ES

SU_4 AND STRONG INTERACTIONS

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ABSTRACT

The group SU_4 is probably the most straightforward and economical generalization of SU_3 which allows for basic triplets of integral charges. In this paper we analyze in detail the predictions of a few models for strong interactions based on SU_4. In addition to the usual (non-peculiar) eightfold way particles new peculiar particles are predicted. However they can be allowed to be created and to decay semi-strongly by the mass breaking interactions. Mass relations are obtained. It is pointed out that some difficulties concerning these mass formulae occur when one tries to attribute the ρ; K^*, ω and φ vector mesons to the regular representation of SU_4.

9076 / TH. 437
17 June 1964

1.

The marked success of the eightfold way version [1] of SU_3 for particles and resonances enhanced the interest of physicists in finding a physical meaning for the basic representation of dimension 3 of such a group. The straightforward application of the eightfold way relation between charge, isospin and hypercharge leads to a triplet with non-integral charge and baryonic number. This model has been proposed by Gell-Mann and Zweig [2]. It is by far the most economical because it does not involve new quantum numbers and implies only one fundamental triplet. It has, however, the disadvantage of predicting the actual physical existence of particles of non-integral charge. This property would make the triplet easy to detect if its mass is not too high.

In order to avoid non-integral charges, more than one fundamental triplet is needed. Thus, the SU_3 group must be enlarged. Several attempts have been made in this direction, mainly along two lines : one based on two triplets [3], one based on a triplet and a singlet [4),5]. It is natural to consider at the beginning rank three groups rather than direct products of simple groups of lower ranks. It has been proposed [4], indeed, to consider SU_4 by assigning the triplet and the singlet to its basic representation (dimension 4). In this paper we want to analyze a few models using SU_4 as the basis for strong interactions. Some of them have already been proposed in the literature.

We shall see that with the normal way of breaking the symmetry [*] it shall be impossible to accommodate the nine vector mesons (ρ, K^*, ω, φ) in the regular representation both for linear and quadratic mass formulae. This is considered to be a weak point for the physical significance of SU_4 assignments unless one changes the mass breaking mechanism or its interpretation.

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*) By normal way, we mean that the mass splitting operator

1) belongs to the regular representation,

2) conserves charge and isospin,

3) has to be taken only in the first order of perturbation.

In SU_4 there are several possibilities (not only one as in SU_3 [6]) which obey these three requirements.

2. MODELS FOR SU_4

In SU_4 we have at our disposal three additive quantum numbers, T_3, Y and Z; they correspond to the three generators which can be simultaneously diagonalized.

The quartet has the following quantum numbers :

$$q \equiv \begin{vmatrix} A \\ A \\ B \\ C \end{vmatrix} \qquad \text{with} \qquad \begin{array}{llll} A & T = \frac{1}{2}, & Y = \frac{1}{3}, & Z = \frac{1}{4} \\ B & T = 0, & Y = -\frac{2}{3}, & Z = \frac{1}{4} \\ C & T = 0, & Y = 0, & Z = -\frac{3}{4} \end{array}$$

In order to have integral charges, it is necessary to introduce in the definition of the charge the baryonic number N in addition to T_3, hypercharge Y and Z. Generally one has

$$Q = T_3 + \frac{Y}{2} + \alpha Z + \beta N$$

With this choice, the usual particles characterized by $\alpha Z + \beta N = 0$ satisfy the eightfold way charge formula.

Moreover, the quartet must have $N = 1$ (or $N = -1$) if we want to build the physical bosons and baryons from product decompositions of fundamental fields *). Bosons are obtained from $\bar{q}q$ products, baryons from $qq\bar{q}$ or $\bar{q}\bar{q}q$ products depending on the baryonic or antibaryonic character of q. Finally, if we require that the charges in the known meson and baryon octets are the usual ones, the C particle has to be neutral.

*) It is also possible to consider models where the baryon number changes within the quartet. However, these models give rise to difficulties similar to those discussed in this paper.

We are led to consider the four typical models :

i) Model 1 : The quartets of representation 4 have $N = +1$ and

$$Q = T_3 + \frac{Y}{2} + \frac{Z}{3} + \frac{N}{4} \tag{1}$$

Then the quartet is of the type (A^+, A^0, B^0, C^0) and their antibaryons belong to $\bar{4}$.

ii) Model 2 : The quartets of representation 4 have $N = -1$ and

$$Q = T_3 + \frac{Y}{2} + \frac{Z}{3} - \frac{N}{4} \tag{2}$$

Then the quartet has the same charges as in the preceding case but it is composed of antibaryons. The corresponding baryons (one negative and three neutrals) will belong to the representation $\bar{4}$.

iii) Model 3 : $N = 1$ for the fundamental quartet

$$Q = T_3 + \frac{Y}{2} - \frac{2}{3}Z - \frac{N}{2} \tag{3}$$

in which case the baryon quartet will be composed of (A^0, A^-, B^-, C^0).

iv) Model 4 : It can be obtained from model 3 by placing the baryon quartet in $\bar{4}$ instead of 4 and changing the sign of N in the charge definition.

Models 3 and 4 are appealing for weak interactions which, however, will not be investigated in this paper.

3. PARTICLE ASSIGNMENTS

There is no, a priori, reason why the particles or resonances observed in nature should belong to the lowest representations. This is, however, an appealing guiding idea, so let us therefore look at the possible particle assignments restricting our discussions to the lower representations (of dimension ≤ 20).

The decompositions of the products of lower dimensional representations of SU_4 are shown in Table 1; their contents in SU_3 submultiplets with the corresponding Z values are given in Table 2.

By starting from fundamental quartets, we shall never obtain for mesons and baryons the representations 6, 10, etc. for which Z is half-integral.

For the models under discussion, the bosons belong to the representations 1, 15, 20" (integral Z) while baryons belong to $\bar{4}$, 20, 20' (for models 2 and 4) and to 4, $\overline{20}$, $\overline{20}$' (for models 1 and 3) (fourth-integral Z).

Let us define the "peculiarness" as

$$W = Q - T_3 - \frac{Y}{2} \qquad (4)$$

We see from Table 2 that besides the non-peculiar ($W = 0$), SU_3 submultiplets, there appear peculiar SU_3 submultiplets. The corresponding particles would be stable with respect to decay into non-peculiar particles if Z is conserved. We shall see, however, that the mass breaking interaction will provide a way to violate semi-strongly Z and Y so as to conserve charge and isospin. Such interactions provide particle mixing which appears in mass formulae and, besides, allow the single production of peculiar particles as well as their decay into non-peculiar (SU_3 eightfold) ones. As will be seen later on, this can happen for models 1 and 2 but not for 3 and 4.

