = T_1, \\ & J_2\ddot{\phi}_2 + f_2\dot{\phi}_2 \\ & \quad + f_2\dot{\psi} + m_2r_2[\ddot{y}\cos\phi_2 + \ddot{x}\sin\phi_2] \\ & \quad + m_2r_2g\cos\phi_2 \\ & \quad - m_2r_2l[\ddot{\psi}\cos(\phi_2 + \psi) \\ & \quad - \dot{\psi}^2\sin(\phi_2 + \psi)] = T_2, \end{aligned} \tag{7}$$

where $f = f_\psi + f_1 + f_2$ and T_i is the torque of the hydraulic motor, $i = 1, 2$.

Equation (6) is a set of differential equations of the motion of the vibrating body and Eq. (7) contains the differential equations of rotation of TERDHM. Considering the situation of small damping and ignoring higher order small terms, from Eq. (6) we can obtain the steady solutions in the nonresonant case,

$$\begin{aligned} x &= a_1 \cos\phi_1 - a_2 \cos\phi_2, \\ y &= b_1 \sin\phi_1 + b_2 \sin\phi_2, \\ \psi &= c_1 \sin\phi_1 + c_2 \sin\phi_2, \end{aligned} \tag{8}$$

where

$$\begin{aligned} a_i &= -m_i r_i / M(1 - \omega_x^2/\dot\phi_i^2), \\ b_i &= -m_i r_i / M(1 - \omega_y^2/\dot\phi_i^2), \\ c_1 &= \frac{[m_1 r_1 + (m_1 - m_2) b_1](-l)}{I(1 - \omega_\psi^2/\dot\phi_1^2)}, \\ c_2 &= \frac{[m_2 r_2 - (m_1 - m_2) b_2](-l)}{I(1 - \omega_\psi^2/\dot\phi_2^2)}, \\ \omega_x^2 &= k_x/M, \quad \omega_y^2 = k_y/M, \quad \omega_\psi^2 = k_\psi/I, \\ \omega_x^2 &> \omega_y^2, \quad \dot\phi_i^2 > \omega_x^2, \quad i = 1, 2. \end{aligned} \tag{9}$$

ROTATING SPEED CHARACTERISTICS OF TERDHM

The torques of two hydraulic motors, T_1 and T_2, in Eq. (7) can be expressed as

$$T_i = \mu q_i p_i, \quad q_i = Q_i/\dot\phi_i, \quad i = 1, 2, \tag{10}$$

where q_i, Q_i, p_i are the displacement, flow rate, and nominal pressure of the hydraulic motor, respectively, here the q_i means the flow rate of the hydraulic motor per rotation; and μ is the torque coefficient, which is constant. In this study hydraulic motors are fixed displacement motors. Let

$$q_1 = q_2 = q. \tag{11}$$

From Eq. (10) we know that flow rates Q_1 and Q_2 vary in their small value, and the rotating speed of rotors driven by hydraulic motors will also have a corresponding variation. Introducing a transform as follows:

$$\phi_i = \tau + \alpha_i, \quad \tau = \overline{\omega} t, \quad \tau \gg \alpha_i, \quad i = 1, 2, \tag{12}$$

where $\overline{\omega}$ is the average value of variables ϕ_i and α_i is the small value of the rotating angle. Representing d/dt with a prime and t. Transforming differential t into τ, one can obtain

$$\dot\phi_i = \overline{\omega}(1 + \alpha_i'), \quad \ddot\phi_i = \overline{\omega}^2 \cdot \alpha_i'', \quad |\alpha_i'| \ll 1,$$

and Eq. (10) can be transformed as

$$T_i = \frac{\mu Q_i}{\overline{\omega}(1 + \alpha_i')} \approx \frac{\mu Q_i}{\overline{\omega}}(1 - \alpha_i'). \tag{13}$$

