CERN 62-30
10 October, 1962.

ORGANISATION EUROPÉENNE POUR LA RECHERCHE NUCLÉAIRE
CERN EUROPEAN ORGANIZATION FOR NUCLEAR RESEARCH

SUPER-CONDUCTIVITY AND THE QUANTIZATION OF MAGNETIC FLUX

by

V.F. Weisskopf

G E N E V A

CERN 62-30
10 October, 1962.

SUPER-CONDUCTIVITY AND THE QUANTIZATION OF MAGNETIC FLUX

by

V.F. Weisskopf

G E N E V A

SUPER-CONDUCTIVITY AND THE QUANTIZATION OF MAGNETIC FLUX

Introduction

In this article some fundamental ideas are presented which lead to the understanding of the phenomenon of super-conductivity. It does not contain any new or original thoughts, on the contrary it mainly deals with ideas that were conceived a long time ago by London* , and which prepared the logical framework for the recent ideas of Bardeen, Cooper and Schrieffer**. It was the work of these latter authors which, for the first time, provided a theoretical understanding of super-conductivity on the basis of interactions between electrons. Nevertheless, this article will treat their work only in a phenomenological way; perhaps in the same way as one would treat the molecular interaction between molecules in a liquid when one speaks about some interesting features of the liquid state.

Our aim is mainly to clarify the concept of a super-current as opposed to an ordinary current and its close interrelation with magnetic fields. It then follows quite naturally that the magnetic flux through a super-conducting ring must be a multiple*** of $\frac{hc}{2e}$, and it becomes clear that the stability of currents in super-conducting loops is intimately related to this quantization of the magnetic flux.

---

*F. London, Superfluids, John Wiley, New York, 1950.

**Bardeen, Cooper and Schrieffer, Phys. Rev., 108, 1175 (1957).

***The reasoning presented here is a greatly simplified version of what was first presented by N. Byers and C.N. Yang, Phys. Rev. Letters, 7, 46 (1961).

I. METALLIC CONDUCTION

A metal is characterized by the fact that the conduction electrons move fairly freely throughout its interior. This is why many electric phenomena in metals can be well described by the extreme assumption that the conduction electrons are forming a gas of free particles within the combines of the metal. Such a gas is highly degenerated at temperatures occurring in solid metals. Hence the energy distribution of electrons is of the characteristic Fermi type: at zero temperature the electrons fill compactly all the energy levels available to free particles, up to a maximum energy E_F, the Fermi energy; at somewhat higher temperature the occupation is compact only for energies that lie below E_F by much more than kT. The occupation is "diffuse" in a zone of the order kT around the Fermi energy.

In view of the strong electric forces exerted on the electrons by the metallic ions, the free motion of electrons through the metal seems paradoxical. But in quantum mechanics the motion through a potential with an exact periodicity is almost identical with motion through a vacuum, except when the wave length fulfils the Bragg-conditions, which is not the case for conduction electrons in a metal. However, in order for the electrons to move freely, the lattice must be ideal. The thermal motion and impurities of the lattice destroy its ideality and produce scattering of the electrons. If an electric field is applied, the electrons are not accelerated indefinitely as they would be in the ideal case of free motion. The scattering has the effect of a friction upon the motion of the electrons and this explains the resistance of metals. There is one phenomenon, however, which remains unexplained: the phenomenon of super-conductivity at very low temperatures.

The explanation of super-conductivity has been found only recently, by Bardeen, Cooper and Schrieffer, although many of the main features of the phenomenon were analysed much earlier. Whereas ordinary resistance comes from the interaction of electrons with the ions, super-conductivity is based upon an interaction of electrons among themselves. There are two kinds of interaction: one is the obvious Coulomb repulsion; the other is an attraction coming from the following effect: whenever an electron moves through the lattice, it deforms it slightly in its immediate neighbourhood. A second electron, coming nearer to the first one, finds itself already in a deformed region, thus experiencing a lowering of its potential energy. This is equivalent to an effective attraction between electrons. The strength and quality of this attraction depends on the properties of the lattice. In some metals it is strong and can overcome the Coulomb repulsion, in particular since the latter one is sometimes effectively weakened by dielectric effects. These metals are the super-conducting ones.

II. THE COMPACT DISTRIBUTION

We are not going to explain in detail how the net attraction between the electrons produces its effects. We only describe its main effect and then show how this effect produces super-conductivity.

Let us look again at a gas of free electrons. In Figure 1 we wee a symbolic presentation of the quantum levels of a single electron in a metal. When N is the number of conduction electrons, the N^{th} level from below is the highest occupied one at zero temperature and has E_F as excitation energy. The energy difference, say, between the N^{th} and the $(N+1)^{th}$ state is inversely proportional to the volume containing the electron gas; it is "microscopically" small, by which we mean smaller than any temperature energy kT, for temperatures even way below the transition point.

Let us now look at the spectrum of the quantum states of the free electron gas as a whole (Figure 2). The lowest state is the state where the electrons occupy compactly all levels up to the Fermi energy*. The next higher

*The term "level" is used for the quantum states of a single electron, whereas the term "state" refers exclusively to the quantum states of the electron gas as a whole.

state of the gas as a whole is the one in which a single electron is lifted to the first level above the Fermi energy, etc. The first excited state of the electron gas is therefore only "microscopically" higher than the ground state. The density of states of the gas at somewhat higher excitation energies is of course much larger than the density of levels of the single electrons. This is the situation for a gas of non-interacting particles.

The most important effect of the attraction between electrons is this: it increases the energy difference Δ between the ground state of the electron gas and its first excited state, such that Δ is no longer microscopically small (see Figure 3). This increase comes from a special stability of what we will call a "compact distribution" of electrons. We define a compact distribution as one in which M electrons occupy all single particle levels from the lowest to the M^{th} level, so that there is no unoccupied level below an occupied one. Here M can be any (even) number of conduction electrons; it can be equal to N or less. Such a compact distribution is especially stable; to remove one electron, and to put it into a higher unoccupied level, needs a definite macroscopic energy Δ, which is also called the "gap".

