International Journal of Service Science, Management, Engineering, and Technology
Volume 14 • Issue 1

Subspace Data-Driven Control for Linear Parameter Varying Systems

Jianhong Wang, School of Engineering and Sciences, Tecnologico de Monterrey, Mexico

Yunfeng Zhang, College of Electrical Engineering and Automation, Jiangxi University of Science and Technology, China*

Ricardo A. Ramirez-Mendoza, School of Engineering and Sciences, Tecnologico de Monterrey, Mexico

Ahmad Taher Azar, College of Computer and Information Sciences, Prince Sultan University, Saudi Arabia & Faculty of Computers and Artificial Intelligence, Benha University, Benha 13518, Egypt

Ibraheem Kasim Ibraheem, Computer Engineering Techniques Department, Al-Mustaqbal University College, Iraq

Nashwa Ahmad Kamal, Faculty of Engineering, Cairo University, Egypt

Farah Ayad Abdulmajeed, College of Technical Engineering, Alfarahidi University, Iraq

ABSTRACT

In this research, a unique subspace data driven control for linear parameter changing system with scheduling parameters is presented. This control paves the way for investigating the nonlinear system based on the results regarding the linear system that are already known. Only the data matrix is utilized to represent the output prediction value in the future various time instants, while the input-output observation data matrix is used to identify Markov parameters in the form of state space forms. The cost function in data-driven control is then adjusted using the output prediction value. The optimal control input value of this quadratic cost function is solved using a parallel distribution technique, and the algorithm's iterative convergence is thoroughly examined. Finally, the DC motor, whose mass distribution factor is considered to be one linear parameter varying system, is controlled using the suggested subspace data driven control approach.

KEYWORDS

Linear Parameter Varying System, Parallel Distributed Algorithm, Subspace Data Driven Control

1. INTRODUCTION

Automatic control system is very important for realizing some automatic operations in practical industry and monitoring of complex industrial processes, such as paper, glass, automotive and aircraft etc (Daraz et al., 2022, 2021; Ibraheem et al., 2020a,b; Abdul-Adheem et al., 2020a,b; Soliman et al., 2020; Gorripotu et al, 2021, 2019; Meghni et al., 2017, 2018). Practical industrial processes require some critical process variables to be maintained around their given values, for example, temperature, flow level, pressure and press. The amplitude fluctuations of process variables need to be carefully chosen and continuously controlled to achieve the products, while keeping the desired quality with the minimum raw materials and energy. Then automatic control technology was developed with

DOI: 10.4018/IJSSMET.321198

*Corresponding Author

this mission, being used to ensure the smooth and safe operation for industrial process or maximum corporate benefit. Generally, at present, automatic control technology has been widely used in electric power, oil refining, paper making chemical industry, energy and other industrial production fields (Mahdi et al., 2022; Sain et al., 2022; Fekik et al., 2022a,b, 2021a,b,c,d, 2020a,b; Ali et al., 2022, 2021a,b; Acharyulu et al., 2021; Ajel et al., 2021). More specifically to obtain the qualified productions during the industrial production, some key process variables are usually required to meet certain performance index. Then this performance index will eventually be realized by deigning one suitable controller. Most of the current controller design methods are model-based, so the accuracy of the considered process used in the controller design will ultimately affect the performance of the control loop system (Toumi et al., 2022; Saidi et al., 2022; Serrano et al., 2022; Humaidi et al., 2022, 2020; Hamida et al., 2022; Ben Njima et al., 2021; Ghoudelbourk et al., 2021; Pilla et al., 2021a,b; Ajeil et al. 2020a,b; Azar and Serrano, 2015). Model mismatch will deteriorate the performance of the control loop, for example, having larger overshot, longer adjustment time, oscillation, etc. All above factors lead to substandard product quality, bring great harm to the enterprise. Even if the controller designed based on the current model or identified model can meet the performance index, an improved process model can better describe the dynamic changes of the process, then one controller with better performance can be yielded from this improved process model, so that the raw material and energy consumption are reduced. Actually, most industrial processes in practice present complex nonlinear or parameter varying characteristics. Typically, controllers are designed according to a single, locally linearized model at a certain operating point, in order to achieve the specified performance requirements (Mittal et al., 2021; Al-Qassar et al., 2021a,b; Ammar et al., 2020, 2019, 2018; Kazim et al., 2022, 2021a,b; Najm et al., 2021a,b, 2020; Vaidyanathan et al., 2019, 2018a,b, 2017a,b,c; Radwan et al., 2018; Abdelmalek et al., 2018; Ouannas et al., 2020, 2017a,b,c,d, 2016a,b; Azar et al., 2018a,b; Singh et al., 2021, 2017). However practical industrial process usually has a wide operating range, and process operating points are caused by production planning, such as product grade changes, economic optimization etc., or work environment changes. But a single locally linearized model cannot describe the global dynamic behavior of the considered process, which will greatly deteriorate the performance of controller, designed based on this locally linearized model. Even this approximated linear model leads to the instability of the closed loop system or process. It can be seen that good model used to describe the global dynamic behavior of the process, is a prerequisite for the latter control design or other interesting tasks.

Generally, the mechanism model from the conservation laws of mass, momentum, heat and energy reflects the inherent laws of process dynamic changes and can describe the global dynamic behavior of the process. But with the increasing scale and complexity of modern industrial processes, it is very difficult or almost impossible to construct their mechanism models, even if the mechanism models are derived for the considered plants, the real mechanism models correspond to complex nonlinear equations or nonlinear systems, i.e. all the plants are nonlinear forms. Consider these nonlinear systems, i.e. all the plants are nonlinear terms. Consider these nonlinear equations, their analytical solutions cannot derive, only resorting to the numerical algorithms to get the approximated solutions. It makes very difficult to design controllers based on nonlinear systems. As the process data contains rich information of the considered plant or process, the model of the process can be directly extracted from the process data. This idea of modeling method is named as system identification. The essence of system identification is to select a model from the model classes according to one certain criterion, so that it can best fit the dynamic characteristics of the actual plant. The main steps of system identification include optimal input signal design, model structure selection, parameter identification and model validation. Therefore, from the point of view of system identification, it is not necessary to obtain a process model such as mechanism model whose parameters have clear physical meaning, but only a model that describes the global dynamic behavior of the process is required for the application purpose.

To satisfy the requirements of the modeling nonlinear or parameter varying process, researchers proposed many model structures, for example, linear parameter varying system considered in this paper, orthonormal basis functions, Hammerstein-Wiener model Gaussian process regression, neural networks, support vector machine. Among above mentioned model structures, linear parameter varying system has attracted extensive attention of researchers because of its linear structure and ability to accurately approximate complex nonlinear or time varying property. Linear parameter varying system was originally proposed by Shamma and Athans in their work on gain scheduling controller design. Specifically, linear parameter varying system has a linear structure and its model parameters are expressed as functions of one or more measurable or computable time varying signal. This time varying signal here is the scheduling parameter or scheduling variable, which reflects the working model of the process, so the dynamics of linear parameter varying system depends on the time varying scheduling variable. When the value of the scheduling variable is fixed, the dynamical system is also determined to be one linear time invariant system. Linear parameter varying system is regarded as a tradeoff between the linear system and nonlinear system. It has not only a simple linear structure, but also the ability to accurately describe the nonlinear or time varying parameters due to its time varying model parameters. At present research on linear parameter varying system concentrate on controller design, filtering, fault detection and other combinations with information fusion. This interesting research is widely applied in industrial practices, for example, wind turbines, inverse heat exchangers, aircraft, robot, servo system, magnetic bearing system and super-cavitating vehicle, etc. Although great progress has been achieved in the research fields of linear parameter varying system for industrial application, the study on the identification and control design have been seriously lacked for our considered linear parameter varying system.

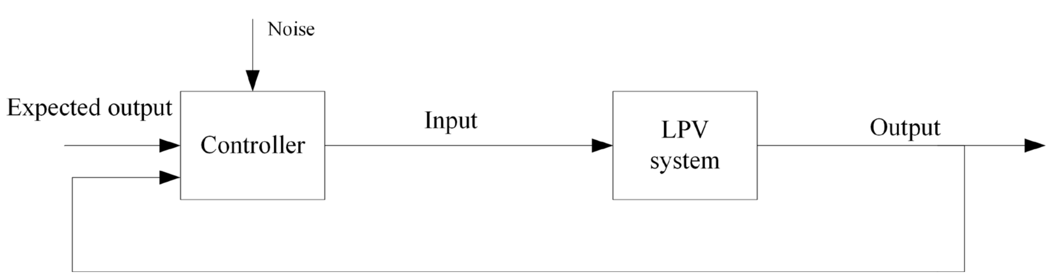
Subspace data driven control is one novel direct data driven control strategy, being from the subspace system identification for state space forms. Specifically, subspace system identification is to apply the past and future input-output data sequence to identify each system matrix, existing in the original state space form, i.e. the singular value decomposition corresponding to the data matrix is implemented to obtain each system matrix. It is well known that subspace data driven control combines the common properties between direct data driven control strategy and subspace system identification, so that the one sequence of output predictions at different time instants are constructed based on the collected past and future input-output data sequence. After comparing these output predictions and their expected or desired output values, a cost function is established within the framework of data driven idea. Consider this cost function with equality or inequality constraints as one constrain optimization problem, the future control input sequence is yielded through some minimization operations, and then the first element from this future control input sequence is chosen as the start point for the next minimization operation. This iterative process is called as the roll horizon process, which is more widely studied and applied in some practice. As subspace data driven data control strategy applies the past input-output observed data sequence to construct the future output predictions at some different future time instants directly without modeling all the system matrices for the original state space form, so this nice subspace data driven control simplifies the three main steps for the classical linear Gaussian regulator, i.e. identification, filtering and control, and this is the main reason why in recent years, lots of research concern on this direct data driven control strategy.

