Eur. Phys. J. A (2015) **51**: 27
DOI 10.1140/epja/i2015-15027-x

THE EUROPEAN PHYSICAL JOURNAL A

Letter

SU(4) symmetry of the dynamical QCD string and genesis of hadron spectra

L.Ya. Glozman[a]

Institute of Physics, University of Graz, A-8010 Graz, Austria

Received: 20 January 2015
Published online: 4 March 2015

Communicated by S. Hands

Abstract. A large degeneracy of mesons of a given spin has recently been discovered upon reduction of the quasi-zero modes of the Dirac operator in a dynamical lattice simulation. Here it is shown that a symmetry group $SU(4) \supset SU(2)_L \times SU(2)_R \times U(1)_A \times \mathcal{C}_i$ is consistent with the observed degeneracy. It is argued that this symmetry group is a symmetry of the dynamical QCD string. Implications of this picture for a genesis of light hadron spectra are discussed.

1 Introduction

A large degeneracy of mesons of a given spin has recently been discovered in a dynamical lattice simulation upon reduction of the lowest-lying eigenmodes of the manifestly chirally invariant overlap Dirac operator from the quark propagators [1,2] (for a previous lattice study with the not chirally invariant Wilson Dirac operator see refs. [3,4]). Of course, after such a truncation the correlators do not correspond to a local quantum field theory[1]. Despite that fact the correlators turned out to be very interesting: They have demonstrated a very clean exponential decay for all $J = 1$ channels and showed intriguing symmetry patterns. A similar degeneracy is seen in the observed highly excited mesons [5,6].

The quasi-zero eigenmodes of the Dirac operator are directly related to the chiral symmetry breaking quark condensate via the Banks-Casher relation [7]. Consequently, if hadrons survive this artificial restoration ("unbreaking") of chiral symmetry one expects that hadrons should fall into chiral multiplets.

The complete set of all possible $\bar{q}q$ chiral multiplets of the $J = 1$ mesons is given in table 1 [5,6].

Upon unbreaking of the chiral symmetry the states within each independent chiral multiplet get degenerate. However, what is completely unexpected, not only a degeneracy within chiral multiplets is seen, but actually a degeneracy of all eight $J = 1$ mesons. This degeneracy is obviously not accidental and tells us something important about the underlying dynamics.

Table 1. The complete set of $\bar{q}q$ $J = 1$ states classified according to the chiral basis. The symbol $\leftrightarrow$ indicates the states belonging to the same representation r of the parity-chiral group that must be degenerate in the $SU(2)_L \times SU(2)_R$ symmetric world. Mesons belonging to the singlet representation $(0,0)$ have no chiral partners.

r	Mesons
$(0,0)$	$\omega(I=0,1^{--})$; $f_1(I=0,1^{++})$
$(1/2,1/2)_a$	$\omega(I=0,1^{--}) \leftrightarrow b_1(I=1,1^{+-})$
$(1/2,1/2)_b$	$h_1(I=0,1^{+-}) \leftrightarrow \rho(I=1,1^{--})$
$(0,1)\oplus(1,0)$	$a_1(I=1,1^{++}) \leftrightarrow \rho(I=1,1^{--})$

A degeneracy of four mesons from the $(1/2,1/2)_a$ and $(1/2,1/2)_b$ representations indicates a restoration of the $SU(2)_L \times SU(2)_R \times U(1)_A$ symmetry [5,6]. This symmetry does not connect, however, these four mesons with other mesons from table 1. Consequently, a degeneracy of all mesons from table 1 implies a larger symmetry, that includes $SU(2)_L \times SU(2)_R \times U(1)_A$ as a subgroup. Our primary purpose is to establish this new symmetry. Given this new symmetry we discuss the physics implications for the highly degenerate system that is observed and various ramifications, in particular a genesis of light hadron spectra.