Let us try to assign the known bosons to SU_4 representations. It is seen from Table 2 that one has a non-peculiar octet + singlet in 15 and an octet alone in 20". In 20", however, when one breaks the mass (asking the mass splitting to transform as components of the regular representation 15) one finds that no splitting is obtained in the octet. This is due to the fact that in 20" × 20" the representation 15 appears only once (see Table 1) and in the antisymmetric combination which automatically gives zero contribution for bosons. Therefore, we shall try to place the two well-known SU_3 octets (pseudoscalar and vector mesons) and the SU_3 singlet (vector meson) in the representation 15.

The representation 15 can be easily visualized in the three dimensional spaces with axes T_3, Y, Z as in Fig. 1. In the figure, we designate the members of the multiplet by the usual notations for vector mesons. We see that besides the usual SU_3 octet $(\rho_0, K_0^*, \omega_0)$ and singlet (φ_0), we find a triplet (3) (λ_0^*, an isospin singlet and ξ_0^*, an isospin doublet) with $Z = 1$ and its antitriplet $(\bar{3})$ $(\bar{\lambda}_0^*, \bar{\xi}_0^*)$ with $Z = -1$.

For the sake of future reference, let us write the regular representation 15 in matrix form (see Table 3)

$$V = \begin{vmatrix} \frac{\rho_0^0}{\sqrt{2}} + \frac{\omega_0^0}{\sqrt{6}} + \frac{\varphi_0^0}{2\sqrt{3}} & \rho_0^- & K_0^{*-} & \overline{\xi_0^*} \\ \rho_0^+ & -\frac{\rho_0^0}{\sqrt{2}} + \frac{\omega_0^0}{\sqrt{6}} + \frac{\varphi_0^0}{2\sqrt{3}} & \overline{K_0^{*0}} & \overline{\xi_0^*} \\ K_0^{*+} & K_0^{*0} & -\frac{2\omega_0^0}{\sqrt{6}} + \frac{\varphi_0^0}{2\sqrt{3}} & \overline{\lambda_0^*} \\ \xi_0^* & \xi_0^* & \lambda_0^* & -\frac{3\varphi_0^0}{2\sqrt{3}} \end{vmatrix} \tag{5}$$

In the matrix V, all particles appear with a sub-index zero to denote that they are the members of the unbroken SU_4 multiplet. The upper index describes the charge which is model dependent for ξ^* and λ^*. When SU_4 is broken in order to split the masses, one has mixings so that the physical particles are linear combinations of the particles appearing in V.

Let us now turn to the baryons. One sees that in 20' (or in $\overline{20}$') there is a non-peculiar SU_3 octet in which one can try to accommodate the known baryons and $D_{3/2}$ resonances. We do not expect a SU_3 baryon singlet (which appears only in representation 36); one could, however, find a reasonable place for the $Y_0^*(1405\text{ MeV}, 1/2^+)$ as a peculiar particle. In Figs. 2 and 3, the weight diagrams of representations 20 and 20' are plotted. The $P_{3/2}$ resonances are in a non-peculiar SU_3 decuplet. The lowest representations of SU_4 containing a decuplet are 20, 56, and 60. For models 1 and 3, the baryons are in $\overline{20}'$ ($4\otimes 4\otimes \bar{4} \supset \overline{20}'$). The $P_{3/2}$ resonances have to be placed in representations appearing in the product $15\otimes\overline{20}' = 4 + \overline{20} + \overline{20}' + \overline{20}' + 36 + 60 + \overline{140}''$. Consequently, the $P_{3/2}$ resonances have to be put in the representation 60. For models 2 and 4, the baryons are in 20' ($4\otimes\bar{4}\otimes\bar{4} \supset 20'$). The $P_{3/2}$ can be placed in representation 20 which appears in the product $15\otimes 20'$ and which contains representation 10 of SU_3. This result highly favours models 2 and 4 over models 1 and 3.

In brief, we see that if we restrict ourselves to lower dimensional representations of SU_4 (up to dimension 20) models 2 and 4 allow for non-peculiar SU_3 singlets and octets, for mesons, and for octets and a decuplet (not $\overline{10}$) for baryons. We note that the representation 20, containing a decuplet of SU_3, does not appear in $4\otimes\bar{4}\otimes\bar{4}$ but is indeed coupled to bosons (15) and $P_{1/2}$ baryons (20') as can be seen from the multiplication Table 1. Moreover, one could expect the peculiar particles to exist in natura (even if produced less abundantly except if in pairs) and with lifetimes somewhat longer than non-peculiar particles (except if their decay products contain another peculiar particle). Mass formulae relating peculiar and non-peculiar particles will exist in some cases.

4. MASS FORMULAE

Let us now discuss how the masses of the initially degenerated members of the SU_4 multiplets can be split by breaking the group. As successfully done in SU_3, we shall begin by asking the mass splitting operator to transform as components of the regular representation, in our case, representation 15. The choice of the components is limited by charge and isospin conservation and by the requirement of hermiticity of the mass matrix. The most general form for the mass operator is therefore

$$\mathcal{M} = m_0 + a\lambda_{15} + b\,\lambda_8 + c\,\lambda_{13} \tag{6}$$

where λ_8, λ_{13} and λ_{15} are generators of SU_4 and where a, b and c are real parameters which depend on the SU_4 representation under consideration. The matrix form of the generators is given in Table 3; the mass matrix is :

$$M = \begin{vmatrix} m_0 + \frac{a}{2\sqrt{3}} + \frac{b}{\sqrt{6}} & 0 & 0 & 0 \\ 0 & m_0 + \frac{a}{2\sqrt{3}} + \frac{b}{\sqrt{6}} & 0 & 0 \\ 0 & 0 & m_0 + \frac{a}{2\sqrt{3}} - \frac{2b}{\sqrt{6}} & c \\ 0 & 0 & c & m_0 - \frac{3a}{2\sqrt{3}} \end{vmatrix} \tag{7}$$

The mass splitting due to the parameter a separates the different SU_3 submultiplets. The parameter b splits the masses inside the SU_3 submultiplets (this is the usual breaking in SU_3) and provides mixing between particles of the same hypercharge, isospin and peculiarness (ω, φ mixing for example). The parameter c mixes particles of equal isospin and charge but different hypercharge and Z with the condition

$$3\,\Delta Y = -2\,\Delta Z \tag{8}$$

We see from equations (1), (2) and (3) that for models 1 and 2 λ_{13} conserves charge while it does not for models 3 and 4. This implies $c = 0$ for models 3 and 4.

