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Sum Rule for the Magnetic Moment of the Dirac Particle

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Recently sum rules for the magnetic moments of baryons have been discussed by many authors.[1] We present here an exact sum rule which relates only experimentally measurable quantities. The technique used in deriving our sum rule is the same as that of Cabbibo and Radicati.[1] Although we discuss the case of the proton, our sum rule holds for any kind of Dirac particle.

Let us consider the following equal time commutation relation:[2]

$$\frac{1}{2}[D_1+iD_2,\ D_1-iD_2] = -i[D_1,\ D_2]=0. \qquad (1)$$

D_i is the electric dipole moment operator defined by

$$D_i=\int d^3x x_i j_0(x), \qquad (2)$$

where $j_0(x)$ is the time component of the total (the iso-scalar part + the third component of the iso-vector part) electromagnetic current. If we take the matrix element of Eq. (1) between the proton states of momentum p and p' along the third axis and consider the limit of the matrix element for p going to infinity, we obtain

$$\left(\frac{\mu}{2m}\right)^2=\frac{1}{8\pi^2\alpha} \times\int_{\omega_0}^{\infty}\frac{d\omega}{\omega}[\sigma_-(\omega)-\sigma_+(\omega)], \qquad (3)$$

where μ is the anomalous magnetic moment[3] of the proton, $\sigma_+(\omega)$ $[\sigma_-(\omega)]$ is the total inelastic cross section for the collision of gamma and proton whose helicities have the same [opposite] signs, ω is the energy of gamma in the laboratory system and ω_0 is the threshold energy for inelastic scattering. When Eq. (3) is applied to the Λ particle, one must add the contribution of the Σ^0 state to Eq. (3). Since the target must be polarized, we have no experimental data with respect to the right-hand side of Eq. (3). However, we expect that Eq. (3) can be checked in the near future.

If we approximate the integral of the right-hand side of Eq. (3) by the contribution of the N^* (1236) resonance to the proton and the neutron we get

$$\mu_n^2=\mu_p^2, \qquad (4)$$

since the N^* contributions are exactly the same for both cases. The experimedtal value of $(\mu_n/\mu_p)^2$ is about 1.14.

Equation (3) is expected to be applicable to the spin 1/2 nucleus. Therefore, one can check by Eq. (3) whether or not the spin 1/2 nucleus can be regarded as a Dirac particle.

1) B. W. Lee, Phys. Rev. Letters **14** (1965), 676.
R. F. Dashen and M. Gell-Mann, Phys. Letters **17** (1965), 142, 145.
N. Cabbibo and L. A. Radicati, Phys. Letters **19** (1965), 697.
S. Fubini, G. Furlan and C. Rossetti, Nuovo Cim. **40A** (1965), 1171.
V. S. Mathur and L. K. Pandit, Phys. Letters **20** (1966), 308.
F. Buccella, G. Veneziano and R. Gatto Nuovo Cim. **42A** (1966), 1019.

2) We do not know of a general proof of Eq. (1). But we also do not know of a counter example for Eq. (1).

3) The left-hand side of Eq. (3) is the contribution from the one proton intermediate state. In the work of Cabbibo and Radicati there appears the total magnetic moment because they mixed the contribution from $\int d^3x\, x_1^2 j_0(x)$.

Note added in proof: After completing this work, we learned that the same sum rule as ours has independently been obtained by Drell and Hearn. Since the commutation relation (1) also holds for the third component of the isovector current or for the isoscalar current alone, we have the same sum rule as Eq. (3) for the squares of the isovector and isoscalar anomalous magnetic moments. Therefore, Eq. (8) in the work of Drell and Hearn holds exactly.

S. D. Drell and A. C. Hearn, Phys. Rev. Letters **16** (1966), 908.