2 Parity-chiral $\bar{q}q$ multiplets

In order to proceed we need to construct the parity-chiral $\bar{q}q$ multiplets of states of any spin. The chirally symmetric $\bar{q}q$ states can be specified with the following set of

[a] e-mail: leonid.glozman@uni-graz.at
[1] Nonlocality turns out to be very small, however, because a tiny amount of modes is removed, of order 10 from more than one million.

quantum numbers: $r; IJ^{PC}$, where r is an index of the parity-chiral group and all other quantum numbers are isospin (I), spin (J), spatial and charge parities (P and C). The $\bar{q}q$ states with $J \geq 1$ fill out the following possible irreducible representations of the parity-chiral group $SU(2)_L \times SU(2)_R \times \mathcal{C}_i$, where a group $\mathcal{C}_i$ consists of the space inversion and identity transformation (a product with this group is required to construct states of definite parity):

i) $(0,0)$:

$$|(0,0);\pm;J\rangle = \frac{1}{\sqrt{2}}|\bar{R}R \pm \bar{L}L\rangle_J. \qquad (1)$$

Here the isospin $I = 0$, R denotes the right-handed fundamental $SU(2)_R$ vector, $R^T = (u_R, d_R)$, while L describes the left-handed $SU(2)_L$ one, $L^T = (u_L, d_L)$. The index J means that a definite spin J and its projection M are ascribed to the given quark-antiquark system according to the relativistic spherical helicity formalism [8,9]:

$$|\lambda_q \lambda_{\bar{q}}\rangle_J = D^{(J)}_{\lambda_q - \lambda_{\bar{q}}, M}(\boldsymbol{n}) \sqrt{\frac{2J+1}{4\pi}} |\lambda_q\rangle| - \lambda_{\bar{q}}\rangle, \qquad (2)$$

where $D^{(J)}_{MM'}(\boldsymbol{n})$ is the standard Wigner D-function describing rotation from the quantization axis to the quark momentum direction $\boldsymbol{n} = \boldsymbol{p}/p$ and λ_q ($\lambda_{\bar{q}}$) are the quark (antiquark) helicities; the quark chirality and helicity coincide, while for the antiquark they are just opposite. The parity of the quark-antiquark state is then given as

$$\hat{P}|(0,0);\pm;J\rangle = \pm(-1)^J|(0,0);\pm;J\rangle. \qquad (3)$$

ii) $(1/2,1/2)_a$ and $(1/2,1/2)_b$:

$$|(1/2,1/2)_a;+;I=0;J\rangle = \frac{1}{\sqrt{2}}|\bar{R}L + \bar{L}R\rangle_J, \qquad (4)$$

$$|(1/2,1/2)_a;-;I=1;J\rangle = \frac{1}{\sqrt{2}}|\bar{R}\boldsymbol{\tau}L - \bar{L}\boldsymbol{\tau}R\rangle_J, \qquad (5)$$

and

$$|(1/2,1/2)_b;-;I=0;J\rangle = \frac{1}{\sqrt{2}}|\bar{R}L - \bar{L}R\rangle_J, \qquad (6)$$

$$|(1/2,1/2)_b;+;I=1;J\rangle = \frac{1}{\sqrt{2}}|\bar{R}\boldsymbol{\tau}L + \bar{L}\boldsymbol{\tau}R\rangle_J. \qquad (7)$$

In these expressions $\boldsymbol{\tau}$ are isospin Pauli matrices. The parity of every state in these representations is determined as

$$\hat{P}|(1/2,1/2);\pm;I;J\rangle = \pm(-1)^J|(1/2,1/2);\pm;I;J\rangle. \qquad (8)$$

Note that a sum of the two distinct $(1/2,1/2)_a$ and $(1/2,1/2)_b$ irreducible representations of $SU(2)_L \times SU(2)_R$ forms an irreducible representation of the $U(2)_L \times U(2)_R$ or $SU(2)_L \times SU(2)_R \times U(1)_A$ groups.

iii) $(0,1) \oplus (1,0)$:

$$|(0,1)+(1,0);\pm;J\rangle = \frac{1}{\sqrt{2}}|\bar{R}\boldsymbol{\tau}R \pm \bar{L}\boldsymbol{\tau}L\rangle_J, \qquad (9)$$

the isospin $I = 1$ and parities

$$\hat{P}|(0,1)+(1,0);\pm;J\rangle = \pm(-1)^J|(0,1)+(1,0);\pm;J\rangle. \qquad (10)$$

For the $J = 0$ states the representations $(0,0)$ and $(0,1)+(1,0)$ are impossible, because the total spin projection onto the momentum direction of the quark for these representations is ± 1.

