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Super-BMS$_3$ invariant boundary theory from three-dimensional flat supergravity

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Abstract: The two-dimensional super-BMS$_3$ invariant theory dual to three-dimensional asymptotically flat $\mathcal{N} = 1$ supergravity is constructed. It is described by a constrained or gauged chiral Wess-Zumino-Witten action based on the super-Poincaré algebra in the Hamiltonian, respectively the Lagrangian formulation, whose reduced phase space description corresponds to a supersymmetric extension of flat Liouville theory.

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1 Introduction

A prime example of duality between a three-dimensional and a two-dimensional theory is the relation between a Chern-Simons theory in the presence of a boundary and the associated chiral Wess-Zumino-Witten (WZW) model: on the classical level for instance, the variational principles are equivalent as the latter is obtained from the former by solving the constraints in the action [1–3].

In the case of the Chern-Simons formulation of three-dimensional gravity [4, 5], the role of the boundary is played by non trivial fall-off conditions for the gauge fields. For anti-de Sitter or flat asymptotics, a suitable boundary term is required in order to make solutions with the prescribed asymptotics true extrema of the variational principle. Furthermore, the fall-off conditions lead to additional constraints that correspond to fixing a subset of the conserved currents of the WZW model [6, 7]. The associated reduced phase space description is given by a Liouville theory for negative cosmological constant and a suitable limit thereof in the flat case [8, 9]. This procedure was also implemented in the context of three dimensional higher spin gravity without cosmological constant, where a flat limit of Toda theory is recovered [10].

In this paper, we apply the construction to three-dimensional asymptotically flat $\mathcal{N} = 1$ supergravity, whose algebra of surface charges has been shown to realize the centrally extended super-BMS$_3$ algebra [11]. The non-vanishing Poisson brackets read

$$\begin{aligned} i\{\mathcal{J}_m, \mathcal{J}_n\} &= (m-n)\mathcal{J}_{m+n} + \frac{c_1}{12} m^3 \delta_{m+n,0}\,, \\ i\{\mathcal{J}_m, \mathcal{P}_n\} &= (m-n)\mathcal{P}_{m+n} + \frac{c_2}{12} m^3 \delta_{m+n,0}\,, \\ i\{\mathcal{J}_m, \mathcal{Q}_n\} &= \left(\frac{m}{2} - n\right) \mathcal{Q}_{m+n}\,, \\ \{\mathcal{Q}_m, \mathcal{Q}_n\} &= \mathcal{P}_{m+n} + \frac{c_2}{6} m^2 \delta_{m+n,0}\,, \end{aligned} \tag{1.1}$$

where the fermionic generators $\mathcal{Q}_m$ are labeled by (half-)integers in the case of (anti)periodic boundary conditions for the gravitino, and the central charges are given by

$$c_1 = \mu \frac{3}{G}\,, \qquad c_2 = \frac{3}{G}\,. \tag{1.2}$$

Here, G and μ stand for the Newton constant and the coupling of the Lorentz-Chern-Simons form, respectively.

The resulting two-dimensional field theory admits a global super-BMS$_3$ invariance. By construction, the associated algebra of Noether charges realizes (1.1) with the same values of the central charges. We provide three equivalent descriptions of this theory: (i) a Hamiltonian description in terms of a constrained chiral WZW theory based on the three-dimensional super-Poincaré algebra, (ii) a Lagrangian formulation in terms of a gauged chiral WZW theory and (iii) a reduced phase space description that corresponds to a supersymmetric extension of flat Liouville theory.

Besides the extension to the supersymmetric case, previous results in the purely bosonic sector are also generalized. This is due to the inclusion of parity-odd terms in the action, which suitably modifies the Poincaré current subalgebra, and consequently, turns on the additional central charge c_1 in (1.1).

2 Brief review of (minimal) $\mathcal{N} = 1$ flat supergravity in 3D

As in the case of pure gravity, minimal $\mathcal{N} = 1$ supergravity in three dimensions [12–14] with vanishing cosmological constant admits a Chern-Simons formulation [15]. Different extensions of this theory have been developed in e.g., [16–29]. Hereafter we consider the most general supergravity theory with $\mathcal{N} = 1$ that is compatible with asymptotically flat boundary conditions, and leads to first order field equations for the dreibein, the spin connection and the gravitino [19](see also [11, 30]). The standard minimal $\mathcal{N} = 1$ supergravity theory is recovered for a particular choice of the couplings (see below). The gauge field $A = A_\mu dx^\mu$ is given by

$$A = e^a P_a + \hat{\omega}^a J_a + \psi^\alpha Q_\alpha\,, \tag{2.1}$$

where e^a, ω^a and ψ^α stand for the dreibein, the dualized spin connection $\omega_a = \frac{1}{2}\epsilon_{abc}\omega^{bc}$, and the (Majorana) gravitino, respectively; while $\hat{\omega}^a := \omega^a + \gamma e^a$ and the set $\{P_a, J_a, Q_\alpha\}$

spans the super-Poincaré algebra,

$$\begin{aligned} &[J_a, J_b] = \epsilon_{abc} J^c\,, \qquad &&[J_a, P_b] = \epsilon_{abc} P^c\,, \qquad &&[P_a, P_b] = 0\,, \\ &[J_a, Q_\alpha] = \frac{1}{2}\left(\Gamma_a\right)^\beta{}_\alpha Q_\beta\,, \quad &&[P_a, Q_\alpha] = 0\,, \qquad &&\{Q_\alpha, Q_\beta\} = -\frac{1}{2}\left(C\Gamma^a\right)_{\alpha\beta} P_a\,, \end{aligned} \tag{2.2}$$

