Super Twisting Disturbance Observer-Based Fixed-Time Sliding Mode Backstepping Control for Air-Breathing Hypersonic Vehicle

YUNJIE WU, FEI MA, XIAOCEN LIU, YUEYANG HUA, XIAODONG LIU, AND GUOFEI LI

1State Key Laboratory of Virtual Reality Technology and Systems, Beihang University, Beijing 100191, China
2School of Automation Science and Electrical Engineering, Beihang University, Beijing 100191, China
3Science and Technology on Aircraft Control Laboratory, Beijing 100191, China
4China Academy of Launch Vehicle Technology, Beijing 100076, China
5Beijing Aerospace Automatic Control Institute, Beijing 100854, China

Corresponding author: Fei Ma (mylovelylover26@163.com)

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ABSTRACT This paper investigates the velocity and altitude tracking control problem for air-breathing hypersonic vehicle (AHV) under external disturbances and uncertainties. An improved smooth super-twisting based disturbance observer (SSTDOB) is proposed to estimate the unknown external disturbances. With the assistance of SSTDOB, an effective fixed-time sliding mode backstepping control (FSMBC) is designed to guarantee the tracking errors converge to a small neighbor of the origin. Meanwhile, a fixed-time tracking differentiator (FTD) is employed to estimate the virtual control inputs, which can eliminate the differential explosion problem. The overall stability of the closed-loop system is analyzed by utilizing Lyapunov stability theory. Simulation results demonstrate the effectiveness of the composite method.

INDEX TERMS Air-breathing hypersonic vehicle, backstepping control, disturbance observer, fixed-time sliding mode control, super twisting algorithm.

I. INTRODUCTION

Air-breathing hypersonic vehicle (AHV) usually flies at more than 5 Mach numbers in the near space region [1]. It has attracted tremendous attentions and numbers of researches due to the advantages of global response, strong penetration ability and great potential in military and civilian applications [2]. Compared with the conventional aircrafts, AHV can adopt scramjet engine as its main power and combine it with the body to realize the propulsion-airframe integration configuration [3]. Consequently, this integration would lead to heavy couplings among the elastic airframe, the propulsion system and the structural dynamics [4]. Moreover, the unmanageable nonlinear dynamics, the uncertainty of flight aerodynamic parameters and the severe external disturbances during the flight envelope would make the control of AHV more challenging [5]. Therefore, stability and robustness are still a focus issue of AHV.

In previous literatures, control approaches for the longitudinal dynamics of AHV can usually be divided into two parts: linear approaches and nonlinear approaches. The linearization technique plays an important role in classical flight control. With this technique, the linearized model of AHV about a specific trim condition can be obtained [6]. Sigthorsson et al. [7] proposed a robust linear output-feedback controller for AHV in the presence of model uncertainties and varying flight conditions. Gibson et al. [8] designed a control architecture containing gain-scheduling and integral control based nominal controller with adaptive strategy for AHV model under thrust and actuator uncertainties, which had shown the superior tracking performance of this controller. Besides, an improved linear-quadratic regulator (LQR) with fractional-order sliding mode control based tracking controller presented in [9] also exhibited excellent robustness for AHV under uncertainties.

However, the traditional linear control methods may show poor capability of providing desired control effect when the flight condition deviates far from the given trimming point.
This drawback can be overcome by some advanced nonlinear control methods, include fuzzy control [10]–[14], neural network control [15]–[21], sliding mode control [22]–[30] and backstepping control [18], [23], [24], [27], [38].

A T-S fuzzy model was adopted by Li et al. [10] to approximate the nonlinear dynamics of AHV. Then a robust adaptive fuzzy based tracking controller for HFV was developed to guarantee the stability of whole system with parameter uncertainty and unmodeled dynamics [11]. Furthermore, the mixed $H_2/H_\infty$ robust fuzzy controller [13] and prescribed performance guaranteed cost fuzzy tracking control [14] for AHV were also investigated. In addition, Xu and Bu proposed a series of neural based controller for the longitudinal dynamics of hypersonic flight vehicle (HFV) [15]–[18]. With the help of these researches, many scholars extend this approximation technique to the controller design of HFV. For example, Xu et al. [20] combined the global neural control with dynamic surface control to achieve the stability of closed-loop HFV system and effectiveness of control scheme, where nonlinear functions of the HFV were approximated by the neural networks. Both fuzzy based controllers and neural based controllers can exhibit excellent approximation strength for unknown nonlinear functions.

Sliding mode control possesses excellent performance in convergence time and disturbance rejection. Thus, Xu et al. [23] combined the adaptive control strategies with sliding mode control, which can provide good tracking performance for the HFV under parametric uncertainty. In Zong et al. [24], a quasi-continuous high-order sliding mode controller based on full state feedback was designed for the longitudinal dynamics of flexible air-breathing hypersonic vehicles (FAHV), where the chattering problem was alleviated by introducing the quasi-continuous high-order sliding mode. To achieve the finite time stability of the control system, Sun et al. [27] proposed a finite time sliding mode control with disturbance observer for AHV. Besides, Yang et al. [28] developed a new nonsingular terminal sliding mode control (NTSMC) with backstepping strategy for FAHV, which can guarantee the finite time convergence and the steady-state precision. Moreover, since Polyakov [31] raised the fixed-time stability, this topic has been rigorously studied [32]–[34]. Zuo and Tie [35] addressed the fixed-time stable of first-order multi-agent systems. Basin et al. [36] considered the application of super-twisting based controller with fixed-time stability. Wang et al. [37] adopted the fixed-time backstepping scheme for AHV with external disturbances. For now, few literatures consider the fixed-time stability of AHV [38], [39], which would lead such problem still challenging.

