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Super Twisting-Based Nonlinear Gain Sliding Mode Controller for Position Control of Permanent-Magnet Synchronous Motors

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ABSTRACT This paper proposes a super twisting-based nonlinear gain sliding mode controller (STNGSMC) to achieve the position control of permanent-magnet synchronous motors (PMSMs). Nonlinear gain is developed to improve the position tracking performance of a super twisting sliding mode controller (STSMC). The inclusion of nonlinear gain in the STSMC reduces chattering, and the stability of the closed loop is mathematically proven using the Lyapunov theorem considering load torque. In the proposed method, chattering is analyzed using the describing function method under unmodeled dynamics, such as that corresponding to the quantization effect of the digital sensor, sensor resolution, and pulse-width modulation (PWM) switching noise, in PMSM position control systems. Consequently, the STNGSMC can improve the position tracking performance in steady-state responses. The performance of the proposed method is verified using simulations. The experimental results demonstrate that chattering can be reduced by the STNGSMC, consequently improving the position tracking performance.

INDEX TERMS Super twisting algorithm, sliding mode controller, position control, permanent-magnet synchronous motors.

I. INTRODUCTION

PERMANENT-magnet synchronous motors (PMSMs) are widely used in industrial applications owing to their high power density, high efficiency, and enhanced reliability. In addition, classical control methods such as proportional-integral (PI) control and proportional-integral-derivative control are used for PMSMs owing to their ease of implementation [1], [2]. However, these classical control methods cannot achieve good tracking performance owing to system nonlinearities and parameter variations over a wide operating range or external disturbances.

Various nonlinear control methods have been studied to improve the control performance of PMSMs. A nonlinear control method for field-weakening control and field-oriented control was proposed in [3]. An adaptive control algorithm was developed to enable speed tracking and minimize torque ripple [4]. In addition, an internal model principle-based controller was proposed to reduce sideband harmonics in

PMSMs with low-switching-frequency inverters [5]. In [6], an adaptive law with type-2 fuzzy logic systems was designed to compensate for interconnection effects, reconstruction errors, and unknown functions. In [7], an adaptive fuzzy controller was designed to compensate for the dynamic uncertainty and external load effect in the speed loop of PMSM drives. A backstepping control law with an extended state observer was designed with input-output linearization in [8]. A nonlinear disturbance observer-based robust backstepping compensator was designed for a position controller under a lumped unknown disturbance in [9]. In [10], a predictive algorithm using integration was designed with respect to time step and ramp reference signals considering constraints such as field weakening and current limitations. Model predictive direct speed control was studied to overcome the limitations of cascaded linear controllers and online predictions in [11]. A position tracking controller was designed to minimize the quadratic index, and a recurrent wavelet-based Elman neural

network was developed to improve control performance and achieve robustness in [12]. These control methods have been improved to achieve tracking performance in PMSMs from different perspectives.

Sliding mode control (SMC) methods have been widely implemented in PMSM control systems because of their robustness and fast response [13]–[17]. However, chattering is a major disadvantage associated with SMC, and various methods have been studied to eliminate or reduce chattering. The signum function is replaced by certain smooth approximations such as tangent, saturation, and hyperbolic functions. A singular-perturbation-theory-based SMC was applied to the position tracking control of PMSMs in [18]. A boundary layer integral SMC was designed based on a quasi-linearized and decoupled model in [19]. These studies used approximation functions in the control scheme to reduce chattering. However, asymptotic stability cannot be ensured using approximation functions. Thus, the convergence of the steady-state error to zero cannot be mathematically verified. Adaptive SMC and iterative learning control were designed to ensure fast response and robustness and to reduce periodic torque ripples in [20]. However, the adaptive SMC method is required because of the the long learning time required to achieve robustness and fast response. To improve both chattering and finite reaching time, high-order sliding mode control (HOSMC) was developed in [21]. In particular, super twisting algorithm-based HOSMC has been commonly used to reduce chattering and achieve a finite reaching time [22], [23]. Several super twisting sliding mode controllers (STSMCs) have been developed to improve the control of the PMSM [24]–[26]. The STSMC mathematically verifies the reduction in chattering, but chattering still appears owing to the unmodeled dynamics, the quantization effect of the digital sensor, sensor resolution, and PWM switching noise. Thus, several STSMC methods have been developed to reduce chattering using the upper-bound function of the disturbance [27], [28]. However, determining the upper-bound function of the disturbance is difficult.

This paper proposed a super twisting-based nonlinear gain sliding mode controller (STNGSMC) to achieve position control of PMSMs. A nonlinear gain is developed to improve the position tracking performance of the STSMC. The inclusion of nonlinear gain in the STSMC reduces chattering, which is analyzed using the describing function method under various unmodeled dynamics in PMSM position control systems. Consequently, the STNGSMC can improve the position tracking performance in the steady-state responses. We mathematically prove the stability of closed-loop systems using the Lyapunov theorem and verify the performance of the proposed method via simulations and experiments.

II. SUPER TWISTING NONLINEAR GAIN SLIDING MODE CONTROLLER IN PMSM

Using a direct-quadrature transformation, the mathematical model of the PMSM can be represented in the state-space

form [1] as follows:

$$\begin{aligned}\dot{\theta} &= \omega \\ \dot{\omega} &= -\frac{B}{J}\omega + \frac{K_m}{J}i_q - \frac{\tau_L}{J} \\ \dot{i}_d &= \frac{1}{L}(-Ri_d + P\omega Li_q + v_d) \\ \dot{i}_q &= \frac{1}{L}(-Ri_q - K_E\omega - P\omega Li_d + v_q)\end{aligned}\quad (1)$$

where v_d and v_q are the direct and quadrature voltages [V], respectively; i_d and i_q are the direct and quadrature currents [A], respectively; θ is the rotor (angular) position [rad]; ω is the rotor (angular) velocity [rad/s]; B is the viscous friction [N·m·s/rad]; J is the inertia of the motor [kg·m²/rad]; R is the phase resistance [Ω]; L is the phase inductance [H]; P is the pole pair; K_m is the torque constant [rad/s/A]; K_E is the back-EMF constant [V·s/rad]; and τ_L is the load torque [N·m] and is assumed to be constant.

