Suburban bus route design

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Abstract
Rural and suburban areas have a low density of population. Hence, the bus services have unique features that enable the design of suburban bus route a practical and interesting research topic. We formulate a suburban bus route design problem and prove its NP-hardness. A dynamic programming approach is developed to obtain the optimal solution. A case study is reported to demonstrate the applicability of the proposed model and solution method.

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Abstract: Rural and suburban areas have a low density of population. Hence, the bus services have unique features that enable the design of suburban bus route a practical and interesting research topic. We formulate a suburban bus route design problem and prove its NP-hardness. A dynamic programming approach is developed to obtain the optimal solution. A case study is reported to demonstrate the applicability of the proposed model and solution method.

Keywords: Public transport; Suburban bus route design; Dynamic programming

1. INTRODUCTION

Buses are an integral part of transport for every country. Common public depends a lot on bus transport as the prime medium of transportation. Therefore, transport authorities and bus companies seek to provide better public transport services. Bus operations planning can be classified into 5 levels: network design, setting frequencies, timetable development, bus scheduling, and driver scheduling (Ceder and Wilson, 1986).

Network design is an initial and fundamental step for bus operations planning. The essential inputs for bus network design include road network and demand for travel. Bus network design is complicated due to the following reasons. (i) Travelers have other choices of transport mode, such as private car, taxi, bicycles, walking, and train. Hence, the number of travelers on buses depends on the properties of the bus services such as transit time, fare, degree of comfortable. However, the service properties depend on the network, frequency, timetabling, etc. (ii) Bus transport may be part of the overall intermodal transport system. (iii)

Even within the bus transport network, travelers may have many choices to travel and travelers may transfer from one bus to another. Consequently, when planning bus network, one has to estimate how travelers will travel in the network.

The bus network design problem has attracted much research attention (e.g., Ceder and Wilson, 1986; Baaj and Mahmassani, 1995; Pattnaik et al., 1998; Mauttone and Urquhart, 2009; Ciaffi et al., 2012; Cipriani et al., 2012). Because of the difficulty of the problem, researchers usually fix part of the decisions and apply heuristic methods to optimize the rest decisions.

In general, a bus network is fully connected and each component of the bus network cannot be analysed or optimized independently. However, there do exist some bus routes that are highly separated from other bus routes, and may be examined without considering the effects of the whole network. Typical examples are rural bus routes and suburban bus routes. A rural bus route connects some rural areas with a city, and a suburban bus route connects one or more suburbs to a city. Because they share common features, in the sequel we only examine suburban bus routes.

Consider the bus routes shown in Fig. 1, where the section between point A and point B is the center of the city of Wollongong. Bus route 39 mainly serves its suburbs of Figtree and Mt Keira; bus route 11 mainly serves Mangerton; and bus route 10 mainly serves Gwynneville. The three circles drawn in Fig. 1 are residential areas where only one bus route is available. We consider the section of bus route 39 in the circle. It has the following properties:

- All residents take the bus route to city center (or from city center to home).
- The demand for travel is generally fixed. The distance is beyond walking capability; the travelers on buses do not have cars; and taking a taxi is very expensive.
- The bus is seldom full. Hence, bus capacity limit is not a constraint.
- The bus route has a number of detours to collect travelers.
- There is little congestion on roads.