Backstepping control (BC) is another effective nonlinear control design scheme. A series of aforesaid methods and the BC logic can be merged into an integral control framework with excellent ability to handle AHV’s higher order nonlinear system. However, we cannot ignore the weak robustness and “explosion of complexity” of conventional BC. To enhance the robustness of controller, many disturbance observer (DOB) based techniques were investigated, such as fuzzy-based observer [12], [14], nonlinear disturbance observer [26], [40], [41], super twisting algorithm based observer [29], [30], extended state observer (ESO) [42], [43], etc. Li and Li [12] utilized a novel fuzzy-based approximator to estimate the total uncertainties of velocity subsystem and altitude system. Wu et al. [26] proposed a strictly-lower-convex-function constructing nonlinear disturbance observer (SDOB) based backstepping controller for HFV. Wang et al. [29] developed a conventional super-twisting algorithm to estimate the composite disturbances and uncertainties. An active disturbance rejection control (ADRC) based robust controller was employed for the AHV autopilot, where the ESO [42] was applied to estimate the parametric perturbations and atmospheric disturbances. In order to handle the differential explosion problem in BC, the dynamic surface control strategy was adopted by Xu et al. [20]. Bu et al. [21], [44] utilized a low-pass filter to transform the non-affine system to affine system so as to avoid the virtual control laws complexity involved in traditional BC strategy. Besides, some filter-based algorithms and differentiator-based algorithms [45], [46] can also get introduced to estimate the derivatives of virtual controls. Yu et al. [46] presented a novel finite-time command filter for the backstepping scheme, which can guarantee the finite time convergence property. Compared with conventional BC, these improved BC methods possess both higher tracking accuracy and better disturbance rejection ability.

Motivated by the above analysis, this study proposes a composite controller which consists of smooth super-twisting algorithm based disturbance observer (SSTDOB) and fixed-time sliding mode backstepping control (FSMBC) for AHV. Compared with the above literatures, the key innovative points of this paper are summarized as follows:

1. The proposed SSTDOB is introduced for the equivalent disturbances of AHV, which has more general formulation for slow varying disturbance estimation and smoother outputs than that of conventional super twisting algorithms.

2. The FSMBC has the merits of both fixed-time sliding mode control and dynamics surface control. The improved sliding surface can strengthen the states convergence speed and the bound of convergence time will be independent of AHV’s initial conditions. The singularity problem caused by the derivative of virtual control is avoided by introducing a switching logic. Besides, all the system states are bounded and the “explosion of complexity” problem is avoided by utilizing a fixed-time tracking differentiator (FTD).

3. The SSTDOB based FSMBC (SSTDOB-FSMBC) can guarantee the overall fixed-time stability of closed-loop system with faster response and higher tracking precision in the presence of external disturbances and uncertainties.

The remainder of this paper is organized as follows. Problem description section presents the longitudinal dynamics of AHV and the decomposed altitude subsystem and velocity subsystem. Then the SSTDOB is developed for the AHV system with the relevant stability analysis at the same time.
The next section shows the main design process of SSTDOB-FSMBC, and the stability of composite method is also analyzed in detail. Besides, we implement two groups of simulation comparisons on an AHV. Eventually, the conclusion and future works are provided.

![Schematic diagram for the longitudinal model of AHV.](image)

**FIGURE 1.** Schematic diagram for the longitudinal model of AHV.

## II. PROBLEM DESCRIPTION

### A. LONGITUDINAL DYNAMIC MODEL OF AHV

As is shown in Figure 1, the longitudinal dynamic model of a generic AHV [26], [29] consists of the following differential equations

\[ \dot{h} = V \sin \theta \]  
\[ \dot{\theta} = \frac{L + T \sin \alpha}{mV} - \frac{(\mu - V^2 (h + R_E)) \cos \theta}{V (h + R_E)^2} + d_2 \]  
\[ \dot{\omega}_e = \frac{M_e}{J_e} + d_4 \]  
\[ \dot{V} = \frac{T \cos \alpha - D}{m} - \frac{\mu \sin \theta}{(h + R_E)^2} + d_5 \]  
\[ \alpha = \beta - \theta \]

where \( h, \theta, \dot{\theta}, \omega_e \) and \( V \) represent the altitude, path angle, pitch angle, pitch rate and velocity of the AHV respectively. \( \mu, \alpha, m, R_E \) and \( J_e \) denote the gravitation constant, angle of attack, mass of the AHV, radius of the earth and moment of inertia around AHV \( z \) axis, respectively. Besides, the terms \( d_i, i = 2, 4, 5, \) will indicate the external disturbances of each corresponding channel.

The thrust \( T \), drag \( D \), lift \( L \) and pitching moment \( M_e \) are expressed as follows

\[ T = C_T q S \]  
\[ D = C_D q S \]  
\[ L = C_L q S \]  
\[ M_e = C_m q l \]

where \( \rho, q = \frac{1}{2} \rho V^2, S \) and \( l \) represent the density of air, dynamics pressure, reference area and mean aerodynamic chord, respectively.

The atmospheric force and moment coefficients are given by

\[ C_T = c_{T0} + c_T^\beta \beta \]  
\[ C_D = c_{D0} + c_D^\alpha \alpha + c_D^\omega \omega^2 \]  
\[ C_L = c_L^\omega \omega \]  
\[ C_m = m_{z0} + m_{z}^\alpha \alpha + m_{z}^\beta \beta^2 + m_{z}^\omega \omega \zeta + m_{z}^{\delta_e} \delta_e \]

where \( \beta \) is the throttle setting, \( \delta_e \) is the elevator deflection angle.

