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Substructure deletion method: a boundary element approach for elastodynamic problems

R. Betti
Department of Civil Engineering and Engineering Mechanics, Columbia University, New York, N.Y. 10027, USA

ABSTRACT

A Boundary Element formulation of the Substructure Deletion Method for the dynamic analysis of three–dimensional inclusions of arbitrary shape, subjected to external forces and embedded in a homogeneous viscoelastic half-space, is presented in this study. The analysis relies on the BEM for solving both the problem regarding the finite inclusion and the problem of the flat half-space. Such a method has been tested in the computation of the impedance matrix of square embedded foundations, with different values of the embedment ratio. The results are consistent with those obtained in previous studies.

INTRODUCTION

Many studies have been done to investigate the dynamic response of rigid bodies embedded in an infinite half–space subjected to external excitations. Particular attention to this problem has been given in the area of earthquake engineering for the evaluation of the soil-structure interaction effects on the response of embedded foundations.

Due to the complexity of the problem, most of the soil–structure interaction studies regarding the response of three–dimensional foundations restrict their analyses to the case of surface foundations or limit their investigations to case of embedded foundations of relatively simple geometry, such as cylindrical and hemispherical (Luco and Wong [6]) ones.

For the analysis of embedded foundations of arbitrary shape, different numerical techniques have been applied using either the Boundary Element Method (BEM) or the Finite Element Method (FEM). The FEM can easily handle complex foundation geometries and soil properties but requires the use of special devices in order to overcome the inability of the method to represent the extended earth (Kausel [5]).

In contrast, the BEM appears to be well–suited for the analysis of soil–structure interaction problems, especially the three–dimensional ones, since it only requires the discretization of the contact surface between the foundation and the surrounding soil (Dominguez [3], Karabalis and Beskos [4], and Antes and von Estorff [1]). Dasgupta [2] and Mita and Luco [7] have developed different "hybrid" approaches in both the time and frequency domain, using the finite element method for the analysis of finite portions of soil.

In this study, a new technique for the determination of the dynamic frequency–dependent impedance matrix for a three–dimensional rigid foundation of arbitrary shape, embedded in a homogeneous, visco–elastic half–space, is presented. This procedure follows an alternative formulation of the Substructure Deletion method using BEM to solve both the problem regarding the analysis of the soil inclusion or deleter (interior problem) and the problem of the flat half–space (exterior problem).

SUBSTRUCTURE DELETION METHOD: A BEM APPROACH

The present approach starts from the basic consideration of the substructure deletion method [2] that, if a volume V' (fig. 1) of the same material of the excavated half–space is inserted into the cavity occupied by the foundation and the continuity conditions are imposed along S_e, the flat, unexcavated half–space is recreated (fig. 1). Hence, the solution of the radiation problem in the case of embedded foundations can be derived from the solution of a flat, homogeneous half–space (exterior problem) and from the analysis of the removed portion of soil V' (interior problem).

Consider the finite portion of soil V', bounded by an external surface S ($S = S_e \cup S_h$ fig. 1). The Betti-Rayleigh theorem, applied to the volume V', can be written as:

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$$\epsilon(\mathbf{x_p})u_i(\mathbf{x_p}) + \int_S T^n_{ji}(\mathbf{x_p};\mathbf{x})u_j(\mathbf{x})dS(\mathbf{x}) = \int_S U_{ji}(\mathbf{x_p};\mathbf{x})t^n_j(\mathbf{x})dS(\mathbf{x}) \ , \quad (i,j=1,2,3) \tag{1}$$

where $\mathbf{x} \in S$ and

$$\epsilon(\mathbf{x_p}) = \begin{cases} 1, & \text{if } \mathbf{x_p} \in V'; \\ 1/2, & \text{if } \mathbf{x_p} \in S; \\ 0 & \text{otherwise.} \end{cases}$$

The components $u_j(\mathbf{x})$ and $t_j(\mathbf{x})$ are the displacement and traction components along the j–th direction at point $\mathbf{x}$ on the external surface S. The functions $U_{ji}(\mathbf{x_p};\mathbf{x})$ and $T^n_{ji}(\mathbf{x_p};\mathbf{x})$ represent the displacement and the traction components, respectively, in the j–th direction at $\mathbf{x}$ due to a point load in the i–th direction at $\mathbf{x_p}$. In this study, Green's functions of the displacement and traction for the viscoelastic infinite space are used for the evaluation of the integrals in eqs. (1) taking into account the material damping of the soil. The material properties are expressed in terms of the complex Lame's constants μ and λ:

$$\mu = \mu^\star(1 + 2iq_\beta)$$

$$\lambda + \mu = (\lambda^\star + \mu^\star)(1 + 2iq_\alpha)$$

where $\mu^\star$ and $\lambda^\star$ represent the real parts of the Lame's constants, q_α and q_β the damping ratios of the P- and S-wave, respectively, and $i = \sqrt{-1}$.