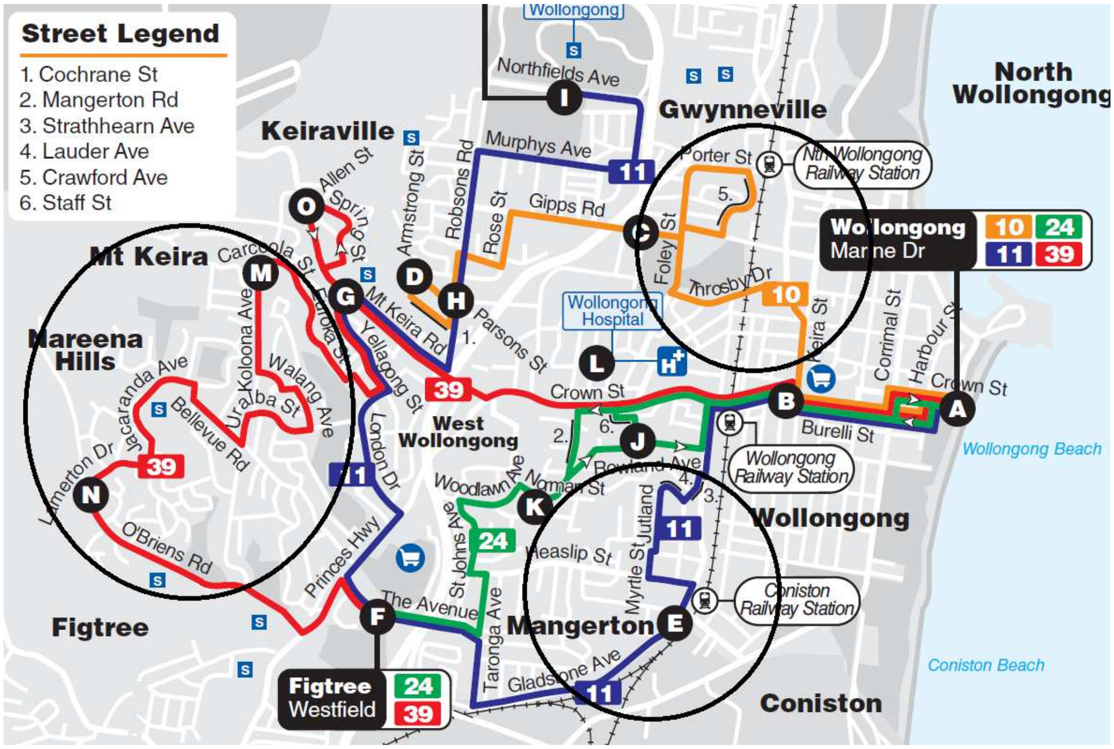

Fig. 1 Bus services near Wollongong, Australia (Source: http://www.premierillawarra.com.au/)

Due to these properties, the design of a suburban bus route such as bus route 39 in Fig. 1 (more exactly, the part of the bus route 39 in the circle) is different from general bus network design problems. The suburban bus route design problem is a new and practical research topic that has not drawn much attention. In general, the suburban bus route design problem is easier than general bus network design problems, and hence we expect that exact solution methods may be developed to design practical suburban bus route.

In this study, we examine the suburban bus route design problem. We assume that we are given the origin and destination of a bus route, and a number of intermediate bus stops to visit. How to determine the intermediate bus stops is beyond the scope of the study. In reality, transport authorities and bus companies should choose the locations of the bus stops such that the distance of a resident's house to the nearest bus stop does not exceed a certain number (e.g., 300 m). The objective is to find a shortest tour that visits all the bus stops. It should be mentioned that there are usually two bus stops at a location, one bus stop on each side of a road. For example, bus stop 1 and bus stop 1′ in Fig. 2. Bus stop 1 is visited in one direction and bus stop 1′ is visited in the other direction. In reality, because suburban roads are very narrow, buses cannot make a U-turn in the middle of a road. Therefore, as shown in Fig. 2, the distances from one point (e.g., bus stop 0) to the two opposite bus stops are different (Note that drivers are required to drive on the left side in Fig. 2). Consequently, to determine the shortest bus route, we not only need to optimize the sequences of bus stops to visit, but also which bus stop of the two opposite bus stops to visit.

Fig. 2 Different distances to the two opposite bus stops

The main contribution of the study is that we address a new and practical suburban bus route design problem. We develop an optimization model and a dynamic programming approach that obtains the optimal solution. The proposed solution approach is applied to two case studies.

