SUMMATED RESPONSE OF THE RETINA TO LIGHT ENTERING DIFFERENT PARTS OF THE PUPIL

DISSERTATION

Presented in Partial Fulfillment of the Requirements
for the Degree Doctor of Philosophy in the
Graduate School of the Ohio State
University

By

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The Ohio State University
1956

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ACKNOWLEDGEMENT

To my wife and parents without whose loyalty, encouragement and affection, past, present and future achievement could not have been contemplated.

To Dr. Glenn Fry, my graduate adviser, for the untold hours of guidance, and sincere advice, and the many helpful hints and ideas, which have made my graduate training and dissertation so rich and rewarding.

To my subjects, Bradford Wild, Ralph Vasa, and Alfred Manfredi, for their cooperative spirit and many hours of assistance.

To the Ohio State University, for a grant in aid from funds allocated by the Ohio State University for support of fundamental research. Without this assistance this research would not have been possible.

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I. INTRODUCTION

Stiles and Crawford (1), in 1933, discovered the now famous effect, which bears their name, as part of an experiment designed to develop a photometric method to measure pupil size. In the course of this experiment they found that light entering the pupil at different points did not produce equal sensations of brightness. Stiles and Crawford formulated the concept that allowance had to be made for this effect in predicting the retinal response for a beam which fills an extended area in the pupil. However, is this the only variable influencing the summated response of the retina to light entering varying regions of the pupil? It is the purpose of this paper to answer this question. Before proceeding further it would be wise to clarify terminology used in the discussion.

Repeated references will be made to beams of light transmitted through the pupil of the eye. A beam is a bundle of rays filling the whole pupil or part of the pupil and which is bounded in object space by an aperture in a diaphragm which constitutes the field stop. The patch of luminance in the field of view which corresponds to the field stop will be referred to as a field or a patch.

By placing the eye so that its entrance pupil falls in the same plane as the exit pupil of the optical instrument, one can manipulate the exit pupil to change the size of the beam entering the eye and also the location of the point of entrance.

It is also necessary to distinguish between Maxwellian and non-Maxwellian beams. In the case of a Maxwellian beam an image of a luminous surface like a ribbon filament is focused on a small aperture and an image of this aperture is then focused in the plane of the entrance pupil of the eye.

In the case of a non-Maxwellian beam the image of a luminous surface is formed in the plane of the field stop, so that each point in the plane of the field stop may be treated as an independent point source which produces a Fraunhofer image of itself on the retina. The kind of image formed on the retina is the same as that encountered in the ordinary use of the eyes.

There is a question which may be asked, is the response of the retina to a non-Maxwellian beam affected in the same manner by the angle of incidence of light at the retina as the response to a Maxwellian beam? Stiles and Crawford (1) have demonstrated that the luminous efficiency of a Maxwellian beam is markedly affected by the angle of incidence at the retina. This angle is also called the acceptance angle.

Furthermore, it is important to know whether the retinal effects produced by the elemental parts of a non-Maxwellian beam summate in the same way as in the case of Maxwellian beam.

One of the purposes of this study is to determine whether results obtained with Maxwellian beams can be applied to problems involving the ordinary use of the eyes.

The problem is one of more than theoretical interest, because several authors have already made use of the assumption that direct transfer exists between the Maxwellian and the ordinary method of viewing; for example, Koomen, Skolnik, and Tousey (2), in their studies on night myopia; Boynton, Enoch, and Bush (3), in discussing physical measurements of glare functions; etc.

Numerous authors have presented formulas including an integration of the Stiles-Crawford effect, as customarily measured with Maxwellian beams in considering various theoretical problems involving pupil size, predicted perceived brightness, etc., for example, Moon and Spencer (4), LeGrand (5), Arnulf (6), Bartley (7), etc.

Prior to a discussion of the research presented in this paper, a review of the available literature pertaining to the topic at hand will be presented in chronological order.

Troland (10,11), during the early part of World War I, derived a simplified equation for the expression of retinal illuminance

$$E_R = B_o \quad A_o$$

This relationship simply states that retinal illuminance (E_R) is a function of the luminance (c/m^2) in field of view measured at the pupil (B_o) and the area (mm^2) of the entrance pupil (A_o). Note that throughout this paper the subscript "R" will refer to retina, and the subscript "o" will refer to the plane of the entrance pupil.

The unit of retinal illuminance was called the photon by Troland, but in later years it has become known as the troland, in honor of the originator. Other authors had previously specified values for retinal illuminance in ordinary units prior to this, but Troland's simple relationship has formed the basis for considerations of this nature since it was first described.

Troland assumed that perceived brightness is related to retinal illuminance without reference to the way in which the light is distributed over the pupil. It may be stated that Troland assumed that all elemental parts of a beam entering the pupil of the eye have the same luminous efficiency. This will be referred to as Troland's additivity principle. To the writer's knowledge, Troland never tested this hypothesis experimentally, nor, in his defense, was there any reason to doubt the validity of his assumption.

In 1933, Stiles and Crawford (1) devised a scheme to measure puple size by a photometric method in which the brightness produced by a beam filling the whole pupil is matched to the brightness produced by a narrow beam directed through the center of the pupil. At first they assumed the same principle of additivity as that of Troland but the discrepancies they uncovered in the course of this experiment led them to investigate the variations in luminous efficiency of rays entering the eye pupil at different points. This variation in luminous efficiency has since become known as the Stiles-Crawford effect.

Of particular interest is this initial aspect of the study is the fact that they found that the perceived brightness for a pupil of 8.0 millimeters diameter never exceeded that predicted for a 5.5 millimeter diameter pupil, for a non-Maxwellian field conjugate with the retina. The white object viewed was positioned approximately twenty centimeters from the eye of the observer. One may assume that they used a mydriatic to dilate the pupil to eight millimeters, and that accommodation was intact. As will be shown later, this implies that peripheral regions of the pupil contributed relatively less to the brightness of the field than the central part.

The variations in luminous efficiency of rays entering the pupil at various points were measured by making a brightness match between the images produced by two narrow Maxwellian beams penetrating different parts of the pupil. One beam was centered in the entrance pupil of the eye and the second beam was varied across the pupil.

The results were expressed in terms of an efficiency function. The value ηr proposed by Stiles and Crawford, designates relative luminous efficiency for a ray entering the eye at any point in the entrance pupil at a radial distance r along some given meridian from the center of the entrance pupil or the point of maximum luminous efficiency, whichever is used as the reference point.

$$\eta r = \frac{Br}{Bro}$$

Br is the luminance of the comparison field required to make it match the test field in brightness when the test beam enters the entrance pupil at the distance r from the reference point at which r = o. Bro is the value of Br when r = o.

Obviously $\eta_r = f(r)$. More properly however, $\eta_r = g(\theta)$, where θ is the angle of incidence or acceptance at the retinal plane and where $\theta = h(r)$.

If one considers Gullstrand's schematic eye, a simple relationship is derived for θ,

$$\tan \theta = .045r$$

and for small values of θ,

$$\theta = .045r.$$

As pointed out by several authors, this corresponds roughly to 2.5° per millimeter.

Figure 1 shows the Stiles-Crawford data with the point of maximum luminous efficiency being used as the reference point. These values were taken from Moon and Spencer (4). The significance of the superimposed theoretical curve will be discussed in connection with their paper. The curve conforms to the following equation proposed by the author for the Stiles-Crawford effect.

$$\eta_r = .25\,(1 + \cos 9.5\,\theta)^2$$

The axis of ordinates is expressed as a logarithmic scale of η_r and the axis of abscissas as the radial distance in the entrance pupil of the eye from the point of maximum luminous efficiency.

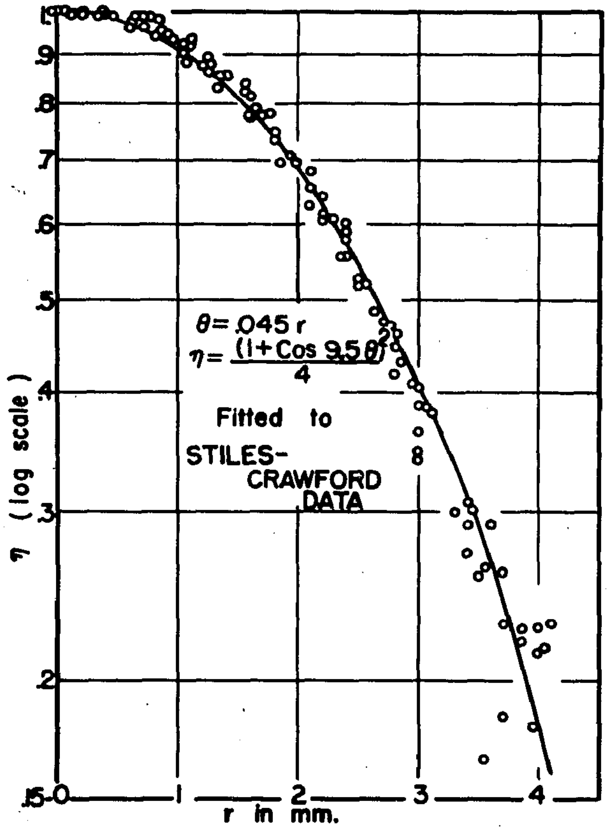

Fig. 1- Stiles and Crawford's luminous efficiency data, maximum at r = 0.

As part of this study, Stiles and Crawford investigated the principles of additivity. They varied the area of a Maxwellian beam in the plane of the pupil and equated the brightness of its field with the brightness of a second field. Light from the second field entered the eye via a narrow beam projected through the center of the pupil.

Stiles and Crawford's experimental data on the additivity effect for a variable size Maxwellian beam in the entrance pupil are plotted in Figures 2 and 3. Their original data have been replotted in a form which will be utilized through out the remainder of the paper. The axis of abscissas represents the area of the test beam in the entrance pupil and the axis of ordinates is a measure of log relative luminance. Relative luminance represents the ratio of the luminance of the comparison field to that of the test field required to make the two fields match each other in brightness.

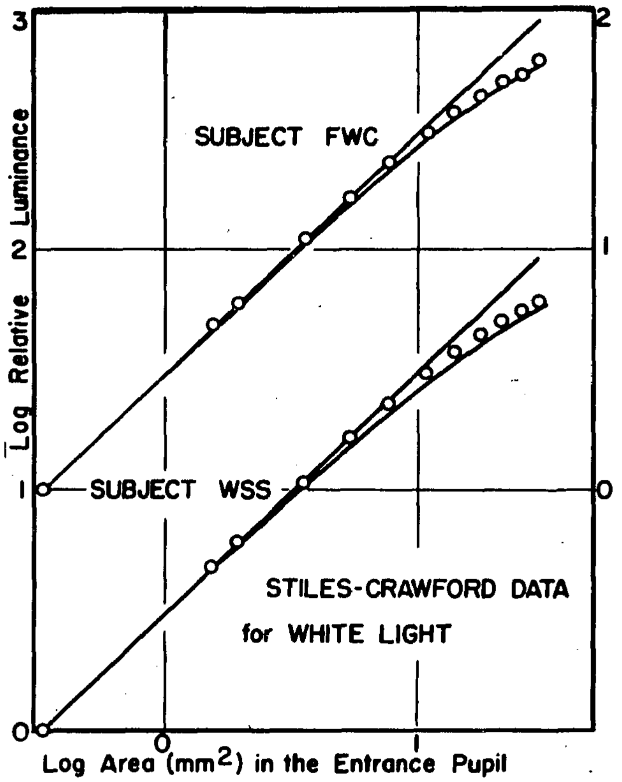

Fig. 2- Stiles and Crawford's additivity data for a variable size Maxwellian beam. The curve represents predicted values, and the straight line the input.

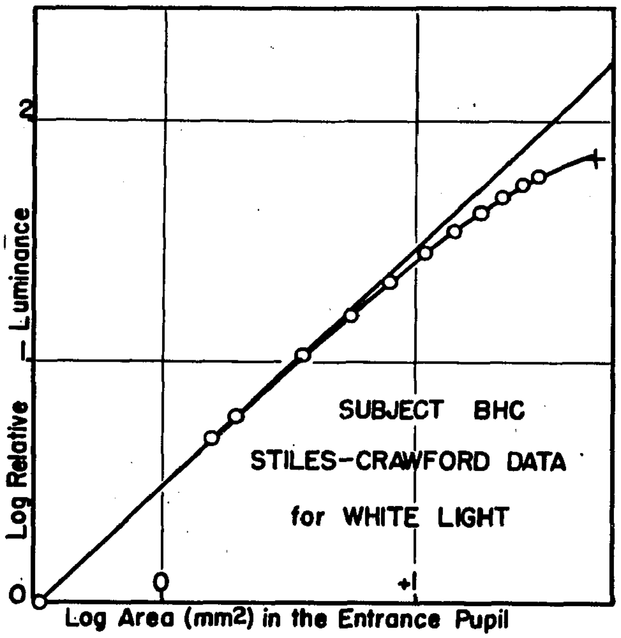

Fig. 3- Stiles and Crawford's additivity data. Circles indicate variable size Maxwellian beam, the cross represents the maximum reading for a variable non-Maxwellian beam.

The straight line which appears in both Figures 2 and 3 represents values predicted if Troland's relationship were valid. One may also think of this line as the input, in terms of an efficiency analysis, and the data as the output. The curved line, which appears in each of the two figures, gives the values obtained when allowance is made for the Stiles-Crawford effect in intergrating the contributions of the different parts of the pupil.

A theoretical expression for the additivity effect, in which allowance is made for the Stiles-Crawford effect, may be derived in the following manner.

The effect of a luminous surface in the field of view upon the photoreceptors may be expressed in terms of the rate (V) of dissociation of molecules of the photosensitive substance per photoreceptor. The relation between the luminance (B) of the surface, and V may be formulated as follows,

$$V = B\,k' \int_0^{2\pi} \int_0^{\bar{r}} \eta(r)\, r\, dr\, d\phi.$$

The polar coordinates in the plane of the entrance pupil are r and ϕ. $\eta(r)$ is the relative luminous efficiency for light passing through an element of the entrance pupil having an area of r dr dϕ. The constant includes a concentration factor.

For the special case of a round aperture stop imaged in the plane of the entrance pupil and centered at the point of maximum luminous efficiency, and assuming that $\eta(r)$ is not a function of ϕ,

$$V = 2\pi B\,k' \int_0^{\bar{r}} \eta(r)\, r\, dr.$$

In an experimental investigation of the additivity effect one compares a test field involving a variable aperture stop and a fixed value of B with a comparison field having a small fixed aperture stop and a variable luminance. For each value of $\bar{r}_T$ the subject varies B_C to obtain a match in brightness.

When the two match in brightness it may be assumed that

$$V_T = V_C .$$

Since

$$V_T = 2\pi \, B_T \, k' \int_0^{\bar{r}_T} \eta(r) \, r \, dr,$$

and

$$V_C = 2\pi B_C \, k' \int_0^{\bar{r}_C} \eta(r) \, r \, dr,$$

$$\frac{B_C}{B_T} = \frac{\int_0^{\bar{r}_T} \eta(r) \, r \, dr}{\int_0^{\bar{r}_C} \eta(r) \, r \, dr} .$$

The ratio (B_C/B_T) is called the relative luminance of the comparison field.