Mechanical tracking errors are defined as follows:

$$\begin{aligned}e_\theta &= \theta_d - \theta \\ e_\omega &= \omega_d - \omega\end{aligned}\quad (2)$$

where θ_d is the desired position, and $\omega_d = \dot{\theta}_d$ is the desired velocity. From (1) and (2), the tracking error dynamics are obtained as follows:

$$\begin{aligned}\dot{e}_\theta &= e_\omega \\ \dot{e}_\omega &= \dot{\omega}_d + \frac{B}{J}\omega - \frac{K_m}{J}i_q + \frac{\tau_L}{J}.\end{aligned}\quad (3)$$

A sliding surface, s , is defined as follows:

$$s = k_1 e_\theta + k_2 e_\omega \quad (4)$$

where k_1 and k_2 are positive constants, and the derivative of the sliding surface is derived as follows:

$$\begin{aligned}\dot{s} &= k_1 \dot{e}_\theta + k_2 \dot{e}_\omega \\ &= k_1 e_\omega + k_2 \left(\dot{\omega}_d + \frac{B}{J}\omega - \frac{K_m}{J}i_q + \frac{\tau_L}{J} \right).\end{aligned}\quad (5)$$

A. WITHOUT LOAD TORQUE

If the load torque is assumed to be zero, the control input of the proposed controller for the mechanical dynamics is designed as follows:

$$i_q = u_{eq} + \frac{J}{K_m} \left(\frac{\lambda_1}{k_2} \sqrt{s^2 + \gamma|s|} \operatorname{sgn}(s) - u_i \right) \quad (6)$$

where

$$\begin{aligned}u_{eq} &= \frac{J}{K_m} \left(\frac{k_1}{k_2} e_\omega + \dot{\omega}_d + \frac{B}{J}\omega \right) \\ \dot{u}_i &= -\lambda_2 \operatorname{sgn}(s).\end{aligned}\quad (7)$$

λ_1 and λ_2 are positive constant numbers, and $0 < \gamma < 1$.

Substituting the control inputs, (6) and (7), into (5), the derivative of the sliding surface becomes

$$\begin{aligned}\dot{s} &= -\lambda_1 \sqrt{s^2 + \gamma|s|} \operatorname{sgn}(s) + u_i \\ \dot{u}_i &= -\lambda_2 \operatorname{sgn}(s).\end{aligned}\quad (8)$$

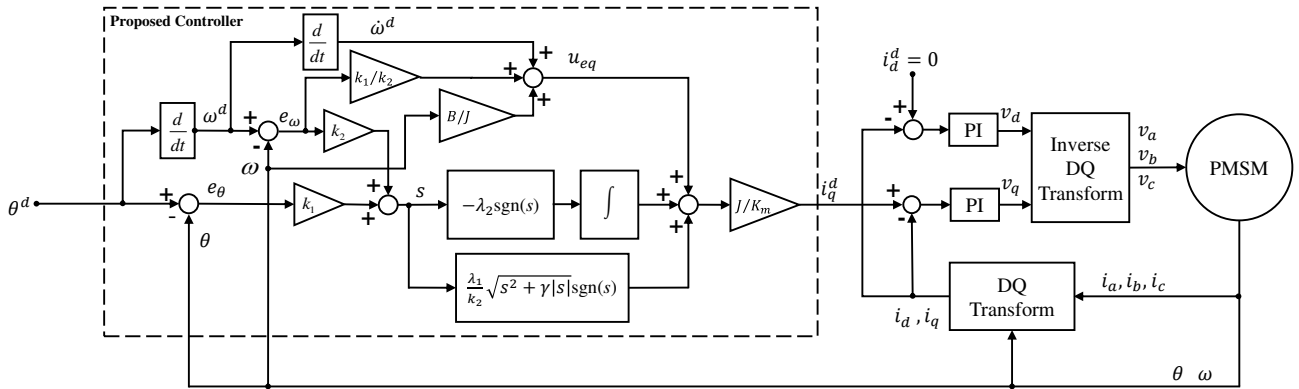


FIGURE 1: Block diagram of the overall system model

The stability of the proposed method was proven using the Lyapunov theory. The auxiliary state variables are defined as follows:

$$\begin{aligned}\zeta_1 &= \sqrt{s^2 + \gamma|s|} \operatorname{sgn}(s) \\ \zeta_2 &= u_i.\end{aligned}\quad (9)$$

The dynamics of auxiliary state variables are defined as follows:

$$\begin{aligned}\dot{\zeta}_1 &= \frac{2|s| + \gamma}{2\sqrt{s^2 + \gamma|s|}} (-\lambda_1 \sqrt{s^2 + \gamma|s|} \operatorname{sgn}(s) + \zeta_2) \\ \dot{\zeta}_2 &= -\lambda_2 \operatorname{sgn}(s).\end{aligned}\quad (10)$$

The dynamics of auxiliary state variables are rewritten as follows:

$$\dot{\zeta} = \frac{1}{|\zeta_1|} A \zeta + \frac{|s|}{|\zeta_1|} R \zeta \quad (11)$$

where

$$A = \begin{bmatrix} \frac{-\gamma\lambda_1}{2} & \frac{1}{2} \\ -\lambda_2 & 0 \end{bmatrix}, \quad R = \begin{bmatrix} -\lambda_1 & 1 \\ 0 & 0 \end{bmatrix}. \quad (12)$$

λ_1 and λ_2 are selected such that A is Hurwitz. The Lyapunov candidate function, V , is defined as follows:

$$V = \zeta^T P \zeta \quad (13)$$

where P is a positive definite such that $A^T P + PA = -I$. The derivative of (13) is derived as follows:

$$\begin{aligned}\dot{V} &= \frac{1}{|\zeta_1|} \zeta^T A^T P \zeta + \frac{1}{|\zeta_1|} \zeta^T P A \zeta \\ &+ \frac{|s|}{|\zeta_1|} \zeta^T R^T P \zeta + \frac{|s|}{|\zeta_1|} \zeta^T P R \zeta \\ &= \frac{1}{|\zeta_1|} \zeta^T (A^T P + PA) \zeta + \frac{|s|}{|\zeta_1|} \zeta^T (R^T P + PR) \zeta.\end{aligned}\quad (14)$$

In (13), P is designed as follows:

$$P = \begin{bmatrix} p_{11} & p_{12} \\ p_{12} & p_{22} \end{bmatrix} \succ 0. \quad (15)$$

where

$$p_{11} = \frac{1 + 2\lambda_2}{\lambda_1 \gamma}, \quad p_{12} = -1, \quad p_{22} = \frac{\lambda_1 \gamma}{2\lambda_2} + \frac{1 + 2\lambda_2}{2\lambda_1 \gamma \lambda_2}. \quad (16)$$