Using a BEM formulation for the solution of eqs. (1), the surface S is discretized into M subregions S_k (k = 1,2,..,M) and uniform tractions and displacements are assumed within each element. Eqs. (1) are then transformed into a system of linear algebraic equations

$$\left[\frac{1}{2}[I]\delta_{pk} + [\bar{T}_{pk}]\right] \mathbf{u}_k = [\bar{U}_{pk}]\mathbf{t}_k \qquad ; \qquad (p,k = 1,2,.....M) \tag{2}$$

where δ_{pk} is the Kronecker delta, $\mathbf{u}_k = \{u_{1k}, u_{2k}, u_{3k}\}^T$ and $\mathbf{t}_k = \{t^n_{1k}, t^n_{2k}, t^n_{3k}\}^T$ represent the displacement and traction vectors for the k-th element of S. Indicating with $\{u_h\}$ and $\{t_h\}$ the displacement and traction components for the elements on S_h and with $\{u_e\}$ and $\{t_e\}$ the displacement and traction components for the elements on S_e (fig. 1), the compliance representation of eqs. (2) becomes:

$$\begin{Bmatrix} \{u_h\} \\ \{u_e\} \end{Bmatrix} = \begin{bmatrix} C_{hh} & C_{he} \\ C_{eh} & C_{ee} \end{bmatrix} \begin{Bmatrix} \{t_h\} \\ \{t_e\} \end{Bmatrix} . \tag{3}$$

When the finite portion of soil is inserted into the cavity occupied by the foundation and the equilibrium conditions and the compatibility conditions for the displacements are imposed over the entire embedded surface S_e, the flat, homogeneous half–space is recreated (exterior problem).

Considering the same discretization for S_h used in the interior problem, the BEM formulation of the representation theorem applied to the flat surface S_h is reduced to:

$$u^f_{ip} = \sum_{k=1}^{N} t^{fn}_{jk} \left[\int_{S_k} U^f_{ji}(\mathbf{x_p}; \mathbf{x}) dS(\mathbf{x}) \right] , \qquad (i,j = 1,2,3) \tag{4}$$

with $p, k = 1, 2, ..., N$, where N is the total number of surface elements. The functions $U^f_{ji}(\mathbf{x_p}; \mathbf{x})$ represent the displacement Green's functions for the flat half–space while u^f_{jk} and t^{fn}_{jk} represent the j-th component of the displacement and surface traction in the k-th surface element, respectively. Eqs. (4) can be conveniently written in a matrix form as :

$$\{u^f_h\} = [C^f_{hh}]\{t^f_h\} \tag{5}$$

where the matrix $[C^f_{hh}]$ is the " compliance matrix " for the flat surface S_h.

After both the interior and exterior problem have been analysed (eqs. (3) and (5)), the force–displacement relationships and the compliance matrix for the embedded surface S_e

$$\{\hat{u}_e\} = [\hat{C}]\{\hat{t}_e\}, \tag{6}$$

can be obtained by applying the Substructure Deletion method. The compatibility conditions for the displacements and the equilibrium conditions over S_e can be expressed as:

$$\{\hat{u}_e\} = \{u_e\} \qquad ; \qquad \{\hat{t}_e\} + \{t_e\} = \{0\} \tag{7}$$

The combined medium becomes a homogeneous, flat half–space so that:

$$\{u^f_h\} = \{u_h\} \ ; \ \{t^f_h\} = \{t_h\} \tag{8}$$

After matrix manipulation, it is possible to obtain:

$$[\hat{C}] = -[C_{eh}] \left[[C^f_{hh}] - [C_{hh}] \right]^{-1} [C_{he}] - [C_{ee}] \tag{9}$$

which represents the compliance matrix for the embedded surface S_e in the excavated half–space. The impedance matrix $[\hat{K}]$ can be obtained by inverting the compliance matrix $[\hat{C}]$.

ANALYSIS OF THE RESULTS

The proposed method has been tested by analysing an embedded rigid foundation, with base dimensions $2a \times 2a$ and depth h. The particular shape of the rigid foundation reduces the dynamic frequency–dependent impedance matrix for the embedded surface S_e to a 6×6 symmetric complex matrix $[K_0]$, whose terms can be expressed as:

$$K_{ij} = \mu a(k_{ij} + i a_0 c_{ij}).$$

The parameter a_0 represents the dimensionless frequency, defined as:

$$a_0 = \frac{\omega a}{v_\beta}$$

where ω is the frequency of the external excitation and v_β is the shear wave velocity. k_{ij} represent the stiffness and inertia effects of the soil while c_{ij} the material and radiation damping.