The existence of this gap suggests an analogy between a compact distribution and a condensed phase. The energy Δ plays the rôle of a binding energy. It takes an energy Δ to remove an electron from the condensed to the gaseous phase. However the removal here is not a removal in space as the removal of an atom from a liquid, but a removal in the momentum space from the compact distribution to a higher momentum or energy. This analogy leads us to expect a transition temperature T_c below which the compact phase exists in equilibrium with the gaseous phase. If that temperature is raised above T_c, the compact distribution "evaporates" into the ordinary one. Clearly T_c is connected with the gap energy Δ by the relation $kT_c \approx \Delta$.

We must make a further assumption regarding the properties of the compact distribution. We assume that the effects of the attractive forces are so weak that the electrons can be considered to move almost like free electrons also in the condensed phase. The interaction changes only the energy relations by producing the energy gap Δ ; the dynamical relations, such as the quantization of the momentum, the relation between momentum and velocity, remain in all essential points the same as for free electrons.

We are not going to explain why and how the attraction between electrons produces such a condensation into the compact distribution, although it is not unplausible that such an effect could be caused by attractive forces. Let us be reminded that the details of the condensation of gases into liquids are far from being known and understood. We will assume the "gap" as existing and we will discuss the phenomena which it produces.

III. MAGNETIC FIELD AND CURRENT

Under ordinary conditions, when there is no magnetic or electric field present, the electron distributions, compact or not, have no net current. For every electron state which moves in a given direction, say, with a momentum $\vec{p}$, there is another of equal energy with $-\vec{p}$; if one is occupied so is the other and their currents cancel out.

Let us represent the electron levels of a metal in the following simplified way. We assume the metal to have cylindrical symmetry, in fact we will only deal with either a solid cylinder or a cylinder with a hole along the axis (Figure 4 a, b). We introduce cylindrical co-ordinates z , r , φ and we are, of course, mainly interested in the currents flowing in the φ-direction. For the sake of simplicity we disregard therefore the co-ordinates z , r and consider only the variable φ at a radius r . We then deal with a one-dimensional problem and our circular electron states must be such that there is an integer number of wave lengths along the circumference $2\pi r$.

Hence we get electron levels whenever the momentum is $p_n = \frac{n\hbar}{r}$, where n is an integer. The energy and the velocity of these states are given by

$$E_n = n^2 \frac{\hbar^2}{2mr^2} \quad , \qquad v_n = n \frac{\hbar}{mr} \tag{1}$$

We therefore get the parabolic energy-velocity plot shown in Figure 5 where the quantum levels are those points whose velocity is an integer multiple of $\frac{\hbar}{mr}$. In this simplified presentation a compact distribution would be one in which all states with $|n| < \frac{N-1}{2}$ being occupied*, for positive and negative n . A non-compact "gaseous" distribution of electrons in thermal equilibrium would be one in which there are some free levels below the Fermi energy and some occupied ones above. The situation for positive and for negative values of n is symmetrical in either case; for every positive n there is a level of equal energy with negative n and opposite velocity. Hence the total current is zero in the compact distribution and in any other distribution at thermal equilibrium.

How does an electric field produce a current under these conditions? The electrons are accelerated in the direction of the field. It means in quantum language that the electrons perform transitions to neighbouring levels under the influence of the field; this change of occupation destroys the symmetry of the distribution and produces a current. When the field disappears, electron scattering re-establishes the original distribution. Such transitions are possible only in a non-compact distribution. When the distribution is compact any change of levels requires at least an energy Δ ; this is much more than ordinary electric fields can provide. Hence currents in a compact distribution are of different nature.

Now let us introduce a constant magnetic field $\mathcal{H}$ parallel to the z-axis. This field can be described by a vector potential $\vec{A}$ such that

---

* We neglect here and in what follows the existence of the electron spin which would double all states.

$\vec{A}_\varphi = \frac{1}{2} r |\mathcal{H}|$, $\vec{A}_r = \vec{A}_z = 0$. In the presence of a magnetic field the relation between momentum and velocity is changed. We now get

$$\vec{v} = \frac{1}{m} (\vec{p} - \frac{e}{c} \vec{A}) \qquad (2)$$

In classical physics this change is not of great importance. Only the velocity is a physical magnitude, and eqn. (1) is a redifinition of momentum. In quantum physics, however, the momentum has more physical significance; it is the reciprocal wave length and therefore determines the quantum states since there must be an integer multiple of wave lengths on a circular path. In order to get rid of apparent arbitrariness because of the gauge of $\vec{A}$, one can introduce the magnetic flux ϕ through a loop with radius r : $\phi = 2\pi r A_\varphi$ and we get for the velocity of the quantized orbits from (1) and (2)

$$v_n = \frac{1}{m} (p_n - \frac{e}{2\pi r c} \phi) = \frac{\hbar}{m r} (n - \frac{e}{2\pi \hbar c} \phi) \qquad (3)$$

The switching on of a magnetic field changes the velocities in all quantum orbits. Of course, this is exactly the change one expects classically from the induction current caused by the switching on of the magnetic flux. According to Ehrenfest's adiabatic principle the quantum orbits change in the same way as a corresponding classical orbit.

What becomes of the distribution of Figure 5 in the presence of a magnetic field? The electron levels are no longer found at values $v_n = \frac{\hbar}{mr}$; the allowed values of v are shifted by ϕ/ϕ_o where $\phi_o = \frac{2\pi \hbar c}{e}$, as indicated in Figure 6 for $\phi/\phi_o \sim 2/3$.

We now can see easily that a compact distribution no longer can have a net zero-current; positive and negative v_n do no longer compensate. The current which we get that way is of different origin than the current produced by an electric field. It is not caused by transitions to levels of different velocity; the electrons remain in their original levels, but the velocity in each level is changed in the presence of a magnetic field.