With (15) and (16), \dot{V} becomes

$$\dot{V} = -\frac{1}{|\zeta_1|} \|\zeta\|^2 + \frac{|s|}{|\zeta_1|} \zeta^T (R^T P + PR) \zeta \quad (17)$$

where

$$R^T P + PR = \begin{bmatrix} r_1 & r_2 \\ r_2 & r_3 \end{bmatrix} \quad (18)$$

$r_1 = -\frac{2(1+2\lambda_2)}{\gamma}$, $r_2 = \lambda_1 + \frac{1+2\lambda_2}{\gamma\lambda_1}$, and $r_3 = -2$. The symmetric matrix $R^T P + PR$ is semi-negative definite if and only if

$$\begin{aligned}r_1 &\leq 0 \\ r_1 r_3 - r_2^2 &\leq 0.\end{aligned}\quad (19)$$

Condition (19) can be rewritten as

$$\begin{aligned}-\frac{2(1+2\lambda_2)}{\gamma} &\leq 0 \\ -\left(\lambda_1 - \frac{1+2\lambda_2}{\gamma\lambda_1}\right)^2 &\leq 0.\end{aligned}\quad (20)$$

The first condition can be satisfied with $(1+2\lambda_2) \geq 0$ because $0 < \gamma < 1$. The second condition is always satisfied for both λ_1 and λ_2 . Consequently, if λ_1 and λ_2 are selected such that A is Hurwitz, and $(1+2\lambda_2) \geq 0$, we have

$$\dot{V} \leq -\frac{1}{|\zeta_1|} \|\zeta\|^2. \quad (21)$$

Thus, ζ converges to zero in a finite time.

B. WITH LOAD TORQUE

In this section, the stability is proven for the load torque. With the load torque, the dynamics of the sliding surface are derived as follows:

$$\begin{aligned} \dot{s} &= k_1 \dot{e}_\theta + k_2 \dot{e}_\omega \\ &= k_1 e_\omega + k_2 \left(\dot{\omega}_d + \frac{B}{J} \omega - \frac{K_m}{J} i_q + d \right). \end{aligned} \quad (22)$$

where d is the load term, $d = -\frac{\tau_L}{J}$. Thus, (22) with (6) can be rewritten as follows:

$$\begin{aligned} \dot{s} &= -\lambda_1 \sqrt{s^2 + \gamma |s|} \operatorname{sgn}(s) + u_i + d \\ \dot{u}_i &= -\lambda_2 \operatorname{sgn}(s). \end{aligned} \quad (23)$$

Let us define $z := u_i + d$; then, $\dot{z} := \dot{u}_i + \dot{d}$ and the system can be written as

$$\begin{aligned} \dot{s} &= -\lambda_1 \sqrt{s^2 + \gamma |s|} \operatorname{sgn}(s) + z \\ \dot{z} &= -\lambda_2 \operatorname{sgn}(s) + \dot{d}. \end{aligned} \quad (24)$$

Because $\dot{d} = 0$, (24) becomes

$$\begin{aligned} \dot{s} &= -\lambda_1 \sqrt{s^2 + \gamma |s|} \operatorname{sgn}(s) + z \\ \dot{z} &= -\lambda_2 \operatorname{sgn}(s). \end{aligned} \quad (25)$$

Because (25) is equivalent to (8), it can be proven that the sliding surface, s , also converges to zero in finite time by the same procedure.

After the sliding surface, s , reaches zero, then

$$0 = k_1 e_\theta + k_2 e_\omega. \quad (26)$$

Consequently, the position tracking error exponentially converges to zero as follows:

$$\dot{e}_\theta = -\frac{k_1}{k_2} e_\theta. \quad (27)$$

A block diagram of the overall system and controller is shown in Fig. 1. The proposed methods (4), (6), and (7), which are shown in Fig. 1, generates the current input i_q . The PI controllers are implemented for current-loop in the PMSMs. The control gain tuning guide is as follows:

- λ_1 and λ_2 are selected such that A is Hurwitz and $(1 + 2\lambda_2) \geq 0$ for the stability.
- The positive constant γ was selected to be less than 1 to reduce chattering.

III. ANALYSIS OF CHATTERING PHENOMENON

This section examines chattering in STSMC, STNGSMC, and SMC under unmodeled dynamics in PMSM control systems. In [29], the chattering analysis is proven by describing the function method with unmodeled dynamics. The block diagram for the sliding surface dynamics, (8), in the presence of the unmodeled dynamics is shown in Fig. 2. In Fig. 2, p is the Laplace operator, μ is the unmodeled dynamics, and the function of Ω is defined as follows:

$$\Omega = \lambda_1 \sqrt{s^2 + \gamma |s|} \operatorname{sgn}(s). \quad (28)$$

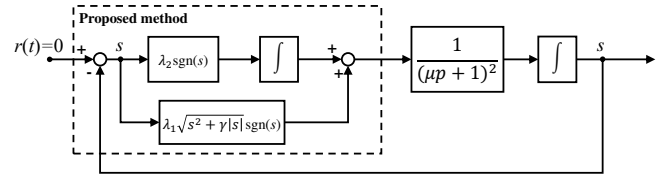


FIGURE 2: Block diagram of proposed method with unmodeled dynamics

Following the describing function method, the chattering of the sliding surface is considered to be $A \sin(\omega t)$, and the input of the nonlinearity is approximated as

$$u = \frac{a_0}{2} + a_1 \cos(\omega t) + b_1 \sin(\omega t). \quad (29)$$

The switching function, $\operatorname{sgn}(\ast)$, is an odd function, and the parameters of (29) are defined as follows:

$$a_0 = a_1 = 0. \quad (30)$$

Therefore, (29) is rewritten as follows:

$$u = b_1 \sin(\omega t) \quad (31)$$

where b_1 is given by the input of the controllers shown in Fig. 2 such as $\lambda_2 \operatorname{sgn}(s)$ and $\lambda_1 \sqrt{s^2 + \gamma |s|} \operatorname{sgn}(s)$. The sliding surface is considered as, $A \sin(\omega t)$, where b_1 is given by

$$\begin{aligned} b_1 &= b_{11} + b_{12} \\ b_{11} &= \frac{1}{\pi} \int_{-\pi}^{\pi} \lambda_2 \operatorname{sgn}(A \sin(\delta)) \sin(\delta) d\delta \\ b_{12} &= \frac{2}{\pi} \int_0^{\pi} \lambda_1 \sqrt{A^2 \sin^2(\delta) + \gamma |A \sin(\delta)|} \operatorname{sgn} \delta \sin(\delta) d\delta. \end{aligned} \quad (32)$$

Using inequalities such as $a^2 + b^2 \geq 2ab$ and $\operatorname{sgn}(A \sin(\delta)) \sin(\delta) = \sin(\delta)$, the equation for b_1 is rewritten as follows:

$$\begin{aligned} b_{11} &= \frac{4\lambda_2}{\pi} \\ b_{12} &> \frac{\lambda_1}{\pi} \int_0^{\pi} 2A \sin^2(\delta) + \gamma \sin(\delta) d\delta = \lambda_1 A + \frac{2\lambda_1 \gamma}{\pi}. \end{aligned} \quad (33)$$