where C is the charge conjugation matrix (see appendix A for conventions). In these terms, the action reads

$$I[A] = \frac{k}{4\pi}\int \langle A, dA + \frac{2}{3}A^2\rangle\,, \tag{2.3}$$

where the bracket $\langle\cdot,\cdot\rangle$ stands for an invariant nondegenerate bilinear form, whose only nonvanishing components are given by

$$\langle P_a, J_b\rangle = \eta_{ab}\,, \qquad \langle J_a, J_b\rangle = \mu\eta_{ab}\,, \qquad \langle Q_\alpha, Q_\beta\rangle = C_{\alpha\beta}\,, \tag{2.4}$$

and the level is related to the Newton constant according to $k = \frac{1}{4G}$. Hence, up to a boundary term, the action reduces to

$$I_{(\mu,\gamma)} = \frac{k}{4\pi}\int 2\hat{R}^a e_a + \mu L(\hat{\omega}) - \bar{\psi}_\alpha \hat{D}\psi^\alpha\,, \tag{2.5}$$

where $\bar{\psi}_\alpha = C_{\alpha\beta}\psi^\beta$ is the Majorana conjugate, and with respect to the connection $\hat{\omega^a}$, the curvature two-form and the covariant derivative of the gravitino are defined as

$$\hat{R}^a = d\hat{\omega}^a + \frac{1}{2}\epsilon^{abc}\hat{\omega}_b\hat{\omega}_c\,, \qquad \hat{D}\psi = d\psi + \frac{1}{2}\hat{\omega}^a\Gamma_a\psi\,, \tag{2.6}$$

respectively, while $L(\hat{\omega}) = \hat{\omega}^a d\hat{\omega}_a + \frac{1}{3}\epsilon_{abc}\hat{\omega}^a\hat{\omega}^b\hat{\omega}^c$ is the corresponding Lorentz-Chern-Simons form.

By construction the action is invariant, up to a surface term, under the following local supersymmetry transformations

$$\delta e^a = -\frac{1}{2}\bar{\epsilon}\Gamma^a\psi\,, \qquad \delta\omega^a = \frac{1}{2}\gamma\bar{\epsilon}\Gamma^a\psi\,, \qquad \delta\psi = D\epsilon + \frac{1}{2}\gamma e^a\Gamma_a\epsilon\,, \tag{2.7}$$

where $D\epsilon = d\epsilon + \frac{1}{2}\omega^a\Gamma_a\epsilon$ is the standard Lorentz covariant derivative of a spinor. The field equations $F = dA + A^2 = 0$, whose general solution is locally given by $A = G^{-1}dG$, decompose as

$$R^a = \frac{1}{2}\gamma^2\epsilon^{abc}e_b e_c + \frac{1}{4}\gamma\bar{\psi}\Gamma^a\psi\,, \quad T^a = -\gamma\epsilon^{abc}e_b e_c - \frac{1}{4}\bar{\psi}\Gamma^a\psi\,, \quad D\psi = -\frac{1}{2}\gamma e^a\Gamma_a\psi\,, \tag{2.8}$$

where R^a, and $T^a = de^a + \epsilon^{abc}\omega_b e_c$ stand for the curvature and torsion two-forms, respectively.

Defining $\hat{\omega} = \frac{1}{2}\hat{\omega}^a\Gamma_a$, $e = \frac{1}{2}e^a\Gamma_a$ and contracting the first two equations in (2.8) with $\frac{1}{2}\Gamma_a$ gives the matrix form $d\hat{\omega} + \hat{\omega}^2 = 0$, $de + [\hat{\omega}, e] = -\frac{1}{4}\psi\bar{\psi}$, so that the decomposition of the general (local) solution is

$$\hat{\omega} = \Lambda^{-1}d\Lambda\,, \qquad \psi = \Lambda^{-1}d\eta\,, \qquad e = \Lambda^{-1}\left(-\frac{1}{4}\eta d\bar{\eta} - \frac{1}{8}d\bar{\eta}\eta\mathbf{1} + db\right)\Lambda\,, \tag{2.9}$$

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where Λ is an $\mathrm{SL}(2,\mathbb{R})$ group element, η a Grassmann-valued spinor and b a traceless 2×2 matrix.

The asymptotic conditions proposed in [11] imply that the gauge field is of the form

$$A = h^{-1}ah + h^{-1}dh\,, \tag{2.10}$$

where the radial dependence is completely captured by the group element $h = e^{-rP_0}$, while

$$a = \left(\frac{\mathcal{M}}{2}du + \frac{\mathcal{N}}{2}d\phi\right)P_0 + du\,P_1 + \frac{\mathcal{M}}{2}d\phi J_0 + d\phi\,J_1 + \frac{\psi}{2^{1/4}}d\phi\,Q_+\,, \tag{2.11}$$

with functions $\mathcal{M}$, $\mathcal{N}$, and the Grassmann-valued spinor component ψ that depend on the remaining coordinates u, ϕ.

The standard supergravity theory with $\mathcal{N}=1$ with its asymptotically flat behaviour is then recovered for $\mu=\gamma=0$. It is also worth pointing out that the fall-off conditions (2.11) can be generalized, along the lines of [31], so as to include a generic choice of chemical potentials [30].