We note for the sake of further reference that λ_8, λ_{13}, λ_{15} transform respectively as ω_0^o, $\lambda_0^{*o} + \bar{\lambda}_0^{*o}$, φ_0^o of model 1 or 2 (5) *).

Mass relations can be derived in various ways. One may write the counterpart of Okubo's SU_3 formula [6]. Because of the λ_{13} term, such a formula needs knowledge of the Clebsh-Gordan coefficients of each representation discussed. One may generalize the U spin of SU_3, to SU_4, or one may use the method based on the d_{ijk}'s coefficients defined as

$$\left[\lambda_i, \lambda_j\right]_+ = \delta_{ij} + 2\, i\, d_{ijk}\, \lambda_k \tag{9}$$

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*) It is interesting to remark that the $(\lambda_0^* + \bar{\lambda}_0^*)$ particle corresponds to the first component of the SU_2 group (H spin) which commutes with the isotopic group. This can easily be shown by considering the matrices of Table 3 as composed of four 2×2 matrices (dashed lines). The T_3 and H_3 directions define a plane corresponding to the SO_4 subgroup of SU_4. (SO_4 is the direct product $SU_2 \otimes SU_2$)

$$\mathcal{H}_3 = -\frac{1}{2\sqrt{3}}\,\lambda_8 + \frac{1}{\sqrt{6}}\,\lambda_{15}$$

This last method can be used only for representations D such that in the product $D \otimes \bar{D}$ the regular representation appears less than three times (this is always the case for representations of dimension lower than 64 [7]).

a) Mesons

The method of traces is best suited for the regular representation to which we ascribed the mesons. Using, for simplicity, the vector meson symbols, the matrix elements of the mass operator are given by

$$\mathrm{Tr}\left\{ \left[\bar{V} V \right]_+ M \right\} \tag{10}$$

where V, $\bar{V}$ and M are defined in (5) and (7).

The symmetric combination appears only (D coupling) because mesons and their antiparticles belong to the same representation. Explicitly one obtains

$$\begin{aligned}
\langle \rho_0 | M | \rho_0 \rangle &= m_0 + 2(a+b) \\
\langle \omega_0 | M | \omega_0 \rangle &= m_0 + 2(a-b) \\
\langle \varphi_0 | M | \varphi_0 \rangle &= m_0 - 4a \\
\langle K_0^* | M | K_0^* \rangle &= m_0 + 2a - b \\
\langle \xi_0^* | M | \xi_0^* \rangle &= m_0 - 2a + b \\
\langle \lambda_0^* | M | \lambda_0^* \rangle &= m_0 - 2(a+b)
\end{aligned} \tag{11}$$

(continued)

$$
\begin{aligned}
&< \varphi_o | M | \omega_o > = \sqrt{2}\, b \\
&< \lambda_o^* | M | \omega_o > = < \bar{\lambda}_o^* | M | \omega_o > = -\sqrt{\tfrac{2}{3}}\, c \\
&< \lambda_o^* | M | \varphi_o > = < \bar{\lambda}_o^* | M | \varphi_o > = -\sqrt{\tfrac{1}{3}}\, c \\
&< K_o^* | M | \xi_o^* > = c
\end{aligned}
\qquad (11)
$$

Remember that $c = 0$ for models 3 and 4.

The physical particles and their masses are the eigenvectors and the eigenvalues of the matrix M.

The first fact we want to stress is that if $c \neq 0$ λ_o^* and $\bar{\lambda}_o^*$ (which are different particles) will mix with ω_o and φ_o. However, using Eq. (11) and defining

$$
\begin{aligned}
\lambda_{o1}^* &= \frac{\lambda_o^* + \bar{\lambda}_o^*}{\sqrt{2}} \\
\lambda_{o2}^* &= \frac{\lambda_o^* - \bar{\lambda}_o^*}{\sqrt{2}}
\end{aligned}
\qquad (12)
$$

which are eigenstates of charge conjugation, one has

$$
\begin{aligned}
&< \lambda_{o2}^* | M | \omega_o > = < \lambda_{o2}^* | M | \varphi_o > = 0 \\
&< \lambda_{o1}^* | M | \omega_o > = -\frac{2}{\sqrt{3}}\, c \\
&< \lambda_{o1}^* | M | \varphi_o > = -\sqrt{\frac{2}{3}}\, c
\end{aligned}
\qquad (13)
$$

This implies that λ^*_{o2} is already an eigenvector of M which we stress by setting

$$\lambda^*_{o2} = \lambda^*_2 \tag{14}$$

Therefore, the particles which appear in nature are not λ^* and $\bar{\lambda}^*$, but λ^*_1 and λ^*_2 which we expect to have distinct masses if $c \neq 0$.

The only other particle which is not mixed is the ρ meson, so that

$$\rho_o = \rho \tag{15}$$

The remaining mass matrix splits into two submatrices, one of rank 2 for ξ^* and K^*,

$$\begin{vmatrix} m_o + 2a - b & c \\ c & m_o - 2a + b \end{vmatrix} \tag{16}$$

and one of rank 3 for λ^*_1, ω, φ

$$\begin{vmatrix} m_o - 2(a+b) & -\frac{2}{\sqrt{3}}\,c & -\sqrt{\frac{2}{3}}\,c \\ -\frac{2}{\sqrt{3}}\,c & m_o + 2(a-b) & \sqrt{2}\,b \\ -\sqrt{\frac{2}{3}}\,c & \sqrt{2}\,b & m_o - 4a \end{vmatrix} \tag{17}$$

The eigenvalues of (16) are the masses of ξ^* and K^*, and those of (17) the masses of $\lambda_1^*, \omega, \varphi$. The masses λ_2^* and ρ being directly given by

$$\begin{aligned} \rho &= m_o + 2(a+b) \\ \lambda_2 &= m_o - 2(a+b) \end{aligned} \tag{18}$$

By eliminating the four parameters m_o, a, b and c in equations (16), (17), (18), one obtains four relations between the physical masses. The first two are linear

$$\lambda_1^* + \omega + \varphi = \frac{5}{2}\left(K^* + \xi^*\right) - 2\rho \tag{19}$$

$$\lambda_2^* = K^* + \xi^* - \rho \tag{20}$$

The third one of degree three reads

$$\left(\lambda_2^* - \lambda_1^*\right)\left(\lambda_2^* - \omega\right)\left(\lambda_2^* - \varphi\right) = 0 \tag{21}$$

This leads to the three solutions *)

$$\lambda_2^* = \lambda_1^*, \quad \xi^* = \frac{2}{3}(\omega + \varphi + \rho) - K^*, \quad \lambda_1^* = \frac{1}{3}(2\omega + 2\varphi - \rho) \tag{22a}$$

$$\lambda_2^* = \varphi, \quad \xi^* = \varphi + \rho - K^*, \quad \lambda_1^* = \frac{1}{2}(3\varphi + \rho - 2\omega) \tag{22b}$$

$$\lambda_2^* = \omega, \quad \xi^* = \omega + \rho - K^*, \quad \lambda_1^* = \frac{1}{2}(3\omega + \rho - 2\varphi) \tag{22c}$$

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*) From Eq. (21) one sees that the three solutions can be obtained one from another by a permutation of the three masses λ_1^*, ω, φ.