To obtain the describing function, b_1 is divided by A , that is given by

$$\begin{aligned} N_{11}(A) &= \frac{4\lambda_2}{\pi A} \\ N_{12}(A) &> \lambda_1 + \frac{2\lambda_1 \gamma}{\pi A}. \end{aligned} \quad (34)$$

The transfer function of the sliding surface with unmodeled dynamics is given by

$$G(p) = \frac{1}{p(\mu p + 1)^2}. \quad (35)$$

Replacing the Laplace parameter p by $j\omega$ yields

$$G(j\omega) = \frac{1}{j\omega(\mu j\omega + 1)^2}. \quad (36)$$

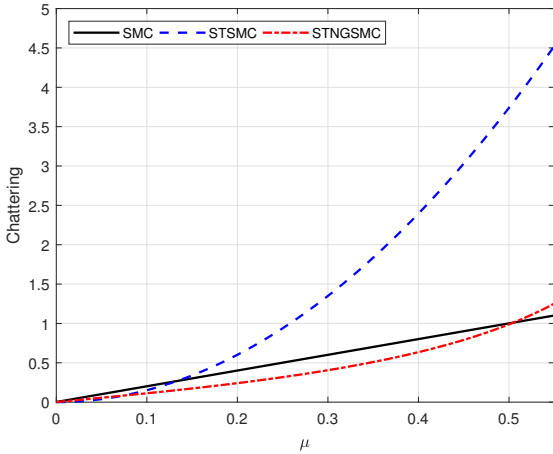


FIGURE 3: Chattering amplitude by unmodeled dynamics

The inverse transfer function of the sliding surface with unmodeled dynamics, $g(j\omega) = \frac{1}{G(j\omega)}$, is given as follows:

$$g(j\omega) = -(2\mu\omega^2) - j(\mu^2\omega^3 - \omega). \quad (37)$$

To complete the analysis, equating $N(A)$ to $-g(j\omega)$ is derived as

$$\frac{4\lambda_2}{\pi A p} + \lambda_1 + \frac{2\lambda_1\gamma}{\pi A} = (2\mu\omega^2) + j(\mu^2\omega^3 - \omega). \quad (38)$$

If the imaginary part of (38) is equal to zero, ω is calculated as follows:

$$\omega = -\frac{1}{3\mu} \left(\frac{27\mu^4 4\lambda_2 + \sqrt{(27\mu^4 4\lambda_2)^2 - 4(3\mu^2)^3}}{2} \right)^{\frac{1}{3}} - \frac{1}{3\mu} \left(\frac{27\mu^4 4\lambda_2 - \sqrt{(27\mu^4 4\lambda_2)^2 - 4(3\mu^2)^3}}{2} \right)^{\frac{1}{3}}. \quad (39)$$

Therefore, A , which is chattering amplitude, is rewritten as

$$A = \frac{2\lambda_1\gamma\omega}{-\lambda_1\pi\omega + 2\mu\pi\omega^3}. \quad (40)$$

Substituting (39) into (40), the chattering amplitude of the sliding surface under unmodeled dynamics is obtained. According to [29], the chattering of SMC and STSMC are calculated as

$$A_1 = \frac{2\mu\lambda_2}{\pi} \quad (41)$$

$$A_2 = \frac{\mu^2(\pi\lambda_1^2 + 16\lambda_2)^2}{4\lambda_1^2\lambda_2^2}$$

where A_1 is the chattering of SMC, and A_2 is the chattering of STSMC.

The chattering of SMC, STSMC, and STNGSMC is represented graphically in Fig. 3. The chattering amplitude of the STSMC is greater than that of the conventional SMC when $\mu > 0.15$. However, the chattering amplitude of the STNGSMC is less than that of the conventional SMC under the condition of $\mu < 0.5$.

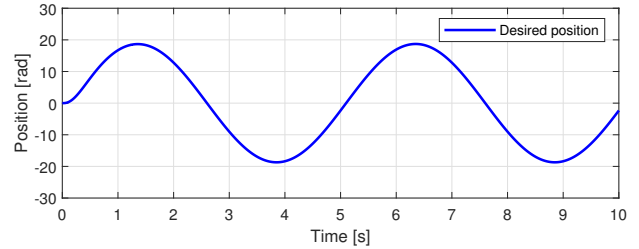


FIGURE 4: Position reference

IV. SIMULATION AND EXPERIMENTAL RESULTS

Simulations and experiments were conducted to validate the performance of the proposed STNGSMC. For comparison, the STSMC input was used for Case 1 as follows:

$$i_{q,1} = u_{eq} + \frac{J}{K_m} \left(\frac{\lambda_1}{k_2} \sqrt{|s|} \operatorname{sgn}(s) + u_i \right)$$

$$u_{eq} = \frac{J}{K_m} \left(\frac{k_1}{k_2} e_\omega + \dot{\omega}_d + \frac{B}{J} \omega \right) \quad (42)$$

$$\dot{u}_i = \lambda_2 \operatorname{sgn}(s).$$

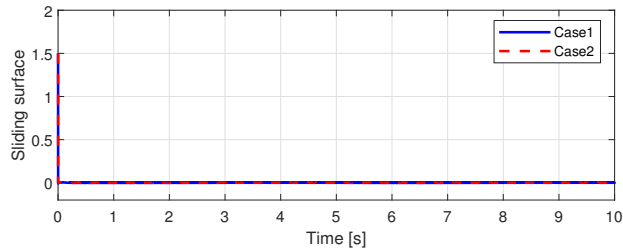
The STNGSMC input, (6) and (7), were implemented for Case 2. The smooth start sinusoidal signal $\theta^d = 6\pi(1 - e^{-5t})\sin(0.4\pi t)$ rad was used as the desired position in the simulations and experiments, as shown in Fig. 4. The desired position is used for the industrial servo system to prevent the peaking phenomenon. The nominal parameters listed in Table 1 were used for the PMSM in the simulations and experiments. The control gains listed in Table 1 were implemented for Cases 1 and 2. An initial position displacement, i.e., $\theta(0) = -0.008$ rad, was set to study the transient response.