3 Chiral constrained super-Poincaré WZW theory

3.1 Solving the constraints in the action

Up to boundary terms and an overall sign which we change for later convenience, the Hamiltonian form of the Chern-Simons action (2.3) is given by

$$I_H[A] = -\frac{k}{4\pi}\int \langle\tilde{A}, du\dot{\tilde{A}}\rangle + 2\langle duA_u, \tilde{d}\tilde{A} + \tilde{A}^2\rangle\,, \tag{3.1}$$

where $A = duA_u + \tilde{A}$.

One of the advantages of the gauge choice in (2.10), for which the dependence in the radial coordinate is completely absorbed by the group element h, is that the boundary can be assumed to be unique and located at an arbitrary fixed value of $r=r_0$. Hence, the boundary generically describes a two-dimensional timelike surface with the topology of a cylinder ($\mathbb{R}\times S^1$). We will also discard all holonomy terms. As a consequence, the resulting action principle at the boundary only captures the asymptotic symmetries of the original gravitational theory. Note also that positive orientation in the bulk is taken as $dud\phi dr$.

The boundary term in the variation of the Hamiltonian action is given by $-\frac{k}{2\pi}du\tilde{d}\langle A_u, \delta\tilde{A}\rangle$. Thus, by virtue of the boundary conditions (2.11), the components of the gauge field at the boundary fulfill

$$\omega^a_\phi = e^a_u\,, \qquad \omega^a_u = 0\,, \qquad \psi^+_u = 0 = \psi^-_u\,, \tag{3.2}$$

so that the boundary term becomes integrable. Consequently, the improved action principle that has a true extremum when the equations of motion are satisfied is given by

$$I_I[A] = I_H[A] - \frac{k}{4\pi}\int dud\phi\,\omega^a_\phi\omega_{a\phi}|^{r=r_0}\,. \tag{3.3}$$

In this action principle A_u are Lagrange multipliers, whose associated constraints are locally solved by $\tilde{A} = G^{-1}\tilde{d}G$ for some group element $G(u,r,\phi)$. Solving the constraints in the action yields

$$I = \frac{k}{4\pi}\left(\int du d\phi\, \left[\langle G^{-1}\partial_\phi G, G^{-1}\partial_u G\rangle - \omega_\phi^a \omega_{a\phi}\right]^{r=r_0} + \Gamma[G]\right)\,, \tag{3.4}$$

where

$$\Gamma[G] = \frac{1}{3}\int \langle G^{-1}dG, (G^{-1}dG)^2\rangle\,. \tag{3.5}$$

Equivalently, in terms of the gauge field components, the action can be conveniently written as

$$I = \frac{k}{4\pi}\left(\int du d\phi\, \left[\omega_\phi^a e_{au} + e_\phi^a \omega_{au} - \omega_\phi^a \omega_{a\phi} + \mu\omega_\phi^a\omega_{au} - \bar{\psi}_u\psi_\phi\right]^{r=r_0} + \Gamma[G]\right)\,, \tag{3.6}$$

with

$$\Gamma[G] = \frac{1}{6}\int (3\epsilon_{abc}e^a\omega^b\omega^c + \mu\epsilon_{abc}\omega^a\omega^b\omega^c - \frac{3}{2}\omega^a(C\Gamma_a)_{\alpha\beta}\psi^\alpha\psi^\beta)\,, \tag{3.7}$$

and the understanding that $A_\mu = G^{-1}\partial_\mu G$. Decomposing this connection according to eq. (2.9) allows one to rewrite this expression in terms of a 2 by 2 matrix trace, so that integrating by parts the first term in $\Gamma[G]$ gives

$$I=\frac{k}{2\pi}\int du d\phi\,\mathrm{Tr}\left[2\dot{\Lambda}\Lambda^{-1}\left(-\frac{\eta\bar{\eta}'}{4}+b'\right)-(\Lambda'\Lambda^{-1})^2+\mu\Lambda'\Lambda^{-1}\dot{\Lambda}\Lambda^{-1}+\frac{\eta'\dot{\bar{\eta}}}{2}\right]^{r=r_0}+\frac{\mu}{3}\int\mathrm{Tr}(d\Lambda\Lambda^{-1})^3\,. \tag{3.8}$$

Furthermore, the boundary conditions (2.10), (2.11) imply that $\partial_\phi A_r = 0$, and hence $G = g(u,\phi)h(u,r)$. More precisely, since in the asymptotic region $h = e^{-rP_0}$, one obtains in particular that $\dot{h}(u,r_0) = 0$. The decomposition in (2.9) is then refined as

$$\begin{aligned}
\Lambda &= \lambda(u,\phi)\,\varsigma(u,r)\,,\\
\eta &= \nu(u,\phi) + \lambda\varrho(u,r)\,,\\
b &= \alpha(u,\phi) + \frac{1}{4}\nu\bar{\varrho}\lambda^{-1} + \frac{1}{8}\bar{\varrho}\lambda^{-1}\nu\mathbf{1} + \lambda\beta(u,r)\lambda^{-1}\,,
\end{aligned} \tag{3.9}$$

where $\dot{\varsigma}(u,r_0) = \dot{\varrho}(u,r_0) = \dot{\beta}(u,r_0) = 0$. Therefore, up to a total derivative in u and ϕ, one finds that the action reduces to that of a chiral super-Poincaré Wess-Zumino-Witten theory,

$$\begin{aligned}
I[\lambda,\alpha,\nu] = \frac{k}{2\pi}\int du d\phi\,\mathrm{Tr}&\left[2\dot{\lambda}\lambda^{-1}\alpha' - (\lambda'\lambda^{-1})^2 + \mu\lambda'\lambda^{-1}\dot{\lambda}\lambda^{-1} + \frac{1}{2}\nu'\dot{\bar{\nu}} - \frac{1}{2}\dot{\lambda}\lambda^{-1}\nu\bar{\nu}'\right]\\
&+\frac{\mu}{3}\int\mathrm{Tr}(d\Lambda\Lambda^{-1})^3\,.
\end{aligned} \tag{3.10}$$