A comparison of the results with those obtained by Mita and Luco [7] is carried out for different values of the foundation embedment ratio h/a (h/a = 0, 0.333, 0.5, 1.0). Figs. 2 and 3 show the values of the vertical impedance functions for two different values of the embedment ratio. For embedment ratios less or equal to 0.5, the results obtained with the present approach show an excellent agreement with the previous results. For higher values of h/a, the results present some significant discrepancies. Such differences can be related to the surface discretization used in the calculations that, for deep embedments, become insufficient and an improvement in the results can be obtained by applying a more refined mesh.

The proposed formulation has also been tested in comparison with other methods that use classical BEM formulations. In fig. 4, the response of a deep rigid foundation to a vertical harmonic load is computed and the results are compared with those obtained by Karabalis and Beskos [4]. The comparison of the results show a relatively good agreement between the present approach and previous formulations.

CONCLUSIONS

The present BEM formulation of the Substructure Deletion method can be considered an efficient approach for the three–dimensional analysis of rigid inclusions of any geometrical shape embedded in an visco–elastic half–space. For embedment ratios less or equal to 0.5, the results show excellent agreement with those obtained in previous studies. For larger values of h/a, some differences arise which can be improved by adopting a finer mesh.

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REFERENCES

1. Antes, H., and von Estorff, O.,:"Transient Behavior of Strip Foundations Resting on Different Soil Profiles by a Time Domain BEM", *Ground Motion and Engineering Seismology, Developments in Geotechnical Engineering*, Computational Mechanics Publications, Elsevier, Amsterdam, Vol. 44, 1987, pp. 291-305.
2. Dasgupta, G.,:"Foundation Impedance Matrices by Substructure Deletion", *Journal of the Engineering Mechanics Division*, ASCE, Vol. 106, 1980, pp. 517-524.
3. Dominguez, J.,:"Dynamic Stiffness of Rectangular Foundations", Report R78-20, Department of Civil Engineering, MIT, Cambridge, MA., 1978.
4. Karabalis, D.L., and Beskos, D.E.,:"Dynamic Response of 3–D Embedded Foundations by the Boundary Element Method", *Computer Methods in Applied Mechanics and Engineering*, Vol. 56, 1986, pp. 91-119.
5. Kausel, E.,:"Local Transmitting Boundaries", *Journal of Engineering Mechanics*, ASCE, Vol. 114, 1988, pp. 1011-1027.
6. Luco, J.E., and Wong, H.L.,:"Response of Hemispherical Foundation Embedded in Half–Space", *Journal of Engineering Mechanics*, ASCE, Vol. 112, 1986, pp. 1363-1374.
7. Mita, A., and Luco, J.E.,:"Dynamic Response of a Square Foundation Embedded in an Elastic Half–Space", *Soil Dynamics and Earthquake Engineering*, Vol. 8, 1989, pp. 54-67.