TABLE 1: Nominal PMSM parameters and control gains

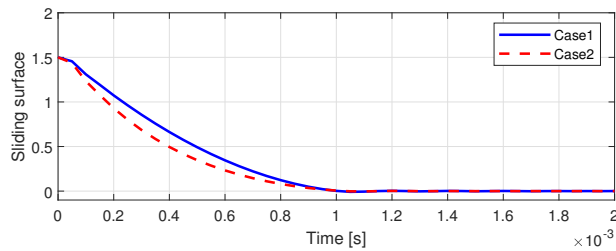
Parameter	Value	Gain	Value
J	4.675×10^{-4} [kg·m ² /rad]	k_1	200
B	3.7×10^{-3} [N·m·s/rad]	k_2	0.5
R	0.2 [Ω]	λ_1	2100
L	0.4×10^{-3} [H]	λ_2	1000
K_m	0.102 [V·s/rad]	γ	0.5

A. SIMULATION RESULTS

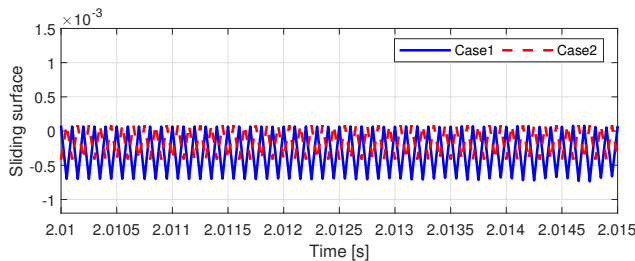
Simulations were conducted using MATLAB/Simulink to validate the performance of the proposed method. The control gains and parameters are listed in Table 1. In the simulation, the parameter uncertainties, which are a maximum of $\pm 20\%$ from the nominal value of Table 1, were applied to verify the robustness of STSMC and STNGSMC. The sliding surfaces of Cases 1 and 2 are shown in Fig. 5. The initial position displacement generated a transient response, where it was observed that the finite reaching time of STSMC is as shown in Fig. 5 (b). After observing the finite reaching time, chattering was observed for STSMC and STNGSMC, as shown in Fig. 5 (c). The chattering in Case 2 was reduced



(a) Sliding surfaces s

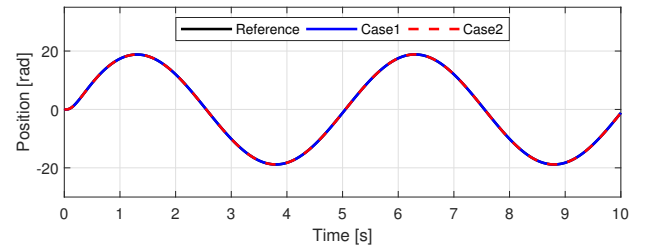


(b) Enlarged sliding surface in the transient response

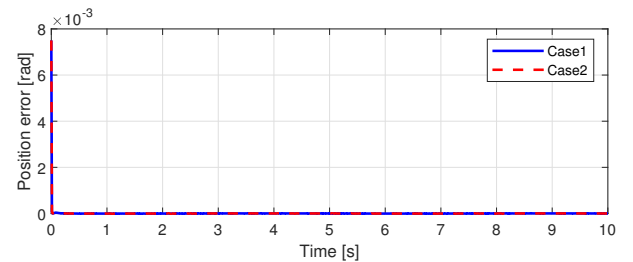


(c) Enlarged sliding surface in the steady-state response

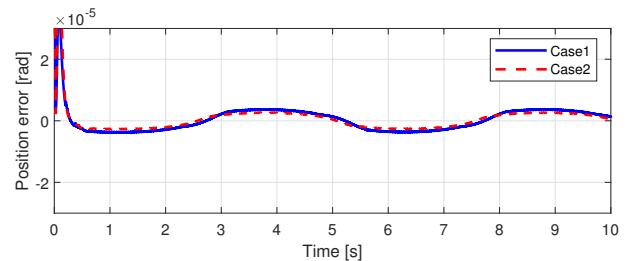
FIGURE 5: Sliding surfaces in simulations



(a) Position tracking performances



(b) Position tracking errors



(c) Enlarged position tracking errors

FIGURE 6: Position tracking performances in simulations

by STNGSMC. The position tracking performances of Cases 1 and 2 are shown in Fig. 6. The chattering of the sliding surface near the origin affected the position tracking performances of Cases 1 and 2. Therefore, we can observe that the ripple of the position of Case 2 was smaller than that of Case 1, as shown in Fig. 6 (c). The current inputs for Cases 1 and 2 are shown in Fig. 7. There were fewer fluctuations in the current input in Case 2 owing to the reduced chattering.

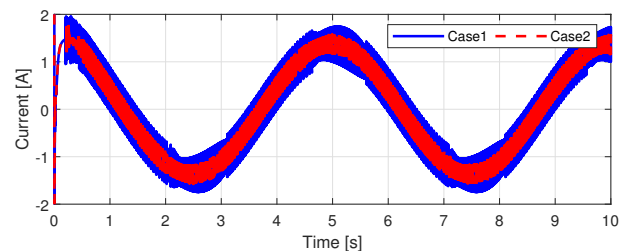


FIGURE 7: Current inputs in simulations

B. EXPERIMENTAL RESULTS

Experiments were performed using a PMSM testbed, which consisted of ControlDesk, two RapidPro units, and a SCALEXIO real-time system, as shown in Fig. 8, to validate the performance of the proposed method. We generated a real-time control code in MATLAB/Simulink to apply ControlDesk. The control algorithm was applied to the same simulink block used in the simulation. ControlDesk manages the overall experimental conditions, control parameters, and data acquisition. RapidPro was equipped with two “PS-HCHBD 2/2” power-stage modules. The maximum ratings

of the power-stage modules were 30 V_{DC} and 19 A_{rms}. An incremental optical encoder (2500 lines/r) was used to measure the position. The control sampling rate was set to 20kHz. The powder brake generates a load torque against the velocity with a maximum value of 2N·m.

The experiments were conducted for the three conditions as follows:

- Condition 1: Without disturbances
- Condition 2: With the parameter uncertainties
- Condition 3: With parameter uncertainties and the load torque.