The field equations are then obtained by varying (3.10) with respect to α, ν, λ, which gives

$$\begin{aligned}
(\dot{\lambda}\lambda^{-1})' &= 0\,,\\
D_u^{-\dot{\lambda}\lambda^{-1}}\nu' &= 0\,,\\
D_u^{-\dot{\lambda}\lambda^{-1}}\alpha' + (\mu\partial_u - \partial_\phi)(\lambda'\lambda^{-1}) - \frac{1}{4}\dot{\nu}\bar{\nu}' - \frac{1}{8}\bar{\nu}'\dot{\nu}\mathbf{1} + \frac{1}{4}\dot{\lambda}\lambda^{-1}\nu\bar{\nu}' + \frac{1}{8}\bar{\nu}'\dot{\lambda}\lambda^{-1}\nu\mathbf{1} &= 0\,,
\end{aligned} \tag{3.11}$$

respectively. The general solution of these equations is given by

$$\begin{aligned}
\lambda &= \tau(u)\kappa(\phi)\,,\\
\nu &= \tau(\zeta_1(u)+\zeta_2(\phi))\,,\\
\alpha &= \tau\left(a(\phi)+\delta(u)+u\kappa'\kappa^{-1}-\mu[\ln\tau,\ln\kappa]+\frac{1}{4}\zeta_1\bar{\zeta}_2+\frac{1}{8}\bar{\zeta}_2\zeta_1\mathbf{1}\right)\tau^{-1}\,.
\end{aligned} \tag{3.12}$$

3.2 Symmetries of the chiral WZW model

By using the Polyakov-Wiegmann identities, the action (3.10) can be shown to be invariant under the gauge transformations

$$\lambda\to\Xi(u)\lambda\,,\qquad \nu\to\Xi\nu\,,\qquad \alpha\to\Xi\alpha\Xi^{-1}\,. \tag{3.13}$$

Moreover, it is also invariant under the following global symmetries

$$\begin{aligned}
&\lambda\to\lambda\,, && \nu\to\nu\,, && \alpha\to\alpha+\lambda\Sigma(\phi)\lambda^{-1}\,,\\
&\lambda\to\lambda\Theta^{-1}(\phi)\,, && \nu\to\nu\,, && \alpha\to\alpha-u\lambda\Theta^{-1}\Theta'\lambda^{-1}\,,\\
&\lambda\to\lambda\,, && \nu\to\nu+\lambda\Upsilon(\phi)\,, && \alpha\to\alpha+\frac{1}{4}\nu\bar{\Upsilon}\lambda^{-1}+\frac{1}{8}\bar{\Upsilon}\lambda^{-1}\nu\mathbf{1}\,,
\end{aligned} \tag{3.14}$$

whose associated infinitesimal transformations read

$$\begin{aligned}
&\delta_\sigma\lambda=0\,, && \delta_\sigma\nu=0\,, && \delta_\sigma\alpha=\lambda\sigma(\phi)\lambda^{-1}\,,\\
&\delta_\vartheta\lambda=-\lambda\vartheta(\phi)\,, && \delta_\vartheta\nu=0\,, && \delta_\vartheta\alpha=-u\lambda\vartheta'\lambda^{-1}\,,\\
&\delta_\gamma\lambda=0\,, && \delta_\gamma\nu=\lambda\gamma(\phi)\,, && \delta_\gamma\alpha=\frac{1}{4}\nu\bar{\gamma}\lambda^{-1}+\frac{1}{8}\bar{\gamma}\lambda^{-1}\nu\mathbf{1}\,.
\end{aligned} \tag{3.15}$$

The Noether currents associated to a global symmetry, whose parameters are collectively denoted by X_1, generically read $J^\mu_{X_1}=-k^\mu_{X_1}+\frac{\partial\mathcal{L}}{\partial_\mu\phi^i}\delta_{X_1}\phi^i$, with $\delta_{X_1}\mathcal{L}=\partial_\mu k^\mu_{X_1}$. Hence, in the case of global symmetries spanned by (3.15), the corresponding currents are given by $J^\mu_\sigma=2\delta^\mu_0\mathrm{Tr}[\sigma P]$, $J^\mu_\vartheta=2\delta^\mu_0\mathrm{Tr}[\vartheta J]$, $J^\mu_\gamma=2\delta^\mu_0\mathrm{Tr}[\gamma Q]$, where

$$\begin{aligned}
P &= \frac{k}{2\pi}\lambda^{-1}\lambda'\,,\\
J &= -\frac{k}{2\pi}\left[\lambda^{-1}\alpha'\lambda-u(\lambda^{-1}\lambda')'+\mu\lambda^{-1}\lambda'-\frac{1}{4}\lambda^{-1}\nu\bar{\nu}'\lambda-\frac{1}{8}\bar{\nu}'\nu\mathbf{1}\right]\,,\\
Q &= \frac{k}{4\pi}\bar{\nu}'\lambda\,.
\end{aligned} \tag{3.16}$$