In addition, the second-order engine dynamics can be described as

\[ \ddot{\beta} = -2\xi \omega_n \beta - \omega_n^2 \beta + \omega_n^2 \beta_c + d_7, \]

where \( \beta_c \) is the demand of throttle setting, \( \xi \) is the damping ratio of the engine dynamics, \( \omega_n \) is the undamped natural frequency, \( d_7 \) is the external disturbance of the throttle.

**Remark 1:** In fact, the disturbances of \( h, \dot{\theta} \) and \( \dot{\beta} \) are not concerned in this study for the reason of their accurate mathematics deduction. Besides, the disturbance terms \( d_i, i = 2, 4, 5, 7 \), can be regarded as the equivalent disturbances consisting of external disturbances, internal uncertainties and model errors which can be well tackled by the disturbance observers.

### B. MODEL TRANSFORMATION

In the process of controller designing, the AHV system is required to be expressed in the strict feedback formulations. Before the model transformation, following assumptions are introduced.

**Assumption 1** [26], [29], [40]: The thrust term \( T \sin \alpha \) in (2) can be omitted, since it is commonly much smaller than \( L \).

**Assumption 2** [26], [29], [40]: The flight path angle \( \theta \) maintains a small value, which means \( \sin \theta \approx \theta \).

To depict the AHV longitudinal model (1)-(6) and (15) more explicitly, the system states and the control inputs are defined as \( x_1 = h, x_2 = \theta, x_3 = \dot{\theta}, x_4 = \omega_e, x_5 = V, x_6 = \beta, x_7 = \dot{\beta}, u_1 = \delta_e, u_2 = \beta_c \), respectively.

Thus, considering Assumption 1 and 2, the AHV longitudinal system can be divided into the altitude subsystem and the velocity subsystem as follows.

\[ \dot{x}_1 = g_1 x_2, \]
\[ \dot{x}_2 = g_2 x_3 + f_2 + d_2, \]
\[ \dot{x}_3 = g_3 x_4, \]
\[ \dot{x}_4 = g_4 u_1 + f_4 + d_4, \]
\[ \dot{x}_5 = g_5 x_6 + f_5 + d_5 \]
\[ \dot{x}_6 = g_6 x_7 \]
\[ \dot{x}_7 = g_7 u_2 + f_7 + d_7 \]

where \( f_j, j = 2, 4, 5, 7 \), and \( g_i, i = 1, 2, \ldots, 7 \), represent the dynamics of relevant channels for AHV longitudinal model.
which can be expressed as
\[ g_1 = x_5, g_2 = \frac{c_2^p \rho S x_5}{2m}, \]
\[ f_2 = -\frac{(\mu - x_2^2(x_1 + R_E)) \cos x_2}{x_2^2(x_1 + R_E)^2} - g_2 x_2, \]
\[ g_3 = 1, g_4 = \frac{m_s^x \rho S x_2^2}{2J_z}, \]
\[ f_3 = \frac{\rho S x_2^2}{2m}(m_0 + m_e^d(x_3 - x_2) + m_e^o(x_3 - x_2)^2 \]
\[ + m_e^o x_4), \]
\[ g_5 = \frac{\rho S x_2^2}{2m} \cos(x_3 - x_2), \]
\[ f_3 = -\frac{\rho S x_2^2}{2m}(c_D 0 + c_D (x_3 - x_2) + c_D (x_3 - x_2)^2 \]
\[ + c_T \rho S x_2^2 \cos(x_3 - x_2) - \frac{\mu \sin x_2}{(x_1 + R_E)^2}, \]
\[ g_6 = 1. \]
\[ g_7 = \omega_n^2, f_1 = -2\xi \omega_n x_7 - \omega_n^2 x_6. \]  
\[ (18) \]

Assumption 3: [37], [47]: The functions \( g_i, i = 1, 2, \ldots, 7 \), satisfy \( |g_i| < g_i \), where \( g_i > 0 \).

This assumption can ensure that the control inputs are nonsingular and bounded.

III. SMOOTH SUPER TWISTING ALGORITHM BASED DISTURBANCE OBSERVER DESIGN
In this section, the smooth super twisting algorithm based disturbance observer (SSTDOB) is developed for the AHV system, which can compensate the equivalent disturbances. Besides, the convergence of the disturbance observation error is also analyzed.

A. OBSERVER DESIGN
It is shown that disturbances \( d_i, i = 2, 4, 5, 7 \), are respectively involved in the differential equation for \( x_2, x_4, x_5 \) and \( x_7 \) without couplings. Thus, observer for \( d_i \) can be designed independently. Before the design of this SSTDOB, the following assumption is made.

Assumption 4: [26], [29], [40] The disturbances \( d_i, i = 2, 4, 5, 7 \), are considered to be bounded, that is to say \( |d_i| \leq \bar{d}_i \), \( |d_i| \leq \bar{d}_i \), where \( \bar{d}_i > 0 \) and \( \bar{d}_i > 0 \).