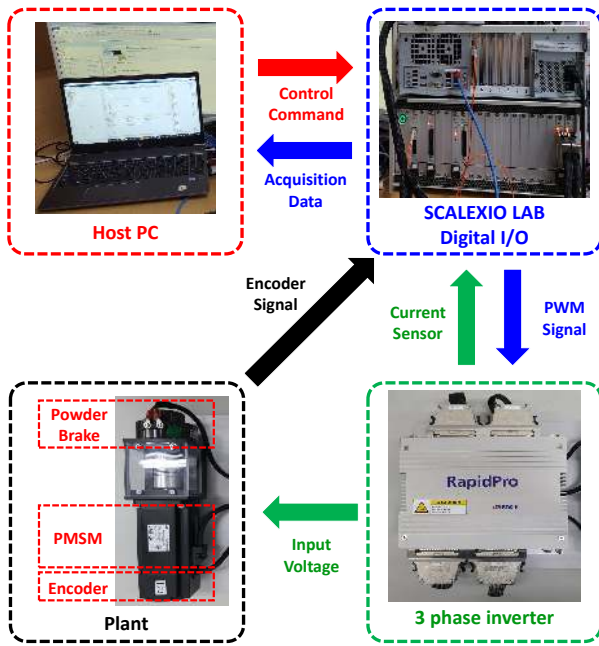
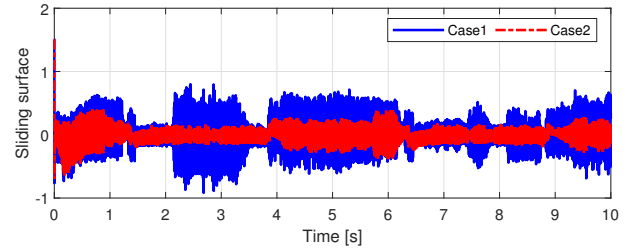
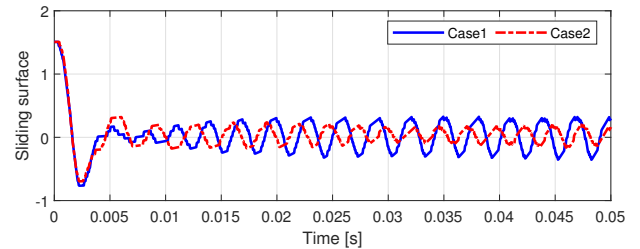


FIGURE 8: PMSM experimental setup



(a) Sliding surface s



(b) Enlarged sliding surfaces

FIGURE 9: Sliding surfaces in Condition 1

1) Condition 1: Without Disturbances

In these experiments, the nominal values listed in Table 1 were used for PMSMs. The sliding surfaces of Cases 1 and 2 are shown in Fig. 9. The chattering was more reduced by the STNGSMC in Case 2 than in Case 1. The frequency spectra of the sliding surfaces are presented in Fig. 10. We observe that near 200 Hz, the chattering of Case 2 was 3 dB smaller than that of Case 2. For the secondary harmonics frequency i.e., 460 Hz, the chattering was also reduced by 15 dB. The position tracking performances of Cases 1 and 2 are shown in Fig. 11. The ripple of the position in Case 2 was smaller than that in Case 1 because the chattering of the sliding surface in Case 2 was smaller than that in Case 1. The input currents are shown in Fig. 12. The input ripple in Case 2 is smaller than that in Case 1.

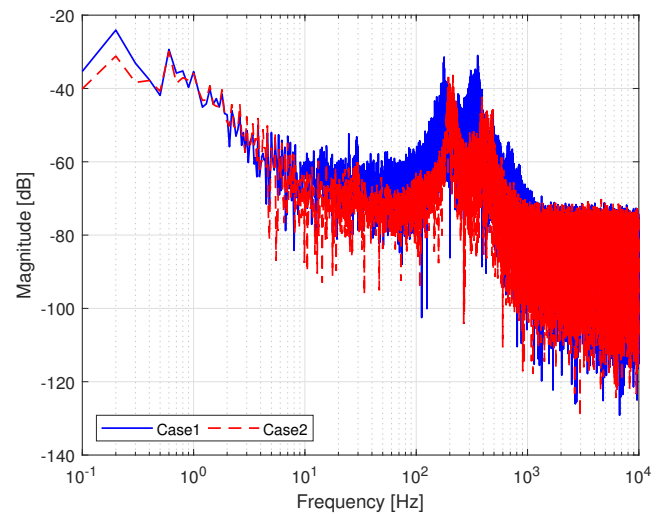


FIGURE 10: Frequency spectra of sliding surfaces in Condition 1

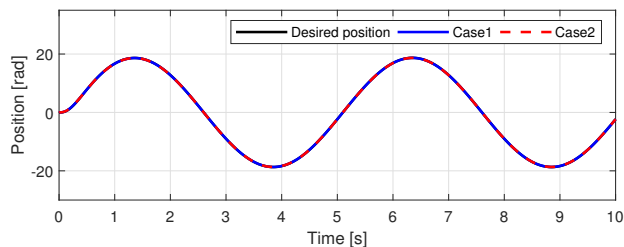
2) Condition 2: With the Parameter Uncertainties

To evaluate the robustness of the proposed method against parameter uncertainties, the experiments were tested with parameter uncertainties. The parameter uncertainties, which are a maximum of 20 % from the nominal value of Table 1, were applied, and the sliding surfaces are shown in Fig. 13. The chattering was more reduced by the STNGSMC in Case 2 than in Case 1. The frequency spectra of the sliding surfaces are shown in Fig. 14. We see that at 200 Hz, the chattering of Case 2 was 3 dB smaller than that of Case 2. For the secondary harmonics frequency, i.e., 460 Hz, the chattering was also reduced by 15 dB. The position tracking performances of Cases 1 and 2 are shown in Fig. 15. The

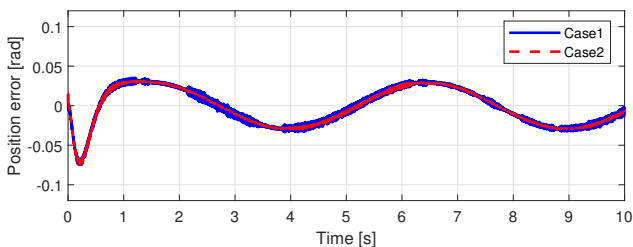
ripple of the position in Case 2 was smaller than that in Case 1 because the chattering of the sliding surface in Case 2 was smaller than that in Case 1. The input currents are shown in Fig. 16. The input ripple in Case 2 is smaller than that in Case 1.