For the Noether $n-1$-forms $j_{X_1}=J^\mu_{X_1}(d^{n-1}x)_\mu$, the current algebra can then be worked out by applying a subsequent symmetry transformation δ_{X_2}, so that

$$\delta_{X_2}j_{X_1}=j_{[X_1,X_2]}+K_{X_1,X_2}+\text{“trivial”}\,, \tag{3.17}$$

where $[\delta_{X_1},\delta_{X_2}]=\delta_{[X_2,X_1]}$, and K_{X_1,X_2} denotes a possible field dependent central extension, and “trivial” stands for exact $n-1$ forms plus terms that vanish on-shell. Furthermore, general results guarantee that, in the Hamiltonian formalism, this computation corresponds to the Dirac bracket algebra of the canonical generators of the symmetries, i. e., $\delta_{X_2} J^0_{X_1} = \{J^0_{X_1}, J^0_{X_2}\}^*$, see e.g. [32–35]. Once applied to the components of the currents, given by

$$P_a(\phi) = \mathrm{Tr}[\Gamma_a P]\,, \qquad J_a(\phi) = \mathrm{Tr}[\Gamma_a J]\,, \qquad Q_\alpha(\phi) = -\frac{k}{2\pi}\bar{\nu}'_\beta \lambda^\beta_\alpha\,, \tag{3.18}$$

this yields

$$\begin{aligned}
\{P_a(\phi), P_b(\phi')\}^* &= 0\,,\\
\{J_a(\phi), J_b(\phi')\}^* &= \epsilon_{abc} J^c \delta(\phi-\phi') - \mu\frac{k}{2\pi}\eta_{ab}\partial_\phi\delta(\phi-\phi')\,,\\
\{J_a(\phi), P_b(\phi')\}^* &= \epsilon_{abc} P^c \delta(\phi-\phi') - \frac{k}{2\pi}\eta_{ab}\partial_\phi\delta(\phi-\phi')\,,\\
\{P_a(\phi), Q_\alpha(\phi')\}^* &= 0\,,\\
\{J_a(\phi), Q_\alpha(\phi')\}^* &= \frac{1}{2}(Q\Gamma_a)_\alpha \delta(\phi-\phi')\,,\\
\{Q_\alpha(\phi), Q_\beta(\phi')\}^* &= -\frac{1}{2}(C\Gamma^a)_{\alpha\beta} P_a \delta(\phi-\phi') - \frac{k}{2\pi} C_{\alpha\beta}\partial_\phi\delta(\phi-\phi')\,,
\end{aligned} \tag{3.19}$$

which is the affine extension of the super-Poincaré algebra (2.2).

3.3 Super-BMS$_3$ algebra from a modified Sugawara construction

In order to recover the super-BMS$_3$ algebra (1.1) from the affine extension of the super-Poincaré algebra in (3.19), it can be seen that the standard Sugawara construction has to be slightly improved. Indeed, let us consider bilinears made out of the currents components P_a, J_a, Q_α, given by

$$\mathcal{H} = \frac{\pi}{k} P^a P_a\,, \quad \mathcal{P} = -\frac{2\pi}{k} J^a P_a + \mu\mathcal{H} + \frac{\pi}{k} Q_\alpha C^{\alpha\beta} Q_\beta\,, \quad \mathcal{G} = 2^{3/4}\frac{\pi}{k}\left(P_2\, Q_+ + \sqrt{2}\, P_0\, Q_-\right), \tag{3.20}$$

for which the current algebra (3.19) implies

$$\begin{aligned}
\{\mathcal{H}(\phi), P_a(\phi')\}^* &= 0\,, & \{\mathcal{P}(\phi), P_a(\phi')\}^* &= P_a(\phi)\delta'(\phi-\phi')\,,\\
\{\mathcal{H}(\phi), J_a(\phi')\}^* &= -P_a(\phi)\delta'(\phi-\phi')\,, & \{\mathcal{P}(\phi), J_a(\phi')\}^* &= J_a(\phi)\delta'(\phi-\phi')\,,\\
\{\mathcal{H}(\phi), Q_\alpha(\phi')\}^* &= 0\,, & \{\mathcal{P}(\phi), Q_\alpha(\phi')\}^* &= Q_\alpha(\phi)\delta'(\phi-\phi')\,,
\end{aligned} \tag{3.21}$$

$$\begin{aligned}
\{\mathcal{G}(\phi), P_a(\phi')\}^* &= 0\,,\\
\{\mathcal{G}(\phi), J_a(\phi')\}^* &= -\frac{\pi}{2^{1/4}k}(\epsilon_{abc}(Q\Gamma^b)_+ P^c + P_a Q_+)\delta(\phi-\phi') - \delta'(\phi-\phi')\frac{1}{2^{1/4}}(Q\Gamma_a)_+(\phi')\,,\\
\{\mathcal{G}(\phi'), Q_\alpha(\phi')\}^* &= -\frac{1}{2^{1/4}}\mathcal{H} C_{\alpha+}\delta(\phi-\phi') + \delta'(\phi-\phi')\frac{1}{2^{1/4}}(C\Gamma_a)_{\alpha+} P^a(\phi)\,.
\end{aligned} \tag{3.22}$$

When expressed in terms of modes, the algebra of the generators $\mathcal{H}$, $\mathcal{P}$ corresponds to the pure BMS$_3$ algebra without central extensions, i.e., the bosonic part of (1.1) with $c_1 = 0 = c_2$. This does however not hold for the mode expansion of the full set $\mathcal{H}$, $\mathcal{P}$, $\mathcal{G}$ whose algebra disagrees with the non-centrally extended super BMS$_3$ algebra given in (1.1). It reflects the fact that the non-constrained super-WZW model (3.10) is invariant under

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global BMS$_3$ transformations, but not under the full super-BMS$_3$ symmetries, in the sense that there are no (obvious) superpartners to $\mathcal{H}$, $\mathcal{P}$ that would close with them according to the (non centrally extended) super-BMS algebra (see [7] for an analogous discussion in the case of the superconformal algebra).