Although lots of research on direct data driven control strategy exist in recent years, but few references are seen about our considered subspace data driven control, which only is suited for state space forms. For example, reference (Brett & Hakan, 2005) constructs the basic and important output prediction value from the point of subspace system identification, and the constructed output prediction value is compared with the other output prediction value within the iterative correlation control strategy in (Cristian, 2010). Reference (Erik & Campi, 2017) proposes the Bayesian framework for the subspace data driven control strategy, then the ellipsoidal optimization algorithm is proposed to solve that cost function for the subspace data driven control strategy, whose optimization problem corresponds to one uncertain optimization problem in (Hakan,2011). Furthermore, the level set from convex analysis theory is applied to construct the original ellipsoid for the above-mentioned ellipsoidal optimization algorithm. Consider the realization problem for this subspace data driven control strategy, reference (Hakan, 2005) applies it in vibration suppression of active noise, which effectively suppresses the flutter of small helicopters in the hovering state. Reference (Jakob,2011) uses a fast gradient algorithm to estimate faults under the constraints of upper and lower fault limits. The overall framework of subspace data driven control is shown in Figure 1, where the controller is designed using subspace data driven control.

The models that appear in nature and industrial mass production are all nonlinear models. Due to the complexity of nonlinear models and their unknown forms, nonlinear system identification and nonlinear control are still under investigation. A common method for nonlinear model analysis is to use Taylor series method to expand the original nonlinear system at a certain operating point or equilibrium stable point, and to approximately replace the original nonlinear system with a linear model that ignores high-order terms. The disadvantage of this linearization method is that when a certain operating point is changing, the linearization model is also changing all the time. In order to make up for the shortcomings of the linearization method, a compromise model is sought between the nonlinear model and the linear model. Although this compromise model is a nonlinear model, it also has the characteristics of a linear model. In the process of modeling a certain motor, it is found that a time-varying scheduling parameter sequence needs to be introduced to reflect the time-varying matrix of the linear state space system. The time-varying model is called a linear parameter changing system in controller design. Reference (Bravo, Alamo, &Vasallo, 2017) proposes a variety of identification strategies for the identification of linear parameter change systems. The identification method is designed based on the given form of the linear parameter change system, such as state space equation form or transfer function form. However, the closed-loop controller design of this system is still in the traditional linear quadratic Gaussian control, and it is still realized by the three major processes of first identification, then filter design and then controller parameter selection.

Based on above mentioned references or contributions on linear parameter varying system, this new paper continues to study the data driven strategy for linear parameter varying system identification and linear parameter varying controller design simultaneously, i.e., the unknown system and controller are all linear parameter varying forms. Roughly the mission of data driven strategy is to extract some important knowledge from data, being collected or measured by sensors. It means the knowledge are included in this data record, then our task is to apply some methods to extract this useful knowledge successfully. To be different with other contributions, the considered closed loop system has one unknown system or plant and the other controller, which are all linear parameter varying forms. Due to the final goal is to design that linear parameter varying controller, data driven idea and system identification are combined to achieve our goal. More specifically, the input-output data record or sequence are used to get the linear parameter varying system and linear parameter varying controller through using the power spectral theory. To give a complete analysis,

Figure 1. Structure of subspace data driven control



iterative identification method is proposed to generate two kinds of parameter estimations, existing in those two linear parameterized system and controller well. For the sake of completeness, optimal input signal is designed for linear parameter varying system, while making the error signal between the real closed loop output and its parameterized closed loop output be zero as small as possible. All mathematical derivations are related with matrix theory, power spectral, numerical optimization and variation analysis, so the optimal input spectral is derived though our calculation. As our above contributions concern on linear parameter varying controller and linear parameter varying system through the data driven and system identification, i.e., the linear parameter varying system and linear parameter varying controller are all needed. It is the reason about why iterative method is used to yield them simultaneously. Moreover, to avoid the modeling process for that linear parameter varying system, data driven idea is also improved to design the linear parameter varying controller without any knowledge of the linear parameter varying system. From the theoretical perspective, this paper reviews some existed theories, such as classical model reference design and numerical optimization.

In this paper, subspace data driven control is applied to the controller design for linear parameter varying system with the scheduling parameters. The input-output observation data matrix is used to identify Markov parameters in the form of state space forms, and only the data matrix is used to represent the output prediction value at the future different time instants. Then the output prediction value is applied to the cost function in data driven control. For this quadratic cost function, a parallel distribution algorithm is used to solve its optimal control input value, and the iterative convergence of the parallel distribution algorithm is analyzed in detail.

2. LINEAR PARAMETER VARYING SYSTEM

Consider the following linear parameter varying system with the scheduling parameter:

$$\begin{cases} x\left(k+1\right) = \sum_{i=1}^{n_\mu} \mu_k^{(i)} \left(A^{(i)} x\left(k\right) + B^{(i)} u\left(k\right) + K^{(i)} e\left(k\right)\right) \\ y\left(k\right) = Cx\left(k\right) + e\left(k\right) \end{cases} \tag{1}$$

where in equation (1), $x\left(k\right) \in R^n, y\left(k\right) \in R^l, u\left(k\right) \in R^m$ are state variable, input variable and control input variable respectively. Noise signal or innovation signal $e\left(k\right)$ is one white noise with zero mean and variance σ_e^2. $\mu_k^{(i)} \in R$ corresponds to the scheduling parameter or weight value for each local model, $\left\{A^{(i)}, B^{(i)}, K^{(i)}\right\}$ is the i th local model. n_μ is the total number of all local models, k is time instant for this control problem. When all the scheduling parameters are all equal to be number 1, i.e. $\mu_k^{(i)} \equiv 1$, then we define:

$$A = \sum_{i=1}^{n_\mu} A^{(i)}, B = \sum_{i=1}^{n_\mu} B^{(i)}, K = \sum_{i=1}^{n_\mu} K^{(i)}$$

Then equation (1) is reduced to the following linear time invariant system:

$$\begin{cases} x\left(k+1\right) = Ax\left(k\right) + Bu\left(k\right) + Ke\left(k\right) \\ y\left(k\right) = Cx\left(k\right) + e\left(k\right) \end{cases} \tag{2}$$

According to some related definitions from subspace system identification theory, the following time varying matrices are set as:

$$A_k = \sum_{i=1}^{n_\mu} \mu_k^{(i)} A^{(i)}, B_k = \sum_{i=1}^{n_\mu} \mu_k^{(i)} B^{(i)}, K_k = \sum_{i=1}^{n_\mu} \mu_k^{(i)} K^{(i)} \tag{3}$$

Innovation signal $e(k)$ existing in output equation (1) is rewritten as:

$$e(k) = y(k) - Cx(k) \tag{4}$$

substituting equation (4) into state equation (1), we have:

$$\begin{aligned} x(k+1) &= A_k x(k) + B_k u(k) + K_k e(k) \\ &= \left[A_k - K_k C\right] x(k) + B_k u(k) + K_k y(k) \end{aligned} \tag{5}$$

Based on the existed notation from subspace system identification, set the stable observation matrix ϕ_k as that:

$$\begin{aligned} &\phi_k = A_k - K_k C = \sum_{i=1}^{n_\mu} \mu_k^{(i)} \phi^{(i)} = \sum_{i=1}^{n_\mu} \mu_k^{(i)} \left(A^{(i)} - K^{(i)} C\right) \\ &\phi^{(i)} = A^{(i)} - K^{(i)} C \end{aligned} \tag{6}$$

Combining equation (1), (5) and (6), the closed form for that formal linear parameter varying system is changed as follows:

$$\begin{cases} \hat{x}(k+1) = \phi_k \hat{x}(k) + B_k u(k) + K_k y(k) \\ y(k) = C\hat{x}(k) + e(k) \end{cases} \tag{7}$$

It is different with the mathematical derivations from subspace data driven control for linear time invariant system, that here the state transition matrix is one time varying form, denoting as that:

$$\varphi_{j,k} = \varnothing_{k+j-1} \varnothing_{k+j-2} \cdots \varnothing_k = \prod_{\tau=0}^{j-1} \varnothing_{k+\tau}, \forall j \geq 1, \varphi_{0,k} = I \tag{8}$$

3. SUBSPACE DATA DRIVEN CONTROL STRATEGY

During the following system identification and controller design process, s and f are the number of past data and future data, N is the number of all observed data, t is the time instant for the identification problem. In order to use equation (7), we combine some observed data as one vector form, i.e. define the following some vector forms as:

$$y_{[k,k+f)} = \left[y(k)^T \ y(k+1)^T \cdots y(k+f-1)^T \right]^T,$$
$$u_{[k,k+f)} = \left[u(k)^T \ u(k+1)^T \cdots u(k+f-1)^T \right]^T$$
$$e_{[k,k+f)} = \left[e(k)^T \ e(k+1)^T \cdots e(k+f-1)^T \right]^T,$$
$$z_{[k-s,k)} = \left[u(k-s)^T \ y(k-s)^T \cdots u(k-1)^T \ y(k-1)^T \right]^T$$

where $y_{[k,k+f)}$ represent the output values at future time instants, and $u_{[k,k+f)}$ are the control input signals at future time instants, i.e. the future control inputs. $e_{[k,k+f)}$ is innovative sequence, $z_{[k-s,k)}$ is one vector, being full of past input-output observation data at the past time instants.

