Super-resolution Orbital Angular Momentum based Radar Targets Detection

Mingtuan Lin, Yue Gao, Peiguo Liu and Jibin Liu

Radiating twisted beams, Orbital Angular Momentum (OAM) based radar provides a new perspective for present radar techniques. However, estimation methods now used has a demerit of resolution. Thus, we raised Multiple Signal Classification (MUSIC) algorithm to improve resolution ability based on this innovative concept. The echo model based on uniform circular array (UCA) for MUSIC was first built. In contrast to uncorrelated signals in classical MUSIC algorithm, echo signals from targets are fully coherent with each other. Spatial smoothing technique was subsequently utilized in OAM regime to tackle it. Simulation results showed the super-resolution capacity of MUSIC to detect objectives compared to the traditional Fast Fourier Transform (FFT) method.

Introduction: Orbital Angular Momentum (OAM) has been widely studied in optic regime regarding imaging, microscopic particle and communication [1,2]. While radio OAM had not been explored until Bo. Thide et al. first proved the effectiveness of OAM generating in a low-frequency band in [3]. The numerous orthogonal helical beams entitle OAM a larger freedom degree, benefiting high capacity of communication and enriching radar targets more information. Recent work on radio OAM consists of two parts, i.e. antennas design to produce twisted beams and OAM multiplexing in communication system. Several approaches including circular array [4-6], spiral phase plate [7-8], higher resonate mode [9] are employed to engender electromagnetic vortex. Meanwhile, OAM multiplexing from simulations to outdoor experiments [10-12] were conducted to attain a high-capacity communication.

Radar based on this concept had not attracted much attention until Guo et al. [13] first proposed OAM based target detection model, opening a new perspective for existing radar techniques. Such system generates various twisted beams to illuminate targets as shown in Fig.1 and receive echo signals in a reverse way. By making a Fast Fourier Transform (FFT) or back projection of received signals, this scheme manifests azimuthal resolution ability without relative moves or beams scanning. Additionally, no complex waveform is needed for this scenario. Later, [14] further extended the detection model to Multiple-in-multiple-out (MIMO) and Multiple-in-single-out (MISO) modes . Besides, two-dimension imaging method by exploiting OAM and frequency diversity was studied as well. Both papers utilized traditional methods like FFT and black projection to make estimations of azimuth angle. To gain a high resolution, numerous samples in OAM domain are required, thus increasing complexity of the system. Therefore, super-resolution techniques based on OAM radar detection are in demand. This paper built the model and derived spatial smoothing multiple signal classification (MUSIC) [15] algorithm to gain super-resolution estimations of targets.

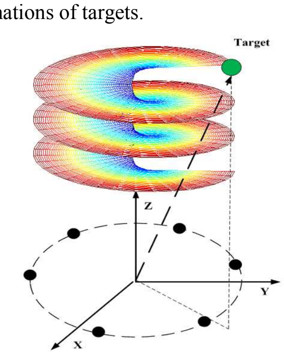


Fig.1 *Illuminate target with twisted beams.*

Detection model:

For MISO mode, the normalized real-time echo signals from M objectives in the received terminal can be extended from [14] and written as:

$$E(\alpha,t) \approx \sum_{m=1}^{M} \sigma_m \frac{e^{j2kr_m}}{r_m^2} e^{i\alpha\varphi_m} J_\alpha(ka\sin\theta_m) s(t) + n(\alpha,t) \tag{1}$$

where s(t) is real-time transmitted signal, $n(\alpha,t)$ is the noise in α regime, a corresponds to the radius of uniform circular array (UCA), α denotes the OAM topological order, k is wave number, σ_m, r_m, θ_m and φ_m link to radar cross section (RCS), distance and direction information of mth target, J_α refers to α th first kind Bessel function. Similar FFT transform relation between α and φ domain can be observed from (1). Based on this, existing means to estimate azimuthal information just make a FFT transform or back projection [13,14]. According to [13], such OAM based radar technique has no capacity to identify elevation angles of targets.