According to the fall-off of the gauge field in (2.11), the remaining boundary conditions that have to be taken into account imply that $[\lambda^{-1}\lambda']^1 = 1$, $[\lambda^{-1}\nu']^- = 0$, $[\lambda^{-1}(-\frac{1}{4}\nu\bar{\nu}' - \frac{1}{8}\bar{\nu}'\nu\mathbf{1} + \alpha')\lambda]^1 = 0$. In terms of the currents, this amounts to imposing the following first class constraints

$$P_0 = \frac{k}{2\pi}\,, \qquad J_0 = -\frac{\mu k}{2\pi}\,, \qquad Q_+ = 0\,. \tag{3.23}$$

The super-BMS$_3$ invariance of our model with the correct values of the central charges is recovered only once the constraints (3.23) are imposed. The generators of super-BMS$_3$ symmetry in the constrained theory are given by

$$\begin{aligned}\tilde{\mathcal{H}} &= \mathcal{H} + \partial_\phi P_2,\\ \tilde{\mathcal{P}} &= \mathcal{P} - \partial_\phi J_2,\\ \tilde{\mathcal{G}} &= \mathcal{G} + 2^{3/4}\partial_\phi Q_+(\phi),\end{aligned} \tag{3.24}$$

which are representatives that commute with the first class constraints (3.23), on the surface defined by these constraints. Furthermore, on this surface, the Dirac brackets of the generators are given by

$$\begin{aligned}&\{\tilde{\mathcal{H}}(\phi), \tilde{\mathcal{H}}(\phi')\}^* = 0\,,\\ &\{\tilde{\mathcal{H}}(\phi), \tilde{\mathcal{P}}(\phi')\}^* = (\tilde{\mathcal{H}}(\phi) + \tilde{\mathcal{H}}(\phi'))\partial_\phi\delta(\phi-\phi') - \frac{k}{2\pi}\partial_\phi^3\delta(\phi-\phi')\,,\\ &\{\tilde{\mathcal{P}}(\phi), \tilde{\mathcal{P}}(\phi')\}^* = (\tilde{\mathcal{P}}(\phi) + \tilde{\mathcal{P}}(\phi'))\partial_\phi\delta(\phi-\phi') - \frac{\mu k}{2\pi}\partial_\phi^3\delta(\phi-\phi')\,,\\ &\{\tilde{\mathcal{H}}(\phi), \tilde{\mathcal{G}}(\phi')\}^* = 0\,,\\ &\{\tilde{\mathcal{P}}(\phi), \tilde{\mathcal{G}}(\phi')\}^* = (\tilde{\mathcal{G}}(\phi) + \frac{1}{2}\tilde{\mathcal{G}}(\phi'))\partial_\phi\delta(\phi-\phi')\,,\\ &\{\tilde{\mathcal{G}}(\phi), \tilde{\mathcal{G}}(\phi')\}^* = \tilde{\mathcal{H}}(\phi)\delta(\phi-\phi') - \frac{k}{\pi}\partial_\phi^2\delta(\phi-\phi')\,,\end{aligned} \tag{3.25}$$

so that, once expanded in modes according to

$$\mathcal{P}_m = \int_0^{2\pi} d\phi\, e^{im\phi}\tilde{\mathcal{H}}\,, \qquad \mathcal{J}_m = \int_0^{2\pi} d\phi\, e^{im\phi}\tilde{\mathcal{P}}\,, \qquad \mathcal{Q}_m = \int_0^{2\pi} d\phi\, e^{im\phi}\tilde{\mathcal{G}}\,,$$

the super-BMS$_3$ algebra (1.1) with central charges given in (1.2) is recovered.

4 Reduced super-Liouville-like theory

In order to obtain the reduced phase space description of the action (3.10) on the constraint surface defined by (3.23), it is useful to decompose the fields according to

$$\lambda = e^{\sigma\Gamma_1/2}e^{-\varphi\Gamma_2/2}e^{\tau\Gamma_0}\,, \qquad \alpha = \frac{\eta}{2}\Gamma_0 + \frac{\theta}{2}\Gamma_2 + \frac{\zeta}{2}\Gamma_1\,, \tag{4.1}$$

where $\sigma, \varphi, \tau, \eta, \theta, \zeta$ stand for functions of u, ϕ. The constraints (3.23) then become

$$\begin{aligned} \sigma' &= e^{\varphi}\,, \\ \zeta' &= \mu(e^{\varphi} - \sigma') + \frac{1}{2}\sigma^2\eta' + \sigma\theta'\,, \\ \nu^{-\prime} &= \frac{1}{\sqrt{2}}\sigma\nu^{+\prime}\,, \end{aligned} \tag{4.2}$$

and hence, by virtue of (4.1) and (4.2), the reduced chiral super-WZW action (3.10) is given by

$$I_R = \frac{k}{4\pi}\int du d\phi \left[\xi'\dot{\varphi} - \varphi'^2 + \mu\varphi'\dot{\varphi} + \frac{1}{\sqrt{2}}\chi\dot{\chi}\right]\,, \tag{4.3}$$