Applying the time varying Markov parameters, and iterating the above equation (7), the future output predictions at future time instants are described as follows:

$$y_{[k,k+f)} = b_x + H_{s,z} z_{[k-s,k)} + \tau_{f,u} u_{[k,k+f)} + \tau_{f,y} y_{[k,k+f)} + e_{[k,k+f)} \tag{9}$$

where all matrices in above equation (9) are formulated as follows:

$$b_x = \begin{bmatrix} C\varphi_{s,k-s}\hat{x}(k-s) \\ C\varphi_{s,k-s+1}\hat{x}(k-s+1) \\ \vdots \\ C\varphi_{s,k-s+f-1}\hat{x}(k-s+f-1) \end{bmatrix},$$
$$H_{s,z} = \begin{bmatrix} E_0 \\ E_1 \\ \vdots \\ E_{f-1} \end{bmatrix}$$
$$\tau_{f,u} = \begin{bmatrix} 0 & & & & \\ CB_k & 0 & & & \\ C\varphi_{1,k+1}B_k & CB_{k+1} & 0 & & \\ \vdots & & & \vdots & \\ C\varphi_{f-2,k+1}B_k & C\varphi_{f-3,k+2}B_{k+1} & \cdots & CB_{k+f-2} & 0 \end{bmatrix},$$
$$\tau_{f,y} = \begin{bmatrix} 0 & & & & \\ CK_k & 0 & & & \\ C\varphi_{1,k+1}K_k & CK_{k+1} & 0 & & \\ \vdots & & & \vdots & \\ C\varphi_{f-2,k+1}K_k & C\varphi_{f-3,k+2}K_{k+1} & \cdots & CK_{k+f-2} & 0 \end{bmatrix}$$

where Markov parameter is defined as follows, for $i = 0, \cdots f-1$, it holds that:

$$E_i = \left[0 \cdots 0 \,, C\varphi_{s-1,k-s+i+1} B_{k-s+i} \,, C\varphi_{s-1,k-s+i+1} K_{k-s+i} \cdots C\varphi_{i,k} B_{k-1}, C\varphi_{i,k} K_{k-1}\right]$$

In case of the internal stability for equation (10, then:

$$\left\| \hat{x}\left(k-s+i\right) \right\|_2 < \infty, i = 0 \cdots f-1$$

If number or time horizon s is sufficiently large, based on the stability of that time varying transition matrix, it guarantees that:

$$\left\| \varphi_{s,\tau} \right\|_2 < \varepsilon \ll 1, \forall \tau \geq 0$$

where ε is a small positive number, and biased term b_x is neglected.

Set the scheduling parameter vector be that:

$$\mu_k = \left[\mu_k^{(1)} \quad \cdots \quad \mu_k^{(n_\mu)}\right]^T$$

For $i = 1 \cdots f-1, j = 1 \cdots s$, define the following matrices:

$$P_{j/k+i-j}^z = \mu_{k+i-1} \otimes \mu_{k+i-2} \otimes \cdots \otimes \mu_{k+i-j} \otimes I_{m+l} = \left(\bigotimes_{\tau=k+i-j}^{k+i-1} \mu_\tau \right) \otimes I_{m+l}$$
$$P_{j/k+i-j}^u = \left(\bigotimes_{\tau=k+i-j}^{k+i-1} \mu_\tau \right) \otimes I_m, P_{j/k+i-j}^y = \left(\bigotimes_{\tau=k+i-j}^{k+i-1} \mu_\tau \right) \otimes I_l$$

where in above equation $\otimes$ is matrix direct product operation.

Collecting the n_j possible multiply operations of the local parameter matrices, we get:

$$\left(\prod_{\tau=1}^{j-1} \phi^{(n_\tau)} \right) \gamma^{(n_j)} = \left(\prod_{\tau=1}^{j-1} \phi^{(n_\tau)} \right) \left[B^{(n_j)} \quad K^{(n_j)} \right] \tag{10}$$

where $\phi^{(n_\tau)}, \gamma^{(n_j)}$ and index n_j are defined as follows respectively:

$$\phi^{(n_\tau)} = A^{(n_\tau)} - K^{(n_\tau)} C, \quad \gamma^{(n_j)} = \left[B^{(n_j)} \quad K^{(n_j)} \right]$$
$$n_1 \cdots n_{j-1} \; n_j \in \left\{ 1 \cdots n_\mu \right\}$$

Then recursively compute to yield that:

$$L_j = \left[\phi^{(1)} L_{j-1} \quad \phi^{(2)} L_{j-1} \quad \cdots \quad \phi^{(n_\mu)} L_{j-1}\right], \\ L_1 = \left[\gamma^{(1)} \quad \gamma^{(2)} \quad \cdots \quad \gamma^{(n_\mu)}\right] \tag{11}$$

Using our constructed all vectors and matrices, the time varying Markov parameter in equation (9) is rewritten as the following explicit form with the scheduling parameter vector:

$$C\varphi_{j-1,k+i-j+1}\gamma_{k+i-j} = CL_j P^z_{j/k+i-j}, \gamma_{k+i-j} = \left[B_{k+i-j} \quad K_{k+i-j}\right] \tag{12}$$

The purpose of subspace system identification for linear parameter varying time with scheduling parameters is to estimate the Markov parameters in (9), so we collect the past input-output observation data sequence and scheduling parameter to establish the following information matrix:

$$z^{2k+f-2}_{k-s} = \left[N^s_{k-s} z_{[k-s,k)}, N^s_{k-s+1} z_{[k-s+1,k+1)}, \cdots N^s_{2k+f-s-1} z_{[2k+f-s-1,2k+f-1)}\right] \tag{13}$$

where matrix N^s_{k-s+i} is defined as that:

$$N^s_{k-s+i} = \begin{bmatrix} P^z_{s/k+i-s} & & & \\ & P^z_{s-1/k+i-s+1} & & \\ & & \ddots & \\ & & & P^z_{1/k+i-1} \end{bmatrix}$$

Based on equation (9), the future output prediction data sequence at future time instant is represented as that:

$$y_{[k,k+f)} = E_0 z^{2k+f-2}_{k-s} + e_{[k,k+f)} \tag{14}$$

when matrix z^{2k+f-2}_{k-s} is a full row rank matrix, the least squares solution of Markov parameters is that:

$$\hat{E}_0 = y_{[k,k+f)} \left(z^{2k+f-2}_{k-s}\right)^+ = \left[CL_s \quad \cdots \quad CL_1\right] \tag{15}$$

where $\left(z^{2k+f-2}_{k-s}\right)^+$ in above equation is the pseudo-inverse operation.

On the basis of the above least squares estimation $\hat{E}_0$, we can construct the output prediction value at future time instant for our considered subspace data driven control strategy. After substituting the estimation in equation (15) into equation (9) and neglecting the biased term b_x or unknown innovative sequence $e_{[k,k+f)}$, we determine the deterministic future output prediction value at future time instant, i.e.:

$$\hat{y}_{[k,k+f)} \approx H_{s,z} z_{[k-s,k)} + \tau_{f,u} u_{[k,k+f)} + \tau_{f,y} \hat{y}_{[k,k+f)} \tag{16}$$

Equation (9) is one closed observation form. As the unknown term $\hat{y}_{[k,k+f)}$ is included in both sides of equation (16), so we need rewrite the output prediction $\hat{y}_{[k,k+f)}$ as the other predictive form, being related with $u_{[k,k+f)}$ and $z_{[k-s,k)}$. Then the output prediction value for that linear parameter varying system with scheduling parameter is given as follows:

$$\begin{aligned} \hat{y}_{[k,k+f)} &= \begin{bmatrix} \hat{y}(k) \\ \hat{y}(k+1) \\ \vdots \\ \hat{y}(k+f-1) \end{bmatrix} = \begin{bmatrix} \Gamma_0 \\ \Gamma_1 \\ \vdots \\ \Gamma_{f-1} \end{bmatrix} z_{[k-s,k)} + \begin{bmatrix} \Lambda_{1,1} & 0 & \cdots & 0 \\ \Lambda_{2,1} & \Lambda_{2,2} & \cdots & 0 \\ \vdots & \vdots & \vdots & \vdots \\ \Lambda_{f-1,1} & \Lambda_{f-1,2} & \cdots & \Lambda_{f-1,f-1} \end{bmatrix} u_{[k,k+f-1)} \\ &= \Gamma z_{[k-s,k)} + \Lambda u_{[k,k+f-1)} \end{aligned} \tag{17}$$

where parameters $\left\{\Gamma_i, \Lambda_i \,/\, i = 1 \cdots f-1, 1 \le j < i\right\}$ can be generated recursively to be:

$$\begin{cases} \Gamma_i = E_i + \sum_{\tau=0}^{i-1} \left(CL^y_{i-\tau} P^y_{i-\tau/k+\tau} \right) \Gamma_\tau, \Gamma_0 = E_0 \\ \Lambda_{i,j} = CL^u_{i-j+1} P^u_{i-j+1/k+j-1} + \sum_{\tau=1}^{i-j} \left(CL^y_{i-j+1-\tau} P^y_{i-j+1-\tau/k+\tau+j-1} \right) \Lambda_{\tau+j-1,j} \\ \Lambda_{i,i} = CL^u_1 P^u_{1/k+i-1} \end{cases} \tag{18}$$

By comparison, it can be seen that the estimated value of the output prediction of the linear parameter varying system is more complicated than the estimated value of the output prediction of the linear time-invariant system. Equation (17) is abbreviated as:

$$\hat{y}_{[k,k+f)} = \Gamma z_{[k-s,k)} + \Lambda u_{[k,k+f-1)} \tag{19}$$

To establish the idea of subspace data driven control strategy, we assume the desired or expected output trajectory r is known, i.e.:

$$r_{[k,k+f)} = \left[r^T(k) \quad r^T(k+1) \quad \cdots \quad r^T(k+f-1) \right]^T \tag{20}$$

The future control input sequence at future time instant is designed through minimizing the following quadratic cost function or performance function:

$$J_k\left(u_{[k,k+f-1)}, \mu_{[k,k+f-1)}\right) = \left\| r_{[k,k+f)} - \hat{y}_{[k,k+f)} \right\|_Q^2 + \left\| \Delta u_{[k,k+f-1)} \right\|_{R_1}^2 + \left\| u_{[k,k+f-1)} \right\|_{R_2}^2 \tag{21}$$

where weighted matrices Q, R_1, R_2 are all positive matrices, and the second term is added in equation (21) to show the control input rate of change. The purpose is not to make large jumps in the control input sequence, and always maintain a smooth control input.