If the distribution is not compact, a total current zero can always be established, by having a few electrons on the side where the v_n are higher (right side in Figure 6) be scattered into some of the holes on top of the side where the v_n are lower (left side in Figure 6) and compensate for the asymmetry. This is what happens in a normally conducting metal when a magnetic flux is switched on. The induction produces a current, which decreases to zero because of the resistance in the metal. That decrease is caused by the scattering described above. No such decrease would take place, however, in a super-conducting metal because of the compact electron distribution. An electron at the high side of the distribution cannot be scattered to the low side because it would have to be broken off the compact distribution. This needs the energy Δ which is much higher than the energy they would gain by going over to the low side of the distribution. *

IV. CONDUCTING CYLINDER IN MAGNETIC FIELD

Let us now see in greater detail what happens in a metallic cylinder (Figure 4 a) when a magnetic field parallel to the z-axis is switched on. If the temperature is above the transition point, the switching on process creates circular currents which soon die out. The end state is one without currents and a constant magnetic field throughout the metal, but a state with a different

---

*There seems to be a logical flaw in this explanation. One could argue that the topmost electron on the high side might be scattered to the lowest unoccupied state on the low side and, thus, form again a compact distribution, so that the energy Δ is gained in the end state. This is not so, however. In the compact distribution, electron pairs are interacting with each other, in particular the pairs of equal and opposite quantum numbers n. If an electron were shifted from the top of the right side to the top of the left side, it would not find any partner on the other side to interact, and hence cannot integrate into a new compact distribution except if all electrons change their partners. Such a complete reorganization of the compact distribution has a very low probability (low overlap integral).

electron distribution than before the magnetic field was switched on. If the temperature is below transition point, the compact electron distribution cannot change and therefore maintains a circular current in the presence of a magnetic field. This current has the direction induced by the switching on of the magnetic field and produces a magnetic field of its own which, inside the circular current, is opposed to the direction of the original field. As we will see immediately, the current induced near the surface of the cylinder is strong enough to cancel the original field over most of the inside of the conductor. Outside of the surface the magnetic field produced by the current adds to the original one. Hence we get qualitatively a current and field distribution as a function of the radius which is shown in Figure 7.

Let us be more quantitative. According to (2) each electron gets an additional velocity $\frac{e}{mc}\vec{A}$ in the presence of a magnetic field. The total induced current density j is then given by $\vec{j} = -\nu \frac{e^2}{mc}\vec{A}$ where ν is the number of electrons per unit volume in the compact distribution. Taking the curl of this relation we obtain the London equation:

$$\text{curl}\,\vec{j} = -\nu \frac{e^2}{mc}\vec{\mathcal{H}} = -\frac{c}{\ell^2}\vec{\mathcal{H}} \qquad (4)$$

Another curl gives us (remember curl curl $j = -\nabla^2 j$ since $\vec{\nabla}\cdot\vec{j} = 0$ and curl $\vec{\mathcal{H}} = \vec{j}$)

$$\nabla^2\vec{j} = \nu \frac{e^2}{mc^2}\vec{j} \qquad (5)$$

From this follows that, if j depends on one co-ordinate x, for example the distance from the surface, it must be exponential $|j| \sim e^{\pm\frac{x}{\ell}}$ where $\ell = \left(\frac{mc^2}{e^2\nu}\right)^{1/2}$. Hence at any surface of a super-conductor the current must drop towards the inside (it cannot very well increase to infinity) exponentially with a penetration depth $\ell = d\left(\frac{d}{r_o}\right)^{1/2}$ where $d = \nu^{-1/3}$ is the average distance between onduction electrons and r_o is the classical electron radius. Since d is of the order of the Bohr radius, ℓ is approximately 137 d. In fact, it is somewhat larger than that.

Having seen now that the super-current is contained in a very thin layer at the surface, it is better to look at it as a current sheet whose strength is measured by a current J per cm. (It is the current in a strip of the sheet of 1 cm width parallel to the current direction.) If j is the current density in a sheet of thickness ℓ, we get $J = \ell j$. From London's equation (4) it follows * then simply that, whenever there is a magnetic field $\vec{\mathcal{H}}$ along (not perpendicular to) a surface of a super-conductor, there must be a current sheet flowing in that surface at right angles to $\vec{\mathcal{H}}$ with a current per cm $J = c\,|\mathcal{H}|$. This is what one would call the differential law of super-conductivity. The current is just big enough to cancel the magnetic field inside the super-conductor in a depth larger than the thickness ℓ of the sheet.

The situation of a super-conducting cylinder of Figure 4 a in a magnetic field parallel to its axis is then as follows: the magnetic field sets up a current sheet around the cylinder, which increases the original field in the neighbourhood outside the conductor, and cancels the field inside the conductor. Let us keep in mind that this current is <u>not</u> caused by a change in the electron distribution of the metal, as ordinary currents are. It is the same unchanged compact distribution which, in the presence of a magnetic field, gives rise to a current**. This current is confined to the surface region because the current itself prevents the magnetic field from being established inside.

---

*Derivation: we perform a surface integral of equation (4) over a rectangle 1 cm long and Δ cm wide, where $1 \gg \Delta \gg \ell$. Its plane is perpendicular to the surface, in the direction of the current. One long side lies on the surface, the other in the interior of the metal. The surface integral over the left side of (4) is equal to the contour integral of j and gives j. The surface integral over the right side gives $|\mathcal{H}|\, c/\ell$ since $\vec{\mathcal{H}}$ only penetrates as far as ℓ into the metal. Hence we get $J = \ell j = c\,|\mathcal{H}|$.