All the basis vectors (1), (4)–(7) and (9) are the relativistic spherical helicity states in the quark-antiquark system that represent natural relativistic basis for the bound quark-antiquark states with definite chirality. They should not be confused with the plane waves. They are correct relativistic basis vectors that carry complete information about chiral degrees of freedom in a meson with restored chiral symmetry and thus should be sufficient to reconstruct the observed higher symmetry.

One can also construct various local [10] and nonlocal [5,6] composite $\bar{q}q$ operators that have the required chiral symmetry properties.

3 The $\mathrm{SU(4)} \supset \mathrm{SU(2)_L} \times \mathrm{SU(2)_R} \times \mathrm{U(1)_A} \times \mathcal{C}_i$ symmetry

Our task is to find a minimal symmetry group that combines all four irreducible representations $(0,0)$, $(1/2,1/2)_a$, $(1/2,1/2)_b$ and $(0,1)+(1,0)$ of the parity-chiral group into one representation of a larger group. Transformations of this group should connect all basis vectors (1), (4)–(7) and (9) to each other.

Transformations that link (4) with (5) and (6) with (7) are the $SU(2)_L \times SU(2)_R$ transformations, *i.e.*, independent rotations of both right-handed and left-handed fundamental vectors R and L in the isospin space. In order to connect (4) and (5) with (6) and (7) we need in addition the $U(1)_A$ transformation, that links the $(1/2,1/2)_a$ and $(1/2,1/2)_b$ states of the same isospin but opposite parity. The $SU(2)_L \times SU(2)_R \times U(1)_A$ transformations do not connect, however, (4)–(7) with (1) or (9), because both the basis vectors (1) and (9) are selfdual with respect to $U(1)_A$. Consequently, in order to find a symmetry group that connects all basis vectors (1), (4)–(7), (9), we need to find transformations that link the states (4)–(7) with (1) and (9).

Such transformations can be most transparently seen when we use explicit notations for the basis vectors. Consider, as an example, the $Q = -1$ charge states of (7) and (9) of equal parity:

$$\frac{1}{\sqrt{2}}|\bar{u}_R d_L + \bar{u}_L d_R\rangle_J \quad \text{and} \quad \frac{1}{\sqrt{2}}|\bar{u}_R d_R + \bar{u}_L d_L\rangle_J. \qquad (11)$$

A symmetry transformation that connects both these states is $(d_L \leftrightarrow d_R) \otimes (u_L \leftrightarrow u_L) \otimes (u_R \leftrightarrow u_R)$. This cannot be a parity transformation, because the space inversion transforms the left-handed quarks into the right-handed quarks and vice versa for both flavors simultaneously. Such a transformation can be obtained if we perform two independent $SU(2)_U$ and $SU(2)_D$ rotations of two independent fundamental vectors U and D, where $U^T = (u_L, u_R)$ and $D^T = (d_L, d_R)$. Similarly, the $Q = +1$ states of (7) and (9) of the same parity

$$\frac{1}{\sqrt{2}}|\bar{d}_R u_L + \bar{d}_L u_R\rangle_J \quad \text{and} \quad \frac{1}{\sqrt{2}}|\bar{d}_R u_R + \bar{d}_L u_L\rangle_J, \tag{12}$$

transform into each other through $(u_L \leftrightarrow u_R) \otimes (d_L \leftrightarrow d_L) \otimes (d_R \leftrightarrow d_R)$, which again can be accomplished via two independent $SU(2)_U$ and $SU(2)_D$ rotations.

One can check that the same is true for the $Q = 0$ states of (7) and (9) as well as for the $Q = 0$ states (1) and (4).