Considering the observer for \( d_i \), the relevant auxiliary variables \( y_i, i = 2, 4, 5, 7 \), can be constructed as
\[ \hat{y}_i = \psi_i + \hat{d}_i \]
\[ \text{where } \psi_i \text{ can be expressed as} \]
\[
\psi_i = \begin{cases} 
  g_2 x_3 + f_2 & i = 2 \\
  g_4 u_1 + f_4 & i = 4 \\
  g_5 x_6 + f_5 & i = 5 \\
  g_7 u_2 + f_7 & i = 7 \\
\end{cases} 
\]  
\[ (20) \]

Errors between \( x_i \) and \( y_i \) are defined as \( e_i = x_i - y_i \). With the assistance of \( e_i \), observer for \( d_i \) can be designed as
\[ \hat{d}_i = k_{i1} \text{sig}(e_i) \frac{e_i - e_{i1}}{e_{i2}} + k_{i2} e_{i1} \]
\[ + \int_0^t \left( k_{i3} \text{sig}(e_i(t)) \frac{e_i - e_{i1}}{e_{i2}} + k_{i4} e_{i1}(t) \right) d\tau \]  
\[ (21) \]

In (21), \( k_{i1}, k_{i2}, k_{i3}, k_{i4} \) and \( p_i, i = 2, 4, 5, 7 \) are all positive constants and satisfy following conditions
\[ \begin{cases} 
  p_i \geq 2, & k_{i1} > 2 \sqrt{\bar{d}_i}, \quad k_{i2} > 0, \\
  k_{i3} > \bar{d}_i, & k_{i4} > \bar{d}_i \\
\end{cases} \]  
\[ (22) \]

where \( k_{i4} \) is given by
\[ k_{i4} = \frac{(p_{i1} - 1)/p_i}{N_i} (2k_{i2}^2 + 2k_{i2}) - \frac{p_{i1} - 1/k_{i2}}{N_i} (2k_{i2}^2 + 2k_{i2} - k_{i3} + k_{i3} - \bar{d}_i - k_{i3} - \bar{d}_i). \]  
\[ (23) \]

B. STABILITY ANALYSIS FOR SSTDOB
With Assumption 4, the following Theorem 1 of proposed SSTDOB for AHV can be established.

Theorem 1: With the proposed SSTDOB (21), observation errors of \( x_i \), \( i = 2, 4, 5, 7 \), can converge to 0 in finite time.

Proof: Combining (19) and the differential equations for \( x_i, i = 2, 4, 5, 7 \), in (16) and (17), it can be easily acquired that
\[ \ddot{e}_i = d_i - \hat{d}_i \]
\[ (23) \]

Substituting (21) into (23), we have
\[ \ddot{e}_i = d_i - k_{i1} \text{sig}(e_i) \frac{e_i - e_{i1}}{e_{i2}} - k_{i2} e_{i1} \]
\[ - \int_0^t \left( k_{i3} \text{sig}(e_i(t)) \frac{e_i - e_{i1}}{e_{i2}} + k_{i4} e_{i1}(t) \right) d\tau \]  
\[ (24) \]

To facilitate the following deduction, we define \( e_{i2} = d_i - \int_0^t \left( k_{i3} \text{sig}(e_i(t)) \frac{e_i - e_{i1}}{e_{i2}} + k_{i4} e_{i1}(t) \right) d\tau \). Evidently, (24) can be reformed as the following second-order system.
\[ \begin{cases} 
  \dot{e}_{i1} = -k_{i1} \text{sig}(e_i) \frac{e_i - e_{i1}}{e_{i2}} - k_{i2} e_{i1} + e_{i2} \\
  \dot{e}_{i2} = -k_{i3} \text{sig}(e_i) \frac{e_i - e_{i1}}{e_{i2}} - k_{i4} e_{i1} + \hat{d}_i \\
\end{cases} \]  
\[ (25) \]

Consider the Lyapunov candidate function
\[ V_i = 2k_{i3} e_{i1}\frac{e_i - e_{i1}}{e_{i2}} + k_{i4} e_{i1} + \frac{1}{2} \frac{e_{i2}^2}{e_{i2}} \]
\[ + \frac{1}{2} \left( k_{i4} \text{sig}(e_i) \frac{e_i - e_{i1}}{e_{i2}} + k_{i2} e_{i1} - e_{i2} \right)^2 \]
\[ \leq \xi_i^T P_i \xi_i \leq \lambda_{\text{max}}(P_i) \| \xi_i \|^2 \]
\[ (26) \]

where
\[ \xi_i = \begin{bmatrix} 
  \text{sig}(e_i) \frac{e_i - e_{i1}}{e_{i2}} \\
  e_{i1} \\
  e_{i2} \\
\end{bmatrix} \]
\[ (27) \]
\[ P_i = \begin{bmatrix} 
  2k_{i3} + \frac{1}{2} k_{i2}^2 & 1 & \frac{1}{2} k_{i1} k_{i2} & - \frac{1}{2} k_{i1} \\
  \frac{1}{2} k_{i1} k_{i2} & k_{i4} + \frac{1}{2} k_{i2}^2 & - \frac{1}{2} k_{i2} & - \frac{1}{2} k_{i1} \\
  - \frac{1}{2} k_{i1} & - \frac{1}{2} k_{i2} & 1 & 0 \\
\end{bmatrix} \]
\[ (28) \]
We note that the derivative of $\xi_i$ can be expressed as

$$
\dot{\xi}_i = \begin{bmatrix}
\frac{p_i - 1}{p_i} |e_{i1}|^{\frac{1}{p_i}} \dot{e}_{i1} \\
\dot{e}_{i1} \\
\dot{e}_{i2}
\end{bmatrix}
= \begin{bmatrix}
\frac{p_i - 1}{p_i} |e_{i1}|^{\frac{1}{p_i}} \left( -k_{i1} \text{sign} \left( e_{i1} \right)^{\frac{p_i - 1}{p_i}} - k_{i2} e_{i1} + e_{i2} \right) \\
-k_{i1} \text{sign} \left( e_{i1} \right)^{\frac{p_i - 1}{p_i}} - k_{i2} e_{i1} + e_{i2} \\
-k_{i3} \text{sign} \left( e_{i1} \right)^{\frac{p_i - 1}{p_i}} - k_{i4} e_{i1} + \bar{d}_i
\end{bmatrix}
= |e_{i1}|^{\frac{1}{p_i}} A_i \dot{\xi}_i + B_i \xi_i + \begin{bmatrix} 0 \\ 0 \\ \bar{d}_i \end{bmatrix}
$$