By substituting Eqs. (8), (12), and (13) into (17), integrating τ in the region $[0, 2\pi]$, and then calculating the average value, we can obtain the motion equation of TERDHM in terms of a small angle:

$$\begin{aligned} &\alpha_1'' + (e + uQ_1)\alpha_1' + [h \sin(\alpha_1 - \alpha_2)]\alpha_2' \\ &\quad + (e - uQ_1) + \frac{h}{2}\sin(\alpha_1 - \alpha_2) = 0, \\ &\alpha_2'' + (e + uQ_2)\alpha_2' - [h \sin(\alpha_1 - \alpha_2)]\alpha_1' \\ &\quad + (e - uQ_2) - \frac{h}{2}\sin(\alpha_1 - \alpha_2) = 0, \end{aligned} \tag{14}$$

where

$$\begin{aligned} &e = f_i/J_i\overline{\omega}, \quad u = \mu P_i/J_i\overline{\omega}^3, \\ &h = (b_i - a_i - c_i l)/r_i, \quad i = 1, 2. \end{aligned} \tag{15}$$

Notice that Eq. (14) is an autonomous nonlinear equation. Setting

$$\alpha_1' = X_1, \quad \alpha_2' = X_2, \tag{16}$$

Eq. (14) can be rewritten as

$$\begin{aligned} X_1' &= F_1(X_1, X_2) \\ &= -(e + uQ_1)X_1 - [h \sin(\alpha_1 - \alpha_2)]X_2 \\ &\quad - (e - uQ_1) - \frac{h}{2}\sin(\alpha_1 - \alpha_2), \end{aligned} \tag{17}$$

$$\begin{aligned} X_2' &= F_2(X_1, X_2) \\ &= -(e + uQ_2)X_2 + [h \sin(\alpha_1 - \alpha_2)]X_1 \\ &\quad - (e - uQ_2) + \frac{h}{2}\sin(\alpha_1 - \alpha_2). \end{aligned} \tag{18}$$

The singular point equations in the phase plane (X_1, X_2) are

$$\begin{aligned} &-(e + uQ_1)X_1 - [h \sin(\alpha_1 - \alpha_2)]X_2 \\ &\quad - (e - uQ_1) - \frac{h}{2}\sin(\alpha_1 - \alpha_2) = 0, \\ &-(e + uQ_2)X_2 + [h \sin(\alpha_1 - \alpha_2)]X_1 \\ &\quad - (e - uQ_2) + \frac{h}{2}\sin(\alpha_1 - \alpha_2) = 0. \end{aligned} \tag{19}$$

Then

$$\begin{aligned} X_1 &= \overline{X} + \frac{1}{H}\left[eu\Delta Q + \frac{h}{2}(e - 3u\overline{Q})\sin(\alpha_1 - \alpha_2)\right], \\ X_2 &= \overline{X} - \frac{1}{H}\left[eu\Delta Q + \frac{h}{2}(e - 3u\overline{Q})\sin(\alpha_1 - \alpha_2)\right], \\ \overline{X} &= \frac{1}{H}\left[E + \frac{3uh}{4}\Delta Q \sin(\alpha_1 - \alpha_2)\right], \end{aligned} \tag{20}$$

where

$$\begin{aligned} E &= u^2 Q_1 Q_2 - e^2 - (h^2/2)\sin^2(\alpha_1 - \alpha_2), \\ H &= (e + uQ_1)(e + uQ_2) + h^2 \sin^2(\alpha_1 - \alpha_2), \\ \overline{Q} &= (Q_1 + Q_2)/2, \quad \Delta Q = Q_1 - Q_2. \end{aligned}$$

From Eq. (20) we can see that the two angular velocities act on each other and both of them vary in the small range approaching their average value $\overline{X}$.