The fourth relation between known particles ($K^*, \rho, \omega, \varphi$) depends on the chosen solution. It is respectively

$$\left(\varphi - \frac{4K^*-\rho}{3}\right)\left(\omega - \frac{4K^*-\rho}{3}\right) + \frac{2}{9}\left(K^*-\rho\right)^2 = 0 \qquad (23a)\ ^{*)}$$

$$\left(\frac{4K^*-\rho}{3} - \omega\right)\left(\frac{8}{3}K^* + 2\omega - \frac{5}{3}\rho - 3\varphi\right) + \frac{4}{9}\left(K^*-\rho\right)^2 = 0 \qquad (23b)$$

$$\left(\frac{4K^*-\rho}{3} - \varphi\right)\left(\frac{8}{3}K^* + 2\varphi - \frac{5}{3}\rho - 3\omega\right) + \frac{4}{9}\left(K^*-\rho\right)^2 = 0. \qquad (23c)$$

The formulae (23a,b,c) are unfortunately not well satisfied by the actual masses of ω, K^*, ω, φ.

We consider this very unpleasant feature as a strong argument against the validity of the SU_4 group. Nevertheless, if one disregards for a moment the constraints imposed on the known masses of vector bosons by Eq. (23) and if one tries to get some conclusions concerning the masses of the peculiar particles, one obtains some attractive results. The best solution seems to be the solution a. Using (22a), one predicts then a narrow resonance λ_2^* of mass around 950 MeV, $T = 0$, $C = +1$ which could be eventually identified with the recently discovered $\eta\pi\pi$ resonance [8] if it turns out that this particle is a vector. Besides λ_2^* one would also expect a λ_1^* three pion narrow resonance $T = 0$, $C = -1$ in the region of mass 950 MeV and also a narrow πK (ξ^*), $T = 1/2$ resonance around 820 MeV.

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*) The formula (23a) which is also valid for $c = 0$, has been independently obtained for this case by M. Nauenberg (private communication).

It would be very interesting to test it in $\bar{p}p$ annihilation such particles are produced abundantly in pairs.

As was said, we believe relations (23) to be a very serious difficulty. One could think that by an adequate generalization of the mass breaking mechanism, one could eventually escape these unpleasant constraints. Of course, we want to maintain all known properties of SU_3. This will be the case if we consider the λ_{15} term as the most important term in the mass splitting operator.

We have then to compute the effect of this term up to a certain order of perturbation, n, such that the nth contribution of the λ_{15} term is of the same order than the first contribution of the λ_8 and λ_{13} terms. This means that the cross-terms coming from λ_{15} and λ_8 are smaller than the terms λ_{15}^n and λ_8 and therefore can be neglected.

It is interesting to note that at any order of perturbation, the contribution of λ_{15} to the equations (11) consists only in an arbitrary additive term d to the matrix element $\langle \varphi_0 | M | \varphi_0 \rangle$. One can see that Eq. (20) is still valid; this is no longer the case for Eqs. (19), (21) and (23). Consequently, it is always possible to put the nine well-known vector mesons in the representation 15. Unfortunately, no definite prediction can then be made about the peculiar particles *).

*) More precisely the system of equation is

$$d = \lambda_1^* + \omega + \varphi - \frac{5}{2}(K^* + \xi^*) + 2\rho$$

$$4c^2 d = 3(\lambda_2^* - \lambda_1^*)(\lambda_2^* - \omega)(\lambda_2^* - \varphi)$$

$$\xi^* = \lambda_2^* - K^* + \rho$$

$$\frac{\lambda_1^* \omega \varphi}{\lambda_2^*} - 2K^*\xi^* + \left(\frac{K^* + \xi^*}{2}\right)\rho = \frac{d}{3}\left[2K^* + 2\xi^* - \rho + 4(2a-b) - \frac{4c^2}{\lambda_2^*}\right]$$

$$4(2a-b)^2 + 4c^2 = (K^* - \xi^*)^2$$

Giving $\omega, \varphi, \rho, K^*$ and λ_2^* one can calculate in particular c^2, ξ^* and λ_1^*. One has to solve a quadratic equation. The discriminant of this equation and c^2 are positive only for $\lambda_2^* \lesssim 910$ and consequently for $\xi^* \lesssim 780$.

On the other hand, restricting oneself to the case $c = 0$, the formula (23a) is again valid. Therefore, in order to place the nine known vector mesons in the regular representation, c must be different from zero.

Let us now discuss briefly the case of pseudoscalar mesons. They can be treated exactly as were the vector mesons. The mass equations are given by formulae (22) and (23) making the adequate substitutions. Due to the fact that Eqs. (23) were not good at all for vector mesons, one sees no reason to believe in their validity in the case of pseudoscalar mesons. It is therefore impossible, due to the lack of information concerning the existence of a pseudoscalar meson σ corresponding to φ, to obtain any prediction on peculiar pseudoscalar particles.

If, however, one would try to believe for a moment the equations (23), one has the following results [using (mass)^2]

$$\text{(a)} \quad \begin{cases} \sigma \cong 1{,}130 \text{ mev} \\ \lambda_1 \cong \lambda_2 \cong 1{,}020 \text{ mev} \\ \xi \cong 910 \text{ mev} \end{cases}$$

$$\text{(b)} \quad \begin{cases} \sigma \cong \lambda_2 \cong 1{,}020 \text{ mev} \\ \lambda_1 \cong 1{,}130 \text{ mev} \\ \xi \cong 910 \text{ mev} \end{cases} \tag{24}$$

$$\text{(c)} \quad \begin{cases} \sigma \cong 600 \text{ mev} \quad \text{or} \quad 310 \text{ mev} \\ \lambda_1 \cong 310 \text{ mev} \quad \phantom{\text{or}} \quad 600 \text{ mev} \\ \xi \cong 300 \text{ mev} \end{cases}$$

These formulae are very sensitive to the input masses π, K, η.