Furthermore, since $\bar{r}_C$ is a constant, $\frac{1}{k''}$ may be substituted for $\int_0^{\bar{r}_C} \eta(r)\, r\, dr$

and hence

$$\frac{B_C}{B_T} = k'' \int_0^{\bar{r}_T} \eta(r)\, r\, dr.$$

It should be noted that B_C/B_T has a value of unity, when $\bar{r}_C = \bar{r}_T$.

If, as Troland assumed, $\eta(r) = 1$ then the rate of photochemical decomposition reduces to the following expression.

$$V = k' BA$$

and the expression for B_C/B_T reduces to

$$\frac{B_C}{B_T} = k'' \frac{\bar{r}_T^{\,2}}{2} = \frac{A_T}{A_C}.$$

This is a statement of Troland's additivity principle.

If one substitutes the author's proposed relationship for η_r into the equation for relative luminance

$$\frac{B_C}{B_T} = k'' \int_0^{\overline{\theta}_T} \frac{(1 + \cos k\theta)^2}{4} \, r \, dr.$$

Since

$$\tan \theta = .045r$$

the above expression may be transformed to

$$\frac{B_C}{B_T} = \frac{k''}{4(.045)^2} \int_0^{\overline{\theta}_T} (1 + \cos k\theta)^2 \tan \theta \sec^2 \theta \, d\theta.$$

Since θ never exceeds 11° at the retina, little error is introduced by simplifying the expression for θ.

$$\theta = .045r$$

$$\frac{B_C}{B_T} = \frac{k''}{4(.045)^2} \int_0^{\overline{\theta}_T} (1 + \cos k\theta)^2 \, \theta \, d\theta$$

Solving this integral,

$$\frac{B_C}{B_T} = \frac{k''}{4(.045)^2 k^2} \left[\frac{3}{4}(k\theta)^2 + 2k\theta \sin(k\theta) + \frac{k\theta}{4} \sin 2(k\theta) + 2\cos(k\theta) + 1/8 \cos 2(k\theta) - 2.125 \right].$$

The 2.125 term is the evaluation of the previous five terms for the case of $\theta = 0$.

The integral may, as is obvious, be solved for the case of an annulus, by merely computing for the proper limits of integration.

The theoretical curves for the additivity effect in this paper are plots of the above relationship, with $k = 9.5$. Inspecting the data for all three subjects, it will be noted, that the data for two of the observers do not exactly follow the curve predicted from the Stiles-Crawford effect. For these two observers we have what might be considered a negative discrepancy, in that the additivity data do not fully reflect values predicted by integration of luminous flux falling at a given retinal point and corrected for the Stiles-Crawford effect. The significance of these data will be discussed in relation to the author's findings in the discussion section.

It will be noted, on subject BHC's plot that an extra point is plotted, the cross on Figure 3. This corresponds to data found by experimentation using a non-Maxwellian field, and may not be properly assigned to subject BHC. The authors state that the readings for the eight millimeter entrance pupil never exceeded this value. Thus, this is the maximum experimental value obtained using a variable size non-Maxwellian beam in the entrance pupil of the eye. In this instance a slight positive discrepancy exists, ie, an effect greater than that predicted by integration (including correction for the Stiles-Crawford effect). Are these differences due to the fact that in one instance a variable size Maxwellian beam was used, and in the other case a variable size non-Maxwellian beam was employed? As will be seen at a later point, the methodological elements of white light, and flicker photometry versus direct comparison may have some bearing on the problem.

A review of all phases of the Stiles-Crawford effect shall not be attempted in this paper, rather only those papers pertinent to the problem at hand shall be considered. Needless to say, the work of Stiles and Crawford has been confirmed many times and several different aspects have been evaluated.

Bocchino (9), in 1936, apparently without knowledge of the Stiles-Crawford effect, studied the luminous efficiency of a telescope as a function of the size of the exit pupil of the telescope. Inasmuch as the method employed by Bocchino in obtaining the data is not clearly outlined, the writer has not presented the data in this discussion. Assuming the validity of the method, Bocchino's results indicate great positive discrepancies, which are not even approximated by consideration of the Stiles-Crawford effect.

Both Crawford (12) and Stiles (13) in 1937 demonstrated that the retinal direction effect they described earlier (1), was a photopic phenomenon and therefore related to cone vision, the rods apparently not manifesting a Stiles-Crawford effect. Goodeve (14) had previously shown that the phenomenon was at least mediated by cones by eliciting the phenomenon for far red radiation. Further, Stiles (13) and later Flamant (15) demonstrated the effect is not a function of adaptation level in the photopic range. These and other studies therefore, established the Stiles-Crawford effect as probably being physically mediated by the cones.

Stiles (13) in 1937 noted that the Stiles-Crawford effect varies only slightly with wavelength, but changes occur in hue as angle of incidence at the retina varies. Since much of experimental work in this paper requires the use of monochromatic light, and the matching of a bipartite field, to avoid heterochromatic matching problems, a wavelength was chosen which did not manifest marked perceived wavelength changes (552 millimicrons).

Craik (16) in 1940, measured the transmission of light through the media of a cat eye, in order to test the hypothesis that the Stiles-Crawford effect was a retinal phenomena. A 50 watt projection bulb was placed about one meter from a

0.5 mm. aperture placed in front of an excised cat eye. This was traversed across the corneal plane. A window was cut in the rear of the eye and transmission measured with a photocell. The pupil was dilated with adrenalin. He found the transmission to be quite uniform across the entrance pupil, never falling off more than 30% at the border of the pupil. The image of the filament was focused on the retina and remained clear throughout. Since at the edge of the pupil, luminous efficiency drops about a log unit, the Stiles-Crawford effect is of necessity a retinal effect. It should be noted that the illumination does not really start to fall off at all in terms of transmission until the equivalent of a radius of 3.0 mm. is reached, according to Craik's data on the cat.

In 1944, Moon and Spencer (4) attempted to unify the available empirical treatments of the Stiles and Crawford data into a form which could be readily applied. They combined this data with that of several authors on pupil size as a function of luminance. Arnulf (6) made a somewhat similar analysis with a slightly different approach during the same year. It is perhaps wise to pause at this point of the discussion to consider the function used to fit the Stiles-Crawford data.

It will be remembered from previous discussion that relative luminous efficiency,

$$\eta_r = \frac{Br}{Bro},$$

and that, as a consequence, $\eta = f(r)$ or more properly $\eta = g(\theta)$, where θ is the acceptance angle at the retina.

Crawford (12), in 1937, derived the following equation to fit the original data (1, 4),

$$\eta_r = 1.025\, e^{-0.105\,(r + 0.47)^2}$$

When the maximum is made the reference point this equation simplifies to

$$\eta_r = e^{-0.105r^2}$$

or,

$$\eta_r = 10^{-0.0456r^2}$$

when the base is changed. The more generalized form of this equation

$$\eta_r = 10^{-\alpha r^2} = e^{-2.3\alpha r^2} .$$

Moon and Spencer, in their treatment of the same data derived two curves:

$$\eta_r = 1 - 0.085r^2 + 0.002r^4$$

and

$$\eta_r = 0.379 + 0.621 \cos 0.515r$$

which are both essentially the same formula, since the former represents an expansion of the latter, the more generalized form of which is the following:

$$\eta_r = (1 - a) + a \cos b\, r .$$

Although this gives a somewhat better fit than the Crawford equation, it is rather difficult to apply to data, since two arbitrary constants must be determined rather than one.

The following equation and theory is offered as a possible answer to the Stiles-Crawford effect. The relationship, $\frac{(1 + \cos \theta)^2}{4}$ which is known as the "obliquity factor" for intensity in diffraction theory (17) would account for about a two percent loss in luminous efficiency for an entrance pupil having a 9.0 mm. diameter. Obviously, this does not account for the Stiles-Crawford effect, but the classical data of Stiles and Crawford are fit well by the following general form:

$$\eta_r = .25\ (1 + \cos k\theta)^2$$

If k is set equal to 9.5,

$$\eta_r = .25\ (1 + \cos 9.5\theta)^2$$

or,

$$\eta_r = .25\ (1 + \cos 9.5\ (\tan^{-1}(.045r))\ )^2.$$

Thus, only one **arbitrary** constant is needed, ie, k = 9.5, since .25 is a normalizing constant. This curve is plotted with Stiles and Crawford's data in Figure 1. It should be noted, the value 9.5 might be slightly modified to give even a better fit. This constant has been chosen because it offers the best compromise value fitting the author's data and that of the original investigators.

Let us now investigate this relationship. O'Brien (18) in 1946 proposed a theory accounting for the Stiles-Crawford effect. He suggested that the function of the ellipsoid of the cone is to concentrate light in the outer segment of the cell. However, the amount of light concentrated is a function of the angle of incidence of the light at the cone. This relationship is dependant upon the differential index of refraction and the resulting critical angle. In brief, his theory predicts that virtually all light incident normally at the mouth of the cell will be concentrated without loss in the outer segment of the cell, since the energy will be totally reflected a multiple number of times (see Figure 4). This concentration of energy would be proportional to the cross sectional areas of the inner and outer segments. Oblique rays of light would be partially reflected and partially

Fig. 4- The theoretical construct accounting for the Stiles-Crawford effect proposed by O'Brien.

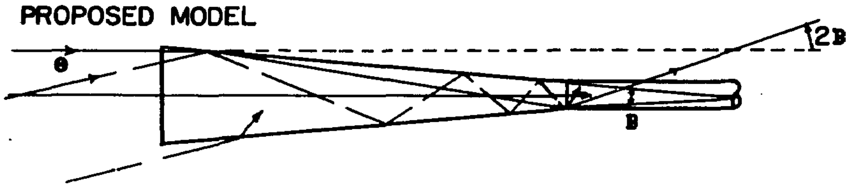

Fig. 5- Proposed theoretical construct to account for Stiles-Crawford data.

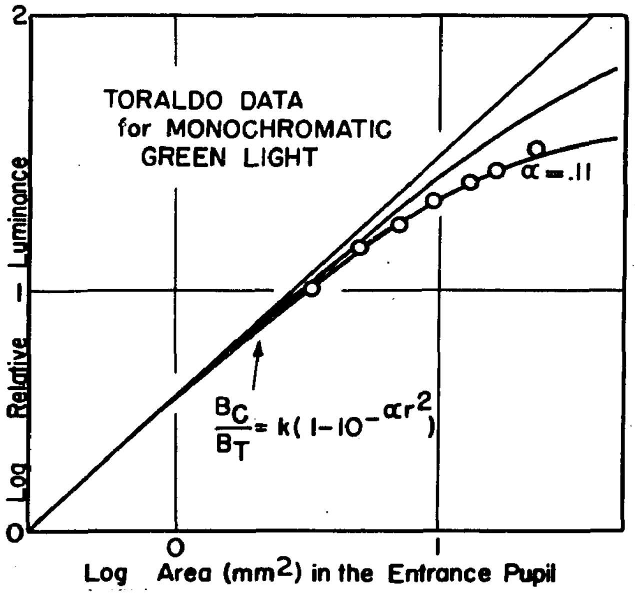

Fig. 6- Toraldo di Francia's additivity data. The curve fitting the data was derived by that author. Input, and predicted curves also appear.

transmitted, thus giving a smaller concentration of energy. Since the index of refraction of the cones and surrounding media is not yet known (19), no true test of this theory can be made. O'Brien tested this theory using a microwave model (20), and found good agreement with the available data. Figure 4 is a copy of O'Brien's theoretical construct.

Let us consider the same model from a slightly different point of view. Assuming for the moment, the disposition of indices is such that every ray incident within the ellipsoid, from a point in the exit pupil of the eye, is totally reflected (ie, all rays are incident at greater than the critical angle), and in the ellipsoid these rays make (n) reflections on an average. Taking an axial section, one has effectively two plane mirrors with an enclosed dihedral angle. Remembering a simple theorem of elementary geometric optics, "If a ray lying in a principal section (any section made by a plane perpendicular to the line of intersection of the plane of a pair of inclined mirrors is called a principal section of a system) is reflected successively at two plane mirrors, it will be deviated from its original direction by an angle equal to twice the dihedral angle between the mirrors" (21). Thus, a mechanism exists which greatly increases the angle of incidence of rays from the exit pupil, at the intersection of the ellipsoid with the outer segment of the cone (see Figure 6), where it is believed the photo-pigments are concentrated. The greater the obliquity of the incident ray the greater the total number of reflections and therefore the greater the deviation from the original angle. Thus, it is hypothesized that the Stiles-Crawford effect is related to the increase, or amplification or magnification of angles of incidence (θ) by the ellipsoid. This would account for the factor k, which would then be a function of the angle and relative index of the cone, and the number of reflections. The Stiles-Crawford effect would therefore reflect the decrease in amplitude (or intensity) of the disturbance imparted by the highly oblique angles of incidence at the outer segment of the cone. Further, the obliquity of light rays being refracted into the cone is increased according to Snell's law (see bottom Figure 6). Thus, at least two mechanisms exist for increasing the obliquity of the incident ray. The proposed relationship thus accounting for the Stiles-Crawford effect would be defined by the equation,

$$\eta_r = .25\ (1 + \cos k\theta)^2$$

where k = f(n,B,R), n being the relative index of refraction between cell and medium, R being the average number of reflections occurring in the average cone having an average apex angle of B. A model for the proposed relationship appears in Figure (6). The number of reflections between two plane mirrors is a function of the angle of incidence and the dihedral angle between them. Therefore,

$$R = h(\theta,\ B)$$

In more general terms,

$$k = f(n, B,\ h(\theta, B)\ ) = 9.5$$

In the case of the rods, where one deals essentially with a cylindrical element, one would predict $k = 1$. The reason being that the mechanisms for acceptance angle amplification are no longer the same. Since the dihedral or apex angle is equal to zero degrees, no increase in angle is afforded by that means and because the morphological structure of the rod is different, the relative contribution afforded by refraction at the cell wall is reduced. If $k = 1$, as was noted previously, only a two percent drop in luminous efficiency would be predicted for a 9.0 mm. pupil. This conforms closely to known relationships, that is, η_r for rods is approximately equal to one. Obviously, one would predict from this statement that at the center of the fovea, where cones approach rods in physical appearance, a reduced Stiles-Crawford effect would be elicited.

A test of this theory as opposed to O'Brien's awaits further data on the index of refraction of the cones (and the ellipsoid in particular) and their surrounding media. This theory would require a greater difference in indices than that reported by O'Brien, in order to provide a smaller critical angle. It is more than likely the true effect is a combination of both relationships, ie, a function including loss due to critical angle considerations should be included. As has been inferred with regard to O'Brien's theory, due to the small dimensions of the cones, one should remember that simple geometric optics and elemental physical optics may not apply. However, the agreement of the proposed formulation with the classical Stiles-Crawford data is quite good. A third alternative was proposed by Toraldo di Francia (22) in 1948. He suggested that the action of the ellipsoid is analogous to that of an impedance matching device, as employed in microwave assemblies, and that the cone itself is a form of dielectric antenna.

One other point in support of the above proposal, is that the ellipsoid of the cone is not given that name by accident. If one observes the morphological aspects of the cone, especially in regions slightly removed from the center of the fovea, where the cones are not as tightly packed, one notes that the ellipsoid is actually an ellipsoid or paraboloid of revolution, rather than the idealized conoid of revolution. If one then thinks of the focus of such an ellipsoid or paraboloid of revolution occurring at or near the mouth of the outer segment within which visual pigments are theoretically located, one may visualize the formation there of a diffraction pattern, if one may speak of such, in apertures approaching the wave length of light. Obviously, the obliquity factor in such a formation would certainly not be negligible, since wall reflections, as in the case of the conoid, provide a mechanism for increasing obliquity.