After expanding the second term to be that:

$$\begin{gathered}\Delta u_{[k,k+f-1)} = S_\Delta u_{[k,k+f-1)} - S_{uz} z_{[k-s,k)} \\ S_\Delta = \begin{bmatrix} I_m & & & \\ -I_m & I_m & & \\ & \ddots & \ddots & \\ & & -I_m & I_m \end{bmatrix}, \quad S_{uz} = \begin{bmatrix} 0 & I_m & 0 \\ & \ddots & \\ 0 & 0 & 0 \end{bmatrix}\end{gathered} \tag{22}$$

That cost function (21) satisfies the following hard constrain condition:

$$\begin{cases} \hat{y}_{[k,k+f)} = \Gamma z_{[k-s,k)} + \Lambda u_{[k,k+f-1)} \\ \mathrm{A}\begin{bmatrix} u_{[k,k+f-1)} & \hat{y}_{[k,k+f)} \end{bmatrix} \le b \end{cases} \tag{23}$$

Consider the optimization problem with the cost function (21) and the equality or inequality constrain condition (23), the following parallel distribution algorithm is applied to solve its optimal solution.

4. PARALLEL DISTRIBUTION ALGORITHM

Expanding the cost function (21) and neglecting those constant terms without control input, we have:

$$\begin{aligned} J_k\left(u,\mu\right) &= u^T\left[\Lambda^T Q \Lambda + S_\Delta R_1 S_\Delta + R_2\right]u + 2\left[\left(r - \Gamma z\right)^T Q\Lambda - \left(S_{uz} z\right)^T\right]u \\ &= \frac{1}{2} u^T \Lambda_1 u - u^T b_1 \end{aligned} \tag{24}$$

where during the above mathematical derivations, the first equality condition in equation (23) is used, then we rewrite the second inequality condition in equation (23) as that:

$$\begin{bmatrix} \mathrm{A}_1 & \mathrm{A}_2 \end{bmatrix}\begin{bmatrix} u \\ \Gamma z + \Lambda u \end{bmatrix} \le b$$

i.e.:

$$\left(\mathrm{A}_1 + \mathrm{A}_2\Lambda\right)u \le b - \mathrm{A}_2 \Gamma z$$

Simplifying it to be that:

$$\mathrm{A}_3 u \leq c \tag{25}$$

Combining equation (24) and (25) to be one quadratic programming problem, which is formulated to be that:

$$\begin{aligned} &\min_u \frac{1}{2} u^T \Lambda_1 u - u^T b_1 \\ &s.\,t.\ \mathrm{A}_3 u \leq c \end{aligned} \tag{26}$$

The dual function of equation (26) is that:

$$q(\lambda) = \inf_u \left\{ \frac{1}{2} u^T \Lambda_1 u - u^T b_1 - \lambda^T \left(\mathrm{A}_3 u - c \right) \right\}$$

That decision variable λ in above dual function is named as the Lagrange multiply, and the dual function is minimized in case of $u = \Lambda_1^{-1} \left(b_1 - \mathrm{A}_3 \lambda \right)$. After substituting this optimal value u into that dual function, we have:

$$q(\lambda) = -\frac{1}{2} \lambda^T \mathrm{A}_3 \Lambda_1^{-1} \mathrm{A}_3 \lambda - \lambda^T c - \mathrm{A}_3 \Lambda_1^{-1} b_1 - \frac{1}{2} b_1 \Lambda_1^{-1} b_1 \tag{27}$$

Through some basic variation transformations, the dual function for that quadratic programming problem is deem as that:

$$\begin{aligned} &\min_\lambda \frac{1}{2} \lambda^T M \lambda + \lambda^T d \\ &s.\,t. \quad \lambda \geq 0, M = \mathrm{A}_3 \Lambda_1^{-1} \mathrm{A}_3, d = c - \mathrm{A}_3 \Lambda_1^{-1} b_1 \end{aligned} \tag{28}$$

We see that if the optimal solution λ^* for that dual function is obtained, then the optimal solution for that original quadratic programming problem is that:

$$u^* = \Lambda_1^{-1} \left(b_1 - \mathrm{A}_3 \lambda^* \right) \tag{29}$$

The parallel distribution algorithm is proposed to solve that dual problem here. Set a_j as the j th column for matrix A_3, and all elements of a_j is not zero. If $a_j = 0$, it is no senseless due to constrain condition $a_j u \leq c$. As weighted matrix Q is positive and definite, so the jth diagonal element of matrix M is that $M_{jj} = a_j' \Lambda_1^{-1} a_j$, and it is positive too. It means that for each j, the dual cost function is strictly convex along the jth coordinate.

Taking the first-order partial derivative with respect to λ_j for that dual cost function, it holds that:

$$d_j + \sum_{k=1}^{m} M_{jk}\lambda_k \tag{30}$$

where M_{jk} and d_j are elements coming from matrix M and vector d respectively.

Set that above partial derivative be zero, the process of unconstrained minimization of the dual cost function from the initial point λ to reach $\tilde{\lambda}_j$ along the j th coordinate is that:

$$\tilde{\lambda}_j = -\frac{1}{M_{jj}}\left(d_j + \sum_{k\neq j} M_{jk}\lambda_k\right) = \lambda_j - \frac{1}{M_{jj}}\left(d_j + \sum_{k=1}^{m} M_{jk}\lambda_k\right) \tag{31}$$

Due to nonnegative constraint $\lambda_j \geq 0$, the iterative form with the j th coordinate is updated is follows:

$$\lambda_j = \max\left\{0, \tilde{\lambda}_j\right\} = \max\left\{0, \lambda_j - \frac{1}{M_{jj}}\left(d_j + \sum_{k=1}^{m} M_{jk}\lambda_k\right)\right\} \tag{32}$$

Based on the first-order partial derivative (30) of the dual cost function with respect to λ_j, equation (32) can be adjusted as:

$$\lambda_j\left(t+1\right) = \max\left\{0, \lambda_j\left(t\right) - \frac{\eta}{M_{jj}}\left(d_j + \sum_{k=1}^{m} M_{jk}\lambda_k\left(t\right)\right)\right\} \tag{33}$$

where $\eta > 0$ is the step size, and this recursive form is suited for parallel distribution algorithm.

From the numerical optimization theory, we see that in order to ensure that the iterative process of (33) can converge to its global minimum, the step size parameter η should be selected to be sufficiently small, that is, the convergence can be achieved for the special case, i.e. $\eta = \frac{1}{m}$.

5. SIMULATION EXAMPLE

Our considered subspace data driven control of linear parameter varying system with scheduling parameter vector is applied to the modeling and controller design of DC motor. The DC motor can be approximated by a linear model while ignoring a variety of external factors. However, when the consideration of the mass distribution is added to the rotating disk, a nonlinear model is required to characterize it. The reconsideration graph of the DC motor mass is shown in Figure 2.

The mass distribution on a homogeneous disk will become inhomogeneous. The mathematical description of the DC motor can be divided into two parts: the motor part and the mechanical part. Some calibration parameters in the DC motor are shown in Table 1. Firstly, we use Kirchhoff's voltage law to get:

$$L_m\dot{I}\left(t\right) = v\left(t\right) - K_i\omega\left(t\right) - R_m I\left(t\right) \tag{34}$$

Figure 2. Structure of DC motor

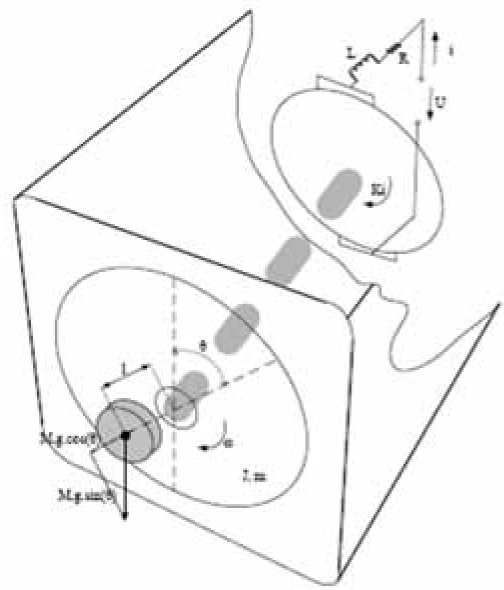


Table 1. Nominal parameters in DC motor

Parameters	Number value
Motor torque constant	$K_i = 53.6 \times 10^{-3} Nm / A$
resistance	$R_m = 9.5\Omega$
damping	$L_m = 0.84 \times 10^{-3} H$
Disc inertia	$J = 2.2 \times 10^{-4} Nm^2$
friction coefficient	$b = 6.6 \times 10^{-5} Nms / rad$
extra quality	$M = 0.07kg$
Mass distribution at the center of the disk	$l = 0.042m$

where $I\left(t\right)$ is current intensity, $v\left(t\right)$ is control input voltage, and $\omega\left(t\right)$ is rotation velocity.

The variable relationship between the motor part and the mechanical part is that:

$$\begin{cases} J\dot{\omega}\left(t\right) = K_i I\left(t\right) - b\omega\left(t\right) + Mgl\sin\left(\theta\left(t\right)\right) \\ \dot{\theta}\left(t\right) = \dot{\omega}\left(t\right) \end{cases} \tag{35}$$

where $\theta\left(t\right)$ is rotation angle.

When an appropriate observation parameter is chosen to replace the nonlinear term, equation (35) will become a linear parameter varying system.