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The solution c) manifestly has to be discarded. Masses given by solutions a) and b) seem to us also not to be very satisfactory.

b) Baryons

Whatever the choice of the representations for the baryons and their excited states may be, the mass splitting operator involves so many parameters that no definite predictions can be made.

However, we will give a general result valid for representations D for which the regular representation appears less than three times in the product $D \otimes \bar{D}$ [7]. This includes all representations up to dimension 60.

Using the H spin already defined, we obtain

$$\begin{aligned} M = {} & m_0 + \alpha Z + \beta Y + \gamma \lambda_{13} \\ & + \alpha' \left(\frac{4}{3} Z^2 - \sum_{i=1}^{8} \lambda_i^2 \right) + \beta' \left[T(T+1) - H(H+1) \right] \\ & + \gamma' \left[\lambda_4 \lambda_9 + \lambda_5 \lambda_{10} + \lambda_6 \lambda_{11} + \lambda_7 \lambda_{12} - \frac{4}{\sqrt{3}} \lambda_{13} \left(\lambda_8 + \sqrt{2} \lambda_{15} \right) \right] \end{aligned} \tag{25}$$

The effective use of this formula implies knowledge of the Clebsh-Gordan coefficients of the group where λ_j are the generators of the representation D.

Let us examine two applications of (25) to the particular case in which there is no λ_{13} breaking term (i.e., W conservation), this means $\gamma = \gamma' = 0$.

1) Representation 20'

According to Table 1 and Fig. 3, the representation 20' decomposes into (charges are given for model 2) :

a) an octet 8, $Z = 3/4$ - N, Λ, Σ, Ξ ;

b) a sextet 6, $Z = -1/4$ - (isotriplet $\sigma_1^{(-0+)}$, isodoublet $\sigma_{1/2}^{(-0)}$, isosinglet $\sigma_0^{(-)}$);

c) a triplet $\bar{3}$, $Z = -1/4$ - (isodoublet $\tau_{1/2}^{(-0)}$, isosinglet $\tau_0^{(0)}$);

d) a triplet 3, $Z = -5/4$ - (isodoublet $t_{1/2}^{(-0)}$, isosinglet $t_0^{(-)}$).

The following relations can be obtained

$$\begin{aligned}
\Xi + N &= \frac{1}{2}(3\Lambda + \Sigma) \\
\Xi - N &= \sigma_0 - \sigma_{1/2} \\
\Sigma - N &= t_0 - t_{1/2} \\
N + t_{1/2} &= \frac{1}{2}(3\tau_0 + \sigma_1) \\
\sigma_{1/2} &= \frac{1}{2}(\sigma_0 + \sigma_1) \\
t_{1/2} + t_0 + \Sigma + \Xi &= 3\tau_{1/2} + \sigma_{1/2} \\
\langle \sigma_{1/2} | M | \tau_{1/2} \rangle &= \frac{\sqrt{3}}{4}(\Lambda - \Sigma)
\end{aligned} \tag{26}$$

The first equation is the Gell-Mann Okubo mass formula which is implied for the SU_3 octet contained in the representation 20' of SU_4 if the peculiarness is conserved. In the case where c is different from zero, it could be possible to consider σ_0 as the Y_0^* (1405) resonance.

2) Representation 20 (Fig. 2)

Representation 20, as are all other representations described by a one row Young diagram, is governed by a three dimensional linear spacing law

$$M = m_o + \alpha Z + \beta Y \qquad (21)$$

In model 2, this representation contains the 3/2 resonances : the usual baryon decuplet $(Z = \frac{3}{4})$, a sextet $(Z = -\frac{1}{4})$, a triplet $(Z = -\frac{5}{4})$ and a singlet $(Z = -\frac{9}{4})$.

If the mass of a peculiar particle is known, all other masses can be deduced. For example, if we take the isospin one of the sextet to be the 1660 Y_1^* resonance, we find

	Decuplet	Sextet	Triplet	Singlet
$T = 0$	Ω^- 1672	$\Omega^{-\prime}$ 1950	$\Omega^{-\prime\prime}$ 2228	$\Omega^{-\prime\prime\prime}$ 2506
$T = \frac{1}{2}$	Ξ^* 1527	$\Xi^{*\prime}$ 1805	$\Xi^{*\prime\prime}$ 2083	
$T = 1$	Y_1^* 1382	$Y_1^{*\prime}$ 1660		
$T = \frac{3}{2}$	N^* 1237			

These mass formulae have only to be considered as rough approximation in a model where the $(\lambda_o^* + \bar{\lambda}_o^*)$ breaking term permits the decay of the peculiar particles into known particles and their single production starting from non-peculiar particles.

5.

In this paper we have discussed in some detail a few models based on the SU_4 group [9]. All physical particles, mesons and baryons were constructed from a fundamental quartet, with baryonic number 1 or -1, belonging to the basic representation of the group.

In this way, one possible classification of elementary particles is obtained which has some pleasant features (e.g., the baryons, baryonic resonances, mesons can be put in relatively small dimensional representations).

The SU_4 group, as do other groups of rank three, predicts the existence of some "peculiar particles". The different ways of breaking the group do not only allow the production of these particles in pairs but also their single production with a somewhat smaller cross-section. The peculiar particles can also decay into ordinary particles of the eightfold way with widths of the order of a few MeV. It might then be possible to consider some of the existing resonances, e.g., $\eta\pi\pi$ (960) or Y_o^* (1405) as peculiar particles; this appears as an attractive possibility.

Unfortunately, we have also seen that in the case of the vector mesons, one obtains from the solutions of the mass equation relations which are constraints on the masses of ω, φ, ρ, K^*. These relations are very badly satisfied, both for linear and quadratic mass relations.

If one really wants to escape this difficulty one is led either to enlarge the mass breaking operator (in this way the theory is less restrictive) or to reinterpret the formulae (for example, if other resonances are discovered).

It is a pleasure to thank Professor M. Nauenberg for many fruitful discussions. We also acknowledge Dr. M. Levine and Professor L. Van Hove for useful remarks.

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9) We have also considered another model based on SU_4 which is connected to the study of the Sp_6 group by L. Van Hove and two of us [3]. This model introduces two triplets of baryons (trions) into the six dimensional representation of SU_4. It is a sixfold-way version of SU_4 (more precisely, $SU_4/Z_2 = SO_6$). Details on the physical implications of this model are being studied at present.