3) Condition 3: With parameter uncertainties and the load torque

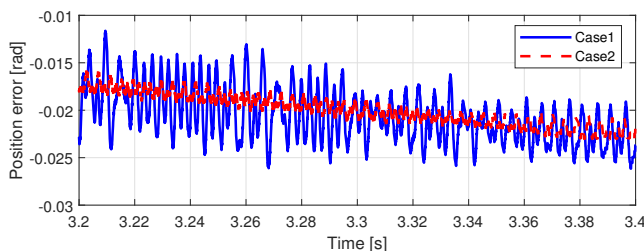
To evaluate the robustness of the proposed method against parameter uncertainties and load torque, the experiments



(a) Position tracking performances



(b) Position tracking errors



(c) Enlarged Position tracking errors

FIGURE 11: Position tracking performances in Condition 1

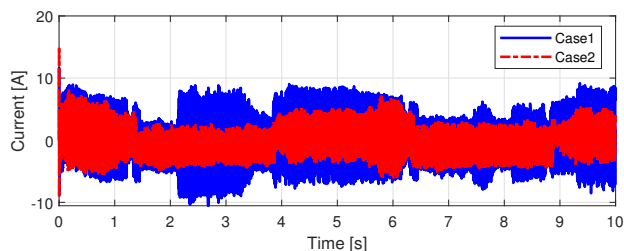
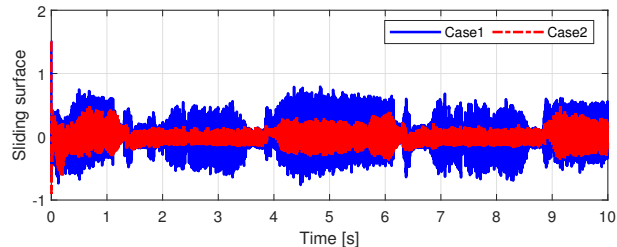
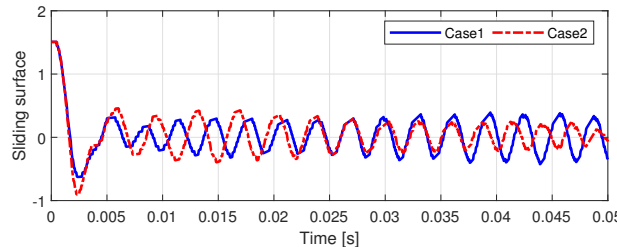


FIGURE 12: Current inputs in Condition 1

were tested with the parameter uncertainties and load torque. As shown in Fig. 17, a large load torque was applied. The sliding surfaces of Cases 1 and 2 are shown in Fig. 18. Owing to the large disturbance, including the parameter uncertainties and load torque, the chattering for Condition 3 increased more than that for Conditions 1 and 2. The chattering was more reduced by the STNGSMC in Case 2 than in Case 1. The frequency spectra of the sliding surfaces are presented in Fig. 14. We see that near 200 Hz, the chattering of Case 2 was 3 dB smaller than that of Case 1. The position tracking performances of Cases 1 and 2 are shown in Fig. 20. The ripple of the position in Case 2 was smaller than that in Case



(a) Sliding surfaces s



(b) Enlarged sliding surfaces

FIGURE 13: Sliding surfaces in Condition 2

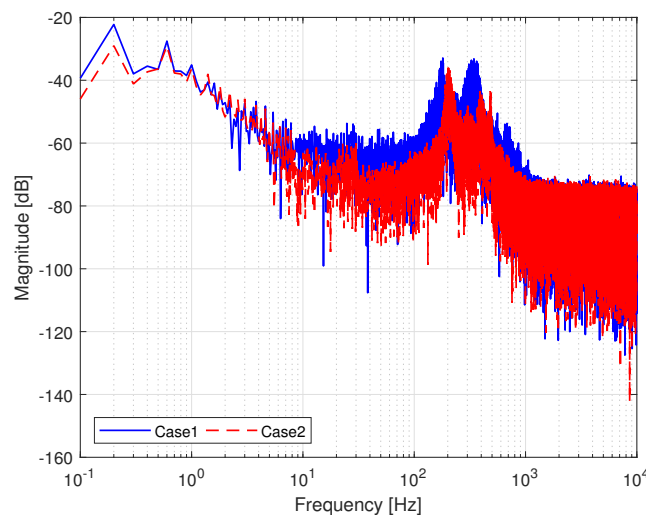
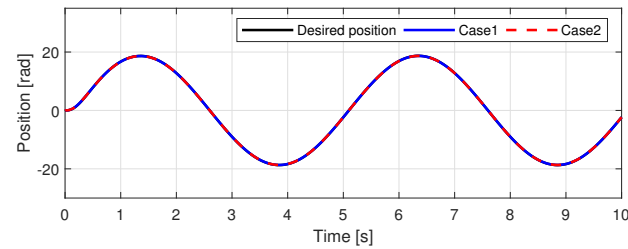


FIGURE 14: Frequency spectra of sliding surfaces in Condition 2

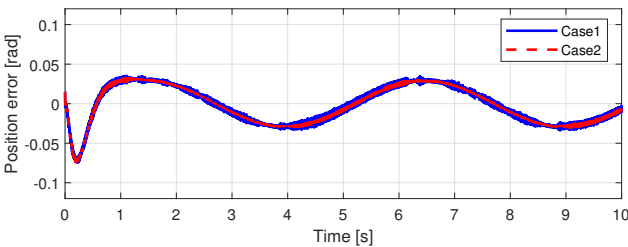
1 because the chattering of the sliding surface in Case 2 was smaller than that in Case 1. The input currents are shown in Fig. 21. The input oscillations are similar between Cases 1 and 2. However, the chattering and oscillation were reduced in Case 2.

V. CONCLUSION

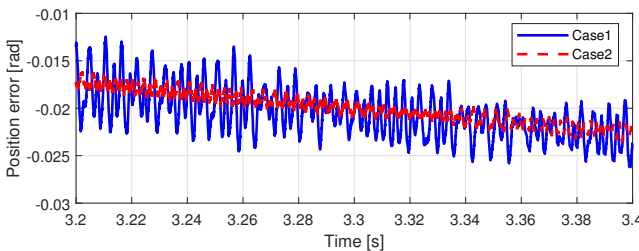
In this paper, we proposed the STNGSMC to improve the position control performance of PMSMs, and the nonlinear gain was developed to improve the position tracking perfor-