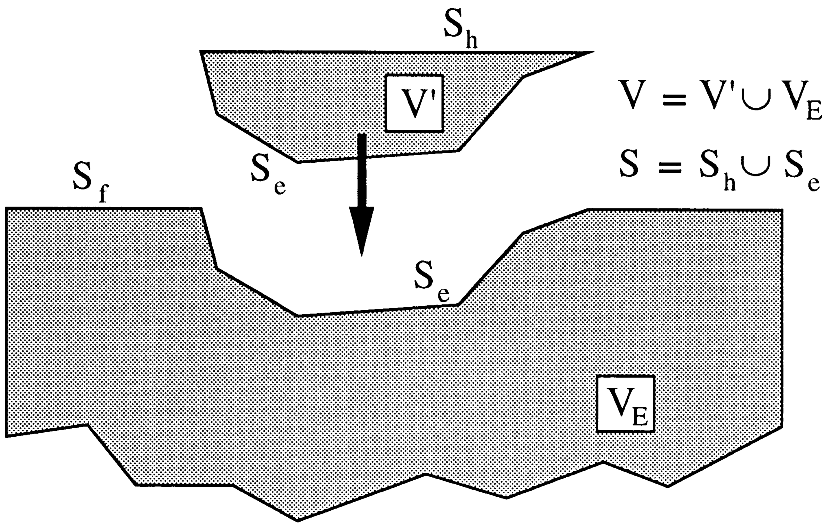

Fig.1: Substructure Deletion Method

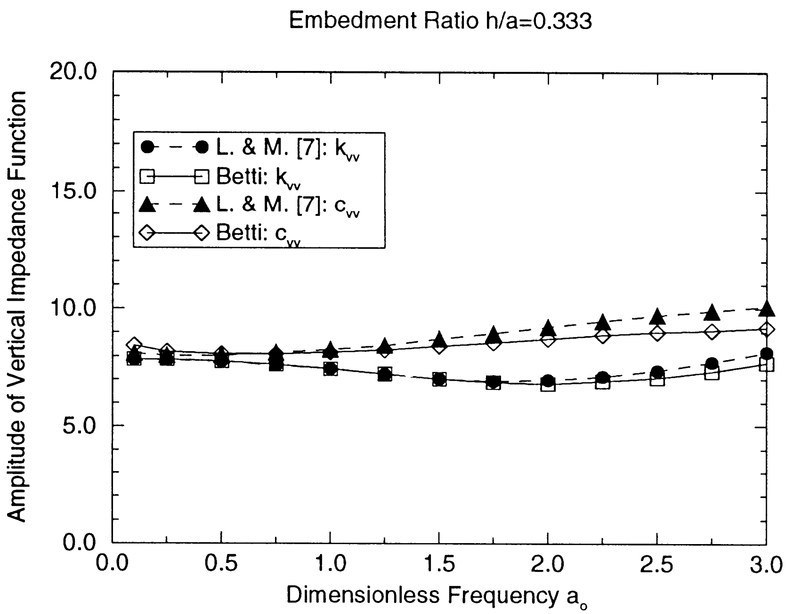

Fig. 2: Vertical Impedance Function: h/a = 0.333

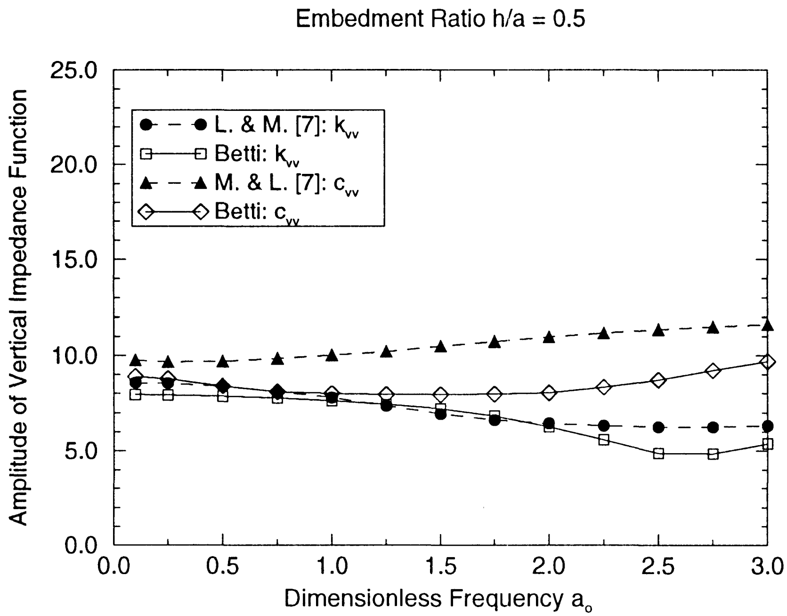

Fig. 3: Vertical Impedance Function: h/a = 0.5

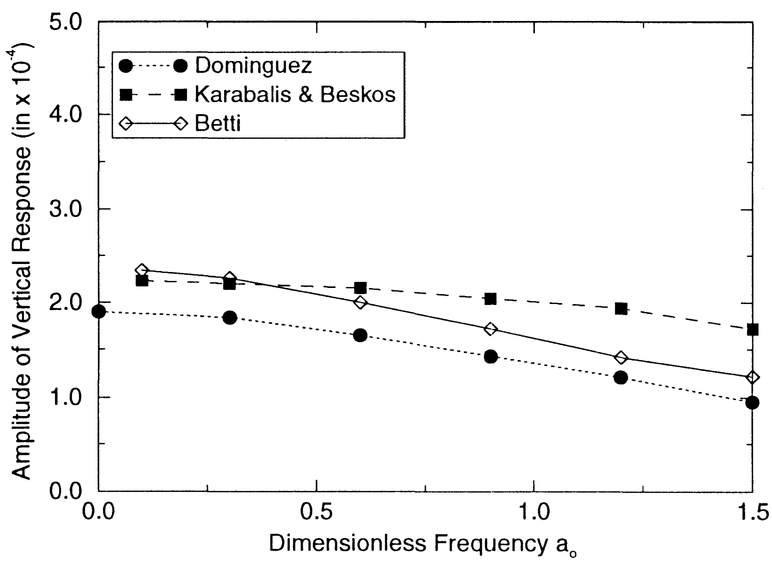

Fig. 4: Response of Embedded Foundation to Vertical Harmonic Force