In 1947, Tolaldo di Francia (8) published a paper dealing specifically with the additivity effect. He had been disturbed by the implications of Bocchino's data, and sought to define a true relationship between pupil size and perceived brightness. His apparatus presented a small comparison beam Maxwellian field, and a non-Maxwellian test field viewed through a telescope.

The size of the entrance pupil of the telescope was varied in order to change the area subtended in the entrance pupil of the eye by the non-Maxwellian beam. Monochromatic green light was used. The data (Figure 6) show a marked positive deviation from the values which would be predicted by the application of the Stiles and Crawford's data. In addition to the input curve and the curve predicted by integration of the Stiles-Crawford curve, a third curve is included in Figure 6 which represents Toraldo's data.

The studies known by the author to have dealt with the Stiles-Crawford effect using non-Maxwellian fields are those of Goldman (23), Alpern and Benson (24), and O'Brien (25). However, none of these may be considered quantitative in terms of perceived brightness.

Goldman performed a rather ingenious experiment using an ophthalmoscope and comparing qualitatively a subject's reports of brightness of the ophthalmoscope's beam as the angle of incidence of light varied on the retina, with that of his own subjective impression while concurrently looking into the subject's eye.

Alpern and Benson studied the size of the pupil as a function of point of entry of a narrow non-Maxwellian beam through the pupil. Since the data are rather scattered, it is difficult to draw any conclusion other than that there is a Stiles-Crawford effect for at least part of the pupillary receptors.

O'Brien showed that apparent light and dark patches on the retina could be interchanged by a shift in position of an aperture. This was interpreted to indicate that the retina was divided randomly in patches with different direction maxima of efficiency.

In summary, where measures have been taken of the additivity effect for the ordinary method of viewing, using both monochromatic and white light sources, the data has revealed varying positive discrepancies from values predicted (1,8,9) when allowance is made for the Stiles-Crawford effect in integrating the contributions of the different parts of the pupil. While on the other hand the additivity data of Stiles and Crawford, using a variable Maxwellian field, adheres fairly closely to, or is less (ie, negative discrepancies) than the predicted values. The question as to the contribution made by Maxwellian versus non-Maxwellian field therefore, needs solution. Should one expect differences on this basis?

Thus, a program of research was carried out in an attempt to uncover the causes of these differences. The experiments were designed to answer the following questions, some of which will be more readily understood at a later point in the paper.

a. What is the true additivity effect and can we expect a difference in additivity effect data obtained using a variable size Maxwellian or a non-Maxwellian beam?

b. Is there a difference between the Stiles-Crawford effect measured by traversing a Maxwellian or a non-Maxwellian beam across the pupil?

c. What is the contribution of blur as caused by (1) spherical aberration, and (2) chromatic aberration?

d. What is the effect of blur on luminance matching techniques in general?

The following experiments were designed to provide answers to these questions:

1. The Stiles-Crawford effect (Maxwellian beam traversed) was measured on the three subjects used throughout. This was measured for white light and for monochromatic green light of wavelength 552 mμ. A bipartite field was used in making the brightness matches.

2. The Stiles-Crawford effect for a traversed non-Maxwellian beam was measured. Only monochromatic green light was employed.

3. On a flicker photometry apparatus, a non-Maxwellian and Maxwellian field, of the same extent and exactly superimposed, were compared as a function of pupil area.

4. The additivity effect was measured for both white and monochromatic green light. The size of a non-Maxwellian beam was varied.

5. Because of the problem of brightness matching in the presence of blur, introduced when the larger pupillary areas were employed in additivity effect investigations, a study was conceived to investigate the effect of blur on matching. The

results showed that there was a consistant underestimation of perceived brightness when blur existed. This apparent decrease in brightness was more evident when a bipartite field was used, than when flicker photometry was employed. This result will be discussed in its proper place. However, much of the remainder of the research program investigated the effect of blur on additivity.

6. The additivity effect was studied under varying amounts of dioptric blur. Only monochromatic green light was used.

7. Annular zones were projected into the pupil of the eye, spherical aberration of the eye was corrested, and brightness matches were made without blur. Only monochromatic green light was used.

8. The additivity effect was tested employing the flicker photometry technique using monochromatic green light.

II. APPARATUS

Three separate instruments were constructed in the course of the research program. These will be known as the direct comparison apparatus, the blur apparatus, and the flicker photometry apparatus. The direct comparison apparatus was used for almost all phases of testing where a bipartite field was employed. This included the studies on the Stiles-Crawford effect, and several aspects of the experiments dealing with the additivity effect. The blur apparatus was applied to the study of the effect of blur on brightness matches, using both the direct comparison and flicker photometry methods. With the aid of the flicker photometer apparatus certain aspects of the additivity effect were investigated.

A. The Direct Comparison Apparatus

The apparatus was designed to incorporate the following features.

1. To measure the Stiles-Crawford effect allowing a small Maxwellian beam to traverse the entrance pupil of the eye. The resulting brightness to be compared with that produced by a second standard patch.

2. To measure, in the same manner, the Stiles-Crawford effect with a non-Maxwellian beam traversing the entrance pupil of the eye.

3. To measure the additivity effect, by admitting into the eye a beam, the dimensions of which may be varied in the entrance pupil of the eye. The resulting brightness to be compared with that produced by a second standard patch.

The following design was employed (Figure 7). A six volt, eighteen ampere, ribbon filament lamp was used as the source (S1). A lens (L1) collimated the beam which was then passed through a filter box (F1), where either a neutral density filter could be introduced to control the luminance level, or interference or other type filters could be placed to control the wavelength composition of the beam. The beam was then divided into two elements by a beam splitter. If one first considers the beam reflected at a right angle by the beam splitter, one finds the light passing through a balanced neutral density wedge (F2,F3), which was used in making all brightness matches. The still collimated beam was then redirected by a penta

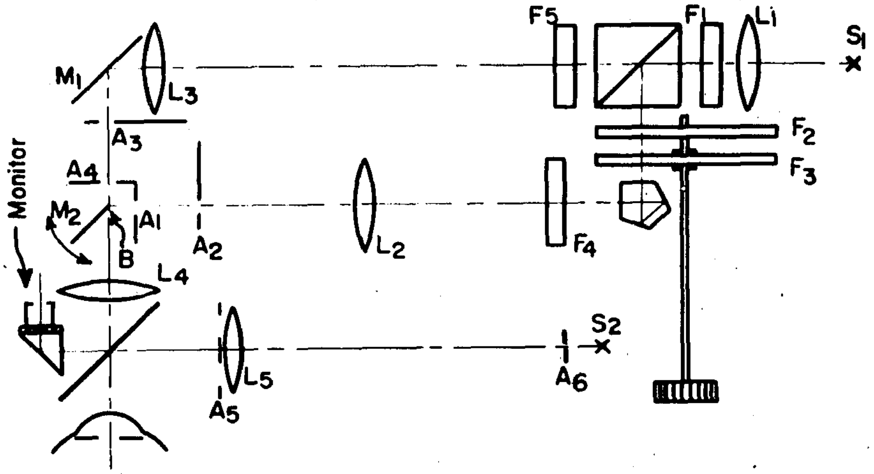

Fig. 7- Direct comparison apparatus used in measures of the Stiles-Crawford and additivity effect.

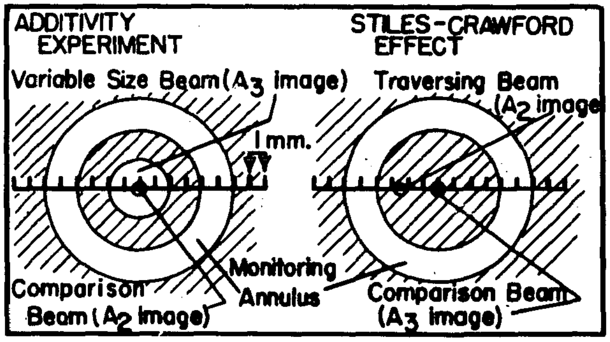

Fig. 8- Patterns seen objectively through the monitoring device for additivity and Stiles-Crawford effect experiments.

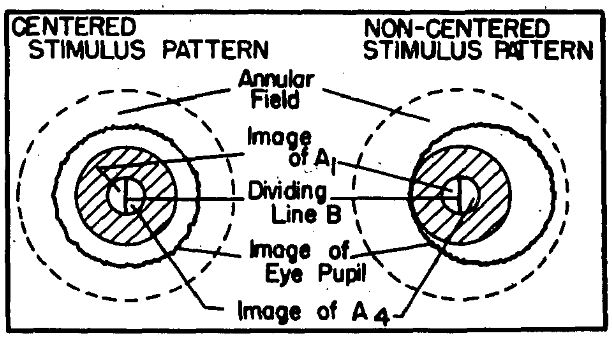

Fig. 9- Subjective pattern seen for all experiments using the direct comparison apparatus, centered and non-centered.

prism through a filter cell (F4) and a lens (L2). The light was focused by this lens (L2) in the plane of a variable size round aperture (A2). Aperture A2 was conjugate with the entrance pupil of the eye, and thus, this beam effectively forms a Maxwellian view. The diverging beam then passed through a fixed round aperture (A1) (equal in size to aperture A4) which was placed as close as possible to the mirror (M2), the edge of which (B) formed the dividing line of a bipartite field consisting of one half of A1 and one half of A4. The total field subtended 1° 36' at the eye. Point B fell at the focal point of lens L4, and as a result, the dividing line was optically imaged at infinity. The center of this line constituted the fixation point. Mirror M2 rotated about this dividing line (B), enabling the experimenter to traverse the image of A2 horizontally across the entrance pupil of the eye. This allowed measurement of the Stiles-Crawford effect. At the same time, the image of aperture (A1) on the retina would remain virtually fixed if the eye was perfectly focused.

Returning to the beam splitter, the transmitted part of the collimated beam passed through a filter cell (F5) and lens L3. The converging beam was reflected at mirror M1, and passed through a variable round aperture (A3), which was conjugate with the entrance pupil of the eye. By means of micro-photometric calibration, it was found that within the range of apertures used in this experiment, the illumination of the beam in the plane of the entrance pupil of the eye was constant. It was possible to vary the size, position and configuration of aperture A3. The ribbon filament was focused by lens L3 at point B through aperture A4; therefore it is conjugate to the retina since B is in the focal plane of lens L4. Aperture A4 acts as field stop. Mirror M2 divides the field.

Two auxilliary systems were introduced which acted as controls. A thin piece of plane glass was placed at an angle of 45^{o} between lens L4 and the eye. Part of each of the two beams was thus reflected and part transmitted through this plate. A right angle prism was introduced to allow vertical viewing, and a piece of ground glass was placed on its upper surface. This unit was placed such that A2 and A3 were focused in the plane of ground glass (which was therefore equivalent to the entrance pupil of the eye). The ground glass was viewed through an eyepiece with a calibrated reticule. Hence, it was possible to have a constant monitor on both the size and position of the apertures in the entrance pupil of the eye. Further, one could check centration of the beams by measuring the point of disappearance of a traversing aperture, eg, by rotation of M2 until the subject reported sudden disappearance of the traversing beam. Similarly, by this means, one could also measure the size of the pupil of the eye.

As a further means of constantly monitoring the beam, both objectively and subjectively, a second control system was introduced. A seven watt frosted glass bulb, having a tungsten filament (S2), was placed behind a punctate aperture (A6), which fell at a considerable distance from the short focus lens (L5). The aperture (A6) was imaged just before the beam reached the glass plate. A circular diaphragm (A5) was attached to the surface of lens L5 and allowed only an annular beam to pass. The portion passing through the glass plate enabled a constant check on the positioning of the annular beam, as well as the positioning of the two experimental fields. The patterns as seen through the monitor for additivity and Stiles-Crawford effect experiments (also duplicated in the entrance pupil of the eye) are shown in Figure 8. The portion of the beam reflected by the glass plate, presented an annular pattern in the plane of the entrance pupil. Since only a point source was used, the effect was that of a shadow. The inner border of the annulus had a diameter of 7.85 mm. in the entrance pupil, while the outer border was much larger. Subjectively the outer border was never seen but was limited by the border of the pupil of the eye itself. If the entrance pupil was eight millimeters or larger, it was possible to center the two rings, the inner one formed by the physical annulus and the outer one formed by the pupil border, such that a concentric annulus was seen. This was used as a constant subjective monitor of positioning. If the subject was off center the pattern appeared as on the right hand side of Figure 9. Not only was this device useful in monitoring subjective position, and facilitating initial alignment, but it also served to guarantee the eyes were constantly fully open, and all light incident in the plane of the entrance pupil entered the observer's eye.

The subject was held rigidly with a bite bar and head rest, provision being made for exact alignment by the use of multiple crosshairs for anterior-posterior positioning. Lateral and vertical position was controlled by use of the device mentioned above, and was checked by measuring pupil position by means of the cut-off effect occurring when a traversing spot leaves the field.

To impart greater versatility to the apparatus, auxilliary instrumentation was introduced at aperture A3. The shaft of the multiple aperture wheel was mounted eccentricly, allowing one to traverse a small image of this aperture across the entrance pupil of the eye. Further, since it was possible to vary the size of the aperture, one could measure the additivity effect. Lastly, a mount was prepared which made it possible to introduce photographic plates with clear annuli at A3, Figure 7. This latter addition made it possible to correct the spherical aberration for a given annular zone in the entrance pupil of the eye, as well as to measure the luminous efficiency for different zones of the entrance pupil. Table I gives the sizes and areas of the images in the plane of the entrance pupil of the eye, of the several apertures used. Table II gives the dimensions, in the entrance pupil of the eye, of the projected annuli. The transmittances of the clear annuli in photographic plates are shown in Column 6 of Table II.

In various phases of the experiment, additional lenses were introduced before the eyes of the observer in a trial frame, at a distance of twelve millimeters from the apex of the cornea.

B. The "Blur Apparatus

A second apparatus (Figure 10) was constructed to investigate specifically the effect of blur of a circumscribing border on perceived brightness of a patch. The design enabled an observer to make a brightness match between two fields, one clear,

TABLE I

Apertures Employed in the Additivity Effect and Stiles-Crawford Effect Experiments.

Non-Maxwellian Field (Apertures placed at A3)		Radius in Entrance Pupil	Area in Entrance Pupil
Number	1	1.01 mm	3.17 mm^2
	2	1.54 mm	7.45 mm^2
	3	2.15 mm	14.52 mm^2
	4	2.30 mm	16.62 mm^2
	5	2.59 mm	21.07 mm^2
	6	2.87 mm	25.88 mm^2
	7	3.09 mm	30.00 mm^2
	8	3.37 mm	35.57 mm^2
	9	3.60 mm	40.60 mm^2
	10	3.70 mm	43.01 mm^2
	11	3.89 mm	47.42 mm^2
	12 *	0.30 mm	0.28 mm^2
Maxwellian Field (Aperture placed at A2)			
Number	12 *	0.43 mm	0.58 mm^2

* Apertures used in both experiments. All other apertures were used in the additivity effect experiments only.

