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Substrate-induced bianisotropy in metamaterials

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We demonstrate that the presence of a supporting substrate can break the symmetry of a metamaterial structure, changing the symmetry of its effective parameters, and giving rise to bianisotropy. This indicates that magnetoelectric coupling will occur in all metamaterials fabricated on a substrate, including those with symmetric designs. © *2010 American Institute of Physics*. [doi:10.1063/1.3486480]

Optical metamaterials usually consist of a patterned metal-dielectric composite structure mounted on a substrate. They are designed to exhibit exotic properties, such as a negative refractive index.[1] The design process usually aims to achieve the maximum possible symmetry, to better approximate an ideal isotropic material.[2] In fabricating optical metamaterials, it is important to have straight side walls, since tapering breaks symmetry and causes bianisotropy. A bianisotropic material is one which develops an electric polarization in response to a magnetic field, and vice-versa, with material parameters of the form $D_i=\epsilon_0\epsilon_{ij}E_j-i\xi_{ij}H_j/c$ and $B_i=\mu_0\mu_{ij}H_j+i\xi^T_{ij}E_j/c$. This complicates the effective medium description, and can inhibit the negative refractive index.[3] The effect of fabrication imperfections has been studied in order to characterize bianisotropy with the aim to reduce its effect.[4,5] However, in this letter we show that *the presence of a substrate intrinsically induces bianisotropy in a metamaterial*, which should be taken into account for accurate characterization and control of the metamaterial properties.

A substrate has previously been shown to significantly influence the plasmonic resonances associated with a negative refractive index.[6] However, to date little attention has been paid to the resultant changes in symmetry,[7] an important exception being "planar chiral" structures.[8] In this case an essentially two-dimensional structure can exhibit optical activity, either when the metamaterial array has low in-plane symmetry, or when the sample is not normal to the incident wave vector. We will show here a different form of substrate-induced symmetry breaking, for three-dimensional symmetric structures illuminated at normal incidence.

The negative index in optical metamaterials arises when there is both a negative electric and magnetic response. The negative electric response occurs naturally in metals, while the negative magnetic response requires a pair of metal structures having an antisymmetric plasmonic mode. Thus we consider here a general model of a pair of plasmonic resonators, which could be the two metal layers of a fishnet, a cut-wire pair or a plasmonic dimer. In Fig. 1(a) we show the structure under consideration, noting that the details of this patterning are unimportant for this argument, except that the pattern is uniform through the layers.

Conceptually, bringing the two identical patterned metallic layers together will cause their resonances to hybridize into symmetric and antisymmetric modes. These will then respond to the imposed electric and magnetic field, respectively. In practice there is substantial coupling of evanescent waves into the dielectric substrate, and this coupling will be different for the two plasmonic layers. Therefore we can consider that the system is formed by the hybridization of two nonidentical plasmonic resonators, as shown in Fig. 1(b). The nominally symmetric mode will have some antisymmetric component, and vice-versa. Each mode will therefore couple to both electric and magnetic fields, resulting in bianisotropy. The argument is essentially identical to that for intrinsically nonsymmetric structures, once it is appreciated that the substrate breaks the symmetry of the system, even if it is outside the boundaries imposed during the retrieval procedure. We will show a specific example of the experimentally important fishnet structure. We consider the case of a semi-infinite substrate; however the results still apply for a finite substrate.

The material parameters of a homogeneous structure can be retrieved from transmission and reflection data.[9] By utilizing the two values of reflection from different sides of the structure, this approach has been extended to asymmetric structures exhibiting bianisotropy.[10–12] After calculating the generalized scattering parameters, we consider the two values of the reflection coefficient when illuminating from free space (S_{11}) or through the substrate (S_{22}). Even for a symmetric structure, these will not be equal, since they are defined with respect to different values of the wave impedance. However, we can transform the scattering matrix S with reference impedances ($\eta_0,\eta_{\mathrm{sub}}$) to S' with both reference impedances being η_0, and we find that for a simple dielectric layer, the reflection coefficients become equal, and this should also be true for a symmetric metamaterial.

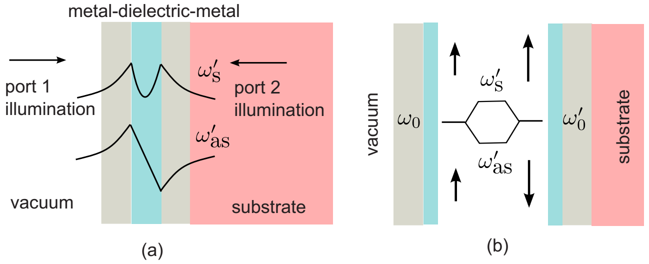

FIG. 1. (Color online) (a) System under consideration and (b) representation as hybridization of nonidentical resonators.