T Á B L E 1

Product decomposition of representations in SU_4

4	$\bar{4}$	6	10	$\overline{10}$	15	20	$\overline{20}$	$20'$	$\overline{20'}$	$20''$	
$6+10$	$1+15$	$\bar{4}+20'$	$20+20'$	$\bar{4}+\overline{36}$	$4+\overline{20'}+36$	$35+45$	$\overline{10}+\overline{70}$	$15+20''+45$	$6+\overline{10}+64$	$\overline{20'}+60$	4
		$1+15+20''$	$15+45$	$\overline{15}+\overline{45}$	$6+10+\overline{10}+64$	$36+84'$	$\overline{36}+\overline{84'}$	$4+\overline{20'}+36+60$	$\bar{4}+20'+\overline{36}+\overline{60}$	$6+50+64$	6
			$20''+35+45$	$1+15+84$	$6+10+64+70$	$56+60+84'$	$\bar{4}+\overline{36}+\overline{160}$	$\overline{20'}+36+60+84'$	$\bar{4}+20'+\overline{36}+140''$	$\overline{10}+64+126$	10
					$1+15+15+20''+45+\overline{45}+84$	$20+20'+120+140''$	$\overline{20}+\overline{20'}+\overline{120}+\overline{140''}$	$\bar{4}+20+20'+20'+\overline{36}+\overline{60}+140''$	$4+\overline{20}+\overline{20'}+\overline{20'}+36+60+140''$	$15+20''+45+\overline{45}+175$	15
						$50+84''+126+140$	$1+15+84+300'$	$64+70+126+140$	$15+45+84+256$	$\overline{36}+140''+224$	20
								$6+10+\overline{10}+50+64+64+70+126$	$1+15+15+20''+45+\overline{45}+84+175$	$\bar{4}+20'+\overline{36}+\overline{60}+140'+140''$	$20'$
										$1+15+20''+84+105+175$	$20''$

T A B L E 1 (continued)

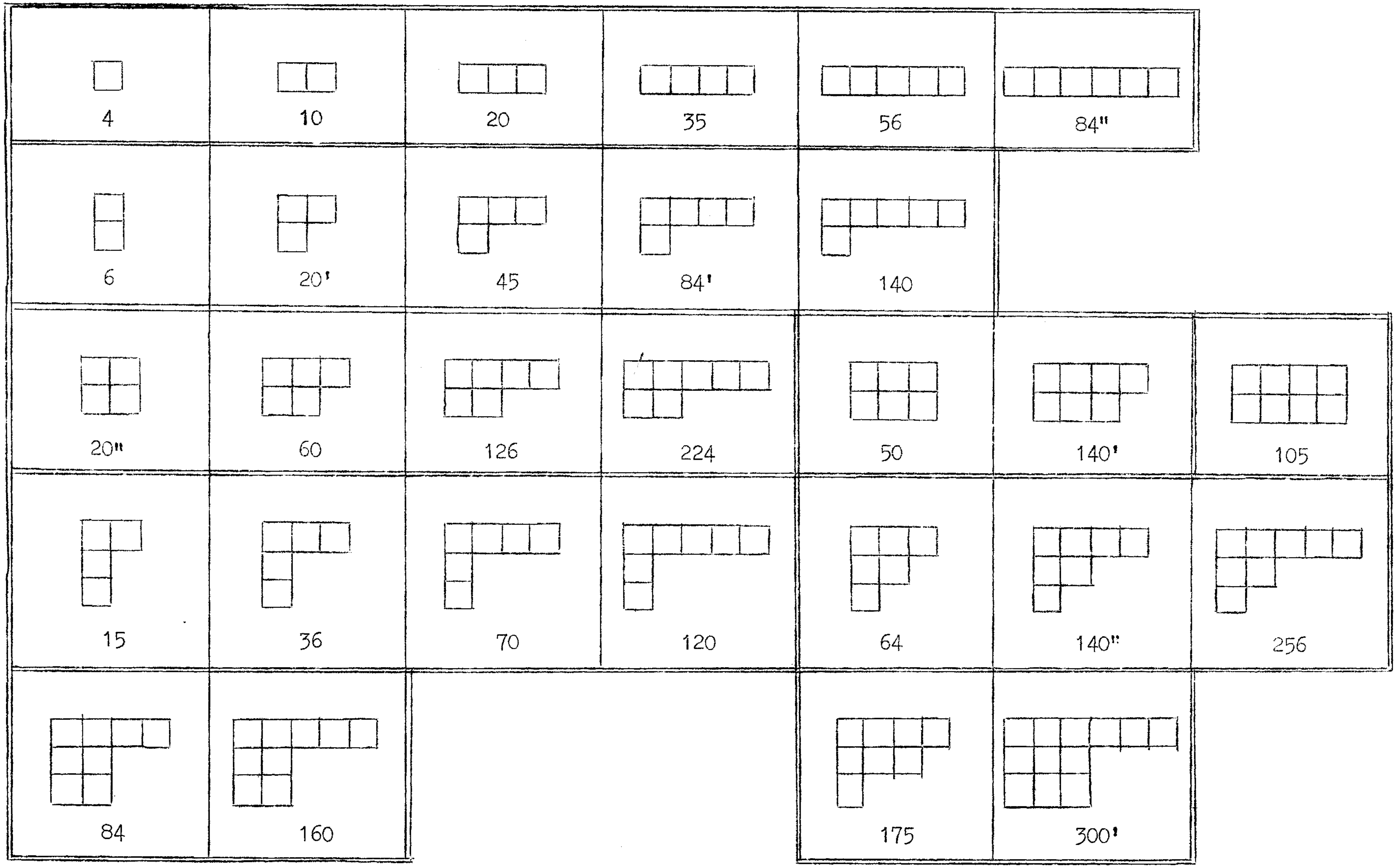
Young diagrams and dimension of the corresponding representations

$$\text{Dimension} = \frac{(a+3)(b+2)(c+1)(a-c+2)(a-b+1)(b-c+1)}{12}$$

where a = number of square blocks in the first row

b = " " " " " " second "

c = " " " " " " third "



4	10	20	35	56	84"	
6	20'	45	84'	140		
20"	60	126	224	50	140'	105
15	36	70	120	64	140"	256
84	160			175	300'	