TABLE II

Annular Zone Dimensions in the Entrance Pupil

(Annuli placed at A3)	Outer Radius	Inner Radius	Average Radius	Area	t*
Number 1	2.05 mm	1.55 mm	1.80 mm	5.66 mm^2	.503
2	2.47 mm	2.05 mm	2.26 mm	5.96 mm^2	.450
3	3.14 mm	2.55 mm	2.84 mm	10.55 mm^2	.398
4	3.41 mm	2.96 mm	3.18 mm	9.00 mm^2	.225

t* = Transmittance of the clear area of the photographic plates.

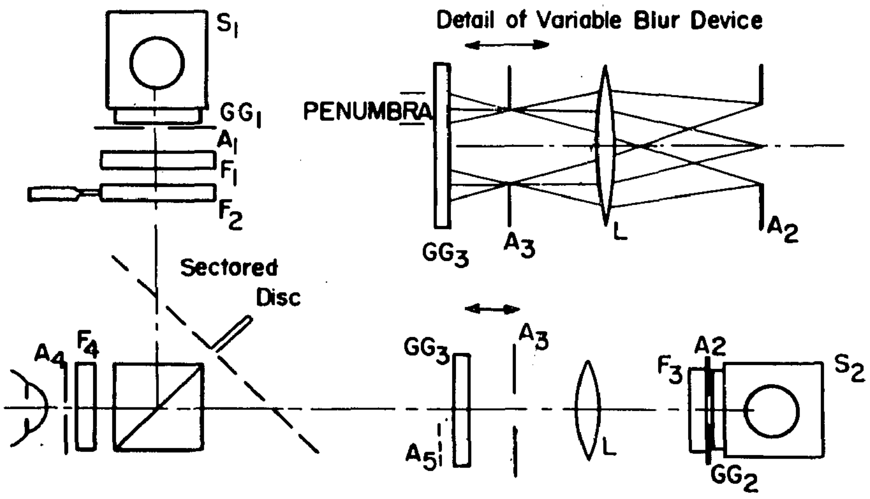

Fig. 10- Blur apparatus used in studies of the effect of blur on photometric matching. Inset shows detail of variable blur device.

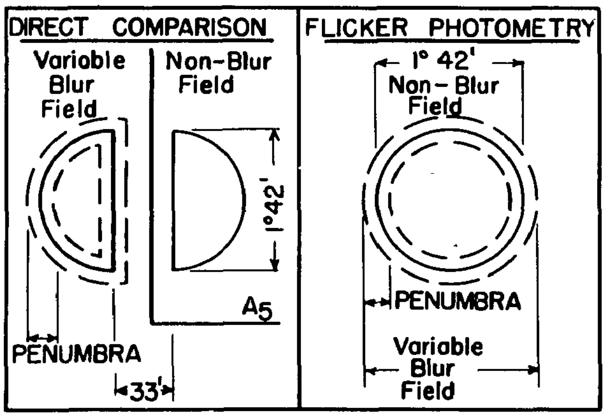

Fig. 11- Subjective patterns viewed using the blur apparatus. The dashed lines indicate the limits of the penumbra for a given test situation. The non-blurred state is included.

and one having the variable blur border. Both the flicker photometry method and the direct comparison method were utilized. The two fields presented for viewing (Figure 11), were made either to overlap for flicker presentations, or by simply blocking off the appropriate parts to form a bipartite field.

The clear or non-blur field will be considered first. A 60 watt tungsten source, having a frosted bulb (S1), was enclosed in a box. Upon the front surface of this box was placed a piece of ground glass (GG1). The field was sharply defined by a round aperture (A1), attached to the ground glass. This aperture subtended 1° 42' at the eye. The light then passed through crossed polaroids (F1,F2), which were used in making the match. This beam was then reflected by a beam splitter through an interference filter (F4) and a 3 mm. artifical pupil (A4) into the eye.

A second source (S2) (also a 60 watt frosted tungsten bulb) provided illumination for the field having controllable blur. As in the first case, the light passed through a diffusing ground glass

(GG2) and a fixed aperture (A2, one inch in diameter). The aperture was placed at the focal point of a + 4.50 diopter lens (L). Thus, the aperture was effectively projected to infinity and subtended an angle of 6° 34' at the lens. A moveable aperture (A3) of the same size at A1, was placed between the lens and a piece of ground glass (GG3). When A3 was in contact with GG3, a sharp border was obtained and since GG3 was at the same distance, ie, forty-five centimeters, from the beam splitter as A1, a field was seen by the eye in the same plane, and of the same size as A1. As A3 moved from GG3, greater blur was visible. The degree of blur was proportional to the width of the penumbra at the eye, which was a function of the distance (x) of aperture A3 from the screen. When A3 is displaced in a fore and aft direction to change the width of the penumbra, the central field luminance is not affected. As in the previous case, the field was seen through an interference filter (F4) and an artificial pupil (A4). A filter (F3) was placed before A2 to approximately equate the effect of the polaroids.

For the flicker condition circular apertures A1 and A3 were completely exposed, and a sectored disc interposed in the path of the beam. For the bipartite match, half of A1 and A3 were covered, the sectored disc removed, and a masking aperture (A5) placed in front of ground glass three (GG3), A1 was projected upon this mask (A5) visually. This later element prevented border effects, halation, and scatter from the ground glass (GG3) from interfering in the judgement of match. This necessitated separation of the two matching fields by 33' of arc. Figure 11 shows the fields as seen for each test condition. The dashed lines reveal the extent of the penumbra in this hypothetical case. The test luminance level as measured with the Macbeth illuminometer was 1.104 millilamberts, and retinal illuminance was 99.51 trolands. The sectored disc (50-50) rotated ten cycles per second.

C. <u>The Flicker Photometry Apparatus</u>

A third apparatus was constructed (Figure 12) to compare, by means of flicker photometry, the Maxwellian and ordinary methods of viewing. Two

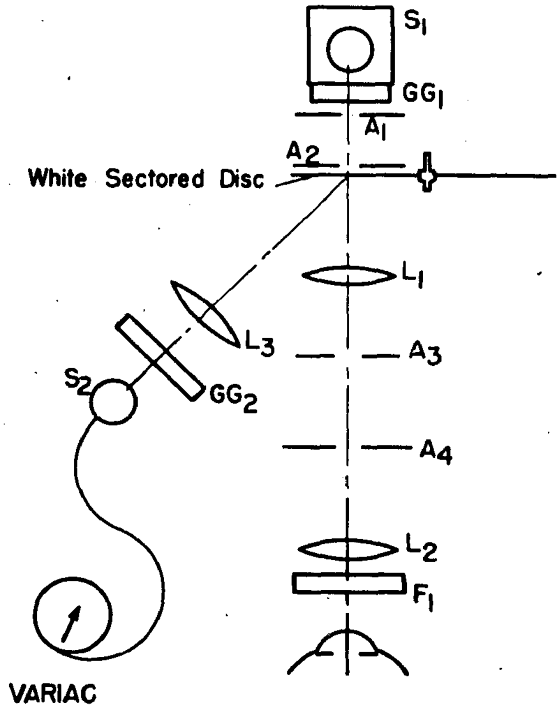

Fig. 12- Flicker photometry appartus, used for studies of the additivity effect, and for the equating of a non-Maxwellian with a Maxwellian field.

experiments were conducted using this equipment. The additivity effect was determined using the flicker method, and an investigation of the effect of varying the size of the Maxwellian and non-Maxwellian beams in unison was undertaken.

Source S1 at the top of the figure (12) was s sixty watt frosted tungsten bulb. This was further diffused at ground glass (GG1). Aperture A1 was imaged by the lens (L1) in the plane of A3. By appropriate settings of the size of A1 and A3, control of the size of the beam entering the entrance pupil of the eye could be obtained. This will be developed further below. Aperture A3 was imaged by lens L2 in the plane of the entrance pupil. If the image of A1 in the plane of A3 was larger at any time than A3, A3 acted as the limiting element in determining the size of the beams from both sources in the entrance pupil of the eye. If A1 was made very small, it determined the size of the beam from source S1. At the same time, varying A3 in size allowed control of the size of the beam originating at source S2 in the entrance pupil, and hence the additivity effect could be investigated.

Source S2 was an eighteen amp, six volt, ribbon filament lamp, the intensity of which could be controlled with a variac (method of matching). An image of the ground glass (GG2) was formed on the white cardboard sectored disc by lens L3. Aperture A2 was placed behind the disc merely to limit the stray light in the system. The sectored disc plane was imaged at aperture A4, which was conjugate with the retina. This aperture acted as the field stop and was seen through lens L2. It subtended a visual angle of 56'.

An interference filter was placed between L2 and the eye, and alignment of the eye was assured by multiple crosshairs and the cut-off effect. The flicker rate was 10 cycles per second. It may be noted that with the apparatus it was possible to superimpose two exactly equal apertures on the entrance pupil and on the retina, allowing comparison between Maxwellian and ordinary methods of viewing under identical circumstances.

Since the sectored disc is conjugate to the retina, this represents the ordinary method of viewing aperture A4; and since GG1 is conjugate to the entrance pupil, this provides Maxwellian viewing of A4.

III PROCEDURE

Three emmetropic subjects, R.V. age 38, B.W. age 28, and A.M. age 24, served as subjects in each of the experiments. In addition to these three regular subjects, four added subjects served in the "penumbra blur" study. The left eye only was used in each of the investigations, except the one performed with the blur apparatus. The pupil of the eye used was dilated in each experiment (except in the variable penumbra blur study) by the use of paredrine hydrobromide, 1% ophthalmic solution. This drug produces mydriasis, with only slight cycloplegia. Experimentation was not undertaken until dilation was complete. One drop was administered every fifteen minutes during each experimental session in order to maintain constancy of dilation. This was intended to prevent the variation of the Stiles-Crawford effect with time after instillation of a mydriatic as was reported by Ronchi (26).

The logical starting point of the investigation was the measurement of the Stiles-Crawford effect. A small circular Maxwellian beam, 0.86 millimeters in diameter, was traversed horizontally across the entrance pupil of the eye, and the brightness of its field was matched with that of a comparison field (non-Maxwellian). The narrow beam of light, having as its source the comparison field, was centered in the entrance pupil. The bipartite field viewed by the subject subtended a visual angle of 1^{o} 36' (Figure 9). The three regular emmetropic subjects served in this experiment and only the left eye was used.

Alignment presented a considerable problem, and hence, great care was employed in the initial positioning of the subject and in the maintenance of his position. Position was checked relative to a fixed zero point both by the disappearance of field technique, and by the constant monitoring provided by the concentric annulus stimulus. These controls were described in the apparatus section of this report. Head position was maintained by the use of a bite bar and head rest, and the subject was asked to maintain his position, once aligned, until the completion of the experiment, or until a suitable break point was reached. Although photopic levels of illumination were used exclusively, dark adaptation of approximately twenty to twenty-five minutes was completed prior to making measurements. Prior to making matches to a new stimulus presentation, a light adaptation time of three minutes was allowed. The aforementioned controls were used in all experiments described in this discussion.

The base or reference luminance level employed on the direct comparison apparatus was 7.74 millilamberts (6.90 trolands retinal illumination) in the plane of the pupil for white light, and 4.35 millilamberts (3.88 trolands retinal illumination) for testing using monochromatic green light. The only exception to this was during the measurement of the Stiles-Crawford effect when the non-Maxwellian field was traversed across the pupil. In this case the base reference luminance level for the monochromatic green light was 54.00 millilamberts (48.18 trolands retinal illumination). In all other experiments, as the stimulus conditions were varied, the comparison field luminance level increased, while in this last mentioned instance, the roles of test and comparison beam were reversed. This shift of base level was employed in order to maintain the readings within the range of the neutral density wedge.

After a suitable practice period prior to readings, subjects were asked to make six matches at each test setting of the traversing Maxwellian beam in measuring the Stiles-Crawford effect. Settings were made every half millimeter across the entrance pupil of the eye starting at the center of the pupil and progressing outward until the border of the pupil was reached. Care was taken not to allow cutoff during extreme readings. At the completion of a traverse from the center to either the right or left edge of the pupil, the Maxwellian beam was again centered in the pupil and a traverse was made in the opposite direction. In a second session, initial direction of traverse was reversed. Mean relative luminous efficiency values (η_r) were computed for each session, and the mean of the results of two sessions were combined to produce the data points. Thus each point in the graphs represents the mean of twelve readings. As alluded to above, white light and a monochromatic green light were used routinely throughout the entire series of experiments. A

Baird interference filter, with a dominant green wavelength 552 $m\mu$ was used, since according to Stiles (13) perceived hue changes little in this region of the spectrum with pupil traverse.

The Stiles-Crawford effect experiment for a traversing non-Maxwellian beam was conducted along exactly the same lines. In this instance, traverse across the pupil was provided by rotating the eccentric shaft on which A3 was mounted.

The same apparatus was employed in the testing of the additivity effect. B.W., R.V., and A.M. served as subjects and the left eye only was used. In this instance, a Maxwellian beam, diameter 0.857 millimeters, was centered at the point of the maximum of the Stiles-Crawford effect as determined in the initial experimental procedure. This policy was adopted in order to simplify the theoretical treatment involving the integration of the Stiles-Crawford effect over the pupillary area. The pupil area of the non-Maxwellian beam was varied by introducting eleven different circular apertures in the plane of A3 (Figure 7, Table I), which were also centered about the horizontal Stiles-Crawford maximum. Care was taken to avoid vignetting at the edge of the pupil. The monitoring annulus, provided a most useful means of accomplishing this, since continous visibility of the peripheral pattern guaranteed the inclusion of all rays within this area of the entrance pupil. Entrance pupil diameters (see Table I) varied from 0.596 mm. to 7.78 mm. with the corresponding areas ranging from .28 $mm.^2$ to 47.42 $mm.^2$ Aperture number ten was used only when aperture number eleven would not fit within the entrance pupil as its size differed little from aperture nine. As in the previous case, both white light and wavelength 552 mμ were used, and each series of readings was conducted twice. Apertures were presented in order from small to large, and ten readings were taken for each aperture at each test session. Data points therefore, represent means of twenty readings. Relative luminance values represent ratios of the luminance of the comparison field to that of the test field. The reader is referred to the introduction of the paper for the exact meaning of luminance in this context. Prior to initiation of an experimental session, ten practice readings were taken routinely, and three minutes of light adaptation was allowed for each change in stimulus presentation. The matches were made using a bipartite field, and the subject was asked to make a luminance match between the two halves of the field with fixation at the midpoint of the line dividing the two fields.

As indicated in the introduction, several important questions arose when the larger sized apertures were used, since they caused considerable blurring of one half of the bipartite field. Several experiments were designed to investigate the effect of blur on matching brightness and the effect of eliminating the blur upon the additivity data.

The first such experiment employed the blur apparatus in which two fields were compared. The degree of blur of the border of one of the fields was variable. Seven degrees of blur were considered. A table superimposed on Figure 20 gives the distance that the variable position aperture was moved along the x axis away from the point of contact with the ground glass screen, and the corresponding angular subtense of the penumbra at the eye. The values of x and angular subtense (ξ) are related by the following expression:

$$\xi = 0.748\ x$$

where ξ is expressed in minutes of arc and x in millimeters of displacement.