Set the scheduling parameter as that:

$$\mu\left(t\right)=\frac{\sin\left(\theta\left(t\right)\right)}{\theta\left(t\right)}$$

When the rotation angle is measurable, the scheduling parameters can be estimated. Select the following state variables as that:

$$x\left(t\right)=\left[\theta\left(t\right) \quad \omega\left(t\right) \quad I\left(t\right)\right]^{T}$$

The state space form of continuous time linear parameter change can be obtained, i.e.:

$$\begin{aligned}&\begin{bmatrix}\dot{\theta}\left(t\right)\\ \dot{\omega}\left(t\right)\\ \dot{I}\left(t\right)\end{bmatrix}=\left(\begin{bmatrix}0 & 0 & 0\\ 0 & -\frac{b}{J} & \frac{K_i}{J}\\ 0 & -\frac{K_i}{L_m} & -\frac{R_m}{L_m}\end{bmatrix}+\begin{bmatrix}1 & 0 & 0\\ \frac{Mgl}{J} & 0 & 0\\ 0 & 0 & 0\end{bmatrix}\mu\left(t\right)\right)\times\begin{bmatrix}\theta\left(t\right)\\ \omega\left(t\right)\\ I\left(t\right)\end{bmatrix}+\begin{bmatrix}0\\ 0\\ \frac{1}{L_m}\end{bmatrix}v\left(t\right)\\ &y\left(t\right)=\begin{bmatrix}1 & 0 & 0\\ 0 & 0 & 1\end{bmatrix}\begin{bmatrix}\theta\left(t\right)\\ \omega\left(t\right)\\ I\left(t\right)\end{bmatrix}\end{aligned} \tag{36}$$

Neglecting the fast motor system, then equation (36) is reduced to be a second order system:

$$\begin{bmatrix}\dot{\theta}\left(t\right)\\ \dot{\omega}\left(t\right)\end{bmatrix}=\left(\begin{bmatrix}0 & 1\\ 0 & -\frac{1}{\tau}\end{bmatrix}+\begin{bmatrix}0 & 0\\ \frac{Mgl}{J} & 0\end{bmatrix}\mu\left(t\right)\right)\begin{bmatrix}\theta\left(t\right)\\ \omega\left(t\right)\end{bmatrix}+\begin{bmatrix}0\\ \frac{K_m}{\tau}\end{bmatrix}v\left(t\right) \tag{37}$$

For formula (37), the discretization method in linear system theory is used. According to the discretization process of continuous linear time-invariant system, h is taken as the period of discrete use. The corresponding relationship between each system matrix between the continuous system and the discrete system is as follows:

$$\begin{aligned}&\begin{bmatrix}\theta\left(k+1\right)\\ \omega\left(k+1\right)\end{bmatrix}=e^{h\left(\begin{bmatrix}0 & 1\\ 0 & -\frac{1}{\tau}\end{bmatrix}+\begin{bmatrix}0 & 0\\ \frac{Mgl}{J} & 0\end{bmatrix}\mu\left(k\right)\right)}\begin{bmatrix}\theta\left(k\right)\\ \omega\left(k\right)\end{bmatrix}+\int_0^h e^{\tau\left(\begin{bmatrix}0 & 1\\ 0 & -\frac{1}{\tau}\end{bmatrix}+\begin{bmatrix}0 & 0\\ \frac{Mgl}{J} & 0\end{bmatrix}\mu\left(t\right)\right)}d\tau\begin{bmatrix}0\\ \frac{K_m}{\tau}\end{bmatrix}v\left(k\right)\\ &y\left(k\right)=\begin{bmatrix}1 & 0\\ 0 & 0\end{bmatrix}\begin{bmatrix}\theta\left(k\right)\\ \omega\left(k\right)\end{bmatrix}\end{aligned} \tag{38}$$

The exponential matrix in the above formula is expanded according to the existing calculation method in the linear system theory, and the linear parameter variation system such as equation (1) can be obtained. This complex expansion process can directly call the matrix calculation program in MATLAB. A pseudo-real-time simulation environment is established for the DC motor. The software driver of the motor is established under the real-time Microsoft Windows XP using the real-time MATLAB/Simulink environment. The communication protocol is realized through the USB interface, and the sampling frequency in the pseudo-real-time condition is set to 20HZ. A series of observation data sets are collected for the DC motor module, and a reference trajectory is selected based on the given scheduling parameters to construct a calibration parameter sequence. The subspace data driven control method is used to realize the tracking of the position and speed of the DC motor. The entire closed-loop control experiment is initiated using data acquisition and offline physical parameter identification.

The least squares algorithm is applied to identify Markov parameter, and at this time, some physical parameters are chosen as $N = 2000, T_s = 0.05s$. When exciting the scheduling parameter, it must satisfy that:

$$\lim_{\theta \to 0} \left\| \mu\left(t\right) A_1 \right\|_2 = 0$$

If the above equation is not satisfied, the original system will lose nonlinear performance. The reference selection of the rotational speed should ensure that the scheduling parameters $\mu\left(t\right)$ are excited sufficiently continuously. Based on the idea of data driven for future output prediction estimates (19) and possible hard constraints (23), the cost function is minimized with respect to the future control input sequence. The rolling time domain strategy is used for the obtained control input sequence, i.e., only the first element is selected each time as the construction basis of the predicted value in the next optimization process. For the optimization problem at each sampling instant, the future time-domain level is chosen as $f = 3$, and each weighted matrix are $Q = 100I, R_1 = 0.01I, R_2 = 0.1I$. That hard constraint about the control input is $-7V \leq v \leq 7V$.

The simulation results using subspace data driven control for this DC motor are shown in Figures 3-5. Figure 3 shows the position tracking curve of the DC motor under the action of the subspace

Figure 3. Position tracking curves for DC motor

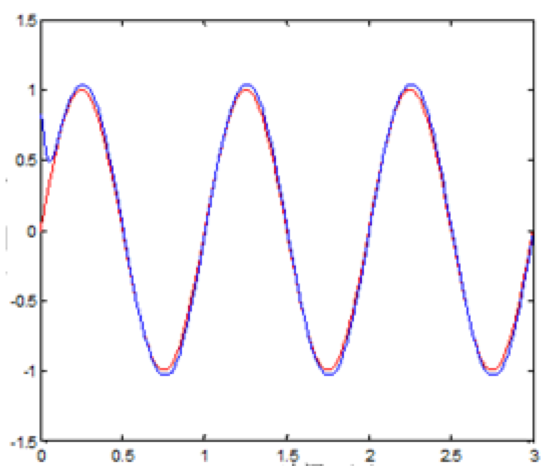


Figure 4. Position tracking error for DC motor

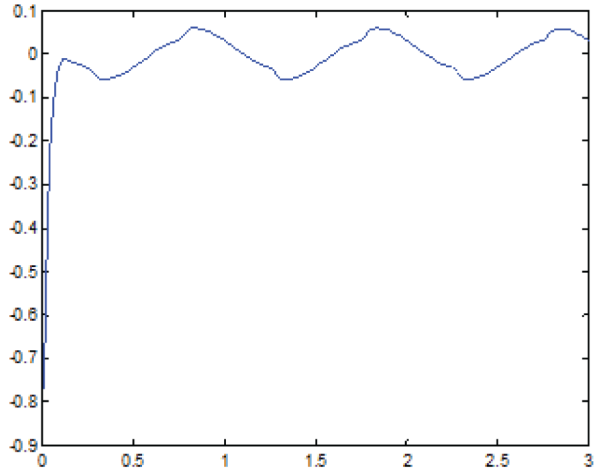
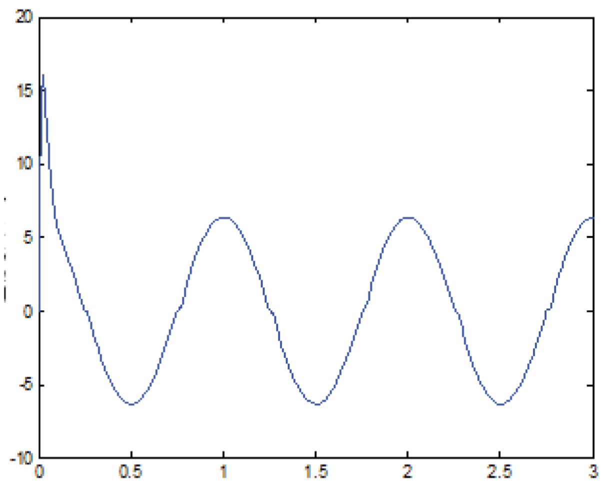


Figure 5. Velocity response for DC motor



data driven controller, where the blue curve is the given reference trajectory, and the red is the actual controller output curve. It can be seen from Figure 3 that there is a large deviation between the two curves at the initial moment, and as time goes on, the deviation between the two curves will gradually decrease, and an approximate effect can be achieved. Figure 4 shows the corresponding DC motor position tracking error curve, which can also better explain the fitting effect of Figure 3. Figure 5 shows the rotation speed response curve of the DC motor, which is a continuous smooth curve, then it indicates that the speed of the DC motor is completely determined by the designed subspace data driven controller.

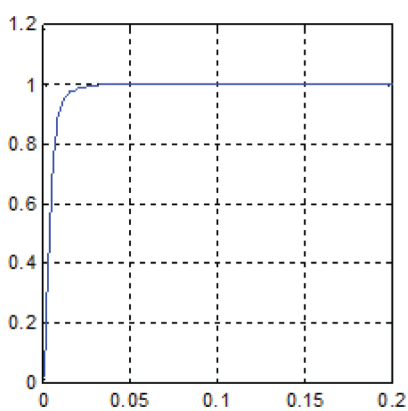
The DC motor shown in Figure 2 is placed in the servo closed-loop control loop of the flight simulation turntable. Flight simulation is a follow-up servo mechanism, which can simulate various flight attitudes of the aircraft in the air. The servo control loop of the flight simulation turntable, as a high-precision servo device, puts forward high requirements for the position follow-up control: on the one hand, it is expected that the position tracking should not have overshoot, and the dynamic response process should be fast and smooth; on the other hand, to ensure the tracking accuracy requires position tracking with a small steady-state error. In the servo control loop used to control the rotation of the flight simulation turntable, the DC motor uses the subspace predictive control strategy to control the rotation of the flight simulation turntable through the position signal fed back by the photoelectric encoder or the code disc.

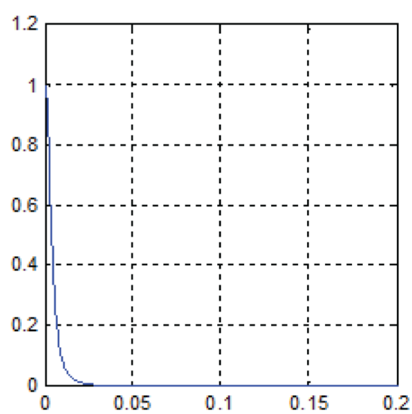
In order to test the subspace predictive controller of the DC motor using the linear parameter change system, the unit step signal is selected as the input signal of the entire flight simulation turntable system, and the output of the system is simulated. The physical parameters of the DC motor still use the values in Table 1. We select the unit step signal with the input signal $r(t) = 1(t)$ in the upper computer, and simulate the system output and error. The simulation results are displayed in the lower computer. The simulation graph is shown in Figure 6. It can be seen from the input signal response curve that when the system input is a unit step signal, the adjustment time of the system is less than $0.2\mathrm{s}$, the output has no overshoot, and the steady-state error is less than 0.05, which is in line with the time domain index of the position system.