This paper is organized as follows. Section 2 describes the problem and formulates an optimization model. Section 3 analyzes the hardness of the problem and develops a dynamic programming approach. Section 4 reports a case study. Conclusions are future work are in Section 5.

2. PROBLEM DESCRIPTION AND MODEL DEVELOPMENT

Consider a suburban bus route that starts from a bus station, denoted by node 0 and ends at another bus station or city centre denoted by node $N+1$. Between these two nodes, a bus needs to visit N bus stops. We let $1, 1', 2, 2' \ldots N, N'$ to represent the bus stops between node 0 and node $N+1$, where bus stops k and k' correspond to two bus stops at the same geographical location but on both sides of a street, $k = 1, 2 \cdots N$. We refer to k as the opposite bus stop of k', and k' as the opposite bus stop of k. We define set $K = \{1, 2 \cdots N\}$, set $K' = \{1', 2' \ldots N'\}$, and set $\overline{K} = K \cup K' \cup \{0, N+1\}$.

We let d_{ij} represent the distance between nodes $i \in \overline{K}$ and $j \in \overline{K}$. d_{ij} is a known parameter. We define x_{ij} as the decision variable which equals 1 if and only if the bus visits node $j \in \overline{K}$ immediately after node $i \in \overline{K}$, and 0 otherwise.

The mathematical model is:

$$min \sum_{i \in \overline{K}} \sum_{j \in \overline{K}} d_{ij} x_{ij} \tag{1}$$

subject to:

$$\sum_{j \in \overline{K}} x_{0j} = 1 \tag{2}$$

$$\sum_{i \in \overline{K}} x_{i,N+1} = 1 \tag{3}$$

$$\sum_{i \in \overline{K}} x_{ij} = \sum_{i \in \overline{K}} x_{ji}, \forall j \in K \cup K' \tag{4}$$

$$\sum_{i \in \overline{K}} (x_{ik} + x_{ik'}) = 1, \forall k \in K \tag{5}$$

$$\sum_{i \in S} \sum_{j \in S} (x_{ij} + x_{ji}) \leq |H| - 1, \forall H \subseteq \overline{K} \tag{6}$$

$$x_{ij} \in \{0,1\}, \forall i \in \overline{K}, \forall j \in \overline{K} \tag{7}$$

$$x_{ii} = 0, \forall i \in \overline{K} \tag{8}$$

$$x_{kk'} = 0, x_{k'k} = 0, \forall k \in K \tag{9}$$

The objective function (1) minimizes the total length of the bus route. Eq. (2) requires that the bus visits exactly one node $j \in \overline{K}$ after node 0. Eq. (3) requires that the bus visits exactly one bus stop before node $N + 1$. Eq. (4) requires that if j is visited from other bus stops then the bus must visit another bus stop after j. Eq. (5) requires that exactly one of k and k' is visited from other bus stops. Eq. (6) is the sub-tour elimination constraints, which must be satisfied for any subset H of $\overline{K}$. Eq. (7) defines x_{ij} as binary variables. Eq. (8) imposes that the bus does not visit a bus stop twice. Eq. (9) imposes that after a bus stop, the bus does not visit its opposite bus stop.

3. ALGORITHM DESIGN

3.1 Hardness of the problem

The decision version of the suburban bus route design problem is in NP, that is, give the bus route it can be determined in polynomial time whether the length of the bus route is shorter than a given constant L. We show the NP completeness of the problem by reducing a well-known NP-complete problem, the *travelling salesman problem (TSP)*, into a suburban bus route design problem.

The decision version of the TSP is defined as follows: Given a set of cities $\{0,1,2 \cdots N\}$, and the distance D_{ij} from city i to j, is there a tour that starts and ends at city 0 and visits each other city exactly once such that the length of the tour is shorter than L?