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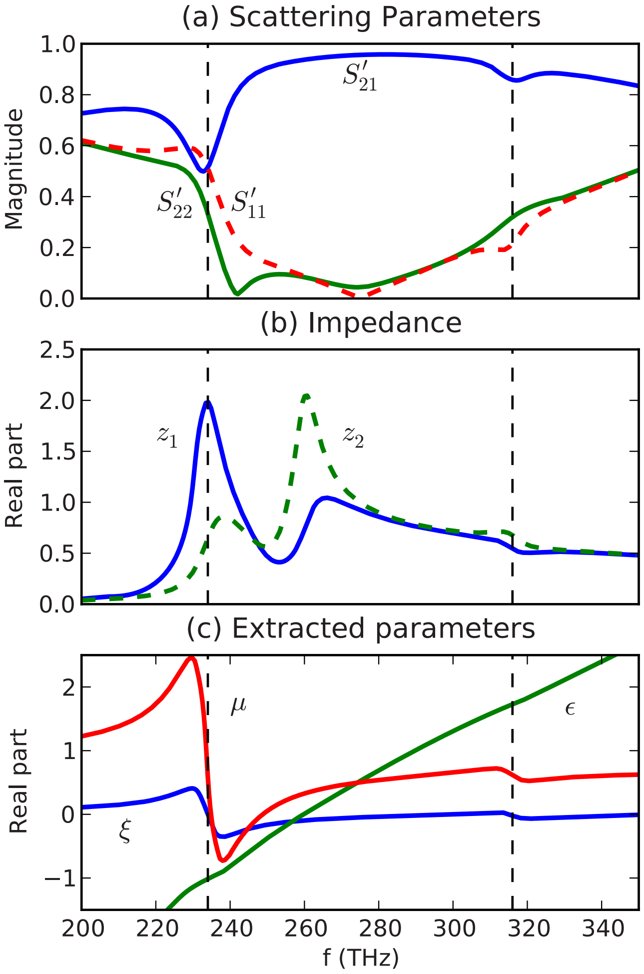

FIG. 2. (Color online) Simulation results of the fishnet structure. (a) Transmission and reflection magnitude, (b) real part of extracted impedance values, and (c) real part of extracted permittivity, permeability, and magnetoelectric coupling. The dashed lines indicate the (approximately) antisymmetric plasmonic resonances.

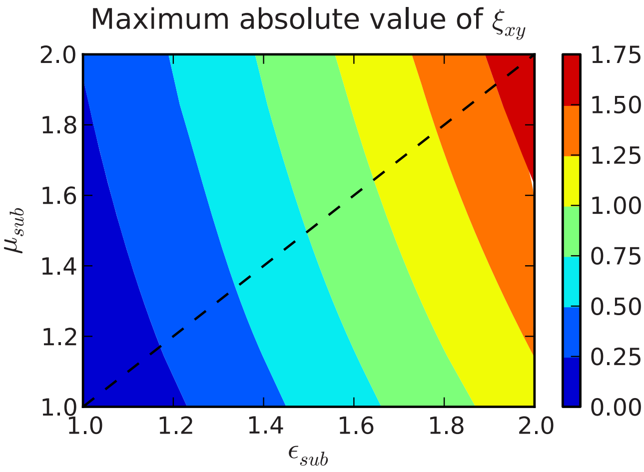

FIG. 3. (Color online) Maximum real part of ξ_{xy} as a function of substrate permittivity and permeability. The dashed line indicates impedance matching to free space.

We then apply this approach to a fishnet structure, with transverse period of 400 nm, consisting of 200 nm × 350 nm holes in metal-dielectric-metal layers with metal thickness 30 nm and dielectric thickness 30 nm, on a substrate of index 1.5 which is impedance matched to free space (chosen for consistency with subsequent results, as justified below). The structure was modeled in CST MICROWAVE STUDIO,[13] with the metal being modeled using a fourth order fit to the parameters of silver, and the dielectric layer having $\epsilon = 2.75$. The transmission of this structure is shown in Fig. 2(a) to exhibit two dips, corresponding to the excitation of approximately antisymmetric plasmonic modes, and a maximum corresponding to the cut-off resonance of the nearly-symmetric hole mode.[14] It can be clearly seen that the two reflection coefficients are different in magnitude, with the most significant differences corresponding to the frequencies where plasmonic modes are excited. We note that in modeling of lossless structures, bianisotropy may not be immediately apparent, since $|S_{11}| = |S_{22}|$ is enforced by energy conservation and reciprocity. However, the phase of these two parameters may be altered by the substrate, hence bianisotropy can still occur.

We apply the method given in Ref. 12 to extract the two values of impedance, which are plotted in Fig. 2(b). Failing to account for the bianisotropy in the extraction results in some error in the effective index, and yields an impedance value close to that of z_1 (not shown). It is clear that the variation in impedance is strongest near the plasmonic features, supporting the argument that the nonsymmetric hybridization of plasmonic modes is the cause of bianisotropy. In Fig. 2(c) we extract the equivalent parameters of the structure using the method from Ref. 10, where for clarity only the real parts are shown. It can be seen that the relevant component of the magnetoelectric coupling, ξ_{xy} is significant around the magnetic resonance. Observing the negative extrema of ξ_{xy} and ϵ at ~237 THz, we see that the magnetic polarization excited by the electric field is about a third as strong as that excited by the magnetic field. Interestingly, at ~260 THz, which corresponds to the cut-off resonance of the holes where $\epsilon \approx 0$ there is a strong variation between impedance values but no corresponding feature in the extracted magnetoelectric coupling.