**The compact distribution in the presence of a magnetic field does no longer exactly fulfil the definition which we gave on page 3. No longer are all levels filled up to a given energy. It is seen in Figure 6 that the levels on one side would be occupied up to a higher level than in the other, if the flux is more than Φ_0. Moreover, without field, the distribution consists of pairs of levels, n and -n, which are exactly opposed to each other (one is the time reversed of the other). The interaction between such pairs of electrons seems to be of particular importance for the stability of the compact distribution. In the region of magnetic field the

(cont'd, see over)

We are now in a position to understand the lack of resistance in a piece of straight super-conducting cylindrical wire of radius R , inserted between two ordinary conductors. The net current I through the wire produces a magnetic field around the wire whose strength at the wire's surface according to an elementary formula is $\mathcal{H} = I/_{2\pi\ Rc}$. This field produces a current sheet $J = c\,\mathcal{H}$ running in the direction of the wire and which adds up correctly to the total current I , since the total current of the current sheet is $2\pi\ RJ = I$. Hence the total current through the super-conducting wire is carried by the surface current sheet. Again, we have the same compact electron distribution all through the wire; it is a distribution which would give no current without a magnetic field, but which gives rise to a surface current in the presence of the circular magnetic field $\mathcal{H}$.

If the temperature of the wire were suddenly raised, the current carrying electron distribution which the magnetic field has produced at the surface would soon be destroyed by scattering, and the current would thereby be reduced. Then an electric potential drop develops between the two ends of the wire, which changes the electron distribution by accelerating the electrons in the direction of the electric field. The balance between this accelerating effect and the slowing down by scattering establishes the ordinary current according to Ohm's law.

The lack of resistance in the super-conducting state comes from the fact that the compact electron distribution produces by itself a surface current in the circular magnetic field which this current is surrounded with. Hence no electric potential drop develops between the ends of the wire. The original unchanged compact electron distribution is able to provide for the current.

**(cont'd)
symmetry between the members of these pairs is destroyed. This is why we must amend our definition here: a compact distribution must meet all requirements as to filling levels up to a given energy and consisting of time reversed pairs, in the interior of the metal only. Near the surface we assume that the compact distribution will remain the same as in the interior in terms of quantum level occupation, in spite of the fact that some of the requirements of our definitions are not fulfilled in the presence of a magnetic field. The lack of symmetry in the pairs does not seem to affect the stability as long as it is only restricted to a very thin surface region.

V. SUPER-CONDUCTING RING IN MAGNETIC FIELD THE QUANTIZATION OF MAGNETIC FLUX

The situation becomes much more interesting when we consider a metallic ring (Figure 4 b). Above transition temperature everything is quite simple: the switching on of a magnetic field parallel to the z-axis produces a current around the ring which dies down after a short time, and then the magnetic field establishes itself all through the conductor.

In the super-conducting state the situation is different. At first one might think that it is impossible to have any magnetic flux at all through the hole of the ring and therefore no net circular current flowing in the metal. One argues as follows: in the super-conducting state there should be currents only at the surfaces and not in the interior of the metal. If there is a magnetic flux through the hole, however, the compact distribution ought to produce a circular current in the interior of the metal; this is because any circular line around the hole within the metal would include the magnetic flux wherever it is put, deep in the interior of the metal or near its surface (Figure 9). According to (3) this should give rise to a current. Hence we arrive at a contradiction. In fact, a flux of a few gauss/cm^2 would give a current density of 10^{10} amps/cm^2 in the interior of a conducting loop of 1 cm radius where the current should be zero. This is an impossible situation since equation (5) has shown that the current density must fall off exponentially towards the interior of a super-conductor within a small distance from the surface.

In order to resolve this contradiction we must re-examine the concept of compact distribution. Let us have another look at Figure 5 and Figure 6. We saw that the electronic quantum levels are shifted when a magnetic field is switched on. An interesting situation arises when the magnetic flux is an integer multiple of $\Phi_0 = \frac{2\pi \hbar c}{e}$. Then, the allowed quantum levels are exactly the same as for zero flux; they are only labelled differently. In fact, in the case of $\Phi = k\,\Phi_0$, k being integer, the situation is similar to the situation in classical physics for any value of Φ: there is no change compared to $\Phi = 0$ except a redefinition of the magnitude p.

What is the physical consequence of this? We have recognized before that the compact distribution which is established for $\phi = 0$ would carry a current at $\phi \neq 0$. But we can see now that, for $\phi = k\,\phi_o$, one can establish a different compact distribution with zero current, simply by occupying the levels symmetrically around the one which has $v_n = 0$, which will not be the level with $n = 0$, but $n = k$. Hence we have resolved the difficulty encountered before. It is possible to have a compact distribution with zero current in the interior of a super-conducting ring including a flux ϕ provided that the flux is an integer multiple of ϕ_o. In fact, this new compact distribution is exactly identical with the one at zero flux. It also consists of pairs of electrons with levels that are symmetrical in respect to time reversal. The only difference is the labelling of the levels. So there can be, in fact, a super-conducting ring including a flux $\phi = k\,\phi_o$ with zero current in the interior of the metal. Of course there will be currents at the surface in order to maintain the magnetic field, but this will be discussed later on.

Before we study the details of that situation in respect to the surface currents, we must add the following important observation: a compact distribution with zero current can also be established for fluxes being half integer multiples of ϕ_o : $\phi = (k + \frac{1}{2})\,\phi_o$. This is easily seen in Figure 10. In this case the allowed velocities v_n are again arranged symmetrically around $v = 0$, and a currentless compact distribution is possible if the levels $k + n + 1$ and $k-n$, $n = 0, 1, 2 \ldots$ are occupied by pairs of electrons. This distribution also consists of pairs of opposite electrons. In this case, of course, the number of electrons is different by one unit from the number in the other compact distributions. But this is irrelevant since there are always some electrons evaporated off the condensed phase.

Even before getting into the details one fact already emerges: the magnetic flux through a super-conducting ring is quantized and can only assume integer and half integer values of* $\phi_o = \frac{2\pi\hbar c}{e}$.