(29)

where

$$
A_i = \begin{bmatrix}
-\frac{p_i - 1}{p_i} k_{i1} & -\frac{p_i - 1}{p_i} k_{i2} & \frac{p_i - 1}{p_i} \\
0 & 0 & 0 \\
-k_{i3} & -k_{i2} & 1 \\
0 & -k_{i4} & 0
\end{bmatrix}
$$

$$
B_i = \begin{bmatrix}
0 & 0 & 0 \\
-k_{i1} & k_{i2} & 0 \\
0 & k_{i1} & 0
\end{bmatrix}
$$

Using Assumption 4 and setting $\kappa_i(t) = \text{sign} \left( e_{i1} \right)$, the time derivative of $V_i$ can become

$$
\dot{V}_i = |e_{i1}|^{\frac{1}{p_i}} \xi_i^T P_i \dot{\xi}_i + \xi_i^T P_i B_i \xi_i + 2 |e_{i1}|^{\frac{1}{p_i}} \lambda \xi_i^T \dot{\xi}_i
t = |e_{i1}|^{\frac{1}{p_i}} \xi_i^T \left( P_i A_i + A_i^T P_i \right) \xi_i + \xi_i^T \left( P_i B_i + B_i^T P_i \right) \xi_i + 2 |e_{i1}|^{\frac{1}{p_i}} \lambda \xi_i^T \dot{\xi}_i
$$

(30)

where

$$
M_i(t) = \begin{bmatrix}
0 & 0 & 0 \\
0 & 0 & 0 \\
0 & 0 & 0
\end{bmatrix}
$$

$$
Q_i = \begin{bmatrix}
Q_{11} & Q_{12} & Q_{13} \\
Q_{21} & Q_{22} & Q_{23} \\
Q_{31} & Q_{32} & Q_{33}
\end{bmatrix}
$$

$$
R_i = \begin{bmatrix}
R_{11} & R_{12} & R_{13} \\
R_{21} & R_{22} & R_{23} \\
R_{31} & R_{32} & R_{33}
\end{bmatrix}
$$

(31)

(32)

(33)

with their elements

$$
Q_{11} = \frac{p_i - 1}{p_i} \left( k_{i1}^2 + 4k_{i1} k_{i3} \right) - k_{i1} (k_{i3} - \kappa_i(t))
$$

$$
Q_{12} = \frac{p_i - 1}{p_i} \left( 2k_{i2} k_{i3} + k_{i1}^2 k_{i2} \right) - \frac{1}{2} k_{i2} (k_{i3} - \kappa_i(t))
$$

$$
Q_{13} = -\frac{p_i - 1}{p_i} \left( k_{i1}^2 + 2k_{i3} \right) + (k_{i3} - \kappa_i(t))
$$

$$
Q_{22} = \frac{p_i - 1}{p_i} k_{i1}^2 k_{i2} - \frac{p_i - 1}{p_i} k_{i1} k_{i2},
$$

$$
Q_{23} = \frac{p_i - 1}{p_i} k_{i1} k_{i2},
$$

$$
Q_{21} = Q_{12},
$$

$$
Q_{31} = Q_{13},
$$

$$
Q_{32} = Q_{23},
$$

$$
Q_{33} = \frac{p_i - 1}{p_i} k_{i1},
$$

$$
R_{11} = k_{i1}^2 k_{i2},
$$

$$
R_{12} = k_{i1} k_{i2} + \frac{1}{2} k_{i1} k_{i4},
$$

$$
R_{13} = -k_{i1} k_{i2},
$$

$$
R_{22} = k_{i2}^2 + k_{i2} k_{i4},
$$

$$
R_{23} = k_{i2}^2,\ R_{33} = k_{i2},
$$

$$
R_{21} = R_{12},
$$

$$
R_{31} = R_{13},\ R_{32} = R_{23}
$$

(34)

(35)

(36)

(37)

(38)

(39)

Due to the fact that $k_{ij}, i = 2, 4, 5, 7, j = 1, 2, 3, 4$ are all positive constants with conditions (22) hold, $\tilde{Q}_i$ and $\tilde{R}_i$ are all positive definite matrices. Therefore, (37) can be reformed as

$$
\dot{V}_i \leq -\lambda_{min} \left( \tilde{Q}_i \right) |e_{i1}|^{\frac{1}{p_i}} \| \xi_i \|_{2}^2 - \lambda_{min} \left( \tilde{R}_i \right) \| \xi_i \|_{2}^2
$$

(40)