Now we are in a position to find a minimal symmetry group that connects all vectors (1), (4)–(7) and (9). This group must contain as subgroups the $SU(2)_L$ and $SU(2)_R$ isospin rotations of quarks of fixed chirality, the $SU(2)_U$ and $SU(2)_D$ chirality rotations of quarks with fixed flavor, the $U(1)_A$, as well as a parity transformation $(u_L \leftrightarrow u_R) \otimes (d_L \leftrightarrow d_R)$. This symmetry transforms the fundamental four-component vector N, $N^T = (u_L, u_R, d_L, d_R)$ and represents the $SU(4)$ group. Vectors (1), (4)–(7) and (9) form a basis set for a dim $= 16$ reducible representation $\bar{4} \times 4 = 15 + 1$ of the group $SU(4)$ in the reduction chain $SU(4) \supset SU(2)_L \times SU(2)_R \times \mathcal{C}_i$.

An important issue is that this new $SU(4)$ symmetry is relevant only to $J \geq 1$ states. For the $J = 0$ states only the basis vectors (4)–(7) are possible and the total symmetry group that combines all possible states of the $J = 0$ mesons is $SU(2)_L \times SU(2)_R \times U(1)_A$.

We stress that this symmetry is not a symmetry of the QCD Lagrangian. It should be considered as an emergent symmetry that appears from the QCD dynamics upon removal of the quasi-zero modes of the Dirac operator.

The ultra-relativistic $SU(4) \supset SU(2)_L \times SU(2)_R \times U(1)_A \times \mathcal{C}_i$ symmetry should not be confused with the nonrelativistic Wigner spin-isospin [11,12] (heavy-quark [13, 14]) $SU(4)_{SI} \supset SU(2)_S \times SU(2)_I$ symmetry. It should also not be confused with the $SU(4)$ Pauli-Gürsey symmetry [15,16] that connects the mesonic and baryonic (diquark) states within $N_c = 2$ QCD.

Finally, we note that a generalization of this symmetry to N_f light flavors is straightforward and the relevant symmetry group in this case is $SU(2N_f)$.

4 Genesis of light meson spectra

Within the potential constituent quark model [17,18], that was a basis for intuition and insights for many years, a gross symmetry of the light hadron spectra is $SU(4)_{SI} \times O(3)$, which is a symmetry of the levels of the confining interquark potential. This symmetry gets broken by the phenomenologically introduced spin-spin, tensor and spin-orbit interactions that are fitted to the experimental levels. As a consequence the $SU(4)_{SI}$ symmetry is lifted. Such a physical picture has a solid basis in the heavy quark mesons but cannot be substantiated in the light quark sector where chiral and $U(1)_A$ symmetries and their breakings are crucially important.

The results presented above suggest that the primary energy level has a symmetry $SU(4) \supset SU(2)_L \times SU(2)_R \times U(1)_A \times \mathcal{C}_i$, not to be confused with the nonrelativistic $SU(4)_{SI}$ symmetry of the constituent quark model. *E.g.*, the former symmetry combines into one multiplet of dim $= 16$ all mesons from table 1, while a dim $= 16$ multiplet of $SU(4)_{SI}$ consists of the $\pi, \eta_2, \rho, \omega$ mesons. The primary energy levels observed in [1,2] contain all degenerate states of both parity from table 1, while the positive and the negative-parity levels of the confining potential of the constituent quark model represent different $SU(4)_{SI} \times O(3)$ multiplets that are strongly splitted (with the harmonic confinement this splitting is $\hbar\omega$).

A genesis of the light quark $\bar{q}q$ mesons could be then viewed as follows: A confining interaction gives rise to the highly degenerate primary levels with the symmetry $SU(4) \supset SU(2)_L \times SU(2)_R \times U(1)_A \times \mathcal{C}_i$ and a dynamics related to the quasi-zero modes of the Dirac operator supplies a breaking of both chiral and $U(1)_A$ symmetries as well as a splitting of the primary confining levels. While both $SU(2)_L \times SU(2)_R$ and $U(1)_A$ breakings are most probably related to the instanton-induced dynamics [19–21], different effective microscopic mechanisms could be at work for splitting of the primary energy levels.