---

* The flux ϕ_o can be defined as follows: an increase of flux through a circular electron orbit by the amount ϕ_o induces an increase of $\hbar$ in the angular momentum of the electron.

4635

VI. SUPER-CONDUCTING RING IN MAGNETIC FIELD THE STABILITY OF THE SUPER-CURRENT

Let us now investigate the details of the situation presented by a super-conducting ring which includes a magnetic flux. Since there is a magnetic field parallel to the z-axis along the interior cylindrical surface, we expect a current sheet running around that surface. This current sheet is in fact the super-current which maintains the flux through the hole*.

How does this current come about according to our ideas of a compact electron distribution in the metal? Let us look at the situation at the interior cylindrical surface. The total flux $\bar{\Phi}$ through the hole ($\bar{\Phi} = n\,\phi_0/2$, n : integer) must be an integer or half integer multiple of ϕ_0 : in order to establish a compact electron distribution with zero current density in the interior of the metal. However the magnetic field penetrates a small distance into the super-conductor. Hence the flux through a circular line which is directly on the inner cylindrical surface would be slightly smaller than $\bar{\Phi}$. The penetrating magnetic field is not included. In fact, the flux through any circle which runs in a region of the metal within the distance ℓ from the inner surface would be a little smaller than $\bar{\Phi}$ (see Figure 9). Any deviation of the magnetic flux from the quantized value $\bar{\Phi}$ in the surface region causes the compact distribution to carry a current. Therefore the compact electron distribution would give rise to a current density in that region although it does not give rise to any current in the interior of the metal.

Now we can understand the stability of a current in a super-conducting ring. Suppose the situation as described above is changed by a small perturbation. The super-current, for example, has decreased by a slight amount. This

---

* There is a much weaker return field in the opposite direction along the outer cylindrical surface. Hence there will be a much weaker counter sheet in that surface, whose effects are not essential. It weakens the net super-current by a small amount.

would decrease the magnetic flux $\bar{\Phi}$ by a small amount $\Delta \bar{\Phi}$. The value $\bar{\Phi} - \Delta \bar{\Phi}$, however, is no longer a quantized value. Hence the compact distribution would carry a current density all over the inner volume of the ring, which would give rise to an enormous net current ΔJ . This current ΔJ would have the <u>same</u> direction as the super-current sheet at the interior surface. The reason is obvious: both are caused by a flux which is <u>smaller</u> than the quantized flux $\bar{\Phi}$. Therefore the additional current ΔJ would quickly make up for the decrease of the super-current which has started the perturbation, and redress the situation of the flux $\bar{\Phi}$.

This mechanism insures the stability of a quantized flux in a super-conducting loop and of the super-current which produces it. Any small change of the flux away from $\bar{\Phi}$ creates very large currents which tend to cancel the change immediately. Hence super-currents last for years unchanged if the temperature is kept below the transition point.

The question that still remains to be answered is this: how is the compact electron distribution established in a ring carrying a super-current? It is not, after all, the same compact distribution which is realized in the metal without magnetic field and current. It is established when the super-current is brought to existence. In fact, no super-current can be induced, once the ring is below the transition temperature. Then that compact electron distribution is established which corresponds to zero flux - it is the ordinary one, realized in zero field - and no change is possible; the zero flux is "trapped". A super-current can be established as follows: the "warm" ring is put in a magnetic field, then the ring is cooled below transition temperature in the magnetic field. In this way a magnetic flux exists during the cooling and is trapped in the ring.

Let us see what happens with the electron distributions. We have four stages:

(1) warm ring, no field
(2) warm ring, magnetic field
(3) cold ring, magnetic field
(4) cold ring, no magnetic field.

In stage (1) there is the ordinary non-compact electron distribution, in which, in the average, the electron levels n and -n are equally occupied. The distribution is symmetrical around $n = 0$ since the levels n and -n have equal energy. (The numbers n refer to the electron levels as defined by equation (1) of our one-dimensional model.) When switching on the magnetic field by going from stage (1) to (2), a current pulse is induced which quickly dies down by metallic resistance. A new non-compact distribution is established which is symmetrical, around $n = \kappa$ if $\phi = \kappa\,\phi_0$ is the magnetic flux through the ring; that means, levels $\kappa + n$ and $\kappa - n$ have the same energy and therefore the same occupation number. (If κ is non integer, which is possible above transition temperature, the pair is $k+n$ and $k-n$ where k is the nearest integer to κ, and the statement is only approximately true.) When going from stage (2) to (3), this new distribution condenses into a compact one, symmetrical around $n = k$, where k is the nearest integer or half integer to κ, thus changing the flux if necessary from $\kappa\,\phi_0$ to $k\,\phi_0$. This distribution gives rise to a current sheet at the surface of the ring near the magnetic fields and cancels the field in the interior of the metal according to the now often discussed mechanism of super-conduction. In this stage there is an equally strong current sheet at the inner surface of the ring and at the outer surface, each running in the opposite direction. One prevents the magnetic field from penetrating into the interior of the ring from the inside, the other from the outside. The net current is zero. When going from stage (3) to (4), the magnetic field outside disappears (apart from the weak one which represents the closing of the field lines of the flux through the inside) and so does the outside current sheet. Its reduction is the effect of switching off of the magnetic field and the consequently induced counter current. The magnetic flux inside the loops cannot disappear, however; it is trapped since, as described before, any change in this flux would give rise to enormous current densities in the interior of the ring which would compensate any change.

A super-current carrying ring has a compact electron distribution which is symmetrical around a quantum number n different from zero. It is established when the electron distribution condenses into its compact phase from an ordinary non-compact one which was asymmetrical itself because it represented the currentless equilibrium distribution in the presence of a magnetic field.

- - - - -

Schematic picture of the quantum levels of a single electron. The level distance is microscopically small.