6. CONCLUSION

Because the linear parameter varying system is a transition model of nonlinear system and linear system, it has both common properties and can approximately describe the real system. The subspace data driven control is applied to a linear parameter varying system with a scheduling parameter vector. According to the structural characteristics of the linear parameter varying system, the whole derivation process of the subspace data driven control are given, while introducing the vector product operator to simplify the derivation. A parallel distribution algorithm is used to solve the optimal control input value for the established cost function under the idea of data driven control. A linear parameter varying system considering the mass distribution factor is added to the DC motor, and the subspace prediction is applied to the control example of the DC motor. Because the system in this paper does not take into account the fault, the subspace data driven control of the linear parameter varying system under the existence of the fault can be regarded as the next research subject.

Figure 6. Unit step response and error simulation curve





ACKNOWLEDGMENT

The authors would like to thank Prince Sultan University, Riyadh, Saudi Arabia for their support. Special acknowledgement to Automated Systems & Soft Computing Lab (ASSCL), Prince Sultan University, Riyadh, Saudi Arabia. In addition, the authors wish to acknowledge the editor and anonymous reviewers for their insightful comments, which have improved the quality of this publication.

REFERENCES

Abdelmalek, S., Azar, A. T., & Dib, D. (2018). A novel actuator fault-tolerant control strategy of DFIG-based wind turbines using Takagi-Sugeno Multiple models. *International Journal of Control, Automation, and Systems*, *16*(3), 1415–1424. doi:10.1007/s12555-017-0320-y

Abdul-Adheem, W. R., Azar, A. T., Ibraheem, I. K., & Humaidi, A. J. (2020b). Novel Active Disturbance Rejection Control Based on Nested Linear Extended State Observers. *Applied Sciences (Basel, Switzerland)*, *10*(12), 4069. doi:10.3390/app10124069

Abdul-Adheem, W. R., Ibraheem, I. K., Azar, A. T., & Humaidi, A. J. (2020a). Improved Active Disturbance Rejection-Based Decentralized control for MIMO Nonlinear Systems: Comparison with The Decoupled Control Scheme. *Applied Sciences (Basel, Switzerland)*, *10*(7), 2515. doi:10.3390/app10072515

Acharyulu, B. V. S., Gorripotu, T. S., Azar, A. T., Mohanty, B., Pilla, R., Kumar, S., Serrano, F. E., & Kamal, N. A. (2021). Automatic Generation Control of Multi-area Multi-source Deregulated Power System Using Moth Flame Optimization Algorithm. In *Communication and Intelligent Systems. Lecture Notes in Networks and Systems* (Vol. 204, pp. 717–729). Springer., doi:10.1007/978-981-16-1089-9_56

Ajeil, F., Ibraheem, I. K., Azar, A. T., & Humaidi, A. J. (2020b). Autonomous Navigation and Obstacle Avoidance of an Omnidirectional Mobile Robot Using Swarm Optimization and Sensors Deployment. *International Journal of Advanced Robotic Systems*, *17*(3), 1–15. doi:10.1177/1729881420929498

Ajeil, F. H., Ibraheem, I. K., Azar, A. T., & Humaidi, A. J. (2020a). Grid-Based Mobile Robot Path Planning Using Aging-Based Ant Colony Optimization Algorithm in Static and Dynamic Environments. *Sensors (Basel)*, *20*(7), 1880. doi:10.3390/s20071880 PMID:32231091

Ajel, A. R., Humaidi, A. J., Ibraheem, I. K., & Azar, A. T. (2021). Robust Model Reference Adaptive Control for Tail-Sitter VTOL Aircraft. *Actuators*, *2021*(10), 162. doi:10.3390/act10070162

Al-Qassar, A. A., Abdulkareem, A. I., Hasan, A. F., Humaidi, A. J., Ibraheem, I. K., Azar, A. T., & Hameed, A. H. (2021a). Grey-Wolf Optimization Better Enhances the Dynamic Performance of Roll Motion For Tail-Sitter VTOL Aircraft Guided and Controlled By STSMC. *Journal of Engineering Science and Technology*, *16*(3), 1932–1950.

Al-Qassar, A. A., Al-Dujaili, A. Q., Hasan, A. F., Humaidi, A. J., Ibraheem, I. K., & Azar, A. T. (2021b). Stabilization Of Single-Axis Propeller-powered System For Aircraft Applications Based On Optimal Adaptive Control Design. [JESTEC]. *Journal of Engineering Science and Technology*, *16*(3), 1851–1869.

Ali, H. H., Elbasuony, G. S., & Kamal, N. A. (2021). (20221b). Maximum power production operation of doubly fed induction generator wind turbine using adaptive neural network and conventional controllers. *International Journal of Computer Applications in Technology*, *65*(2), 173–187. doi:10.1504/IJCAT.2021.114984

Ali, H. H., Elbasuony, G. S., & Kamal, N. A. (2022). Control of the rated production power of DFIG-wind turbine using adaptive PSO and PI conventional controllers. *International Journal of Automation and Control*, *16*(2), 219–249. doi:10.1504/IJAAC.2022.121134

Ali, H. H., Kamal, N. A., & Elbasuony, G. S. (2021a). Two-Level Grid-Side Converter-Based STATCOM and Shunt Active Power Filter of Variable-Speed DFIG Wind Turbine-Based WECS Using SVM for Terminal Voltage. [IJSSMET]. *International Journal of Service Science, Management, Engineering, and Technology*, *12*(2), 169–202. doi:10.4018/IJSSMET.2021030110

Ammar, H., Ibrahim, M., Azar, A. T., & Shalaby, R. (2020). Gray Wolf Optimization of Fractional Order Control of 3-Omni Wheels Mobile Robot: Experimental Study. In *2020 16th International Computer Engineering Conference (ICENCO),* Cairo, Egypt. doi:10.1109/ICENCO49778.2020.9357384

Ammar, H. H., Azar, A. T., Shalaby, R., & Mahmoud, M. I. (2019) Metaheuristic Optimization of Fractional Order Incremental Conductance (FO-INC) Maximum Power Point Tracking (MPPT). Complexity, 1-13. doi:10.1155/2019/7687891

Ammar, H. H., Azar, A. T., Tembi, T. D., Tony, K., & Sosa, A. (2018) Design and Implementation of Fuzzy PID Controller into Multi Agent Smart Library System Prototype. In *The International Conference on Advanced Machine Learning Technologies and Applications (AMLTA2018),* (*vol 723*, pp. 127-137). Springer, Cham. doi:10.1007/978-3-319-74690-6_13

Azar, A. T., Kumar, J., Kumar, V., & Rana, K. P. S. (2018b) Control of a Two Link Planar Electrically-Driven Rigid Robotic Manipulator Using Fractional Order SOFC. In *Proceedings of the International Conference on Advanced Intelligent Systems and Informatics 2017.* Springer, Cham. doi:10.1007/978-3-319-64861-3_6

Azar, A. T., Ouannas, A., & Singh, S. (2018a) Control of New Type of Fractional Chaos Synchronization. In Hassanien A., Shaalan K., Gaber T., Tolba M. (eds) *Proceedings of the International Conference on Advanced Intelligent Systems and Informatics 2017.* AISI 2017. Springer, Cham. doi:10.1007/978-3-319-64861-3_5

Azar, A. T., & Serrano, F. E. (2015). Stabilization and Control of Mechanical Systems with Backlash. In Advanced Intelligent Control Engineering and Automation, Advances in Computational Intelligence and Robotics (ACIR) Book Series. IGI-Global. doi:10.4018/978-1-4666-7248-2.ch001

Ben Njima, C., Benamor, A., & Messaoud, H. (2021). A New Robust Adaptive Sliding Mode Control for Discrete-Time Systems With Time-Varying State Delay: Application to Diesel Engine Control. [IJSSMET]. *International Journal of Service Science, Management, Engineering, and Technology*, *12*(2), 132–153. doi:10.4018/IJSSMET.2021030108

Bravo, J. M., Alamo, T., Vasallo, M., & Gegundez, M. E. (2017). A general framework for predictions based on bounding techniques and local approximations. *IEEE Transactions on Automatic Control*, *62*(7), 3430–3435. doi:10.1109/TAC.2016.2612538

Brett, N., & Hakan, H. (2005). Analysis of the variablity of joint input-output estimation methods. *Automatica*, *41*(6), 1123–1132.

Cristian, R. R. (2010). The cost of complexity in system identification: Frequency function estimation of finite impulse response system. *IEEE Transactions on Automatic Control*, *55*(10), 2298–2309. doi:10.1109/TAC.2010.2063470

Daraz, A., Malik, S. A., Waseem, A., Azar, A. T., Haq, I. U., Ullah, Z., & Aslam, S. (2021). Automatic Generation Control of Multi-Source Interconnected Power System Using FOI-TD Controller. *Energies*, *14*(18), 5867. doi:10.3390/en14185867

Erik, W., & Campi, M. C. (2017). Asymptotic properties of SPS confidence regions. *Automatica*, *82*(1), 287–294.