Theorem 1: *The decision version of the suburban bus route design problem is NP-complete.*

Proof: Suppose that node 0 and node $N+1$ coincide. Hence, $x_{i0} = x_{i,N+1}, x_{0i} = x_{N+1,i}$, for all $i \in K \cup K'$. Suppose further that node k and node k' coincide for all $k \in K$, that is $x_{ik} = x_{ik'}, x_{ki} = x_{k'i}$, for all $i \in \bar{K}$ and all $k \in K$. It follows easily now that if the TSP can be solved by solving a suburban bus route design problem. □

3.2 A dynamic programming approach

To address the suburban bus route design problem, we propose a dynamic programming approach. The key element of the dynamic programming approach is the following function:

$$\begin{aligned} &f(i,S) \\ &\coloneqq \text{the shortest sub-tour given that a bus is at bus stop } i \\ &\in K \cup K' \cup \{0\} \text{ and still has to visit the bus stops in } S \\ &\subseteq K \cup K' \text{ and terminates at bus station } N+1 \end{aligned} \tag{10}$$

It should be mentioned that in above function, set S is a collection of bus stops such that a pair of bus stops $k \in K$ and $k' \in K'$ satisfy either (i) $k \in S$ and $k' \in S$, or (ii) $k \notin S$ and $k' \notin S$. Hence, the number of elements in S must be even. If both $k \in S$ and $k' \in S$, then the bus needs to visit only one of the two bus stops.

Clearly, we have:

$$f(i,S) = d_{i,N+1}, \text{if } S = \emptyset \tag{11}$$

We also have:

$$f(i,S) = \min\{d_{ik} + d_{k,N+1}, d_{ik'} + d_{k',N+1}\}, \text{if } S = \{k,k'\} \tag{12}$$

If S has more than two elements, the function can be evaluated in an iterative manner. To this end, for a bus stop $j \in K \cup K'$, we first let $\varphi(j) \in K$ be the corresponding bus stop in K, and $\psi(j) \in K'$ be the corresponding bus stop in K'. For example, if $j \in K$, then $\varphi(j) = j$ and $\psi(j)$ is the opposite bus stop of j.

$$f(i,S) = \min_{j \in S}\{d_{ij} + f(i, S \backslash \{\varphi(j), \psi(j)\})\}, \text{if } S \neq \emptyset \tag{13}$$

The meaning of Eq. (13) is that, the bus needs to travel from node i to a node $j \in S$ first. Hence, it travels the distance d_{ij}. Then, the bus is at j, and neither bus stop $\varphi(j)$ nor $\psi(j)$ needs to be visited. $f(i,S)$ is determined when the best node $j \in S$ is chosen.

Whenever we invoke Eq. (13) once, the number of elements in S will be decreased by 2.

When $|S| = 0$, that is, when $S = \emptyset$, we can use Eq. (11) to compute the function. Hence, any function $f(i, S)$ can be computed. In particular, we are seeking the value of $f(0, K \cup K')$ and the optimal tour that leads to it.

It should be mentioned that because the suburban bus route design problem is NP-hard, the above dynamic programming approach cannot obtain the optimal solution in polynomial time with regard to the size of the problem. For large-scale applications, meta-heuristics, such as genetic algorithm, can be used to obtain good-quality solutions in reasonable time.

4. CASE STUDY

We applied the above dynamic programming approach to a square-block example shown in Fig. 3. Bus stations 0 and $N + 1$ are located at the bottom-left and upper-right corner, respectively. $N = 3$, which means that the bus needs to visit three bus stops between stations 0 and $N + 1$. The locations of the bus stops are plotted using black solid circle. It should be noted that drivers are required to drive on the left side in Fig. 3. We further assume that (i) buses are not allowed to take U-turns in the middle of a road; (ii) buses are allowed to take U-turns at the end of a road; and (iii) the time/cost/distance for turning is assumed to be 0.