(a) Position tracking performances



(b) Position tracking errors



(c) Enlarged Position tracking errors

FIGURE 15: Position tracking performances in Condition 2

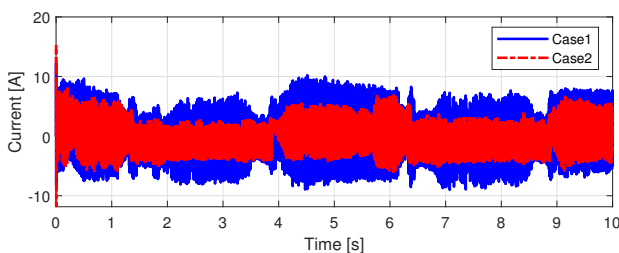


FIGURE 16: Current inputs in Condition 2

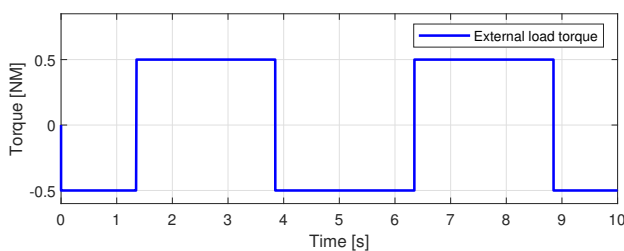
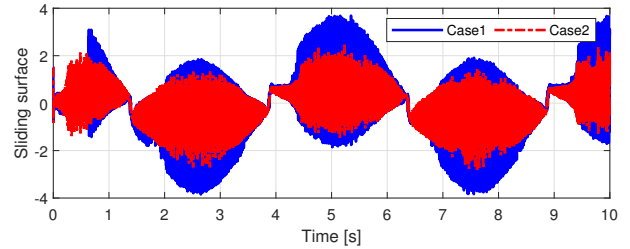
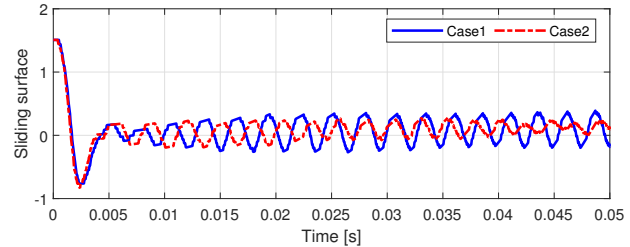


FIGURE 17: Injected load torque in Condition 3



(a) Sliding surfaces s



(b) Enlarged sliding surfaces

FIGURE 18: Sliding surfaces in Condition 3

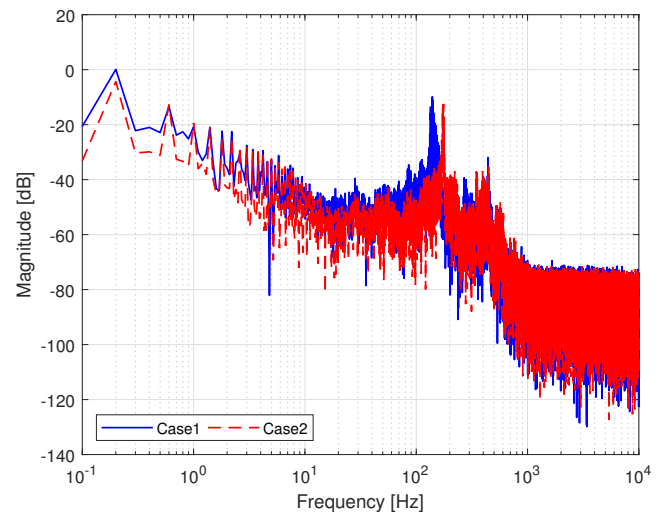


FIGURE 19: Frequency spectra of sliding surfaces in Condition 3

mance of the STSMC. The inclusion of nonlinear gain in the STSMC was demonstrated to reduce chattering. Based

on the results obtained, we proved the stability of closed-loop systems using the Lyapunov theorem, and we experimentally verified the performance of the proposed method. The chattering phenomenon was calculated by describing function methods. The chattering was compared for conventional SMC, STSMC, and STNGSMC along with unmodeled dynamics, such as the PWM switching noise, sensor noise, and quantization effect. In the simulations and experiments, the STNGSMC reduced the chattering phenomenon in SMC. In Condition 1, the sine wave position reference was used without uncertainties. Furthermore, to evaluate the robustness of the proposed method, Condition 2 was conducted under parameter uncertainties, and in Condition 3, the load

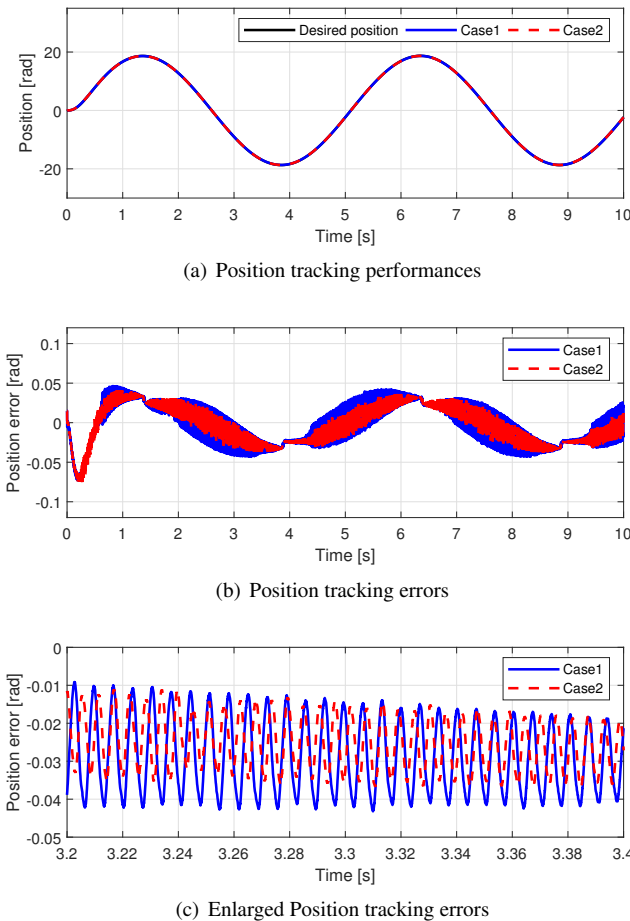


FIGURE 20: Position tracking performances in Condition 3

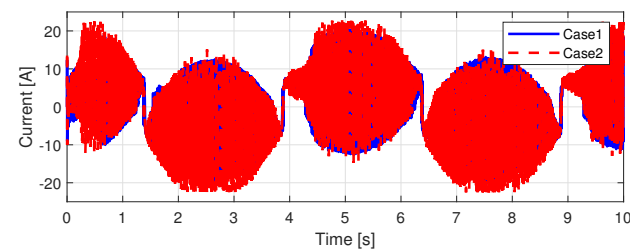


FIGURE 21: Current inputs in Condition 3

torque was applied. The robustness of the proposed method is mathematically proven in Section II. The chattering was reduced in the steady state because of nonlinear gain. Using the analysis of the frequency spectra, the chattering magnitude was reduced by 3 dB on average, which means that the chattering width of the proposed method was reduced by up to half that of STSMC. The simulation and experimental results demonstrated that the chattering width was observed in both the time domain and frequency spectra. The chattering affected the position tracking performance in transient and steady-state responses, and the simulation and experimental results demonstrate that position error ripples are reduced.

A wide chattering width was observed in the experimental results owing to unmodeled dynamics. Therefore, a low oscillation was observed in the control input, which is the quadrature current.

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