For $ka\sin\theta >> 1$, approximation can be made as follows:

$$\begin{aligned} & e^{i\alpha\varphi} J_\alpha(ka\sin\theta) \\ & \approx e^{i\alpha\varphi}\sqrt{\frac{2}{\pi ka\sin\theta}}\cos(ka\sin\theta - \frac{\alpha\pi}{2} - \frac{\pi}{4}) \\ & = [e^{j\alpha(\varphi-\frac{\pi}{2})} \; e^{j\alpha(\varphi+\frac{\pi}{2})}]\begin{bmatrix} \sqrt{\frac{1}{2\pi ka\sin\theta}} e^{j(ka\sin\theta-\frac{\pi}{4})} \\ \sqrt{\frac{1}{2\pi ka\sin\theta}} e^{-j(ka\sin\theta-\frac{\pi}{4})} \end{bmatrix} \end{aligned} \tag{2}$$

Therefore, N discrete samples vector of $E(\alpha,t)$ in α regime can be depicted as:

$$\begin{aligned} \mathbf{E} &= \begin{bmatrix} E(\alpha_1,t) \\ \vdots \\ E(\alpha_N,t) \end{bmatrix} = \mathbf{AS} + \mathbf{n} \\ &= [\mathbf{a}(\varphi_1) \;\; \mathbf{a}(\varphi_2), \cdots \mathbf{a}(\varphi_M)] \begin{bmatrix} \mathbf{S}_1(t) \\ \mathbf{S}_2(t) \\ \vdots \\ \mathbf{S}_M(t) \end{bmatrix} + \begin{bmatrix} n(\alpha_1,t) \\ n(\alpha_2,t) \\ \vdots \\ n(\alpha_N,t) \end{bmatrix} \\ where\ \mathbf{a}(\varphi_i) &= \begin{bmatrix} e^{j\alpha_1(\varphi_i-\frac{\pi}{2})} & e^{j\alpha_1(\varphi_i+\frac{\pi}{2})} \\ \vdots & \vdots \\ e^{j\alpha_N(\varphi_i-\frac{\pi}{2})} & e^{j\alpha_N(\varphi_i+\frac{\pi}{2})} \end{bmatrix}, \\ \mathbf{S}_i(t) &= \begin{bmatrix} \sigma_i \frac{e^{j2kr_i}}{r_i^2}\sqrt{\frac{1}{2\pi ka\sin\theta_i}} e^{j(ka\sin\theta_i-\frac{\pi}{4})} s(t) \\ \sigma_i \frac{e^{j2kr_i}}{r_i^2}\sqrt{\frac{1}{2\pi ka\sin\theta_i}} e^{-j(ka\sin\theta_i-\frac{\pi}{4})} s(t) \end{bmatrix} \quad i \in [1,M] \end{aligned} \tag{3}$$

$\mathbf{A} \in C^{N\times 2M}$ (N>2M) is the steering matrix, $\mathbf{S} \in C^{2M\times L}$ indicates reformed echo signals vector, $\mathbf{a}(\varphi_i)$ and $\mathbf{S}_i(t)$ are modified steering vector and echo signal of ith target, L is the sample length of s(t).The amended model in (3) indicates 2M targets, implying appearance of ambiguities.

For presenting Gaussian noise, covariance matrix of received signals under different orders can be acquired by:

$$\mathbf{R_{EE}} = \mathbf{ARA}^H + \rho_n \mathbf{I} \tag{4}$$

where ρ_n is the noise power, $\mathbf{R}$ involves covariance matrix of reformed echo signals, $\mathbf{I}$ is the unitary matrix. Similar to DOA estimation, columns of $\mathbf{A}$ are linear independently, however echo signals of multiple targets are fully coherent, in contrast to uncorrelated incident signals in classical MUSIC algorithm. For uniform samples in α field with sample rate f_α, front spatial smoothing technique [16] are capable of tackling this problem. According to smoothing theory, divide N samples in α field to p mixed subsample blocks as shown in Fig.2, each block has h samples, then N=p+h-1. To achieve full rank, the number of blocks should meet $p \geq 2M$.

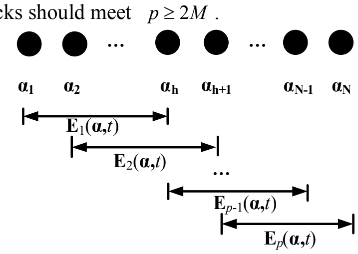


Fig.2 *Front-spatial smoothing*

Subsequently, calculate the modified covariance matrix as follows:

$$\mathbf{R}^f = \frac{1}{p}\sum_{i=1}^{p}\mathbf{R}_i \quad \mathbf{R}^f \in C^{h\times h} \tag{5}$$

where $\mathbf{R}_i$ is covariance matrix of ith sub-block $\mathbf{E}_i(\alpha,t)$.