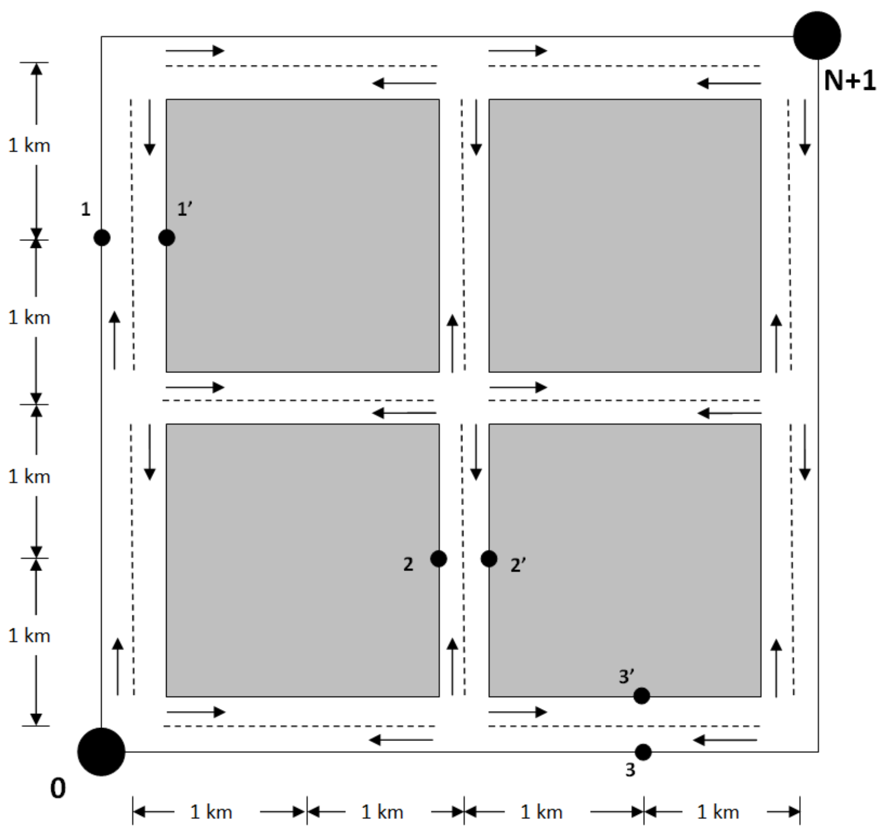

Fig. 3 A square-block example

We first compute the shortest distance between each pair of bus stops in set $\overline{K}$. The result is shown in Table 1.

Table 1 Pairwise distances of bus stops (from the column bus stop to the row bus stop)

→	0	1	1'	2	2'	3	3'	4
0	0	3	9	3	5	9	3	8
1	9	0	2	8	6	10	8	5
1'	3	2	0	6	4	8	6	7
2	5	4	6	0	2	6	4	5
2'	3	6	8	2	0	4	2	7
3	3	6	8	2	4	0	2	7
3'	5	6	8	4	6	2	0	5
4	8	7	5	7	5	5	7	0

We then compute the optimal bus route using the dynamic programming approach. After calculation, we obtain the shortest path: 0→1→2’ →3’→4, which is shown in Fig. 4.