The three regular observers reported that the maximum blur encountered with the blur apparatus was of the same order as, or slightly greater than, that encountered during the additivity effect experiments on the direct comparison apparatus using monochromatic green light. It should be pointed out that matching a clear with a blurred area presents difficult problems. One contends with perceived differences in surface texture, Mach rings, etc., under these conditions. Hence, to try to avoid some of the serious criteria problems arising, subjects were instructed to match the luminances in the central portions of the fields. To avoid the added problems of blur resulting from chromatic aberration and chromatic magnification, only light of wavelength 552 mμ was used. A brightness match was made using both flicker and direct comparison photometry. In the direct comparison match the fields were separated by 33' of arc to avoid any overlapping of gradients. Both regular experimental subjects and four other highly trained observers made these matches. The right eye only was used.

In any one session, readings were taken for both flicker and direct comparison methods. Care was taken to vary the order of experimental presentation, that is, flicker first, bipartite first, most blurred first, and least blurred first. Thus, for each subject, twenty readings were taken for each stimulus presentation. The results may be expressed in terms of the ratio of the luminances of the two fields required for a match. A composite graph of the results of seven subjects is presented. The results indicate that the brightness of a blurred field is underestimated, and that such effects were considerably less using flicker photometry.

In view of the general results found in the above experiment, several new approaches were adopted. Two experiments were designed which were adapted to the direct comparison apparatus. The first was to determine the effect of introducing different lenses before the eye of the observer and noting the effect upon the match. Three different apertures were used at A3 (the "non-Maxwellian side" of Figure 7), numbers 12, 7, and 11, ie, smallest, large, and largest (see Table I for sizes), and on the Maxwellian side a fixed small aperture was employed, 0.857 millimeters in diameter at the entrance pupil of the eye. Positive and negative lenses ranging in diopter steps from + 3 to - 3 diopters were employed. The lenses were placed approximately fifteen millimeters in front of the entrance pupil of the eye.

The lenses not only affect the vergence of the rays incident at the eye, but also affect the size and position of the exit pupil of the apparatus. The head was moved to compensate the displacement of the exit pupil and care was taken to keep the beams entering the pupil centered at the point of maximum luminous efficiency.

The lens changes the size of the exit pupil of the instrument by a factor equal to (1 - hF), where h is the distance (meters), and F is the refracting power of the lens.

Thus, for each size of aperture stop, seven different stimulus presentations were made, ie, no lens, +1, +2, +3, -1, -2, -3. For each one of these presentations, during each test session, ten readings were taken. In addition, as a check on reliability, a second set of ten readings was taken for the no lens situation. Thus, for each aperture, for each session, eighty matches were made, or a total of 240 matches per session. Two sessions were held for each subject in the testing of this phase of the work, and thus each experimental point represents the mean of twenty readings. To minimize criteria problems, sets, and other foreign variables, the ordering of apertures and lenses was completely randomized.

The data (Figures 21,22), which will be discussed in the results section, are plotted in a manner emphasizing the "blur effect". Essentially this may be considered as extending the additivity data (Figures 18,19) in the third dimension, perpendicular to the base line, but not quite perpendicular to the log relative luminance, log area plane. The axis of abscissas now represents the dioptric lens power. The area of the beam in the entrance pupil of the eye varies for the several lenses used, and is slightly greater for negative power lenses than for positive power lenses. This accounts for this plane not being perpendicular to the reference plane.

Since monochromatic light was used, the low power supplementary negative lenses tend to decrease (or eliminate) the blur induced by the spherical aberration of the eye, and thus give a truer luminance match, one which is independent of the blur effect. Higher negative power and all plus power lenses tend to increase the blur of the field.

A second related experiment was conducted. The small circular aperture was maintained at A2 (Figure 7), and was imaged in the plane of the entrance pupil at the point of maximum luminous efficiency. Photographically produced annular apertures were substituted for the round apertures at A3. Images of these apertures were formed in the entrance pupil of the eye, and were made concentric to the point of maximum luminous efficiency. By using these annular apertures it was possible to correct the refraction of the eye for the given zone, and finally to get a measure of the luminous efficiency (η_r) for that zone.

If one makes use of the same principles in predicting the retinal response for a zone as for a circular aperture in the entrance pupil, it may be stated for a zone, the field of which constitutes the test field

$$V_T = k' B_T \, t \, \eta(r_T) \, 2\pi \, r_T (\Delta r_T).$$

In this relationship, t represents the transmittance of the annular zone of the photographic plate, and $2\pi r_T(\Delta r_T)$ represents the area of the zone with the average radius of the zone equal to r_T and the width Δr_T. Similarly for a small circular beam in the entrance pupil, the field of which constitutes the comparison field

$$V_C = k' B_C \eta (r_C) \pi r_C^2 .$$

Since a photographic plate was not employed in the experiment, a transmittance factor is not necessary. For a very small circular field centered at the maximum of luminous efficiency, $\eta (r_C)$ is approximately equal to one. Thus, the expression for V_C reduces to

$$V_C = k' B_C \pi r_C^2 .$$

When the two fields match in brightness

$$V_C = V_T ,$$

and

$$\eta (r_T) = \left[\frac{(r_C)^2}{t \ (2r_T) \Delta r_T} \right] \frac{B_C}{B_T} .$$

As in the previous case, ten readings were taken for each annulus at each of two sessions, with data points again designating the mean of twenty readings. Four different annuli were used; see Table II for their full details, including the correction factor for transmittance.

As indicated in the introductory statements, the flicker photometer apparatus was used for two experiments. The first of these was a confirmation of results obtained using a bipartite field on the additivity effect. In this instance, the image of aperture A1 (Figure 12), in the plane of aperture A3, was just smaller than the smallest diameter used at aperture A3. Thus, the aperture for the Maxwellian beam was aperture A1 rather than aperture A3. However, aperture A3 acted as aperture stop for the non-Maxwellian beam and this was imaged in the plane of the entrance pupil of the eye. Table III gives the diameters and areas in the entrance pupil of the eye for the various settings. Great care was taken to avoid vignetting, especially when the largest aperture stop was employed. The subject was held in place by a bite bar, and

TABLE III

Entrance Pupil Dimensions Used in Conjunction With Additivity Effect Studies on the Flicker Apparatus

	Diameter	Area
Maxwellian	1.23 mm	1.18 mm^2
Non-Maxwellian		
1	1.60 mm	2.01 mm^2
2	4.07 mm	13.02 mm^2
3	6.08 mm	29.05 mm^2
4	8.46 mm	56.21 mm^2

centration was checked by a vignetting method, and by the use of multiple crosshairs for anterior-posterior positioning. For each series of readings, the presentation of the apertures was reversed. The subject took ten settings for each stimulus condition during each of two sessions, after appropriate practice readings and adaptation. Consequently each data point represents a mean of twenty readings. The luminance of the comparison Maxwellian field measured in the plane of the pupil was .425 millilamberts (1.60 trolands retinal illuminance). Only monochromatic light of wavelength, 552 mμ. was used, and the flicker match was made by varying the intensity of the ribbon filament source with a variac. The field viewed on the retina subtended 56' of arc. The flicker rate was maintained at ten cycles per second throughout.

A second experiment performed on the flicker photometer apparatus was a comparison of the luminance of a Maxwellian field with the luminance of a non-Maxwellian field. In this instance the aperture A3 served as the aperture stop for both beams. Matches were made for different entrance pupil areas. Areas in the pupil correspond to those listed for the non-Maxwellian beam, numbers 1,2, and 4 in Table III. In addition, an annular aperture was introduced which was conjugate to the entrance pupil, having the following dimensions in that plane:

Outer diameter	8.00 mm
Inner diameter	6.00 mm
Area	21.992 mm^2

This was used to determine if there was any difference for an annulus presented near the border of the pupil.

As in the case of the first experiment on the flicker apparatus, monochromatic green light was used, the flicker rate was 10 cycles per second, the retinal image size was 56' of arc, and the luminance level in the plane of the pupil was .425 millilamberts. Proper adaptation, centration and practice conditions were maintained, and the data points represent composite means of two experimental sessions of ten readings each. Again, ordering of presentation was reversed for each of the two series. The axis of ordinates is the ratio between the luminance of the fields, while the axis of abscissas gives the area in the entrance pupil of the eye.

IV RESULTS

A. Studies on the Stiles-Crawford Effect

In order to provide a control, and a basis for theoretical analysis, the first experiments conducted were the measurements of the Stiles-Crawford effect for both white light and a monochromatic green light. These studies were completed first to assure the writer that his subjects would yield typical curves for the Stiles-Crawford effect. The reason green light was chosen was that Toraldo's data were collected using a green wavelength, and the particular wavelength band used does not change appreciably in perceived hue with peripheral excursion of the beam in the pupil. The initial data were collected using a traversing Maxwellian beam and a fixed centered non-Maxwellian beam. Although they are reported with the above group of data, data for a non-Maxwellian beam traversing the pupil were obtained several months afterward. The equipment required a major modification in going from the one kind of experiment to the other. Because the maxima of two of the subjects shifted slightly, and because it is desirable to have the maxima superimposed, all data on the Stiles-Crawford effect, except Figure 16, are plotted with the points of maximum efficiency at zero.

Figures 13,14, and 15 are the plots of the mean values for each of the subjects under the three conditions of testing. All data has been taken in these studies on the left eyes of the subjects. Right and left imply traverse right and left in the entrance pupil of that eye and correspond to nasal and temporal. It is found that subjects B.W. and R.V. had a maximum of luminous efficiency at approximately 0.5 mm. right of the center of the pupil, and subject A.M. had a maximum at approximately 0.5 mm. left of the center of the pupil.

The top two curves in each figure represent traverses only of the Maxwellian beam, the blackened circles designating the values for white light, the non-blackened circles designating the values for monochromatic green light. The two bottom curves are a comparison of a Maxwellian beam traverse (circles) and a non-Maxwellian beam traverse (triangles),

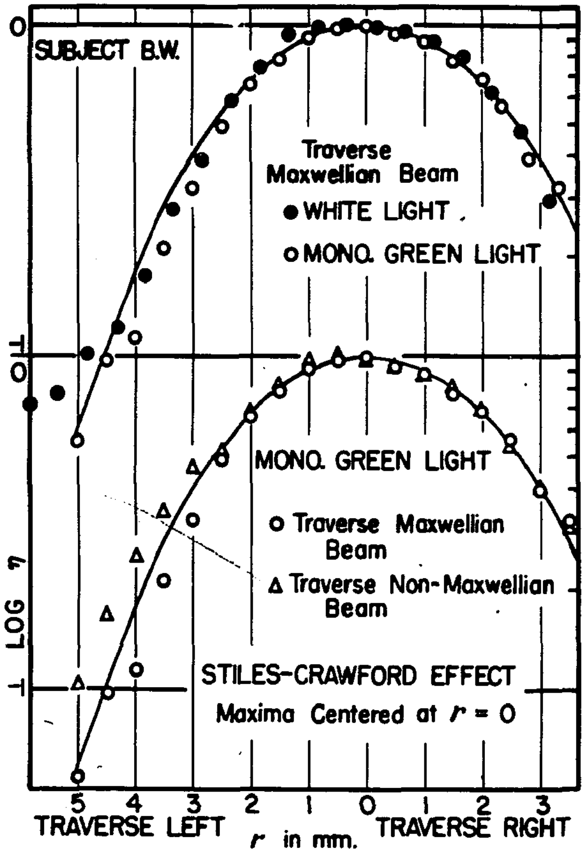

Fig. 13- Stiles-Crawford effect measurements of subject B.W. for three different experimental conditions with maximum at r = 0. The theoretical curve proposed by the author is included.

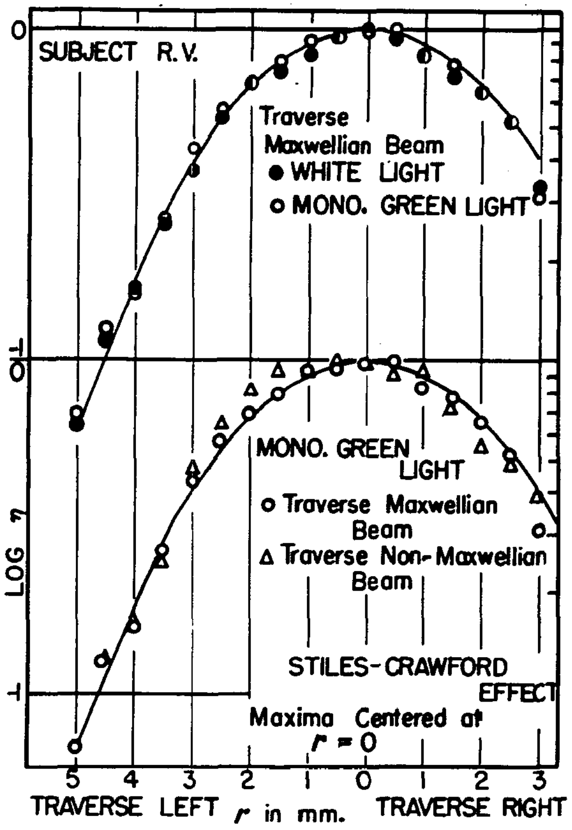

Fig. 14- Stiles-Crawford effect measurements of subject R.V. for three different experimental conditions with maximum at r = 0. The theoretical curve proposed by the author is included.

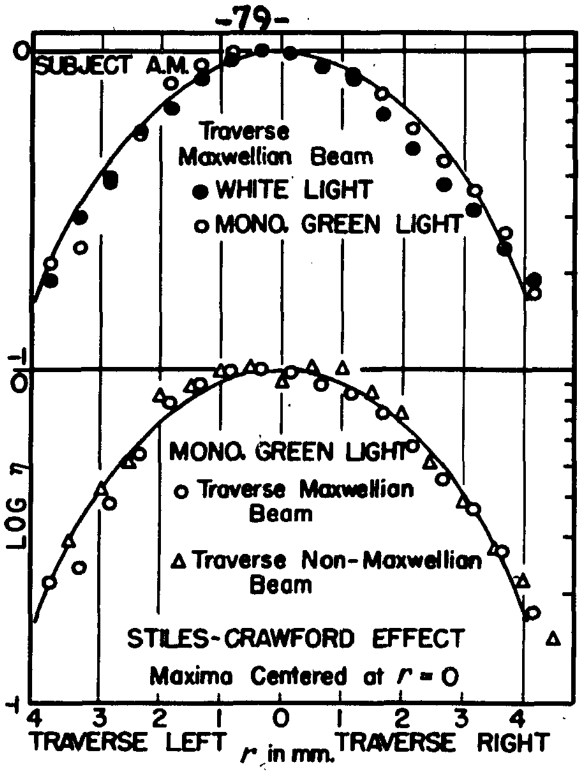

Fig. 15- Stiles-Crawford effect measurements of subject A.M. for three different experimental conditions with maximum at r = 0. The theoretical curve proposed by the author is included.