Fekik, A., Azar, A. T., Denoun, H., Kamal, N. A., Bahgaat, N. K., Gorripotu, T. S., Pilla, R., Serrano, F. E., Mittal, S., Rana, K. P. S., Kumar, V., Vaidyanathan, S., Hamida, M. L., Yassa, N., & Amara, K. (2021d) Improvement of fuel cell MPPT performance with a fuzzy logic controller. In Advances in Nonlinear Dynamics and Chaos (ANDC), Renewable Energy Systems, (pp. 161-181). Academic Press. doi:10.1016/B978-0-12-820004-9.00023-1

Fekik, A., Azar, A. T., Denoun, H., Kamal, N. A., Hamida, M. L., Kais, D., & Amara, K. (2021c). A backstepping Direct Power Control of Three Phase pulse width modulated Rectifier. In *Soft Computing Applications. SOFA 2018. Advances in Intelligent Systems and Computing* (Vol. 1222). Springer. doi:10.1007/978-3-030-52190-5_32

Fekik, A., Azar, A. T., Hamida, M. L., & Kamal, N. A. (2022a). Sliding Mode Control Based on Observation of line side PWM Rectifier Voltage. *2022 2nd International Conference of Smart Systems and Emerging Technologies (SMARTTECH),* Riyadh, Saudi Arabia. doi:10.1109/SMARTTECH54121.2022.00042

Fekik, A., Azar, A. T., Kamal, N. A., Denoun, H., Almustafa, K. M., Hamida, M. L., & Zaouia, M. (2021b). Fractional-Order Control of a Fuel Cell-Boost Converter System. In *Advanced Machine Learning Technologies and Applications. AMLTA 2020. Advances in Intelligent Systems and Computing* (Vol. 1141, pp. 713–724). Springer. doi:10.1007/978-981-15-3383-9_64

Fekik, A., Denoun, H., Azar, A. T., Kamal, N. A., Zaouia, M., Benyahia, N., Hamida, M. L., Benamrouche, N., & Vaidyanathan, S. (2021a). Direct power control of three-phase PWM-rectifier with backstepping control. In *Backstepping Control of Nonlinear Dynamical Systems, Advances in Nonlinear Dynamics and Chaos (ANDC)* (pp. 215–234). Academic Press. doi:10.1016/B978-0-12-817582-8.00017-9

Fekik, A., Denoun, H., Azar, A. T., Kamal, N. A., Zaouia, M., Yassa, N., & Hamida, M. L. (2020b) Direct Torque Control of three phase Asynchronous motor with sensorless speed estimator. *The International Conference on Advanced Intelligent Systems and Informatics AISI 2019.* (*vol 1058*, pp. 243-253). Springer. doi:10.1007/978-3-030-31129-2_23

Fekik, A., Denoun, H., Azar, A. T., Koubaa, A., Kamal, N. A., Zaouia, M., Hamida, M. L., & Yassa, N. (2020a) *Adapted Fuzzy Fractional Order proportional-integral controller for DC Motor*. The First International Conference of Smart Systems and Emerging Technologies (SMART TECH 2020), Riyadh, Saudi Arabia. doi:10.1109/SMART-TECH49988.2020.00019

Fekik, A., Hamida, M. L., Houassine, H., Azar, A. T., Kamal, N. A., Denoun, H., Vaidyanathan, S., & Sambas, A. (2022b). Power Quality Improvement for Grid-Connected Photovoltaic Panels Using Direct Power Control. In *Modeling and Control of Static Converters for Hybrid Storage Systems* (pp. 107–142). IGI Global., doi:10.4018/978-1-7998-7447-8.ch005

Gevers, M., Bazanella, A. S., Bombois, X., & Miskovic, L. (2009). Identification and information matrix: How to get just sufficiently rich. *IEEE Transactions on Automatic Control*, *54*(12), 2828–2840. doi:10.1109/TAC.2009.2034199

Ghoudelbourk, S., Azar, A. T., & Dib, D. (2021). Three-level (NPC) Shunt Active Power Filter Based on Fuzzy Logic and Fractional-order PI Controller. *Int. J. Automation and Control*, *15*(2), 149–169. doi:10.1504/IJAAC.2021.113338

Gorripotu, T. S., Azar, A. T., Pilla, R., & Kamal, N. A. (2021). Impact of Ultra Capacitor on Automatic Load Frequency Control of Nonlinear Power System. In *Intelligent Computing in Control and Communication. Lecture Notes in Electrical Engineering* (Vol. 702, pp. 333–341). Springer. doi:10.1007/978-981-15-8439-8_27

Gorripotu, T. S., Samalla, H., Jagan Mohana Rao, C., Azar, A. T., & Pelusi, D. (2019). TLBO Algorithm Optimized Fractional-Order PID Controller for AGC of Interconnected Power System. In *Soft Computing in Data Analytics. Advances in Intelligent Systems and Computing* (Vol. 758). Springer. doi:10.1007/978-981-13-0514-6_80

Hakan, H. (2005). From experiment design to closed loog control. *Automatica*, *41*(3), 393–438. doi:10.1016/j.automatica.2004.11.021

Hamida, M. L., Fekik, A., Azar, A. T., Kamal, N. A., Ardjal, A., & Denoun, H. (2022). Fuzzy Logic Cyclic Reports Modulation Control For a Five-Cell Inverter. In: *2nd International Conference of Smart Systems and Emerging Technologies (SMARTTECH)*, Riyadh, Saudi Arabia. doi:10.1109/SMARTTECH54121.2022.00043

Hjalmarssion, H. (2011). A geometric approach to variance analysis in system identification. *IEEE Transactions on Automatic Control*, *56*(5), 983–997. doi:10.1109/TAC.2010.2076213

Humaidi, A. J., Ibraheem, I. K., Azar, A. T., & Sadiq, M. E. (2020). A New Adaptive Synergetic Control Design for Single Link Robot Arm Actuated by Pneumatic Muscles. *Entropy (Basel, Switzerland)*, *22*(7), 723. doi:10.3390/e22070723 PMID:33286496

Humaidi, A. J., Sadiq, M. E., Abdulkareem, A. I., Ibraheem, I. K., & Azar, A. T. (2022). Adaptive backstepping sliding mode control design for vibration suppression of earth-quaked building supported by magneto-rheological damper. *Journal of Low Frequency Noise, Vibration and Active Control*, *41*(2), 768–783. doi:10.1177/14613484211064659

Ibraheem, G. A. R., Azar, A. T., Ibraheem, I. K., & Humaidi, A. J. (2020a). A Novel Design of a Neural Network based Fractional PID Controller for Mobile Robots Using Hybridized Fruit Fly and Particle Swarm Optimization. *Complexity*, *3067024*, 1–18. doi:10.1155/2020/3067024

Jakob, K. (2011). A design algorithm using external perturbation to improve iterative feedback tuning convergence. *Automatica*, *47*(2), 2665–2670.

Jianhong, W. (2019). Optimal input signal design for multi UAVs formation anomaly detection. *ISA Transactions*, *91*(6), 157–165. doi:10.1016/j.isatra.2019.01.027 PMID:30799024

Jianhong, W., & Daobo, W. (2011). Global nonlinear separable least square algorithm for the aircraft flutter model parameter identification. *Journal of Vibration and Shock*, *30*(2), 210–213.

Jianhong, W., & Daobo, W. (2011). Bias compensated instrumental variable algorithm for the aircraft flutter model parameter identification. *Electronic Optics & Control*, *18*(12), 70–74.

Jianhong, W., & Ricardo, A. R. (2019). Combing instrumental variable and variance matching for aircraft flutter model parameters identification. *Shock and Vibration*, *28*(6), 1–12. doi:10.1155/2019/4296091

Jianhong, W., & Yanxiang, W. (2021). Synthesis analysis for multi UAVs formation anomaly detection. *Aircraft Engineering and Aerospace Technology*, *93*(1), 180–189. doi:10.1108/AEAT-04-2020-0076

Kazim, M., Azar, A. T., Abdelkader, M., & Koubaa, A. (2022). *Adaptive Backstepping based Linear Parameter Varying Model Predictive Control Multi-rotor UAVs*. In 2022 2nd International Conference of Smart Systems and Emerging Technologies (SMARTTECH), Riyadh, Saudi Arabia. doi:10.1109/SMARTTECH54121.2022.00045

Kazim, M., Azar, A. T., Koubaa, A., Ibrahim, Z. F., Zaidi, A., & Zhang, L. (2021b) Event-driven programming-based path planning and navigation of UAVs around a complex urban environment. In Unmanned Aerial Systems, Advances in Nonlinear Dynamics and Chaos (ANDC), (pp. 531-565). Academic Press. doi:10.1016/B978-0-12-820276-0.00028-5

Kazim, M., Azar, A. T., Koubaa, A., & Zaidi, A. (2021a). Disturbance Rejection Based Optimized Robust Adaptive Controller for UAVs. *IEEE Systems Journal*, *15*(2), 3097–3108. doi:10.1109/JSYST.2020.3006059

Mahdi, S. M., Yousif, N. Q., Oglah, A. A., Sadiq, M. E., Humaidi, A. J., & Azar, A. T. (2022). Adaptive Synergetic Motion Control for Wearable Knee-Assistive System: A Rehabilitation of Disabled Patients. *Actuators*, *11*(7), 176. doi:10.3390/act11070176

Marko, T., Lerenzo, F., & Carlo, N. (2017). Data driven control of nonlinear systems: An on line direct approach. *Automatica*, *75*(1), 1–10.

Meghni, B., Dib, D., Azar, A. T., Ghoudelbourk, S., & Saadoun, A. (2017). Robust Adaptive Supervisory Fractional order Controller For optimal Energy Management in Wind Turbine with Battery Storage. [Springer-Verlag, Germany]. *Studies in Computational Intelligence*, *688*, 165–202. doi:10.1007/978-3-319-50249-6_6

Meghni, B., Dib, D., Azar, A.T., & Saadoun, A. (2018). Effective Supervisory Controller to Extend Optimal Energy Management in Hybrid Wind Turbine under Energy and Reliability Constraints. *International Journal of Dynamics and Control, 6*(1), 369—383. Springer. 10.1007/s40435-016-0296-0

Mittal, S., Azar, A. T., & Kamal, N. A. (2021) Nonlinear Fractional Order System Synchronization via Combination-Combination Multi-switching. In: Hassanien A.E., Slowik A., Snášel V., El-Deeb H., Tolba F.M. (eds) *Proceedings of the International Conference on Advanced Intelligent Systems and Informatics 2020. AISI 2020.* Springer, Cham. doi:10.1007/978-3-030-58669-0_75

Najm, A. A., Ibraheem, I. K., Azar, A. T., & Humaidi, A. J. (2020). Genetic Optimization-Based Consensus Control of Multi-Agent 6-DoF UAV System. *Sensors (Basel)*, *20*(12), 3576. doi:10.3390/s20123576 PMID:32599862

Najm, A. A., Ibraheem, I. K., Azar, A. T., & Humaidi, A. J. (2021b). On the stabilization of 6-DOF UAV quadrotor system using modified active disturbance rejection control. In Unmanned Aerial Systems, Advances in Nonlinear Dynamics and Chaos (ANDC), (pp. 257-287). Academic Press.