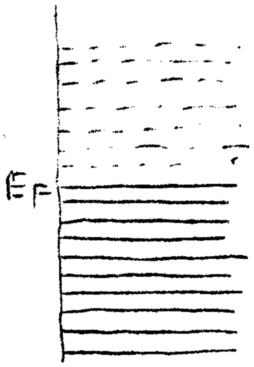

Fig 1

Schematic picture of the quantum states of a free electron gas as a whole. The level distance is microscopically small.

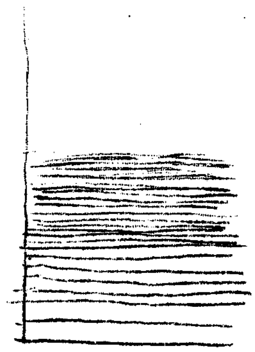

Fig 2

Schematic picture of the quantum states of an electron gas with attractive interaction. The "gap" Δ is macroscopic.

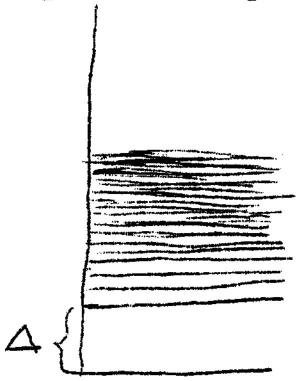

Fig 3

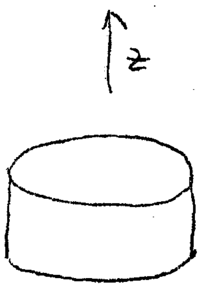

Fig 4a

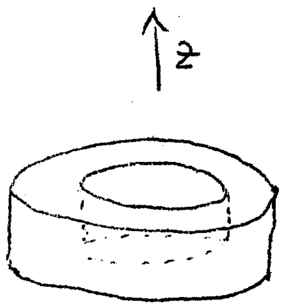

Fig 4B

The two shapes of conductors considered here.

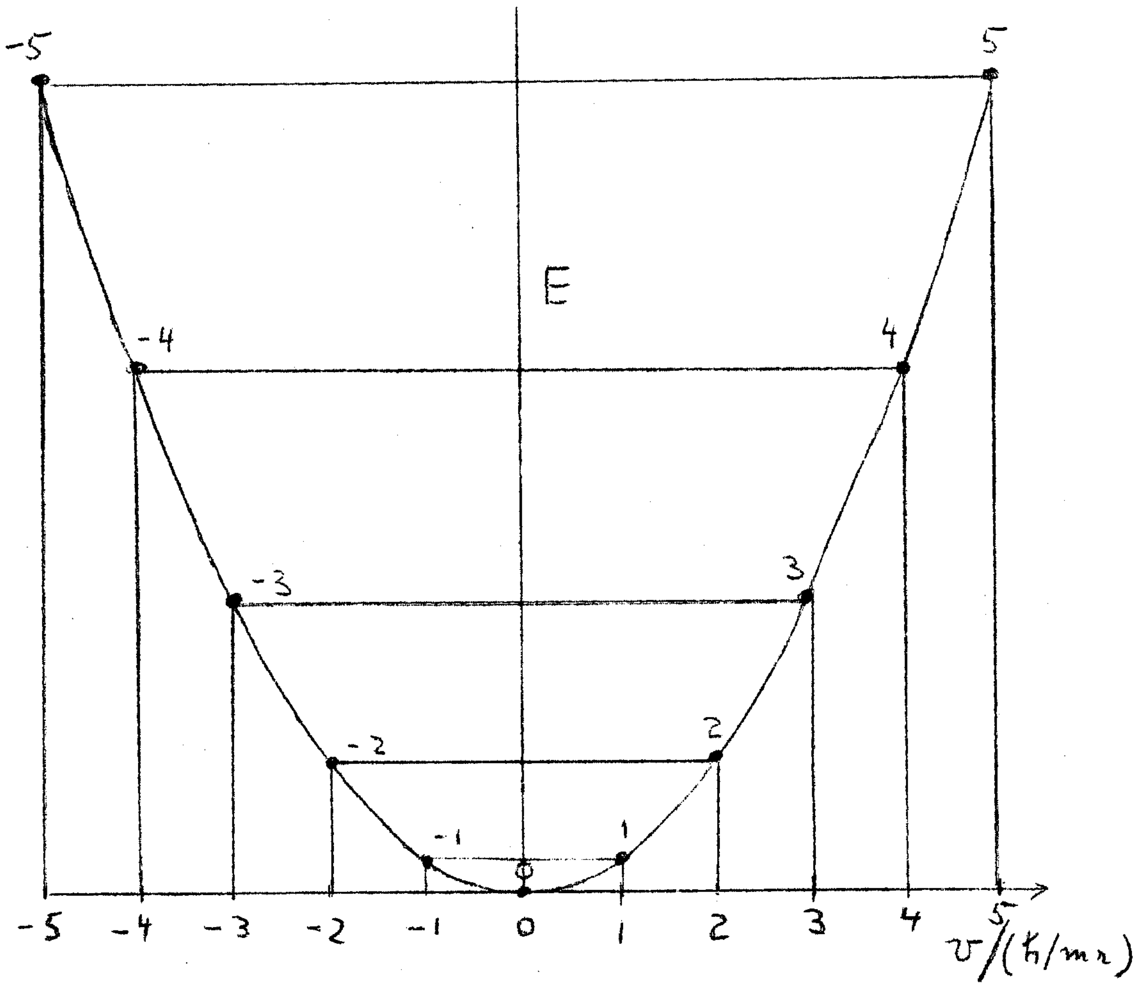

Fig 5

One-dimensional electron levels for zero magnetic flux.
Energy E is plotted against velocity in units of $(\hbar/mr)$.
The dots on the parabolic curve are the allowed levels.
The numbers near the dots are the quantum numbers n .