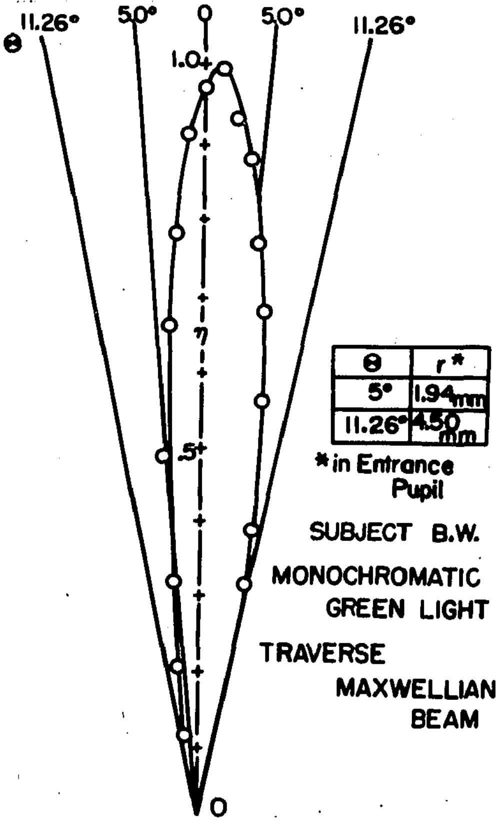

Fig. 16- The Stiles-Crawford effect as a function of acceptance angle (θ).

both using monochromatic green light. The non-blackened circles of both top and bottom sets of data for each figure represent, therefore, the same data. It is felt that the differences between data are of no consequence.

Looking closely at the data, certain things are evident. First, there is little evidence of any upturn on most of the data for peripheral readings. The one major apparent shift in the data occuring on the non-Maxwellian traverse of subject B.W. This was surprising in that he was by far the most experienced observer. This discrepancy appears to be the result of a shift or apparent misalignment of the subject during one of the test sessions, since all the points seem displaced approximately an equal amount from the other data. Any other interpretation would be doubtful in light of the virtually perfect agreement manifest when considering the remainder of the test points.

The curve proposed by the author, ie,

$$\eta_r = .25\,(1 + \cos 9.5\theta)^2$$

where

$$\theta = .045r ,$$

has been fitted to the data. The fit is quite good for all observers, and this data, on this basis would seem to correspond well with that of Stiles and Crawford (Figure 1). Prior to concluding this discussion of results for the Stiles-Crawford effect experiments, to help give the reader an idea of the magnitude of this effect, a curve is plotted (Figure 16) in the fashion suggested by Toraldo (22) showing the same data as represented by the non-blackened circles in figure 13 for subject B.W., but with the maximum not at zero. This is a plot of η_r as a function of angle of incidence at the retina, or what might be called acceptance angle (θ).

Thus, these preliminary experiments showed that little or no difference exists between the Maxwellian and non-Maxwellian case for the Stiles-Crawford effect, and that little or no difference exists between values for white light and for green light of wavelength 552 $m\mu$. The proposed theoretical distribution fits the data well.

B. The Comparison of a Maxwellian With a Non-Maxwellian Beam.

Although the data complied by traversing punctate Maxwellian and non-Maxwellian beams across the entrance pupil was one of the main tests of the luminous efficiency of such beams, it was felt that the more important test was that performed on the flicker photometer apparatus. Here, two beams sharing the same aperture and field stop, therefore having the same dimensions, and differing only in being Maxwellian or non-Maxwellian, were compared for different size entrance pupils. The two stimuli differed only in the nature of their physical distributions on the retina. The area of the entrance pupil was varied from 2.011 to 56.213 mm.^{2} (see Table III). Figure 17 shows the data for three subjects. Certainly if differences existed between the two situations, they would have been manifest in this experiment. As evidenced by the data, they were not. Values greater than one indicate that the Maxwellian image required less flux than the non-Maxwellian image in order to appear equally bright.

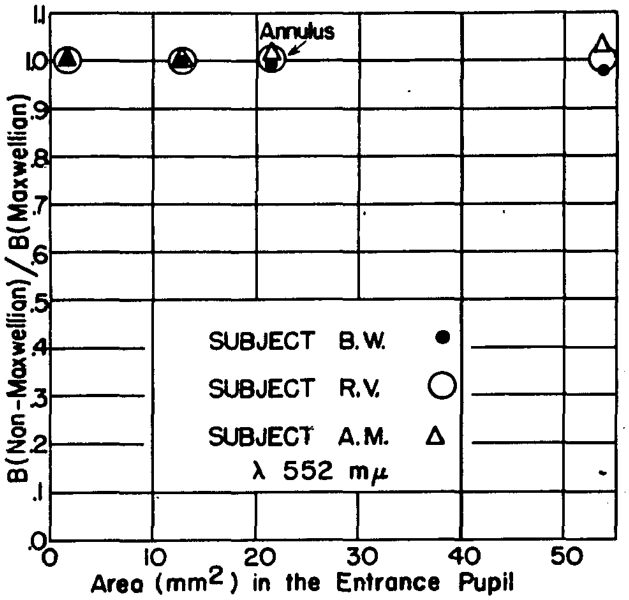

Fig. 17- Comparison of luminance of Maxwellian with non-Maxwellian field using equal but variable size entrance pupils and retinal image.

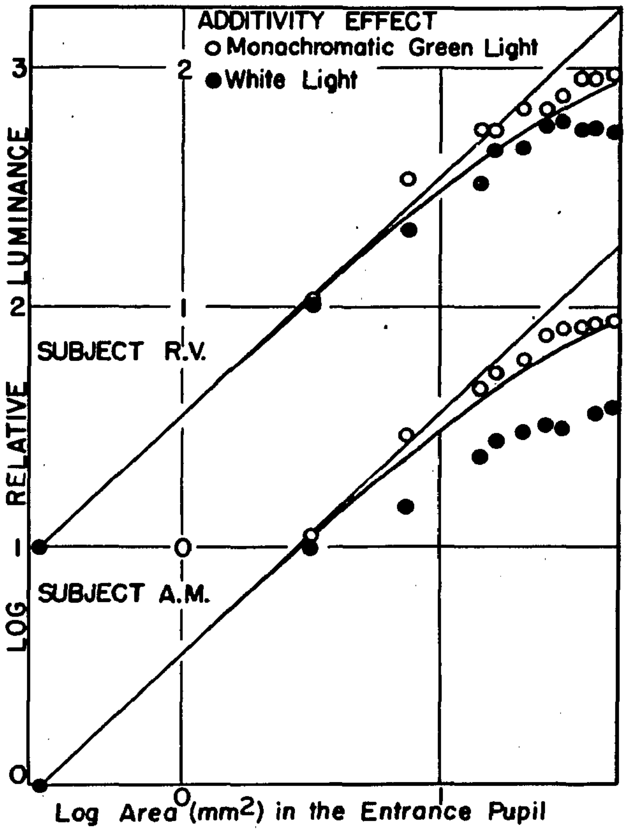

Fig. 18- Data of subject R.V. and A.M. for the additivity effect. Input line and predicted curve are included. Data for white and monochromatic light are presented.

It is apparent that presenting the beam in annulus form makes no difference. Thus, one should not expect to get any difference in relative brightness in an additivity experiment if either a Maxwellian or non-Maxwellian method of viewing is used.

C. Additivity Experiment

The additivity effect was studied under numerous conditions. The size of the non-Maxwellian beam was varied while that of the small reference Maxwellian beam was kept constant. Centration was at the point of the maximum of the Stiles-Crawford effect. It is felt that to this extent previous studies on the additivity effect have erred, for in order to properly integrate the theoretical curve directly, one must follow this course. Data were taken using both green monochromatic light and white light. The initial results using the direct comparison method, were, to say the least, confusing. The spread of the individual curves varied from moderate negative discrepancies (relative to predicted values) to large positive discrepancies of the same degree as those found by Bocchino (9) and Toraldo (10). The problem thus resolved itself into one of defining the causes of these discrepancies and assessing the relative contribution of each variable. In addition to the physical and physiological aspects, criteria problems tended to complicate the issue. It is interesting to note that while one subject (B.W.) reported constantly increasing luminance levels with increasing pupil size, the other two observers noted, especially with white light, that brightness did not increase after a certain point. This aspect is demonstrated in the data. It will be shown, that the discrepancies are referrable largely to criteria problems, and what will be termed the blur effect. Several of the remaining experiments were designed to clarify and delineate this effect.

On the direct comparison apparatus, successively larger apertures were imaged in the entrance pupil of the eye, and brightness matches made for each successive level after appropriate adaptation periods. The data are presented in figures 18 (subject R.V. and A.M.) and 19 (subject B.W.). The blackened circles are the data for white light and the

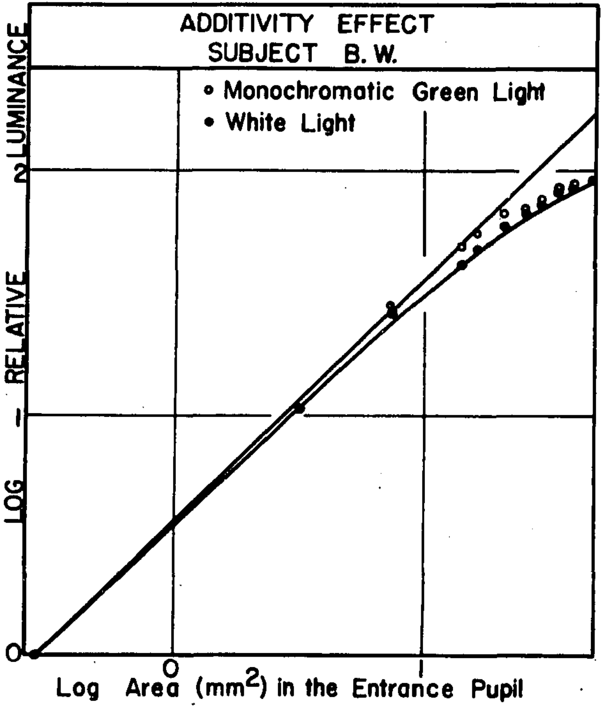

Fig. 19- Data of subject B.W. for the additivity effect. Input line and predicted curve are included. Data for white and monochromatic light are presented.

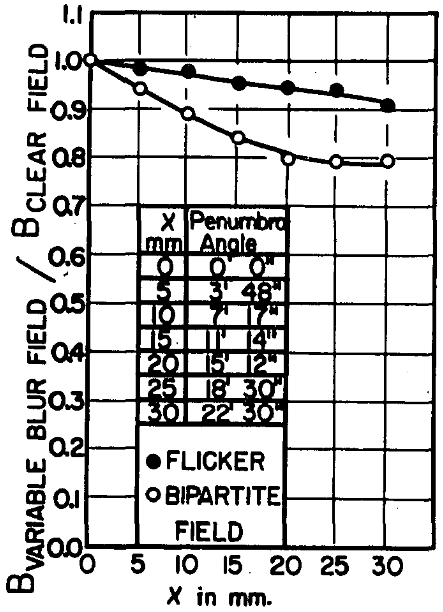

Fig. 20- The effect of varying the angular subtense of the penumbra (blur apparatus) on perceived brightness of a test field.

non-darkened circles are the data points for monochromatic green light.

Certain things will be noted by glancing at Figure 18. First, all data points for white light show less additivity than for the green. This is particularly marked for subject A.M. whose data for white light corresponds roughly to Toraldo's data for monochromatic green light, (Figure 6). Subject A.M.'s data for monochromatic green light follows the predicted curve quite well. It will be noticed that there is a flattening of the curve for the larger apertures. In the case of R.V. a similar picture exists in the white. It will be noted that the white curve becomes flat, indicating no apparent increase of luminance with increase in pupil area, for an entrance pupil diameter greater than five millimeters. Subject R.V. interestingly follows the values predicted by Troland's additivity principle (the "input" curve) for smaller sized apertures, only shifting to the Stiles-Crawford predicted curve for larger apertures.

The data for subject B.W. appear in Figure 19. As in the previous two cases, except for the last point, in each pair of plots the white light data appears to be less bright than that for green light, although the differences are less than those manifest by the other subjects. In order to enhance these differences, the points have been made smaller. Further, subject B.W., in a manner similar to subject R.V., showed a tendency to follow values predicted by the relationship formulated by Troland for smaller apertures. A question arises as to what is the true relationship, since blur first becomes subjectively evident largely in the region where the data seems to shift and settle down to values approximating the curve predicted by inclusion of the Stiles-Crawford effect in the integration.

Certain possible answers are available to the investigator when faced by what appear to be several highly contradictory experimental results. Either, some one (or more than one) variable is uncontrolled, or perhaps the test itself is not a valid measure of the variable being investigated.

Obviously, the latter possibility is not the answer, because the method is inherent in the definition of the additivity effect itself. Rather, as will be seen, the method allows entry of virtually uncontrolled variables, each of which must be separated. However, one cannot cease to consider the method used in this part of the experiment, because it represents the manner in which the eye normally operates. Therefore it becomes requisite to investigate the source of variability. The first major problem is that of criterion employed in making of the match. As the diameter of the projected entrance pupil is increased, one faces a problem of blur due to aberrations of the eye. As will be noted below, the eye does not appreciate low levels of blur. That is, its border sharpening mechanisms clear up a certain amount of the blur which normally exists. Thus, depending upon the level of blur, and individual sensitivity to these effects, the subject is faced with a problem of difference in textures, sharp versus blur borders, Mach rings, overlap of the fields, etc. Thus, obviously the criteria used by subjects for the no blur case, and for a low degree of blur in this experiment, are different from those used for higher orders of blur. In an attempt to minimize criteria problems, the following instructions were given to the subjects:

> "Disregarding the blurredness and the accompanying lack of definition of the stimulus, if blurred, in the right half of the field, vary the luminance of the clear left half of the field so that it appears to match the central portion of the blurred right half of the field while fixating the center of the dividing line."

Obviously, even this rigid statement did not completely rule out criteria problems.

The remainder of the experimental procedures were designed to isolate and determine the role played by the blur and the Stiles-Crawford effects.

D. Blur Experiment

As has been pointed out above, as the size of the aperture imaged in the entrance pupil was increased, one half of the bipartite field became progressively blurred. This was due, in the case of white light to both spherical and chromatic aberration. Thus, an experiment was designed to test the effect of blur on brightness matching.

In this experiment, by optical means, an object was blurred and compared by both flicker and direct comparison techniques to a sharply defined object. Figure 20 shows the data for seven subjects, the three regular experimental subjects, and four highly experienced visual observers. x is the distance of the moveable aperture from the screen and the superimposed table gives the equivalent angle subtended by the penumbra at the eye. It will be noted that blurring the edges of an object decreases its perceived brightness. To prevent any problems arising from overlap, separation between the bipartite fields was made more than adequate. Certain facts become evident when one observes the data. First, the initial decrease in perceived brightness occurs before blur is perceived. Occasionally , in the first few settings the subject would comment on the increase in clarity of the object when, in reality, it had become more blurred. Further, for flicker, over the range tested only a mean decrease of 0.04 log unit in brightness was noted, while for a bipartite field for the same range, apparent decrease in brightness was 0.10 log unit. Note, testing was only conducted with monochromatic green light. The maximum level of blur used in the experiment was qualitatively evaluated by the regular subjects as being of the same order as that experienced with large pupils and green light on the direct comparison apparatus. It should be noted that variability in settings increases slightly with increased blur, this being true also for the additivity experiments.

These results indicate that the role played by blur may help explain (1) the discrepancies found in the data of previous authors, (2) the differences in additivity between monochromatic green and white light, and (3) the flattening out of several of the surves when the larger apertures are used.

The variable penumbra blur experiment thus demonstrated the necessity of eliminating the effect of blur to get a valid measure of the contribution ascribable to the Stiles-Crawford effect for a beam having a large diameter in the plane of the pupil. Two alternatives are possible, ie, either remove blur, or substantially reduce the blur effect by employing flicker photometry. Both of these alternatives were adopted in the experimental program.