Combining (26) and (40), it can be acquired that

$$
\dot{V}_i + \alpha_i \| V_i \|_{P_i} + \beta_i \| V_i \|_{P_i} \leq 0
$$

(41)

where $\alpha_i = \frac{\lambda_{min} \left( \tilde{Q}_i \right)}{\lambda_{max} \left( P_i \right)^{\frac{p_i - 1}{p_i}}} V_i = \frac{2p_i - 3}{2p_i - 2}, \beta_i = \frac{\lambda_{min} \left( \tilde{R}_i \right)}{\lambda_{max} \left( P_i \right)^{\frac{p_i - 1}{p_i}}}, i = 2, 4, 5, 7.$

Thus, according to Yu et al. [46], $V_i$ will converge to 0 in finite time, that is, $e_{i1}$ and $e_{i2}$ converge to 0 in finite time.
In addition, the convergence time $t_i$ can be evaluated by
\[
t_i = \int_{V_i(0)}^{V_i(t)} \frac{1}{V_i} dV_i \leq \int_{V_i(0)}^{V_i(t)} \frac{1}{\alpha_i V_i^{\gamma_i} + \beta_i V_i} dV_i
\]
\[
= \frac{1}{\beta_i (1 - \gamma_i)} \ln \beta_i V_i(0)^{1-\gamma_i} + \alpha_i
\]
(42)

Since $(e_{i1} = 0, e_{i2} = 0)$ is the equilibrium point of (25), $e_{i1}(t) = 0$ is satisfied for all $t > t_i$, which indicates that $\dot{e}_{i1}(t) = 0$ for any $t > t_i$. According to (23), it can be obtained that
\[
\ddot{d}_i(t) = d_i(t) - \ddot{d}_i(t) = 0, \forall t > t_i
\]
(43)

Therefore, observation errors of $d_i, i = 2, 4, 5, 7$, can converge to 0 in finite time, which means the equivalent disturbances can be exactly estimated in finite time. The proof of SSTDOB’s stability is complete.

Remark 2: When $\delta_i = 0$, $i = 2, 4, 5, 7$, $p_i$ can be chosen as $p_i \geq 2$, then the disturbance observation errors can converge to the origin in finite time. When $\delta_i > 0$, two cases can be discussed.

Case A: $0 < \delta_i < \varsigma_i$, where the terms $\varsigma_i$ are small enough positive constants. This indicates that the disturbances are slow varying, that is, $\dot{d}_i \approx 0$. Then, (29) becomes
\[
\dot{x}_i \approx |e_{i1}|^{\frac{1}{2}} A_i \dot{x}_i + B_i \xi_i
\]
(44)
The finite time stability of proposed SSTDOB won’t rely on the boundedness of $\kappa_i(t)$. Therefore, the disturbance observer can still work well when $0 < \delta_i < \varsigma_i$ and $p_i \geq 2$.

Case B: $\delta_i \geq \varsigma_i$. In this situation, the disturbances have higher frequency or larger amplitude. The finite time stability of proposed SSTDOB will depend on the boundedness of $\kappa_i(t)$. Then, setting $p_i = 2$, we yield
\[
|\kappa_i(t)| = |\text{sign}(e_{i1})\dot{d}_i| \leq \delta_i
\]
(45)

With (45), the boundedness of $\kappa_i(t)$ can be guaranteed, which will maintain the finite time performance of the observer. Accordingly, the newly proposed observer will revert to the following equation [48]
\[
\dot{d}_i = k_{i1} \text{sign}(e_{i1})^{\frac{1}{2}} + k_{i2} e_{i1}
\]
\[
+ \int_0^t (k_{i3} \text{sign}(e_{i1}(\tau)) + k_{i4} e_{i1}(\tau)) d\tau
\]
(46)

Therefore, case A will explain the simulation results with slow varying disturbances, that is, $0 < \delta_i < \varsigma_i$ and $p_i \geq 2$. Compared with super twisting algorithm-based observer designed in Nagesh and Edwards [48], the selection of $p_i$ are more flexible, which means that this SSTDOB have more generic formulation for slow varying disturbance estimation. Besides, it will be illustrated that the proposed SSTDOB can achieve excellent estimation ability if we choose $p_i = 2$ for both case A and case B.

Remark 3: As mentioned in Nagesh and Edwards [48], the Filippov solution of (21) cannot stay on $(e_{i1} = 0, e_{i2} = 0)$. There exists some small time interval $T_o$ containing $t_i, e_{i1}$ will monotonically pass through zero. Therefore, (40) holds almost everywhere and the observer will converge to the equilibrium point $(e_{i1} = 0, e_{i2} = 0)$ in finite time.

Remark 4: Since the possible discontinuous function $\text{sign}(e_{i1})^{\frac{p_i-2}{p_i}}$ is hidden in the integral item, $\dot{d}_i$ will be continuous (non-Lipschitzian) and the chattering is eliminated. Besides, calculating the derivative of $\dot{d}_i$ we can yield
\[
\dot{d}_i = k_{i1} \frac{p_i - 1}{p_i} \text{sign}(e_{i1})^{-\frac{1}{p_i}} \dot{e}_{i1} + k_{i2} \dot{e}_{i1}
\]
\[
+ k_{i3} \text{sign}(e_{i1}) \frac{p_i - 2}{p_i} + k_{i4} e_{i1}
\]
(47)

With (25), it is apparent that $e_{i1}, e_{i2}, \dot{e}_{i1}$ and $\text{sign}(e_{i1})^{\frac{p_i-2}{p_i}}$ are continuous. Thus, the continuity of (47) will depend on the terms $\Theta_i = \text{sign}(e_{i1})^{-\frac{1}{p_i}} \dot{e}_{i1}$. Substituting the first formula of (25) into $\Theta_i$, we have
\[
\Theta_i = -k_{i1} \text{sign}(e_{i1}) \frac{p_i - 1}{p_i} |e_{i1}|^{\frac{p_i - 1}{p_i}} - k_{i2} \text{sign}(e_{i1})^{\frac{1}{p_i}} e_{i1}
\]
\[
+ \text{sign}(e_{i1})^{-\frac{1}{p_i}} e_{i2}
\]
(48)