SYNCHRONIZATION AND STABILITY CONDITIONS OF TERDHM

Assuming

$$(\alpha_1 - \alpha_2) = \alpha, \tag{21}$$

subtracting (18) from (17), and noting (16) and (20), the following equation can be obtained:

$$\alpha'' + (e + u\overline{Q})\alpha' + \frac{1}{2}u\Delta Q(X_1 + X_2) + (X_1 + X_2 + 1)h\sin\alpha = 0. \tag{22}$$

Introducing the new variables $d\alpha/d\tau = \theta, \alpha$, Eq. (22) becomes

$$\begin{aligned} \frac{d\alpha}{d\tau} &= \theta, \\ \frac{d\theta}{d\tau} &= -(e + u\overline{Q})\theta - \frac{1}{2}u\Delta Q(X_1 + X_2) \\ &\quad - (X_1 + X_2 + 1)h\sin\alpha. \end{aligned} \tag{23}$$

Hence,

$$\begin{aligned} \frac{d\theta}{d\alpha} &= \frac{D_1(\alpha, \theta)}{D_2(\alpha, \theta)} \\ &= \big[-(e + u\overline{Q})\theta - u\Delta Q(X_1 + X_2)/2 \\ &\quad - (X_1 + X_2 + 1)h\sin\alpha \big]/\theta. \end{aligned} \tag{24}$$

In the phase plane (α, θ), the singular points are defined as

$$D_1(\alpha, \theta) = 0, \quad D_2(\alpha, \theta) = 0. \tag{25}$$

Substituting Eq. (21) into $D_2(\alpha, \theta) = 0$,

$$\alpha_1' - \alpha_2' = X_1 - X_2 = 0. \tag{26}$$

Equation (26) shows that the two angular velocities of TERDHM are equal. That is to say, two eccentric rotors rotate in the synchronous state corresponding to the singular point in the phase plane (α, θ).

Substituting Eq. (20) into (26) one obtains

$$\begin{aligned} \sin(\alpha_1 - \alpha_2) &= \frac{2eu\Delta Q}{(3u\overline{Q} - e)h}, \\ X_1 = X_2 &= \overline{X}. \end{aligned} \tag{27}$$

It can be seen from Eq. (26) that synchronous rotating speed is equal to the average angular velocity of TERDHM and the phase difference between the two rotors depends on the difference ΔQ of the flow rates. If $\Delta Q \neq 0$, then $\sin(\alpha_1 - \alpha_2) \neq 0$, and if $\Delta Q = 0$ only it follows that $\sin(\alpha_1 - \alpha_2) = 0$.

From Eq. (27) the synchronization condition of the rotating motion of TERDHM is found to be

$$\left| \frac{2eu\Delta Q}{(3u\overline{Q} - e)h} \right| \leqslant 1. \tag{28}$$

According to the above analysis we know that the stability of the singular points in the phase plane (α, θ) can determine the stability of the synchronization rotating condition of TERDHM. The properties of the singular points, which are defined by Eq. (25), can be determined by the root λ of the characteristic equation

$$\begin{vmatrix} \dfrac{\partial D_2(\alpha, \theta)}{\partial \alpha} - \lambda & \dfrac{\partial D_2(\alpha, \theta)}{\partial \theta} \\ \dfrac{\partial D_1(\alpha, \theta)}{\partial \alpha} & \dfrac{\partial D_1(\alpha, \theta)}{\partial \theta} - \lambda \end{vmatrix} = 0. \tag{29}$$

Equation (29) can also be written as

$$\lambda^2 + \beta\lambda + \gamma = 0, \tag{30}$$

where

$$\begin{aligned} \beta &= -\left(\frac{\partial D_2(\alpha, \theta)}{\partial \alpha} + \frac{\partial D_1(\alpha, \theta)}{\partial \theta} \right), \\ \gamma &= \frac{\partial D_2(\alpha, \theta)}{\partial \alpha} \frac{\partial D_1(\alpha, \theta)}{\partial \theta} \\ &\quad - \frac{\partial D_2(\alpha, \theta)}{\partial \theta} \frac{\partial D_1(\alpha, \theta)}{\partial \alpha}. \end{aligned}$$

If

$$\beta > 0, \quad \gamma > 0, \tag{31}$$

the characteristic Eq. (30), has roots with negative real parts. In this situation the singular points must be stable.