E. Dioptric Blur and Additivity

Applying, and further testing the conclusions drawn above, the additivity effect was measured for three different aperture sizes, smallest (#12), intermediate (#7), and largest (#11) (See Table I), with a series of six different spherical lenses having dioptric power of +1, +2, +3, -1, -2, -3, and also without any lens. Figure 21, depicts the results for the three subjects. The data have been treated by plotting against the dioptric power of the lenses the ratio of the luminance of the comparison field to that of the test field required to make the two fields match in brightness.

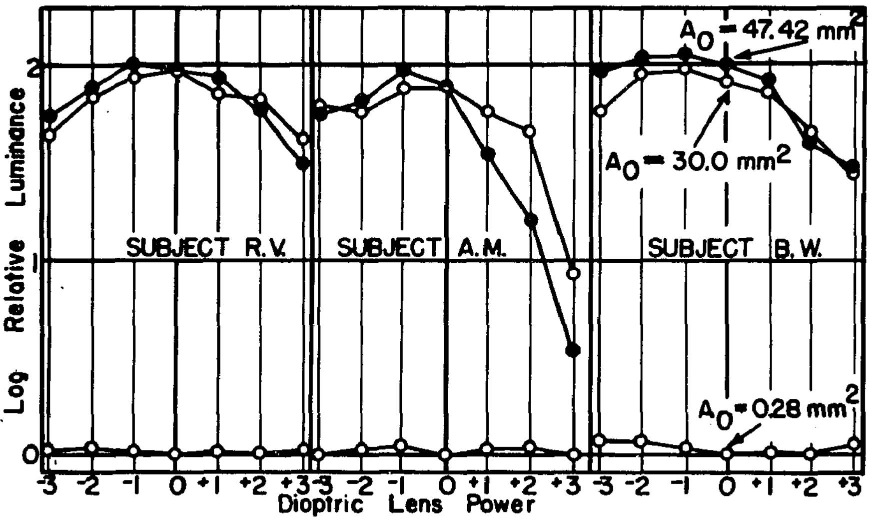

Fig. 21- The blur effect for three different size apertures as a function of dioptric power placed before the eye. Areas of apertures apply only to the zero power case.

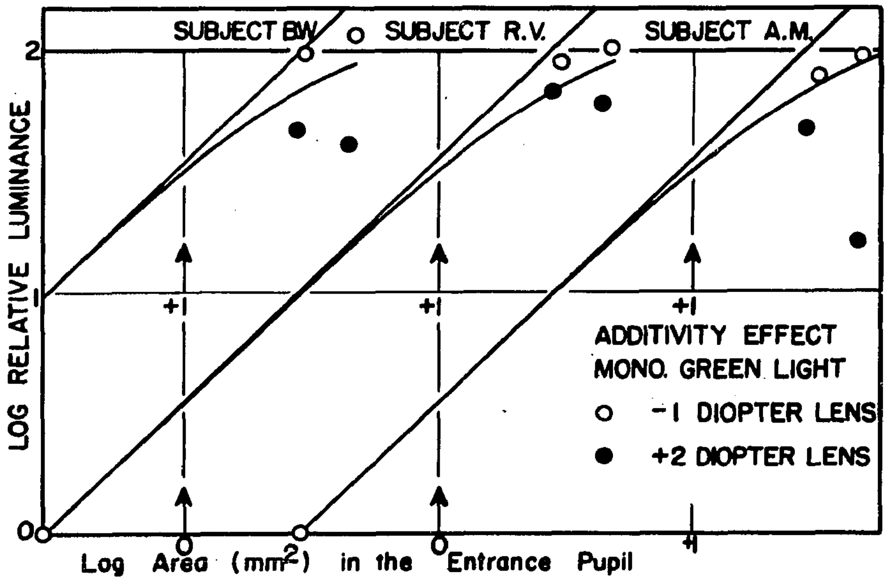

Fig. 22- Additivity effect for three apertures with a -1 and +2 diopter lens placed before the eye. Note, area changes slightly with lens power. Input line and theoretical additivity curve have been included.

As discussed previously, the area of the image of the several apertures projected in the entrance pupil varies slightly with the power of the lens placed before the eye. These measures were made with monochromatic green light.

The results are striking. The perceived brightness remains essentially constant for the small apertures due to the depth of focus of the eye and the fact that the two halves of the bipartite field will blur at approximately the same rate. For the intermediate and larger apertures the effects become marked. The one half on the bipartite field remained clear due to depth of focus, the other half clearing or blurring depending on the lens used. It is evident that when low value minus lenses are used, maximum additivity data for a given size beam are obtained.

This experiment provides answers to the white light variability and discrepancies, as well as explaining the flattening of the additivity data obtained by the direct comparison method for large apertures. The results indicate that these effects are attributable to the combined roles played by spherical and chromatic aberration of the eye.

The magnitude of chromatic aberration of the eye is approximately one and one-half diopters (27). Depending on point of retinal focus, additional blur is therefore contributed by introducing white light. Considering the effect of blur on subject A.M., it is no surprise that he manifests so large a positive discrepancy for white light (Figure 18).

Essentially there is good agreement between these data and those taken for the additivity effect for subject R.V. with the possible exception of data for the intermediate aperture, where it seems the mean for the no lens condition is a bit too high, when considering neighboring and predicted readings. The data for subject B.W., on the additivity experiment (Figure 19) using white light, manifest the expected decrement, due to the blur effect, to only a slight degree.

Figure 22 represents another method of presenting the data. In this figure the additivity data for two lens powers, -1.00 D. and + 2.00 D., have been plotted as a function of area. Note, the black and white circles do not have the same meaning as in Figure 21. If one may assume that the blur effect has been eliminated for the -1.00 dioptric correction, one would expect the data to follow the curve predicted by integration of the Stiles-Crawford effect. A negative discrepancy exists which needs to be explained.

The relatively flat nature of the curves for high power negative lenses (Figure 21), will be treated in the discussion.

Turning again to Figure 22, it will be noted, for all three subjects, when a plus two lens is placed in front of the eye, the field, with the larger aperture focused in the pupil, appears less bright than for the intermediate aperture. In other words, here the first derivative of the blur effect is so great, when blur due to added ocular spherical aberration is increased, even though more luminous flux enters the eye, the field appears less bright.

This experiment thus gives us a measure of the blur effect, a determination of the lens power needed to provide maximum additivity data for a given size entrance pupil, and hence, a measure of the effect of the Stiles-Crawford mechanism on the additivity data. It also gives insight into the discrepancies inherent in the additivity experiments reported both in this paper and in the work of previous investigators.

F. Annular Pupillary Zones

As still another means of testing the part played by the Stiles-Crawford effect in additivity data, independent of blur effects, photographic annular zones were introduced at aperture A3 in the direct comparison apparatus (Figure 7). Thus, they were conjugate with the entrance pupil of the eye. The refraction of the eye was then corrected for each of the zones. Figure 24 gives the curves

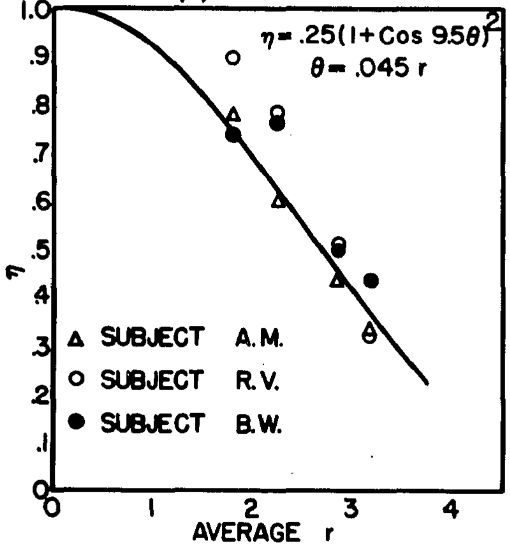

Fig. 23- Additivity effect for aberration corrected annular zones projected in the entrance pupil. This may be considered as a relative efficiency plot. See Table II for dimensions of annuli.

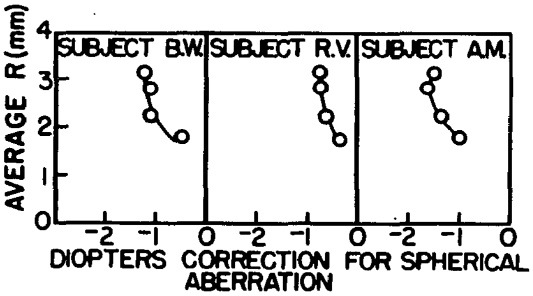

Fig. 24- Correction of spherical aberration for the three subjects.

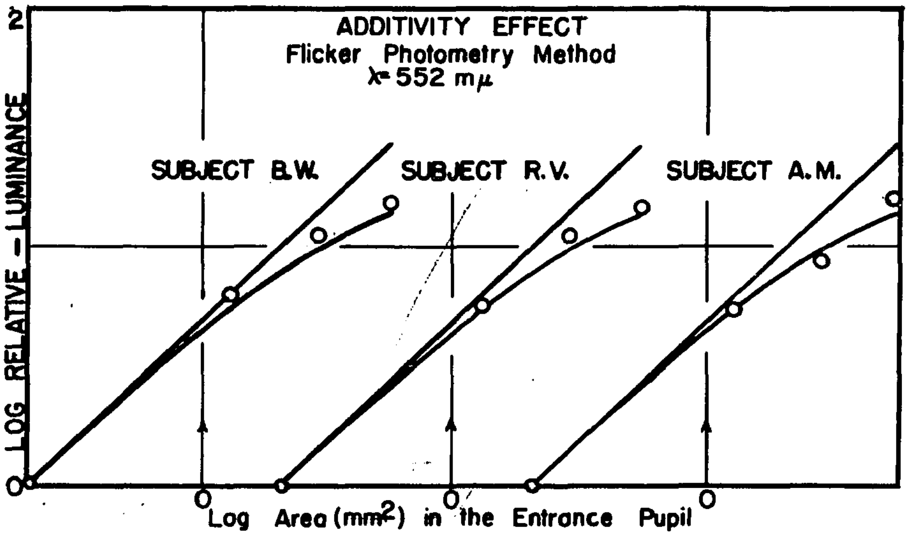

Fig. 25- Additivity effect data for three subjects using flicker photometry. Input line and theoretical additivity curve have been included.

of spherical aberration for the subjects. Lines are not extended to the axis due to lack of data in that region. The brightness of the green monochromatic light was then matched by the subjects. The data are plotted on Figure 23. The method of plotting is discussed at length in the procedure section, and represents a plot of luminous efficiency for annular zones of average radius r. Obviously the points are distributed about the curve, with the average points tending to fall above the curve.

The measurement of transmittance of the photographic plate was difficult. This was partially overcome by doing microdensitometry on the annuli using two different methods and averaging the two results. This is a possible source of error.

G. Additivity Effect Using Flicker Photometry

As a final test, in keeping with the results of the blur experiment, the additivity effect was measured on the flicker photometer apparatus. Selected values were chosen for pupil diameter and matches were made employing monochromatic green light. Figure 25 shows the data for this experiment.

Again, there is a negative discrepancy between the data and the values predicted by the integration of the Stiles-Crawford effect.

In summary, the basic findings are listed below:

1. The perceived brightness of an image produced by a beam of light is the same regardless of whether the beam is Maxwellian or non-Maxwellian. This is true for beams which fill the pupil, or a large part of the pupil, or are confined to a small portion or zone of the pupil.

2. The additivity data, in experiments uncomplicated by the presence of blur, fall between the curve predicted by integration, including the Stiles-Crawford effect, and the input line, favoring the former. This means there is a negative discrepancy between the actual data and the values predicted, and that the test patch is perceived brighter than might be anticipated. Aberrations, which blur the test field and leave the comparison field clear, can convert the discrepancy from negative to positive.

3. Blur from any cause reduces perceived brightness.

4. In matching under blur conditions, flicker photometry gives more consistant results, and is not as sensitive to blur.

5. Criteria problems, as to what constitutes a brightness match, in the presence of blur, present subjective difficulties in both the flicker and direct comparison methods.

V. DISCUSSION

Several aspects of the results obtained need amplification from a theoretical point of view. First, can we expect any systematic difference to result in the data if one employs a Maxwellian or non-Maxwellian type of view? Secondly, although the role played by blur is rather clear cut certain aspects need consideration. In other words, the spherical and chromatic aberration merely act as means of producing blur. A third problem arises, in that the additivity data, when blur is eliminated, do not follow the values which would be predicted by the inclusion, when integrating, of the Stiles-Crawford effect. Rather, it appears as if, in the additivity data, the effect of the obliquity of the rays was somewhat reduced.

If one neglects the differences involved in instrumentation, the real difference between the Maxwellian and the non-Maxwellian type of viewing lies in the distribution of energy in the retinal plane.

Light, having as its source an ultimate radiator, exhibits uniformity of phase, at any instant, across the wave front of the advancing disturbance. This point to point dependence across the wavefront on the same radiator is known as coherence. It is important to note that the implication is not made that neighboring points of a filament or source are in phase with each other.

A wavefront having as its source an ultimate radiator in the field of view, when properly focused by the optical system of the eye, converges to a focus on the retina. Since coherence exists across the converging wavefront, interaction occurs in the image plane between the elemental units of the wavefront as a function of differences in path length. The interaction results in the formation of a Fraunhoffer diffraction image on the retina. This is the case of ordinary or non-Maxwellian viewing.

In the Maxwellian type of view, each point of the source is imaged, not on the retina, but in the entrance pupil of the eye. Thus, each elemental point in the aperture acts as an ultimate source and produces a shadow image of the field stop on the retina. Energy converging to any point on the retina from the image in the entrance pupil is made up, therefore, of the contribution of an infinite number of sources all randomly oriented with regard to phase. Hence, coherence of phase does not exist for this converging bundle, and one would expect merely a summated effect. This occurs since any chance enhancement is balanced by an equally probable destructive effect.

The problem involving the coherence and incoherence of phase across an advancing bundle of rays shall only be considered in terms of the illuminance produced in the retinal plane. To attempt intrareceptor treatment of diffraction presents a difficult problem, especially due to our lack of absolute knowledge of the contours and contents of the cells, or the index variations, and even of the position and orientation of the photochemical substances. Restricting the problem in this manner, can we, therefore, expect on the basis of differences in physical distribution, a difference in retinal illuminance for

Maxwellian and non-Maxwellian type of viewing?

Treating the Maxwellian case first, the problem can be simplified considerably by assuming that the field stop itself, instead of the image of the field stop, lies on the retina. Then each point source in the plane of the exit pupil uniformly illuminates the portion of the retina exposed by the field stop. Thus, in the retinal plane one would expect the illuminance produced by each individual point in the exit pupil to be directly proportional to the intensity of the point source and inversely proportional to the square of the distance from the point to the retina. Of course, this statement neglects transmission and wavelength factors. Assuming that intensity is constant for each point in the pupil, and that the distance to the retina is a constant, the total flux incident at any point on the retina merely represents the sum of the contributions of all infinitesimal elements in the exit pupil.