where $\xi := -2(\theta + \eta\sigma) + \frac{1}{2}(\nu^-\nu^+)$, and $\chi := e^{\varphi/2}\nu^+$. It is worth noting that, in the case of $\mu = 0$, the bosonic part of (4.3) is related to a flat limit of Liouville theory [9]. The super-BMS$_3$ generators (3.24) then reduce to

$$\tilde{\mathcal{H}} = \frac{k}{4\pi}\left(\varphi'^2 - 2\varphi''\right)\,, \quad \tilde{\mathcal{P}} = \frac{k}{4\pi}\left(\xi'\varphi' - \xi'' + \frac{1}{\sqrt{2}}\chi\chi'\right) + \mu\tilde{\mathcal{H}}\,, \quad \tilde{\mathcal{G}} = 2^{1/4}\frac{k}{4\pi}\left(\frac{1}{2}\varphi'\chi - \chi'\right)\,, \tag{4.4}$$

which generate the following transformations

$$\begin{aligned} \delta\varphi &= Y\varphi' + Y'\,, \\ \delta\xi &= 2f\varphi' + \xi' Y + 2f' - 2^{1/4}\epsilon\chi\,, \\ \delta\chi &= Y\chi' + \frac{1}{2}Y'\chi + 2^{-1/4}\epsilon\varphi' + 2^{3/4}\epsilon'\,, \end{aligned} \tag{4.5}$$

with $f = T(\phi) + uY'$, $Y = Y(\phi)$, and $\epsilon = \epsilon(\phi)$. Therefore, by construction, the super-Liouville-like theory turns out to be invariant under (4.5), and the mode expansion of the algebra of Noether charges is again given by (1.1) and (1.2).

5 Gauged chiral super-WZW model

The super-Liouville-like action (4.3), that has been shown to be equivalent to the chiral super-WZW model (3.10) on the constraint surface given by (3.23), can also be described through a gauged chiral super-WZW model. Here we follow the procedure given in [36], where it was shown that Toda theories can be written as gauged WZW models based on a Lie group G. The action is endowed with additional terms involving the currents linearly coupled to some gauge fields that belong to the adjoint representation of the subgroups of G generated by the step operators associated to the positive and negative roots.

Hence, we consider the following action principle

$$\begin{aligned} I[\lambda, \alpha, \nu, A_\mu, \bar{\Psi}] = I[\lambda, \alpha, \nu] & \\ + \frac{k}{\pi}\int du d\phi \, \mathrm{Tr}\Big[& A_u\left(\lambda^{-1}\alpha'\lambda - u\left(\lambda^{-1}\lambda'\right)' - \frac{1}{4}\lambda^{-1}\nu\bar{\nu}'\lambda - \frac{1}{8}\bar{\nu}'\nu\mathbf{1}\right) \\ & + \tilde{A}_u(\lambda^{-1}\lambda') - \mu_M\tilde{A}_u + \left(\frac{1}{4}\lambda^{-1}\nu'\right)\bar{\Psi}\Big]\,, \end{aligned} \tag{5.1}$$

where $I[\lambda,\alpha,\nu]$ is the flat chiral super-Poincaré WZW action (3.10). Here $A_u, \tilde{A}_u$ are along Γ_0, and $\mu_M := \mu\Gamma_1$ with μ an arbitrary constant, while the fermionic gauge field $\bar{\Psi}$ fulfills $[\bar{\Psi}]_+ = 0$ (see appendix B for more details on the construction in the bosonic case).

One can then show that the action (5.1) is invariant (up to boundary terms) under the transformations given in (3.15), where a subset of the symmetries has been gauged by allowing for an arbitrary u dependence of the part of σ, ϑ that belongs to the subspace generated by Γ_0, of the fermionic parameters that belong to the subspace defined by $[\bar{\gamma}]_+ = 0$, $[\lambda\gamma]^- = 0$ and the non-trivial transformations for the gauge fields are

$$\begin{aligned} \delta_\sigma \tilde{A}_u &= -\left(\dot{\sigma} + [A_u, \sigma]\right) \ , \ \delta_\gamma \bar{\Psi} = -\partial_u \bar{\gamma} \, . \\ \delta_\vartheta A_u &= -(\dot{\vartheta} + [A_u, \vartheta]) \, , \ \delta_\vartheta \tilde{A}_u = -[\tilde{A}_u, \vartheta]. \end{aligned} \tag{5.2}$$

Therefore, the reduced theory described by the action in (4.3) is equivalent to the one in (5.1), which corresponds to a WZW model in which the subgroup generated by the first class constraints has been gauged. Indeed, the gauge fields A_u, $\tilde{A}_u$ and Ψ act as Lagrange multipliers for these currents, so that the variation of the action with respect to these non-propagating fields sets them to zero. In other words, solving the algebraic field equations for the gauge fields into the action amounts to imposing the first class constraints (3.23), which shows the equivalence of both descriptions.