5 The dynamical QCD string

In ref. [1] it was speculated that the highly degenerate energy levels observed after subtraction of the lowest Dirac eigenmodes are quantum levels of the dynamical QCD string. Below we precisely formulate arguments that lead to such a conclusion.

Consider a motion of an electrically charged fermion in a static electric field or a relative motion of two charged fermions. In such systems there exist magnetic interactions which manifest themself through the spin-spin, spin-orbit and tensor interactions. In our case we do have a relative motion of two color-charged fermions, however the spin-spin, spin-orbit and tensor interactions are absent. This can be proved as follows.

All relativistic chiral states from table 1 can be decomposed via the unitary transformation into a sum of vectors of the $\{I, {}^{2S+1}L_J\}$ basis [22,23]:

$$|(0,1)+(1,0); 1\ 1^{--}\rangle = \sqrt{\frac{2}{3}}\,|1; {}^3S_1\rangle + \sqrt{\frac{1}{3}}\,|1; {}^3D_1\rangle,$$

$$|(1/2,1/2)_b; 1\ 1^{--}\rangle = \sqrt{\frac{1}{3}}\,|1; {}^3S_1\rangle - \sqrt{\frac{2}{3}}\,|1; {}^3D_1\rangle,$$

$$|(0,0); 0\ 1^{--}\rangle = \sqrt{\frac{2}{3}}\,|0; {}^3S_1\rangle + \sqrt{\frac{1}{3}}\,|0; {}^3D_1\rangle,$$

$$|(1/2,1/2)_a; 0\ 1^{--}\rangle = \sqrt{\frac{1}{3}}\,|0; {}^3S_1\rangle - \sqrt{\frac{2}{3}}\,|0; {}^3D_1\rangle,$$

$$|(0,1)+(1,0);1\ 1^{++}\rangle = |1;{}^3P_1\rangle,$$

$$|(0,0);0\ 1^{++}\rangle = |0;{}^3P_1\rangle,$$

$$|(1/2,1/2)_a;1\ 1^{+-}\rangle = |1;{}^1P_1\rangle,$$

$$|(1/2,1/2)_b;0\ 1^{+-}\rangle = |0;{}^1P_1\rangle. \tag{13}$$

We can invert this unitary transformation and obtain a chiral decomposition of vectors

$$|0;{}^3S_1\rangle,\ |1;{}^3S_1\rangle,\ |0;{}^3D_1\rangle,\ |1;{}^3D_1\rangle, \tag{14}$$

$$|0;{}^1P_1\rangle,\ |1;{}^1P_1\rangle,\ |0;{}^3P_1\rangle,\ |1;{}^3P_1\rangle. \tag{15}$$

Given that all eight states from table 1 are degenerate, we immediately obtain a degeneracy of all eight states (14) and (15). This degeneracy implies absence of the spin-spin, spin-orbit and tensor interactions in the system. Indeed, a nonzero spin-orbit force would split the ${}^3S_1$ and ${}^3P_1$; the ${}^1P_1$ and ${}^3P_1$; etc. terms, a nonzero spin-spin force would split the ${}^1P_1$ and ${}^3P_1$; the ${}^3S_1$ and ${}^1P_1$; etc levels, and a tensor force would split the ${}^3S_1$ and ${}^3D_1$ terms.

We conclude that there are no magnetic interactions in the system. The energy of the system is entirely due to interactions of the color charges via the color-electric field and due to a relativistic motion of the system. We interpret (or, better, define) such a system as a dynamical QCD string (for a simple model see [24]).

One might raise a question why such a system without the color-magnetic field is interpreted by us as a dynamical QCD string? In QED in $3+1$ dimensions a motion of an electric charge in a vacuum induces a magnetic field that lives in a plane that is perpendicular to the charge motion. With one spatial dimension the system becomes, however, pure electric. The absence of the color-magnetic field in our case is natural if the system became effectively one-dimensional. A one-dimensional dynamical color-electric system with chiral quarks at the ends, that is embedded into $3+1$ dimensional space, is natural to call a string. We do not know, however, why such a dimesional reduction should happen in QCD with charges that move with the speed of light.