When $e_{i1} \rightarrow 0^+$, we have $\lim_{e_{i1} \rightarrow 0^+} \Theta_i = 0$; when $e_{i1} \rightarrow 0^-$, we have $\lim_{e_{i1} \rightarrow 0^-} \Theta_i = 0$. Therefore, $\Theta_i$ is continuous and it follows that $\dot{d}_i$ is continuous. Finally, the smoothness of $\dot{d}_i$ can be guaranteed. Besides, it will be illustrated in the simulation results that the proposed SSTDOB can exhibit smoother outputs.

Remark 5: When $e_{i1}$ and $e_{i2}$ converge to zero after finite time $t_i$, (25) becomes
\[
\begin{cases}
\dot{e}_{i1} = e_{i2} = 0 \\
\dot{e}_{i2} = \dot{d}_i
\end{cases}
\]
(49)

With the first formula of (49) and (23), we can make a conclusion that the observation errors of $d_i$ will also converge to zero. As for the second formula of (49), the following inequality holds
\[
|\dot{e}_{i2}| = |\dot{d}_i| \leq \delta_i
\]
(50)

Therefore, the result of (50) is consistent with the boundedness of $\dot{d}_i$ in Assumption 4.

IV. SSTDOB BASED FSMBC FOR AHV SYSTEM

The main objective of this paper is to design a good performance of controller which can track the desired altitude $x_{id}$ and desired velocity $x_{sd}$ respectively. Thus, the SSTDOB based fixed-time sliding mode backstepping control (FSMBC) strategy is proposed for the AHV longitudinal system. The block diagram of this composite controller is shown in Figure 2.

Before the controller designing, some lemmas and assumptions are given as follows

Lemma 1 [37], [47], [49]: Consider a scalar system is expressed as follow
\[
\dot{y} = -a \text{sign}(y)^\alpha - b \text{sign}(y)^\beta, y(0) = y_0
\]
(51)
where $a > 0$, $b > 0$, $\alpha > 1$, $0 < \beta < 1$. System (51) has globally fixed-time stability with convergence time $T$ bounded by

$$T \leq \frac{1}{a \alpha - 1} + \frac{1}{b \beta - 1}$$

(52)

Besides, when system (51) becomes

$$\dot{y} = a \text{sig}(y)^{p} - b \text{sig}(y)^{p} + \Phi, y(0) = y_0$$

(53)

where $\Phi$ is a small positive real number. Then the system (53) will converge to an arbitrarily small neighbor of the origin, i.e., $y \leq 2T$ with $\Phi = a \text{sig}(y)^{p} + b \text{sig}(y)^{p}$, within a fixed-time bounded by

$$T < \frac{1}{a \alpha - 1} + \frac{1}{b (2 \beta - 1) \beta - 1}$$

(54)

Lemma 2 [37], [47], [49]: For $\xi_1, \xi_2, \ldots, \xi_n \geq 0$ and $p > 0$, the following inequality can hold

$$\max(p^{n-1}, 1)(\xi_1^p + \xi_2^p + \cdots + \xi_n^p) \geq (\xi_1 + \xi_2 + \cdots + \xi_n)^p$$

(55)

Lemma 3 [45]: Consider the differentiators are expressed as

$$\begin{align*}
\dot{\sigma}_{i1} &= \sigma_{i2} - \lambda_{i1} \text{sig}(\sigma_{i1} - x_{id}) - \lambda_{i1} \text{sig}(\sigma_{i1} - x_{id}) \beta_i = 0, \\
\dot{\sigma}_{i2} &= -\lambda_{i2} \text{sig}(\sigma_{i2} - x_{id}) - \lambda_{i2} \text{sig}(\sigma_{i2} - x_{id}) \beta_i
\end{align*}$$

(56)

where $e_i = \sigma_{i1} - x_{id}, i = 2, 3, 4, 6, 7, \sigma_{i1}$ and $\sigma_{i2}$ are the estimation of $x_{id}$ and $\dot{x}_{id}$, respectively. $\alpha_i > 1, 0 < \beta_i < 1$. $\alpha_i = 2\alpha_i - 1, \beta_i = 2\beta_i - 1, \lambda_{i1}, \lambda_{i2}, \kappa_{i1}$ and $\kappa_{i2}$ are positive constants such that the matrices $\tilde{\mathbf{A}}_{i1}$ and $\tilde{\mathbf{A}}_{i}$ are Hurwitz.

$$\begin{align*}
\tilde{\mathbf{A}}_{i1} &= \begin{bmatrix} -\lambda_{i1} & 1 \\
-\lambda_{i2} & 0 \end{bmatrix}, \\
\tilde{\mathbf{A}}_{i} &= \begin{bmatrix} -\kappa_{i1} & 1 \\
-\kappa_{i2} & 0 \end{bmatrix}
\end{align*}$$

(57, 58)

With (56) satisfying above conditions, the errors $e_i$ can converge to the origin and $\alpha_{i2}$ will converge to the derivative of virtual control $x_{id}$ in a fixed-time bounded by

$$t_{FTDi} \leq \frac{\lambda_{\text{max}}(\tilde{P}_{i1})}{\lambda_{\text{min}}(\tilde{Q}_{i1})(\alpha_i - 1) \Gamma^{\alpha_i - 1} + \lambda_{\text{max}}^{2-\beta_i}(\tilde{P}_i)}$$