TABLE 2

Lowest representations of SU_4 with their contents in SU_3 submultiplets

Rep. of SU_4	4		6		10			15			20				20′			20″		
Submultiplets of SU_3	3	1	$\bar{3}$	3	6	3	1	3	1+8	$\bar{3}$	10	6	3	1	8	$\bar{3}+6$	3	$\bar{6}$	8	6
Z	$1/4$	$-3/4$	$1/2$	$-1/2$	$1/2$	$-1/2$	$-3/2$	1	0	-1	$3/4$	$-1/4$	$-5/4$	$-9/4$	$3/4$	$-1/4$	$-5/4$	1	0	-1

Rep. of SU_4	35					36				45				50			
Submultiplets of SU_3	15^t	10	6	3	1	6	3+15	1+8	$\bar{3}$	15	8+10	$\bar{3}+6$	3	$\overline{10}$	$\overline{15}$	15	10
Z	1	0	-1	-2	-3	$5/4$	$1/4$	$-3/4$	$-7/4$	1	0	-1	-2	$3/2$	$1/2$	$-1/2$	$-3/2$

Rep. of SU_4	56						60				64			
Submultiplets of SU_3	21	15^t	10	6	3	1	$\overline{15}$	$6+\overline{15}$	8+10	6	8	$\bar{3}+6+\overline{15}$	$3+\bar{6}+15$	8
Z	$5/4$	$1/4$	$-3/4$	$-7/4$	$-11/4$	$-15/4$	$5/4$	$1/4$	$-3/4$	$-7/4$	$3/2$	$1/2$	$-1/2$	$-3/2$

Rep. of SU_4	70					84					84′				
Submultiplets of SU_3	10	6+24	3+15	1+8	$\bar{3}$	6	3+15	1+8+27	$\bar{3}+\overline{15}$	$\bar{6}$	24	$15+15^t$	8+10	$\bar{3}+6$	3
Z	$3/2$	$1/2$	$-1/2$	$-3/2$	$-5/2$	2	1	0	-1	-2	$5/4$	$1/4$	$-3/4$	$-7/4$	$-11/4$

TABLE 3

Generators of SU_4

$$\lambda_1 = \left|\begin{array}{cc|cc} \cdot & 1 & \cdot & \cdot \\ 1 & \cdot & \cdot & \cdot \\ \hline \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot \end{array}\right| \qquad \lambda_2 = \left|\begin{array}{cc|cc} \cdot & -i & \cdot & \cdot \\ i & \cdot & \cdot & \cdot \\ \hline \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot \end{array}\right| \qquad \lambda_3 = \left|\begin{array}{cc|cc} 1 & \cdot & \cdot & \cdot \\ \cdot & -1 & \cdot & \cdot \\ \hline \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot \end{array}\right|$$

$$\lambda_4 = \left|\begin{array}{cc|cc} \cdot & \cdot & 1 & \cdot \\ \cdot & \cdot & \cdot & \cdot \\ \hline 1 & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot \end{array}\right| \qquad \lambda_5 = \left|\begin{array}{cc|cc} \cdot & \cdot & -i & \cdot \\ \cdot & \cdot & \cdot & \cdot \\ \hline i & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot \end{array}\right| \qquad \lambda_6 = \left|\begin{array}{cc|cc} \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & 1 & \cdot \\ \hline \cdot & 1 & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot \end{array}\right|$$

$$\lambda_7 = \left|\begin{array}{cc|cc} \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & -i & \cdot \\ \hline \cdot & i & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot \end{array}\right| \qquad \lambda_8 = \frac{1}{\sqrt{3}}\left|\begin{array}{cc|cc} 1 & \cdot & \cdot & \cdot \\ \cdot & 1 & \cdot & \cdot \\ \hline \cdot & \cdot & -2 & \cdot \\ \cdot & \cdot & \cdot & \cdot \end{array}\right| \qquad \lambda_9 = \left|\begin{array}{cc|cc} \cdot & \cdot & \cdot & 1 \\ \cdot & \cdot & \cdot & \cdot \\ \hline \cdot & \cdot & \cdot & \cdot \\ 1 & \cdot & \cdot & \cdot \end{array}\right|$$

$$\lambda_{10} = \left|\begin{array}{cc|cc} \cdot & \cdot & \cdot & -i \\ \cdot & \cdot & \cdot & \cdot \\ \hline \cdot & \cdot & \cdot & \cdot \\ i & \cdot & \cdot & \cdot \end{array}\right| \qquad \lambda_{11} = \left|\begin{array}{cc|cc} \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & 1 \\ \hline \cdot & \cdot & \cdot & \cdot \\ \cdot & 1 & \cdot & \cdot \end{array}\right| \qquad \lambda_{12} = \left|\begin{array}{cc|cc} \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & -i \\ \hline \cdot & \cdot & \cdot & \cdot \\ \cdot & i & \cdot & \cdot \end{array}\right|$$

$$\lambda_{13} = \left|\begin{array}{cc|cc} \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot \\ \hline \cdot & \cdot & \cdot & 1 \\ \cdot & \cdot & 1 & \cdot \end{array}\right| \qquad \lambda_{14} = \left|\begin{array}{cc|cc} \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot \\ \hline \cdot & \cdot & \cdot & -i \\ \cdot & \cdot & i & \cdot \end{array}\right| \qquad \lambda_{15} = \frac{1}{\sqrt{6}}\left|\begin{array}{cc|cc} 1 & \cdot & \cdot & \cdot \\ \cdot & 1 & \cdot & \cdot \\ \hline \cdot & \cdot & 1 & \cdot \\ \cdot & \cdot & \cdot & -3 \end{array}\right|$$