In Fig. 3 we plot the maximum magnitude of ξ_{xy} over the considered frequency range, as a function of the substrate parameters. While in optical experiments the substrate would have a purely dielectric response, we consider also a substrate with a magnetic response to better understand the substrate influence. Increasing ϵ_{sub} or μ_{sub} results in higher bianisotropy, although ϵ_{sub} has a much greater influence. The dashed line indicates the impedance match between the substrate and free space, and the absence of any features in this region indicates that bianisotropy cannot be attributed to an impedance matching effect. On the other hand, for fixed impedance or permeability, bianisotropy increases monotonically with the index of the substrate. This is consistent with our argument that the change in plasmon dispersion is the cause of this effect.

While the extracted parameters reproduce the original transmission and reflection data up to numerical accuracy, they are nonlocal in nature.[15] Therefore we shall probe the internal field of the structure directly, in order to demonstrate that the magnetoelectric polarization is a real physical effect which does not rely on the homogenizability of the structure. With reference to Fig. 1(a), we excite the structure simultaneously from both directions with normally incident waves. This creates a standing wave pattern, and by adjusting the relative phase of the incident waves it is possible to position the center of the structure at a zero of the electric or magnetic

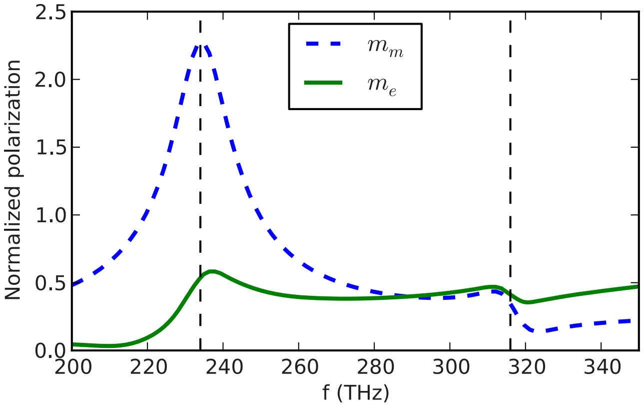

FIG. 4. (Color online) Magnitude of the magnetic dipole moment induced in a fishnet under magnetic (m_m) and electric (m_e) excitation, with dashed lines showing the plasmon resonance frequencies.

field. Given that the thickness of the structure under consideration is much less than the wavelength, we have a good approximation of pure excitation by the magnetic or electric field, respectively. In order to simplify the excitation of the standing wave and the interpretation of the results, we consider the case where the substrate impedance is matched to free space, and the index is 1.5. We calculate the induced magnetic and electric dipole moments by evaluating the integrals $\bar{m}=j\omega/2\int\bar{x}\times(\epsilon-1)\bar{E}d^3x$ and $\bar{p}=\int(\epsilon-1)\bar{E}d^3x$ over the metal and dielectric regions.

The x component of the induced magnetic dipole moment is shown in Fig. 4 for excitation by an electric field m_e, and is compared to m_m, the magnetic polarization induced by the magnetic field. Quantities are normalized to the volume of the unit cell, and to the incident magnetic field, thus they are dimensionally equivalent to polarizabilities χ_{me} and χ_{mm}. It is clearly observable that there are maxima of magnetoelectric coupling corresponding to the frequencies plasmon excitation observable in Fig. 2(a). Significantly, at some frequencies, the magnetoelectric polarization is stronger than the purely magnetic polarization. In addition, there is also some apparent cross-polarization which increases with frequency. This is most likely due to retardation as the structure is no longer subwavelength at higher frequencies.

We expect that the effect predicted and analyzed here should occur in multilayered structures, however in this case the analysis becomes more complicated. In particular, since this is an interface effect, the multivalued nature of the impedance should not strongly depend on the length of the structure. However, inversion of the transmission and reflection assumes homogeneous parameters, so the effect will be incorrectly attributed to bianisotropy throughout the bulk. We note that approaches which account for surface layers of a metamaterial exist[15] but are not widely utilized in the literature. We further propose that at the interface between a bulk metamaterial and free-space, the broken symmetry could also result in local bianisotropy. This would not be observable by any technique based on S-parameter inversion of a finite thickness slab, since the effect on both reflection coefficients would be identical.

In conclusion, we have demonstrated that the substrate breaks the symmetry of structurally symmetric metamaterials, resulting in significant bianisotropy. We have shown that this effect is strongest at the resonance of plasmonic modes, which can be understood as the hybridization of nonidentical resonators to form the metamaterial. Our results suggest that most metamaterials reported in the literature will exhibit bianisotropy, and should be taken into account for accurate analysis of the metamaterial properties.

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