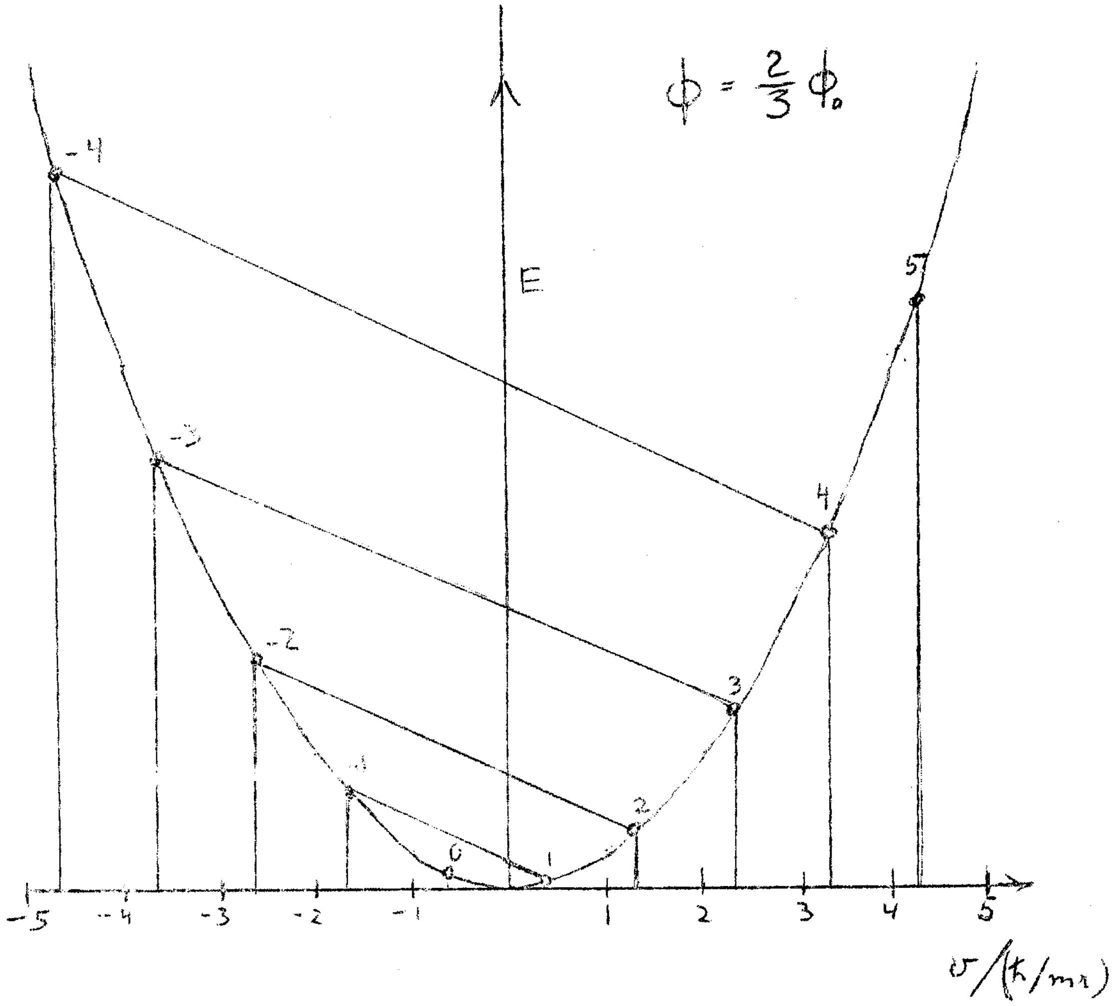

Fig 6

The same as Figure 5 in the presence of a magnetic flux $\phi = \frac{2}{3}\phi_0$.
Levels with n and -n are no longer of the same energy.

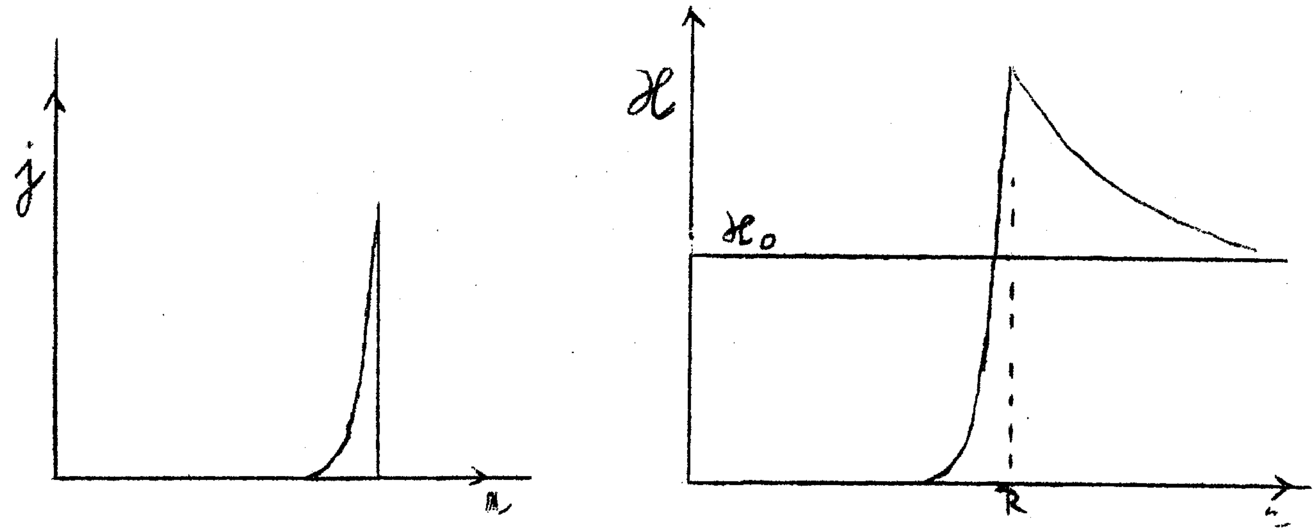

Fig 7. Current density j and magnetic field ℋ as a function of r for a super-conducting cylinder in an external magnetic field $\mathcal{H}_0$. The radius of the cylinder is R .