Najm, A. A., Ibraheem, I. K., Humaidi, A. J., & Azar, A. T. (2021a). *Output tracking and feedback stabilization for 6-DoF UAV using an enhanced active disturbance rejection control*. International Journal of Intelligent Unmanned Systems., doi:10.1108/IJIUS-09-2020-0059

Ohlsson, H., Ljung, L., & Boyd, S. (2010). Segmentation of ARX-models using sum-of-norms regularization. *Automatica*, *46*(6), 1107–1111. doi:10.1016/j.automatica.2010.03.013

Ouannas, A., Azar, A. T., & Radwan, A. G. (2016a) On Inverse Problem of Generalized Synchronization Between Different Dimensional Integer-Order and Fractional-Order Chaotic Systems. The *28th International Conference on Microelectronics*. IEEE. doi:10.1109/ICM.2016.7847942

Ouannas, A., Azar, A. T., & Ziar, T. (2020). On Inverse Full State Hybrid Function Projective Synchronization for Continuous-time Chaotic Dynamical Systems with Arbitrary Dimensions. *Differential Equations and Dynamical Systems*, *28*(4), 1045–1058. doi:10.1007/s12591-017-0362-x

Ouannas, A., Azar, A. T., Ziar, T., & Radwan, A. G. (2017a). Generalized Synchronization of Different Dimensional Integer-order and Fractional Order Chaotic Systems. [Springer-Verlag, Germany.]. *Studies in Computational Intelligence*, *688*, 671–697. doi:10.1007/978-3-319-50249-6_23

Ouannas, A., Azar, A. T., Ziar, T., & Vaidyanathan, S. (2017b). Fractional Inverse Generalized Chaos Synchronization Between Different Dimensional Systems. [Springer-Verlag, Germany.]. *Studies in Computational Intelligence*, *688*, 525–551. doi:10.1007/978-3-319-50249-6_18

Ouannas, A., Azar, A. T., Ziar, T., & Vaidyanathan, S. (2017c). A New Method To Synchronize Fractional Chaotic Systems With Different Dimensions. [Springer-Verlag, Germany.]. *Studies in Computational Intelligence*, *688*, 581–611. doi:10.1007/978-3-319-50249-6_20

Ouannas, A., Azar, A. T., Ziar, T., & Vaidyanathan, S. (2017d). On New Fractional Inverse Matrix Projective Synchronization Schemes. [Springer-Verlag, Germany]. *Studies in Computational Intelligence*, *688*, 497–524. doi:10.1007/978-3-319-50249-6_17

Pilla, R., Gorripotu, T. S., & Azar, A. T. (2021a). Tuning of extended Kalman filter using grey wolf optimisation for speed control of permanent magnet synchronous motor drive. *Int. J. of Automation and Control*, *15*(4-5), 563–584. doi:10.1504/IJAAC.2021.116423

Pilla, R., Gorripotu, T. S., & Azar, A. T. (2021b). Design and analysis of search group algorithm based PD-PID controller plus redox flow battery for automatic generation control problem. *International Journal of Computer Applications in Technology*, *66*(1), 19–35. doi:10.1504/IJCAT.2021.119605

Radwan, A. G., Emira, A. A., Abdelaty, A., & Azar, A. T. (2018). Modeling and Analysis of Fractional Order DC-DC Converter. *ISA Transactions*, *82*, 184–199. doi:10.1016/j.isatra.2017.06.024 PMID:28709651

Saidi, S. M., Mellah, R., Fekik, A., & Azar, A. T. (2022). Real-Time Fuzzy-PID for Mobile Robot Control and Vision-Based Obstacle Avoidance. [IJSSMET]. *International Journal of Service Science, Management, Engineering, and Technology*, *13*(1), 1–32. doi:10.4018/IJSSMET.304818

Sain, C., Banerjee, A., Biswas, P. K., Azar, A. T., & Babu, T. S. (2022). Design and optimisation of a fuzzy-PI controlled modified inverter-based PMSM drive employed in a light weight electric vehicle. *Int. J. Automation and Control*, *16*(3/4), 459–488. doi:10.1504/IJAAC.2022.122603

Serrano, F. E., Azar, A. T., Kamal, N. A., Koubaa, A., & Abdelkader, M. (2022). Robust Dynamic Surface Control of Unmanned Aerial Vehicles with Constrained inputs and Unmodelled Dynamics. *2022 2nd International Conference of Smart Systems and Emerging Technologies (SMARTTECH),* Riyadh, Saudi Arabia. doi:10.1109/SMARTTECH54121.2022.00047

Singh, S., Azar, A. T., Ouannas, A., Zhu, Q., Zhang, W., & Na, J. (2017). Sliding Mode Control Technique for Multi-switching Synchronization of Chaotic Systems. *Proceedings of 9th International Conference on Modelling, Identification and Control (ICMIC 2017).* IEEE. doi:10.1109/ICMIC.2017.8321579

Singh, S., Mathpal, S., Azar, A. T., Vaidyanathan, S., & Kamal, N. A. (2021). Multi-switching synchronization of nonlinear hyperchaotic systems via backstepping control. In *Backstepping Control of Nonlinear Dynamical Systems, Advances in Nonlinear Dynamics and Chaos (ANDC)* (pp. 425–447). Academic Press. doi:10.1016/B978-0-12-817582-8.00024-6

Sippe, G. D. (2008). Validity of the standard cross correlation test for model structure validation. *Automatica*, *44*(4), 1285–1294.

Soliman, M., Azar, A. T., Saleh, M. A., & Ammar, H. H. (2020). Path Planning Control for 3-Omni Fighting Robot Using PID and Fuzzy Logic Controller. In *The International Conference on Advanced Machine Learning Technologies and Applications (AMLTA2019).* (*vol 921*, pp. 442-452). Springer, Cham.

Tang, W., Shi, Z., & Li, H. (2006). Frequency generalization total least square algorithm of aircraft flutter model parameter identification. *Control and Decision*, *21*(7), 726–729.

Torston, S. (2009). A covariance matching approach for identifying errors in variables systems. *Automatica*, *45*(9), 2018–2031. doi:10.1016/j.automatica.2009.05.010

Torston, S. (2009). Relations between bias-elimizating least squares, the Frish scheme and extended compensated least squares method. *Automatica*, *45*(1), 277–282. doi:10.1016/j.automatica.2008.07.007

Toumi, I., Meghni, B., Hachana, O., Azar, A. T., Boulmaiz, A., Humaidi, A. J., Ibraheem, I. K., Kamal, N. A., Zhu, Q., Fusco, G., & Bahgaat, N. K. (2022). Robust Variable-Step Perturb-and-Observe Sliding Mode Controller for Grid-Connected Wind-Energy-Conversion Systems. *Entropy (Basel, Switzerland)*, *2022*(24), 731. doi:10.3390/e24050731 PMID:35626614

Vaidyanathan, S., Azar, A. T., Akgul, A., Lien, C. H., Kacar, S., & Cavusoglu, U. (2019). A memristor-based system with hidden hyperchaotic attractors, its circuit design, synchronisation via integral sliding mode control and voice encryption. [IJAAC]. *Int. J. Automation and Control*, *13*(6), 644–667. doi:10.1504/IJAAC.2019.102665

Vaidyanathan, S., Azar, A. T., & Ouannas, A. (2017a). An Eight-Term 3-D Novel Chaotic System with Three Quadratic Nonlinearities, its Adaptive Feedback Control and Synchronization. [Springer-Verlag, Germany.]. *Studies in Computational Intelligence*, *688*, 719–746. doi:10.1007/978-3-319-50249-6_25

Vaidyanathan, S., Azar, A. T., & Ouannas, A. (2017b). Hyperchaos and Adaptive Control of a Novel Hyperchaotic System with Two Quadratic Nonlinearities. [Springer-Verlag, Germany.]. *Studies in Computational Intelligence*, *688*, 773–803. doi:10.1007/978-3-319-50249-6_27

Vaidyanathan, S., Azar, A. T., Rajagopal, K., Sambas, A., Kacar, S., & Cavusoglu, U. (2018a). A new hyperchaotic temperature fluctuations model, its circuit simulation, FPGA implementation and an application to image encryption. [IJSPM]. *Int. J. Simulation and Process Modelling*, *13*(3), 281–296. doi:10.1504/IJSPM.2018.093113

Vaidyanathan, S., Jafari, S., Pham, V. T., Azar, A. T. E., & Alsaadi, F. (2018b). A 4-D chaotic hyperjerk system with a hidden attractor, adaptive backstepping control and circuit design. *Archives of Control Sciences.*, *28*(2), 239–254.

Vaidyanathan, S., Zhu, Q., & Azar, A. T. (2017c). Adaptive Control of a Novel Nonlinear Double Convection Chaotic System. [Springer-Verlag, Germany.]. *Studies in Computational Intelligence*, *688*, 357–385. doi:10.1007/978-3-319-50249-6_12

Ibraheem K. Ibraheem was born in Baghdad, Iraq, in 1976. He received the B.S. degree in electrical engineering from the University of Baghdad, Baghdad, Iraq in 1998 and M.Sc. & Ph.D. degrees in Computer & Control Engineering from the same university and department, in 2001 and 2007, respectively. In 2019 he became a full Prof. In 2015 he visited High-Performance Computing Lab. in Ashburn / Virginia at George Washington University/ USA for a cooperative research on the hardware implementation of nonlinear control. His research interests include active disturbance rejection control, power control, Robotics, signal processing, nonlinear control and intelligent control applications.