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A Conventions

The orientation has been chosen so that the Levi-Civita symbol fulfills $\epsilon_{012} = 1$, while the tangent space flat metric η_{ab}, with $a = 0, 1, 2$, is assumed to be off-diagonal and given by

$$\eta_{ab} = \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix} .$$

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The Dirac matrices in three spacetime dimensions satisfy the Clifford algebra $\{\Gamma_a, \Gamma_b\} = 2\,\eta_{ab}$, and have been chosen as

$$\Gamma_0 = \sqrt{2}\begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}, \qquad \Gamma_1 = \sqrt{2}\begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix}, \qquad \Gamma_2 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}.$$

The matrices fulfill the following useful properties:

$$\Gamma_a\Gamma_b = \epsilon_{abc}\Gamma^c + \eta_{ab}\mathbf{1}\,, \qquad (\Gamma^a)^\alpha{}_\beta(\Gamma_a)^\gamma{}_\delta = 2\delta^\alpha_\delta\delta^\gamma_\beta - \delta^\alpha_\beta\delta^\gamma_\delta\,, \tag{A.1}$$

where $\alpha = +1, -1$. The Majorana conjugate is defined as $\bar\psi_\alpha = C_{\alpha\beta}\psi^\beta$, where

$$C_{\alpha\beta} = \varepsilon_{\alpha\beta} = C^{\alpha\beta} = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}, \tag{A.2}$$

stands for the charge conjugation matrix, which satisfies $C^T = -C$ and $C\Gamma_a C^{-1} = -(\Gamma_a)^T$. Note that this implies that $\overline{\Lambda^{-1}\psi} = \bar\psi\Lambda$, for any $\Lambda \in \mathrm{SL}(2,\mathbb{R})$. The conjugate of the product of real Grassmann variables is assumed to fulfill $(\theta_1\theta_2)^* = \theta_1\theta_2$.

B Gauged chiral bosonic WZW theory

Let us describe here a way to construct a gauged chiral $\mathfrak{iso}(2,1)$ WZW model associated to (3.10) for the purely bosonic case and $\mu = 0$. The action is given by

$$I(\lambda,\alpha) = \frac{k}{\pi}\int du d\phi\,\mathrm{Tr}\left[\dot\lambda\lambda^{-1}\alpha' - \frac{1}{2}(\lambda'\lambda^{-1})^2\right], \tag{B.1}$$

and it has the following Noether symmetries

$$\begin{aligned} \delta_\sigma\lambda &= 0\,, & \delta_\sigma\alpha &= \lambda\sigma(\phi)\lambda^{-1}\,, \\ \delta_\vartheta\lambda &= -\lambda\vartheta(\phi)\,, & \delta_\vartheta\alpha &= -u\lambda\vartheta'\lambda^{-1}\,. \end{aligned} \tag{B.2}$$

According to (3.23), we are interested in gauging the subset of these symmetries involving the parts of σ and ϑ along Γ_0. These parameters are promoted to depend on both u and ϕ.

One can check that the action

$$I(\lambda,\alpha,A_\mu) = I(\lambda,\alpha) + \frac{k}{\pi}\int du d\phi\,\mathrm{Tr}\left[-A_u\left(\lambda^{-1}\alpha'\lambda - u\left(\lambda^{-1}\lambda'\right)'\right) + \tilde A_u\lambda^{-1}\lambda'\right], \tag{B.3}$$

is invariant under

$$\begin{aligned} &\delta_\sigma\lambda = 0\,, && \delta_\sigma\alpha = \lambda\sigma(u,\phi)\lambda^{-1}\,, && \delta_\sigma A_u = 0\,, && \delta_\sigma\tilde A_u = -\left(\dot\sigma + [A_u,\sigma]\right), \\ &\delta_\vartheta\lambda = -\lambda\vartheta(u,\phi)\,, && \delta_\vartheta\alpha = -u\lambda\vartheta'\lambda^{-1}\,, && \delta_\vartheta A_u = -(\dot\vartheta + [A_u,\vartheta])\,, && \delta_\vartheta\tilde A_u = -[\tilde A_u,\vartheta]\,, \end{aligned} \tag{B.4}$$

with σ and ϑ along Γ_0.

Since the constraints we want to implement set some current components to a constant, the suitable final action is

$$I(\lambda,\alpha,A_\mu) = I(\lambda,\alpha) + \frac{k}{\pi}\int du d\phi\,\mathrm{Tr}\left[-A_u\left(\lambda^{-1}\alpha'\lambda - u\left(\lambda^{-1}\lambda'\right)'\right) + \tilde A_u\lambda^{-1}\lambda' - \mu_M\tilde A_u\right], \tag{B.5}$$

where $\mu_M := \mu\Gamma_1$, with μ an arbitrary constant, and A_u, $\tilde{A}_u$ are along Γ_0. The action (B.5) is indeed still gauge invariant since, as noticed in [36], the variation of $\mathrm{Tr}[\mu_M \tilde{A}_u]$ under a gauge transformation is a boundary term.

Finally, in order to see how the constraints are explicitly implemented, it is useful to parametrize the fields according to

$$\lambda = e^{\sigma\Gamma_1/2} e^{-\varphi\Gamma_2/2} e^{\tau\Gamma_0} \,, \qquad \alpha = \frac{\eta}{2}\Gamma_0 + \frac{\theta}{2}\Gamma_2 + \frac{\zeta}{2}\Gamma_1 \,. \tag{B.6}$$

The field equations for the gauge fields imply that $\sigma' e^{-\varphi} = \mu$ and $\eta'\sigma^2 + 2\theta'\sigma - 2\zeta' = 0$, so that, taking $\mu = 1$, the reduced action is

$$I = \frac{k}{4\pi} \int du d\phi \, \left[\xi' \dot{\varphi} - \varphi'^2\right] \,, \tag{B.7}$$

where $\xi := -2(\theta + \eta\sigma)$, in full agreement with the centrally extended BMS_3 invariant action found in [9].

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