Thus, one would expect, if certain variables are controlled and certain constant factors neglected, retinal illuminance for a variable Maxwellian beam would follow the values predicted by Troland's expression. A more comprehensive discussion of coherence may be found in Morgan (28), and a more complete discussion of retinal illuminance in the case of Maxwellian imagery is available in LeGrand (5).

The case of the pattern of illuminance produced by the non-Maxwellian field on the retina is quite straight forward. The development to be followed in the succeeding paragraphs is based largely upon the discussion appearing in Fry's book "The Blur of the Retinal Image" (29).

If one considers a point S on a spherical wavefront, originally having origin at an ultimate source, emerging from the exit pupil of the eye and converging to a point on the retina, one can give an equation for the disturbance in the plane of the retina in the following form;

$$y = A \sin \Omega$$

The amplitude factor, A, contributed by an infinitesimal area on the wavefront in the exit pupil is proportional to the ensuing relationship;

$$\frac{E_o^{\frac{1}{2}} \quad (r'dr'd\phi)}{\lambda/n \quad \overline{SP'}} \quad t \quad \left(\frac{1 + \cos \theta}{2}\right)$$

In this relationship E_o is the illuminance in the plane of the pupil; $r'dr'd\phi$ is the area of the infinitesimal element in the plane of the exit pupil containing the point S, corresponding to a radial value of r in the entrance pupil of the eye, and the angle ϕ from the zero half-meridian; the wavelength λ is modified by the index (n) of the medium; and according to the inverse square law the amplitude is inversely proportional to the distance from a point arbitrarily selected on the retina, P', to the arbitrary point on the wavefront, S. It should be noted that uniformity of phase exists across the wavefront. The element $\left(\frac{1 + \cos \theta}{2}\right)$ is the obliquity factor of the amplitude which is usually neglected in elementary treatments of diffraction theory. Born (30) in a treatment of Kirchhoff's theory considers this factor. It is this element of the amplitude which is discussed in the introductory part of this paper as perhaps having bearing on the Stiles-Crawford effect. In a treatment such as that which follows, this element can be safely neglected, since theta never exceeds eleven degrees. However, this is not the case when a disturbance passes up a narrow multi-reflecting cone. To simplify the treatment, the transmission (and reflection) factor (t) will be given the value 1.0. The intensity is defined as the square of the amplitude.

The usual expression for the phase angle (Ω) includes an element to account for the phase in the initial position of the wavefront and one giving the total phase change occurring in the distance $\overline{SP'}$.

$$\Omega = (\psi - \psi_0)$$

$$\Omega = 2\pi\left(\frac{\overline{SP'}}{\lambda/n} - \frac{t}{T}\right)$$

Thus the total equation for the resultant disturbance at P' obtained by integrating across the wavefront is as follows;

$$y = \frac{E_o^{\frac{1}{2}}}{\lambda/n\ \overline{S_oP'}} \int_0^{2\pi} \int_0^{\overline{r'}} \sin 2\pi \left[\frac{\overline{SP'}}{\lambda/n} - \frac{t}{T} \right] r'\, dr'\, d\phi.$$

If one solves this integral, it will be found that retinal illuminance can be expressed by the following equation;

$$E_R = \frac{B_o\ A_o}{\frac{\lambda^2}{n^2}\ d^2} \left[\frac{2d\lambda}{A_o np}\ J_1 \left(\frac{A_o np}{\lambda\ d} \right) \right]^2$$

where d is the distance from the pole of the wavefront in the exit pupil to the retina, and p is the distance across the retina from the center of the distribution. If one chooses any point in the distribution the Bessel function $J_1(f\ (p))$ has a fixed value. If $p = 0$, the expression within the brackets is equal to one, and

$$E_R = B_o A_o \text{ (constant).}$$

This, therefore, means that one would expect, for a given absolute point source, the energy at the maximum of the Fraunhoffer pattern is a function following Troland's relationship. Owing to the fact that the several points of the source are in random phase with each other, it is necessary to integrate the resultant effect, produced by each point in the source, in the image plane. Thus, one would not, on the basis of amplitude considerations, expect there to be any difference (other than possibly some constant or fixed difference) between the retinal illumination produced by a Maxwellian versus the non-Maxwellian field.

A question regarding the phase may be raised. In the case of the non-Maxwellian field, there logically would be a surface of constant phase which is parallel to the retinal plane during the transition from a convergent to a divergent wave surface. If this is the case, one might ask, if the phase front is plane, whether any, or at least, a reduced Stiles-Crawford effect might occur?

The fact that no apparent difference exists between the Maxwellian and non-Maxwellian fields, in the experiments here described, tends to negate such a possibility. It remains to be determined whether, at different parts of the Fraunhoffer image, the photoreceptors are affected in the same way by the flux which they intercept.

A question arises when one considers the effect of blur on perceived brightness. Obviously, the first question to be asked is, what apparent differences become manifest to the observer under blur? Interestingly, for low degrees of blur, the subject on occasion reported the image as sharp or sharper than the no blur condition. Since,in the penumbra blur experiment, the blur was mediated by optical means on the object viewed by the eye, the subject's depth of focus did not assist him. No doubt the retinal border-sharpening mechanisms play a role in this instance. The effect of increasing the blur is the appearance of a rather poorly defined border near which is seen a bright Mach ring and within which lies a less luminous surface having a rather matte texture. Thus, the patch no longer appears equally bright, and the textural quality changes.

One perplexing aspect of the blur effect, which cannot be answered without further experimentation, is that of magnitude. The results obtained using the "blur" apparatus reveal that the blur effect tends to level off when the penumbra subtends about one quarter degree for the direct comparison method of matching. On the other hand, the data obtained from the dioptric blur experiment do not exhibit evidence of leveling off with increased blur. If anything, the opposite is the case. In comparing the two conditions, certain differences should be noted. Less blur was produced in the variable penumbra experiment. Further, in the penumbra experiment, the comparison borders were separated so that no overlap occurred, and because only the border was blurred, any central textural changes noted were perceptual and not physical. However, in the dioptric blur experiment, the fields were adjacent (which complicates the problem considerably due to overlap) and textural changes had both a physical and perceptual basis. There seems little doubt however, that it is the blur effect which has led to the marked discrepancies in the work of previous investigators.

One other point worthy of consideration in this context is the role played by accommodation. In the dioptric blur experiment, it will be noted in examining the data (Figure 21), that the curves appear flatter for negative lens power. It may be, that the subjects are able to reduce some of the blur by accommodating, or that accommodation itself induced by the lenses reduces the spherical aberration (31). The maximum obtained on these curves in the minus is mainly due however, to the correction of the spherical aberration of the observer's eyes by the lens itself rather than to reduction in spherical aberration due to accommodation. This aspect is brought to light in the annular zone experiments.

Obviously, it would be important to extend our information concerning the entire blur problem, and certainly further research is needed. Similarly, the several criteria available in such judgments need investigation. From the findings discussed above, it must follow, that care should be taken in visual research where blur is present. Measures must be taken to eliminate it, in order to obtain valid brightness judgments. If it is impossible to correct blur, flicker photometry is the method of choice, since it is less affected by blur and criteria difference problems.

Let us now consider the part played by the Stiles-Crawford effect, and the part played by blur in the additivity data. Two aspects of the problem are immediately evident. First, the total Stiles-Crawford effect does not seem to appear in the additivity experiment. In other words, after correcting blur, the test field is perceived as brighter than one would predict on the basis of the Stiles-Crawford effect. This same result was observed for two of the three Stiles-Crawford observers (Figure 2). One possible explanation could be the following consideration. It is possible, that even for the small fields used in the Stiles-Crawford type experiment, a slight amount of blur is introduced in peripheral matches. As discussed above, this leads to slightly lower brightness readings and hence the value $k = 9.5$ may be too high. This effect is especially noticeable for white light, since chromatic aberration certainly becomes quite visible during testing. However, this may not be the entire answer to the problem.

It is noted that when blur is not present, the additivity data are quite consistant. Marked variability between individuals only enters in the presence of blur. The variability is due to the amount of blur, the subject's sensitivity to blur, and the subject's criterion of matching. In Stiles-Crawford effect experiments, consistancy is greater since blur is less, but as inferred above, some blur exists with beams entering the pupil near its margin, and this no doubt affects the consistancy as well as the validity of the measurement.

VI SUMMARY AND CONCLUSIONS

The data of previous investigators for the additivity effect, defined as the summated response of the retina to light entering different parts of the pupil, have been contradictory. A study was undertaken in an attempt to delineate the variables inherent in these measurements.

The experimental results have demonstrated that additivity data are not affected by coherence or non-coherence of phase in the retinal plane. Thus, the Maxwellian and the ordinary type of viewing may be used interchangeably in Stiles-Crawford and additivity effect experiments.

The experiments conducted in this research have revealed that additivity data are affected by at least the following two factors: (1) the differing degrees of luminous efficiency of rays incident at varying points in the entrance pupil (the Stiles-Crawford effect), (2) the blur of the retinal image.

When blur is eliminated in additivity studies, the Stiles-Crawford effect does not reduce perceived brightness as much as might be predicted.

The blur effect, caused by ocular aberrations, spherical and chromatic aberrations in particular, is a variable dependent upon the observer. This variability is thought to account for the several discrepancies in the data found in the literature. Initial experiments concerning the effect of blur on perceived brightness have been completed and it was shown that blur reduces perceived brightness of a field. Experimentation demonstrated the flicker photometer method to be the method of choice if it is necessary to match a blurred field with a clear one.

A new formula for the Stiles-Crawford effect has been introduced, and a possible mechanism has been suggested which gives theoretical significance to the formula.

VII REFERENCES

1. Stiles, W.S., Crawford, B.H., "The luminous efficiency of rays entering the eye pupil at different points," Proc. Roy. Soc. (London), Ser. B, 112, No. B778, March, 1933, pp 428-450.

2. Koomen, M., Scolnik, R., Tousey, R., "A study of night myopia," J. Opt. Soc. Am., 41, 2, Feb. 1951, pp 80-90.

3. Boynton, R.M., Enoch, J.M., Bush, W.R., "Physical measurements of stray light in excised eyes," J. Opt. Soc. Am., 44, 11, Nov. 1954, pp 875-879.

4. Moon, P., Spencer, D.E., "On the Stiles-Crawford effect," J. Opt. Soc. Am., 34, 6, June 1944, pp 319-329.

5. LeGrand, Y., Optique Physiologique, Tome 2, Editions de la "Revue D'Optique," Paris, 1948, pp 102-107.

6. Arnulf, A., "Le clarte des instruments visuels, influence des variations du diametre pupillaire et de l'effet Stiles-Crawford," Comm. des. Lab. de L'Inst. d'Opt., 1, July 1944, pp 23-35.

7. Bartley, S.H., "Light in decibels," Perceptual and Motor Skills, 5, 4, Dec. 1955, pp 199-200.

8. Toraldo di Francia, G., "Sulle legge integrale dell' effetto Stiles-Crawford," Atti della Fond. G. Ronchi, 2, 1947, pp 100-104.

9. Bocchino, G., "Studio della variazione della luminosita di un cannocchiale at variare della pupilla d' uscita," Ottica, 1, 1936, pp 136-142.

10. Troland, L.T., "Theory and practice of the artificial pupil," Psych. Review, 22, 3, May 1915, pp 167-176.

11. Troland, L.T., "On the measurement of visual sensation intensities," J. Exp. Psych., 2, 1, Feb. 1917, pp 1-34.

12. Crawford, B.H., "The luminous efficiency of light entering the eye pupil at different points and its relation to brightness threshold measurements," Proc. Roy Soc. (London), Ser. B, 124, #B834, Sept. 1937, pp 81-96.

13. Stiles, W.S., "The luminous efficiency of monochromatic rays entering the eye pupil at different points and a new color effect," Proc. Roy. Soc. (London), Ser. B, 123, #B830, June 1937, pp 90-118.

14. Goodeve, C.F., "Relative luminoscity in the extreme red," Proc. Roy. Soc. (London), Ser. A, 155, A886, 1936, pp 664-683.

15. Flamant, F., "Contribution a l'etude du phenomene Stiles-Crawford," Comm. des l'Inst d 'Opt., 2, 1,Mar. 1946, Article 21, pp 47-59.

16. Craik, K.J.W., "Transmission of light by the eye media," J. Physiol., 98, 2,May 1940, pp 179-184.

17. Jenkins, F.A., White, H.E., Fundamentals of Physical Optics, McGraw-Hill, New York, First edition, 1937, pp 113, and 175.

18. O'Brien, B., "A theory of the Stiles and Crawford effect," J. Opt. Soc. Am., 36, 9,Sept. 1946, pp 506-509.

19. Campbell, C.J., A Technique for the Determination of the Index of Refraction of the Rods and Cones of the Retina, Thesis, M.S., Dept. of Optics, U. of Rochester, Rochester, N.Y., 1951.

20. O'Brien, B., "Vision and resolution in the central retina," J. Opt. Soc. Am., 41, 12,Dec. 1951, pp 882-894.

21. Southall, J.P.C., Mirrors, Prisms and Lenses, Third edition, Macmillan, New York, 1933, pp 43-45.

22. Toraldo di Francia, G., "The radiation pattern of retinal receptors," Proc. Phys. Soc. (London), B62, Part 7, July 1949, pp 461-2.

23. Goldmann, H., "Stiles-Crawford effect," Ophthalmologica, 103, 4,April 1942, pp 225-229.

24. Alpern, M., Benson, D., "Directional sensitivity of pupillomotor photoreceptors," Amer. J. Optom. and Arch. Amer. Acad. Optom., 30, 11,Nov. 1953, pp 569-580.

25. O'Brien, B., Miller, N., "A study of the mechanism of visual acuity in the central retina," Wright Air Development Center Technical Report 53-198, May 1953, pp 41-45.

26. Ronchi, L., "Influence d'un mydriatique sur l'effet Stiles-Crawford," Optica Acta, 2, 1, April 1955, pp 47-49.

27. Hartridge, H., Recent Advances in the Physiology of vision, Blakiston Co., Phila., 1950, pp 88.

28. Morgan, J., Introduction to Geometrical and Physical Optics, McGraw-Hill, New York, 1953, pp 180-181.

29. Fry, G.A., Blur of the Retinal Image, Ohio State University Press, Columbus, Ohio, 1956.

30. Born, M., Optik, J. Springer, Berlin, 1933, pp 146-155.

31. Ivanoff, A., Les Aberrations de l'Oeil, Leur Role dans l'Accommodation, Editions de la revue d'Optique, Paris, 1953.

VIII AUTOBIOGRAPHY

I, Jay Martin Enoch, was born in New York City, April 20, 1929. I attended New York City Elementary schools and received my secondary education at the Bronx High School of Science. As a recipient of a New York State Regents Scholarship, I attended Columbia College, Columbia University, for my pre-Optometry undergraduate training, and completed my optometric studies in the Department of Optometry, Columbia University, from which I received the degree Bachelor of Science in 1950. The years 1951-1952 were spent at the Army Medical Research Laboratory, Fort Knox, Kentucky, where an evaluation study of contact lenses was undertaken. Upon leaving the service I sought graduate training in Physiological Optics at the Institute of Optics, University of Rochester, and at the Graduate School, the Ohio State University, the major portion of this education being obtained at the latter institution. During the academic years 1953-1954, 1954-1955, I held the position of Assistant in Optometry at the Ohio State University, and for the academic year 1955-1956, I received appointment as a Research Fellow.