To make use of MUSIC, M is assumed known or already accurately estimated. In accordance with MUSIC algorithm [15], the whole procedure to estimate azimuth angles can be listed as follows:

1) Obtain p original covariance matrix $\mathbf{R}_i$ (i=1, 2,⋯, p);

2) Gain $\mathbf{R}^f$ based on (5).

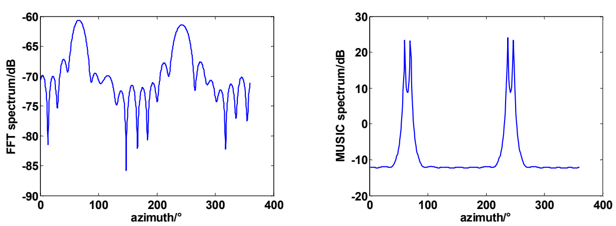
3) Make engine value decomposition of $\mathbf{R}^f$.

4) Search azimuthal spectrum

$$P(\varphi) = \frac{1}{\mathbf{a}^H(\varphi)\mathbf{V}_n\mathbf{V}_n{}^H\mathbf{a}(\varphi)} \tag{6}$$

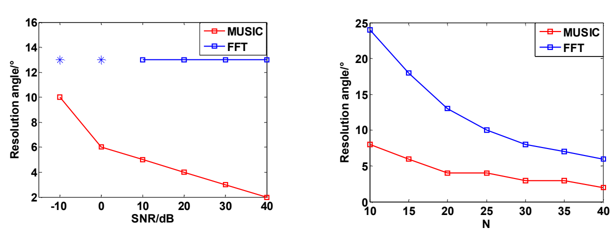
where $\mathbf{V}_n = [\mathbf{q}_{2M+1},..,\mathbf{q}_h]$ $(h \ge 2M)$ is the noise subspace vectors, $\mathbf{a}(\varphi)$=$[e^{j\alpha_1\varphi},\cdots,e^{j\alpha_N\varphi}]^T$ refers to searching steering. It can be predicted from (3) that symmetric peaks at $\hat{\varphi}-90$ and $\hat{\varphi}+90$ directions will be obtained in the spectrum. To acquire a more visible result, replace φ with $\varphi+90^\circ$ in (6). Similar conclusions for MIMO mode can be similarly derived, here we would not state again.

Simulation and results: Use UCA with radius $a = 50\lambda$ to illuminate two targets at distances $r_1 = 782.4\lambda$ and $r_2 = 781.5\lambda$ with directions $(\theta_1,\varphi_1) = (60°,60°)$ and $(\theta_2,\varphi_2) = (70°,70°)$ specifically. Received signals under N=20 discrete orders with $f_\alpha = 1$ are captured in a SNR=20dB environment. Each order, L=500 snapshots of echo signals in time domain are recorded. For MUSIC algorithm, p=2 sub-blocks are divided to solve coherent problem. Estimation based on FFT are implemented on an average of L=500 FFT results, each time 1024-point FFT is conducted. Spectrums of both methods are described in Fig.3, from which we can observe that MUSIC can easily distinguish these two close targets well with two sharp peaks while only one real blunt peak can be viewed by FFT method. For both methods, symmetric ambiguities appear, consistent with previous analysis. Fig.4 demonstrates resolution angle of two methods against SNR and sample size N. As exhibited in Fig4a, the larger SNR, the smaller resolution angle MUSIC method gains, with only 2° resolution ability at SNR=40 dB almost 7 times that of FFT. The asterisk in Fig.4a denotes disability of FFT method to discriminate any two targets when SNR<0 dB. Similar results can be viewed in Fig.4b that the larger size of sample in α regime, the higher resolution ability for both methods, with MUSIC always outperforming FFT specifically.



a b

Fig.3 *Azimuthal estimations of two targets using (a) FFT (b) MUSIC*



a b

Fig.4 *Resolution angle for different (a) SNR and (b) sample size N.*

Conclusion and discussion: This letter put forward a mathematical model for MUSIC to make estimation of targets based on OAM radar. Intrinsic coherent problem induced by model was settled by front smoothing technique. Simulation results illustrated feasibility of MUSIC to realize super-resolution targets detection based on OAM vortex. Benefiting from high-resolution advantage, the same level performance can be achieved with fewer samples by proposed scheme and thus simplify the practical system. Ambiguity problems and estimation deviation issue for small $ka\sin\theta$ should be further investigated. This paper opens a new perspective of super-resolution technique for OAM based radar. Many other high-resolution techniques such as ESPRIT are applicable as well, where further work will focus on.

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