FIG. 1

representation 15

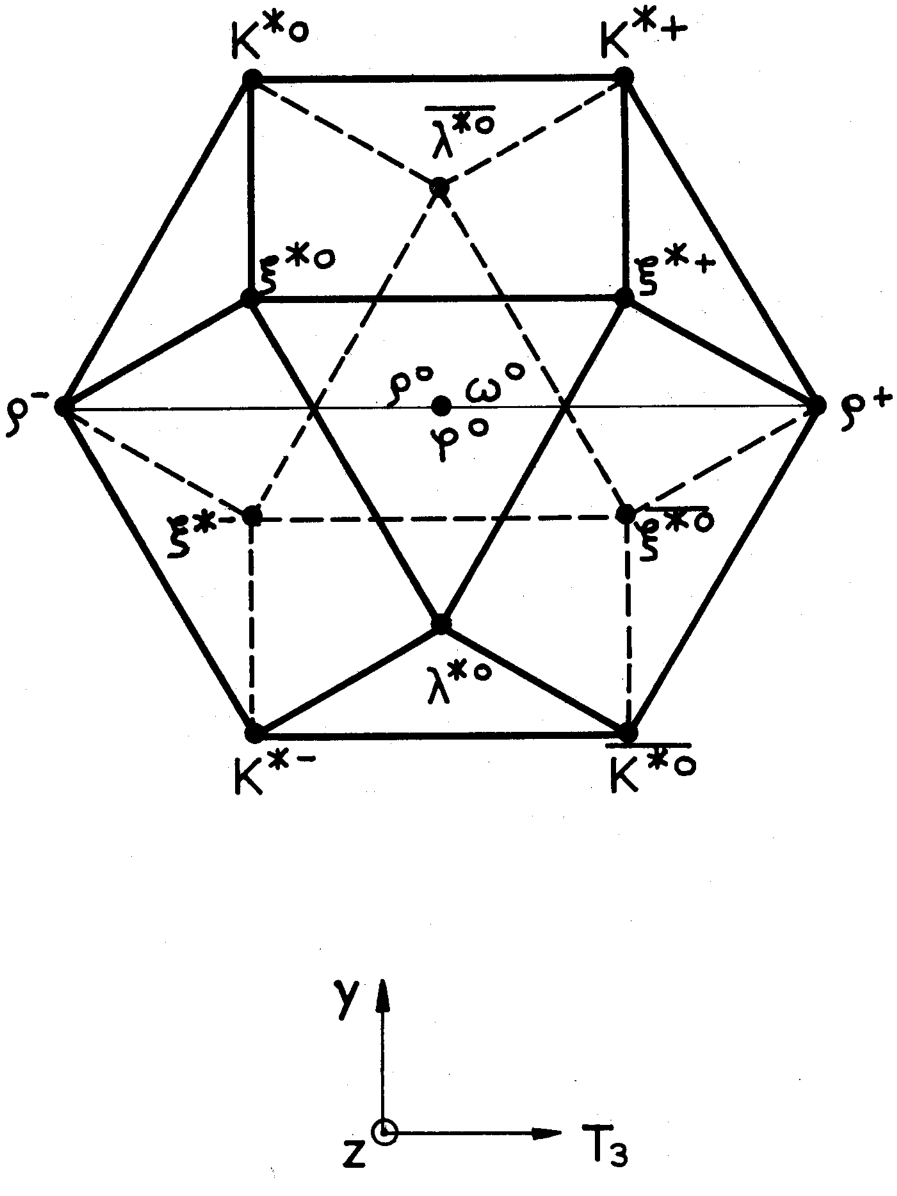


FIG. 2

representation 20

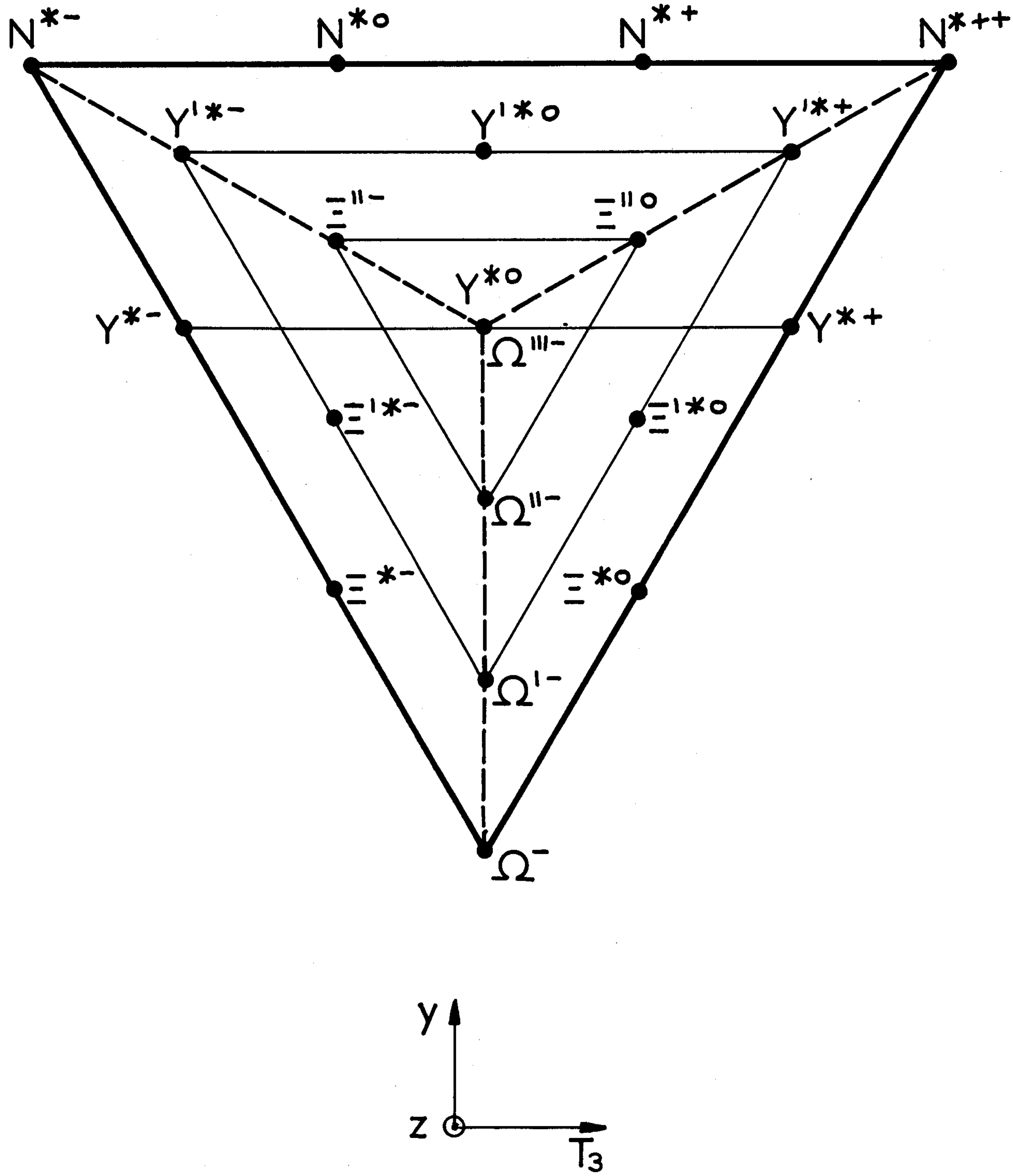


FIG. 3

representation 20'