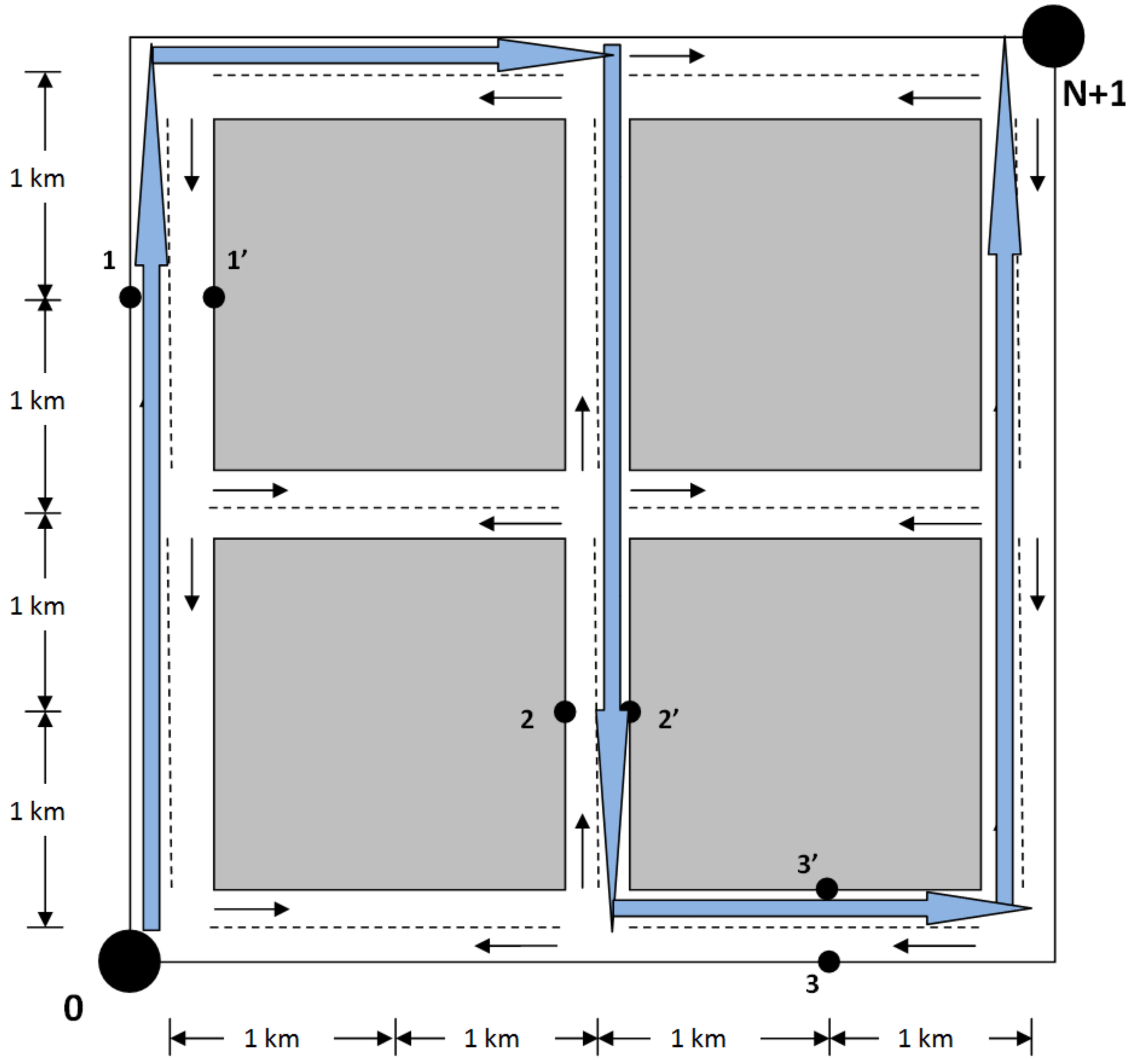

Fig. 4 Shortest bus route that is designed by the dynamic programming approach

5. CONCLUSIONS AND FUTURE WORK

This paper has examined a suburban bus route design problem by taking advantage of its special property. This is a practical research topic that is significant for the public transport services in suburban and rural areas. A mathematical model and a dynamic programming approach are developed. A case study based on a square-block example is conducted to demonstrate the applicability of the proposed model and method.

In future, we will examine the combined bus stop location and bus route design for suburban bus services. In particular, we will assume that there are a number of candidate bus stops to visit, and design a bus route that visits some of the candidate bus stops while ensuring a certain level of service in terms of the shortest walking distance from a house to the nearest visited bus stop.

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