Substituting (9), (15), (24), and β and γ in (30) into (31), and noting that $e > 0, u > 0$, and $\overline{Q} > 0$, one can get the following stability condition

$$Wh\cos(\alpha_1 - \alpha_2) > 0, \tag{32}$$

where

$$W = \left[(e+u\overline{Q})^2 - \left(\frac{u\Delta Q}{2}\right)^2 - h^2\sin^2(\alpha_1-\alpha_2)\right] \times \left[(e+u\overline{Q})^2 - 2(u^2\overline{Q}^2 - e^2)\right] - 4e(e+u\overline{Q})u\Delta Q h \sin^2(\alpha_1-\alpha_2),$$

$$h = \frac{m}{M}\left(\frac{1}{1-\omega_x^2/\overline{\omega}^2} - \frac{1}{1-\omega_y^2/\overline{\omega}^2}\right) + \frac{ml^2}{I}. \quad (33)$$

From (9) and (33) we obtain $h > 0$. In this case the stability condition (32) can be simplified as

$$W\cos(\alpha_1-\alpha_2) > 0. \quad (34)$$

Substituting (33) into condition (34), considering the two possibilities $\cos(\alpha_1-\alpha_2) > 0$, $W > 0$, and $\cos(\alpha_1-\alpha_2) < 0$, $W < 0$, we can see that condition (34) is satisfied only when

$$\cos(\alpha_1-\alpha_2) > 0, \quad W > 0. \quad (35)$$

Thus, we obtain

$$\cos(\alpha_1-\alpha_2) > 0, \quad -u_1(v-v_0)\Delta Q < \sin(\alpha_1-\alpha_2) < u_1(v+v_0)\Delta Q, \quad (36)$$

where

$$u_1 = (e+u\overline{Q})u/h,$$
$$v_0 = \sqrt{v^2 - 1/4(e+u\overline{Q})^2},$$
$$v = 2e/(3u\overline{Q}+e)(u\overline{Q}-e).$$

From condition (36) a quantitative range of stable phase difference is given by

$$-\arcsin[u_1(v-v_0)\Delta Q] < (\alpha_1-\alpha_2) < \arcsin[u_1(v+v_0)\Delta Q]. \quad (37)$$

When $\Delta Q = 0$, from formula (27) we get $\sin\alpha = \sin(\alpha_1-\alpha_2) = 0$. Obviously, the point ($\alpha = 0, \theta = 0$) and the point ($\alpha = \pi, \theta = 0$) in the phase plane (α, θ) are two typical singular points of Eq. (25). Because $\cos\theta > 0$, from condition (36) we know that the singular point ($\alpha = 0, \theta = 0$) is stable. Also, for $\cos\pi < 0$, the singular point ($\alpha = \pi, \theta = 0$) is unstable.

EXAMPLE AND SIMULATIONS

The parameters of a vibrating system are as follows: mass of the vibrating body, $M = 5000$ kg; eccentric masses, $m_1 = m_2 = 100$ kg; eccentric mass moments, $J_1 = J_2 = 16.9$ kg·m^2; average rotating speed of the motors, $\overline{\omega} = 1500$ rpm; spring stiffness of the vibrating system in the x direction, $K_x = 2.418 \times 10^6$ N/m; spring stiffness of the vibrating system in the y direction, $K_y = 1.777 \times 10^6$ N/m; rotating damping coefficients of the rotating shafts, $f_1 = f_2 = 0.12$ N·m·s/rad; torque coefficient of the hydraulic motor, $\mu = 1.59 \times 10^{-3}$, displacements of the hydraulic motors, $q_1 = q_2 = 60$ cm^3; distance between points G and O_i, $l = 0.6$ m.