Surface of a super-conductor. A magnetic field ℋ in the indicated direction produces a current sheet as shown. The magnitude J of the current sheet is the current flowing in a strip of 1 cm width of the sheet. The strip is limited by a pair of dotted lines.

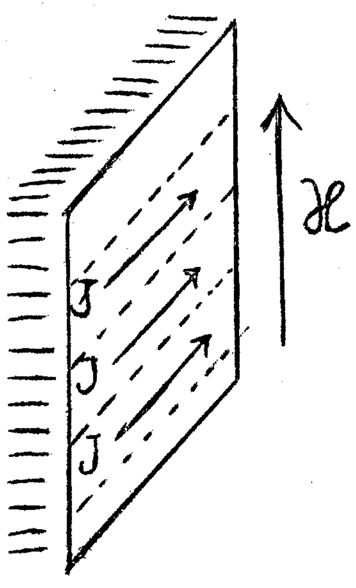

Fig. 8

Super-conducting metal ring of Figure 4 b seen from the z-direction. The outer dotted circle includes the complete magnetic flux through the hole. The inner dotted line includes a smaller flux since the magnetic field penetrates a distance ℓ into the metal.

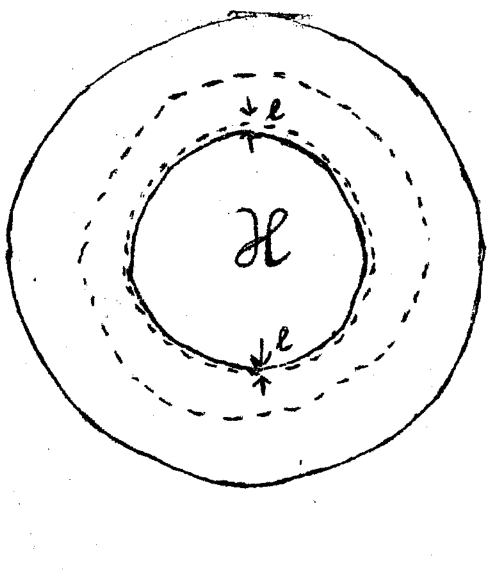

Fig. 9

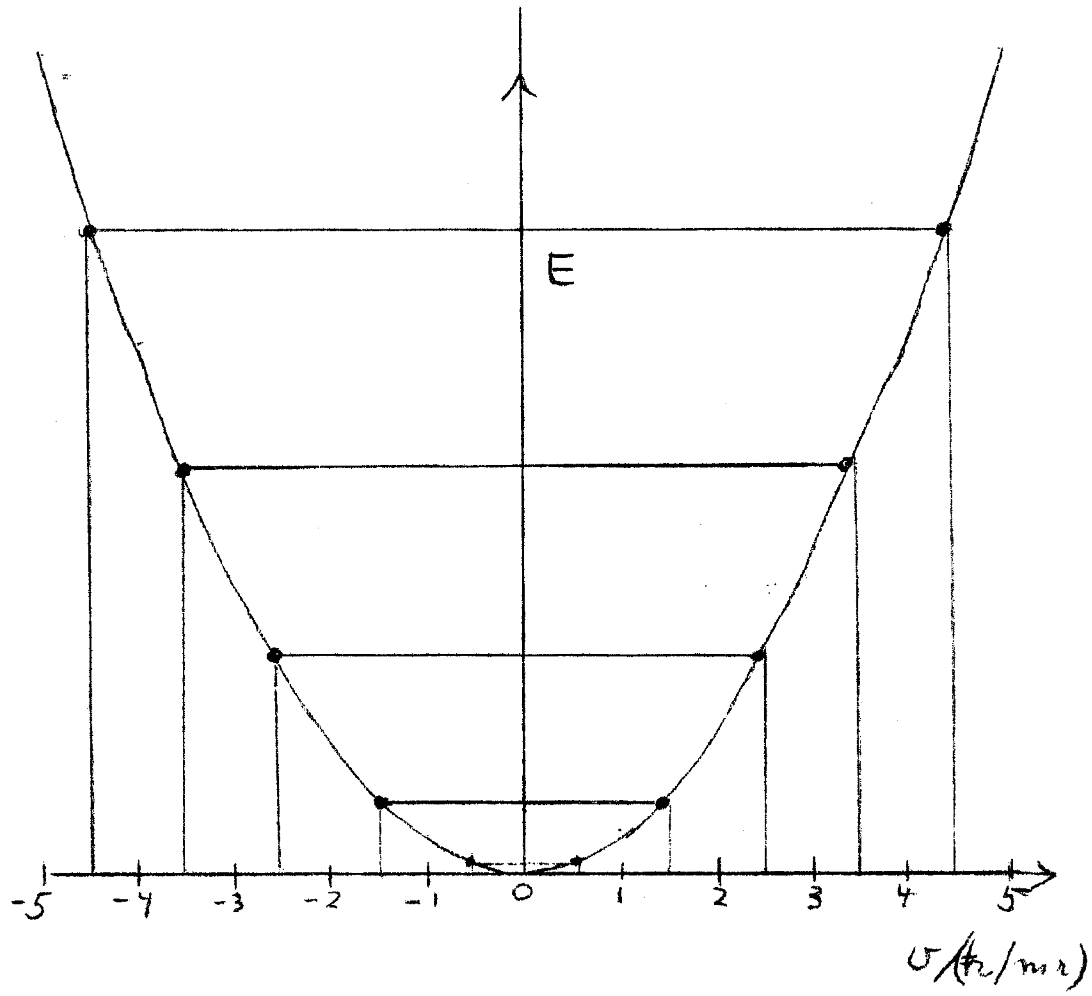

Fig 10

One-dimensional electron levels for a magnetic flux $\phi = (\kappa + \frac{1}{2})\,\phi_0$ where κ is integer. The dots give the energy and velocity of the levels.