6 Summary

We have suggested a new symmetry that is associated with the degeneracy of the energy levels of mesons of a given spin $J \geq 1$ after subtraction of the quasi-zero eigenmodes of the Dirac operator. It is $SU(4) \supset SU(2)_L \times SU(2)_R \times U(1)_A \times \mathcal{C}_i$. It is not a symmetry of the QCD Lagrangian, but is an emergent symmetry that appears from the QCD dynamics upon reduction of the low-lying Dirac modes. We consider this symmetry as a symmetry of the confining interaction in QCD. Actually the symmetry group could be even higher if the energy levels of mesons with different spins will turn out to be degenerate. The latter issue is a subject of the current lattice simulations.

We interpret these highly degenerate energy levels as levels of the dynamical QCD string. We actually define such a system as the dynamical QCD string, because there is no magnetic interaction in the system. The energy of the system comes only from the color-electric interaction and from the relativistic motion of the system. This picture should be contrasted with the well-understood relative motion of two fermions within the local $U(1)$-gauge theory where magnetic interaction is necessarily present.

A genesis of the light meson spectra looks quite different as compared to the constituent quark model. The $\bar{q}q$ spectra could be viewed as a result of the splitting of the primary energy levels of the dynamical QCD string by means of dynamics associated with the quasi-zero modes of the Dirac operator.

We thank C.B. Lang for discussions and careful reading of the manuscript. Partial support from the Austrian Science Fund (FWF) through grant P26627-N27 is acknowledged.

References

1. M. Denissenya, L.Y. Glozman, C.B. Lang, Phys. Rev. D **89**, 077502 (2014).
2. M. Denissenya, L.Y. Glozman, C.B. Lang, Phys. Rev. D **91**, 034505 (2015).
3. C.B. Lang, M. Schröck, Phys. Rev. D **84**, 087704 (2011).
4. L.Y. Glozman, C.B. Lang, M. Schröck, Phys. Rev. D **86**, 014507 (2012).
5. L.Y. Glozman, Phys. Lett. B **587**, 69 (2004).
6. L.Y. Glozman, Phys. Rep. **444**, 1 (2007).
7. T. Banks, A. Casher, Nucl. Phys. B **169**, 103 (1980).
8. M. Jacob, G.C. Wick, Ann. Phys. **7**, 404 (1959) (Ann. Phys. **281**, 774 (2000)).
9. L.D. Landau, E.M. Lifshitz, *Quantum Electrodynamics, Course of Theoretical Physics*, Vol. **4** (Pergamon Press, New York, 1982) sects. 68, 69.
10. T.D. Cohen, X.-D. Ji, Phys. Rev. D **55**, 6870 (1997).
11. E.P. Wigner, Phys. Rev. **51**, 946 (1937).
12. F. Gursey, L.A. Radicati, Phys. Rev. Lett. **13**, 173 (1964).
13. N. Isgur, M.B. Wise, Phys. Lett. B **232**, 113 (1989).
14. N. Isgur, M.B. Wise, Phys. Lett. B **237**, 527 (1990).
15. W. Pauli, Nuovo Cimento **6**, 205 (1957).
16. F. Gursey, Nuovo Cimento **7**, 411 (1958).
17. A. De Rujula, H. Georgi, S.L. Glashow, Phys. Rev. D **12**, 147 (1975).
18. S. Godfrey, N. Isgur, Phys. Rev. D **32**, 189 (1985).
19. G. 't Hooft, Phys. Rep. **142**, 357 (1986).
20. E.V. Shuryak, Nucl. Phys. B **203**, 93 (1982).
21. D. Diakonov, V.Y. Petrov, Nucl. Phys. B **272**, 457 (1986).
22. L.Y. Glozman, A.V. Nefediev, Phys. Rev. D **76**, 096004 (2007).
23. L.Y. Glozman, A.V. Nefediev, Phys. Rev. D **80**, 057901 (2009).
24. L.Y. Glozman, Phys. Lett. B **541**, 115 (2002).