Using Eqs. and expressions (9), (15), (27), and (28), the following results for TERDHM rotating in a stable synchronous state are obtained: 1. if $p = 20$ MPa, then $\Delta Q \leqslant 0.069\overline{Q}$; if $p = 30$ MPa, then $\Delta Q \leqslant 0.046\overline{Q}$. 2. if $p = 20$ MPa, $\Delta Q \leqslant 0.069\overline{Q}$, then $-2.9° < (\alpha_1 - \alpha_2) < 2.9°$; if $p = 20$ MPa, $\Delta Q \leqslant 0.046\overline{Q}$, then $-2.3° < (\alpha_1-\alpha_2) < 2.3°$.

The results of the simulation for the example are shown in Figs. 2, 3, and 4. Figure 2 shows the rotating speed curves of two rotors. It can be seen from the figure that the synchronous rotation of TERDHM can be achieved under the conditions given in this article and the rotating speeds of two rotors correspond to the characteristics described in Eqs. (20) and (27).

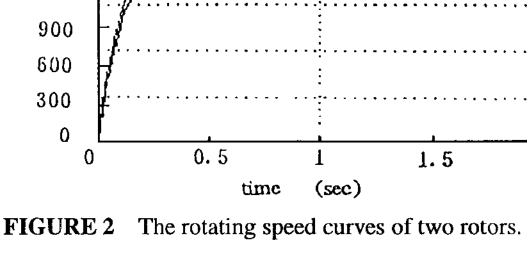

FIGURE 2 The rotating speed curves of two rotors.

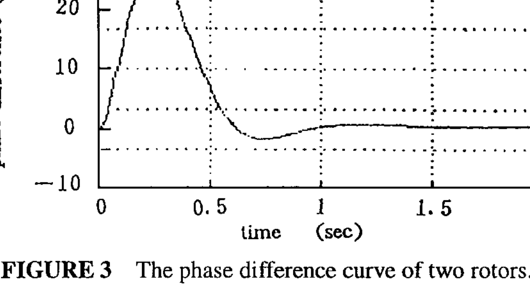

FIGURE 3 The phase difference curve of two rotors.

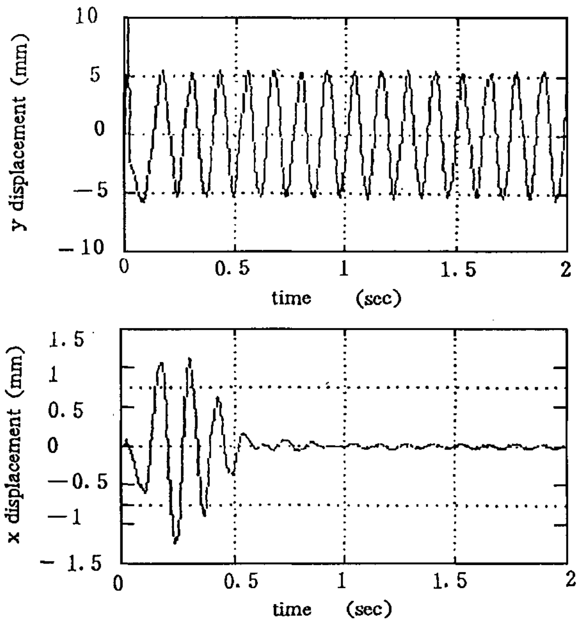

FIGURE 4 The displacement curves in the y, x directions: (a) the displacement curve in the y direction and (b) the displacement curve in the x direction.

Figure 3 shows the phase difference curve of two rotors. It shows that the phase difference in a synchronous rotating condition is very small and stable.

Figure 4 shows the displacement curves of the vibrating body in the x and y directions. Figure 4(a) shows that the vibration in the y direction is a stable forced vibration and it corresponds to the stable synchronous rotation of TERDHM.

CONCLUSIONS

1. The rotating motion of TERDHM is nonlinear.
2. The angular velocities of two rotors act on each other and both of them vary in a small range approaching their average value $\overline{X}$.
3. The synchronous rotating speed of TERDHM is equal to the average angular velocity. The phase difference of two rotors is determined by the flow rate difference ΔQ of the two hydraulic motors.
4. The stable synchronous rotation of TERDHM can be achieved when the synchronous conditions given in (28) and the stable conditions given in (35) are satisfied.

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