(59)

where $0 < \Gamma \leq \lambda_{\text{min}}(\tilde{P}_{i1})$, the symmetric positive matrices $\tilde{P}_{i1}$ and $\tilde{Q}_{i1}$ satisfy

$$\tilde{P}_{i1}\tilde{A}_{i1} + \tilde{A}_{i1}^T\tilde{P}_{i1} = -\tilde{Q}_{i1}$$

(60)

The symmetric positive matrices $\tilde{P}_{i}$ and $\tilde{Q}_{i}$ satisfy

$$\tilde{P}_{i}\tilde{A}_{i} + \tilde{A}_{i}^T\tilde{P}_{i} = -\tilde{Q}_{i}$$

(61)

Remark 6: According to Lemma 3, when $t > t_{FTDi}$, the fixed-time tracking differentiator (FTD) (56) will exhibit an exact estimation of $x_{id}$ and $\dot{x}_{id}$, that is,

$$\begin{align*}
e_i &= 0, \\
\dot{e}_i &= 0 \Rightarrow \begin{cases} x_{id} = \sigma_{i1} \\
\dot{x}_{id} = \dot{\sigma}_{i1} = \sigma_{i2} \end{cases}
\end{align*}$$

(62)
Assumption 5 [40]: The initial tracking errors $z_i(0), i = 1, 2, \ldots, 7$, are bounded satisfying

$$z_i(0) \leq |z_i(0)| < \eta_i \quad (i = 1, 2, \ldots, 7) \quad (63)$$

where $\eta_1, \eta_2, \ldots, \eta_7$ are positive constants.

A. SSTDOB-FSMBC DESIGN FOR ALTITUDE SUBSYSTEM

Step 1.1 (virtual control input for altitude): Define the tracking error of altitude $x_1$ as $z_1 = x_1 - x_{1d}$. Then we select a sliding surface

$$s_1 = l_1 (z_1 - z_1(0) \exp(-\varphi_1 t)) \quad (64)$$

where $l_1 \geq 1, \varphi_1 > 0, z_1(0)$ is the initial error of $x_1$.

Computing the time derivative of (64) we yield

$$\dot{s}_1 = l_1 (g_{1s} x_2 - \dot{x}_{1d} + \varphi_1 z_1(0) \exp(-\varphi_1 t)) \quad (65)$$

Thus, the virtual control input $x_{2d}$ adopted by backstepping control can be given as

$$x_{2d} = -\frac{1}{g_1}(-\dot{x}_{1d} + \varphi_1 z_1(0) \exp(-\varphi_1 t)) + a_1 \text{sig}(s_1)^{\alpha_1} + b_1 F_1(s_1) + c_1 s_1) \quad (66)$$

where $a_1, b_1$ and $c_1$ are positive constants, $\alpha_1 > 1$. $F_1(s_1)$ is constructed as

$$F_1(s_1) = \begin{cases} \text{sig}(s_1)^{\beta_1}, & |s_1| \geq \varepsilon_1 \\ \tau_{11} s_1 + \tau_{12} s_1^2 \text{sig}(s_1), & |s_1| < \varepsilon_1 \\ \end{cases} \quad (67)$$

where $0 < \beta_1 < 1, \varepsilon_1$ is a small enough positive constant, $\tau_{11} = (2 - \beta_1) \varepsilon_1^{-1}, \tau_{12} = (\beta_1 - 1) \varepsilon_1^{-2}$.

To avoid the “explosion of complexity” involved in control law, a fixed-time tracking differentiator (FTD) is introduced as

$$\dot{s}_{21} = \sigma_{21} - \lambda_{21} \text{sig}(\sigma_{21} - x_{2d})^{\alpha_2} + k_{21} \text{sig}(\sigma_{21} - x_{2d})^{\beta_2}$$

$$\dot{s}_{22} = -\lambda_{22} \text{sig}(\sigma_{21} - x_{2d})^{\alpha_2} + k_{22} \text{sig}(\sigma_{21} - x_{2d})^{\beta_2} \quad (68)$$

where $\alpha_2 > 1, 0 < \beta_2 < 1, \lambda_{21}, k_{21} \geq 0, \beta_{2} > 2 - 1$. The selection conditions of $\lambda_{21}, \lambda_{22}, k_{21}$ and $k_{22}$ are same to Lemma 3.

Define $z_2 = x_2 - \sigma_{21}, \varphi_2 = \sigma_{21} - x_{2d}, (65)$ becomes

$$\dot{s}_1 = l_1 (g_1 (z_2 + e_2 + x_{2d}) - \dot{x}_{1d} + \varphi_1 z_1(0) \exp(-\varphi_1 t))$$

$$= l_1 (g_1 z_2 + g_1 e_2 - \alpha_1 \text{sig}(s_1)^{\alpha_1} - b_1 F_1(s_1) - c_1 s_1) \quad (69)$$

Remark 7: Inspired by Zhou et al. [49], we introduce this switching logic in $F_1(s_1)$ to avoid the singularity problem in $\dot{x}_{2d}$. When $|s_1| \geq \varepsilon_1$ calculating the derivative of $x_{2d}$ yields the term $F_1(s_1) = \beta_1 |s_1|^{\beta_1-1} \dot{s}_1$. Since $|s_1| \geq \varepsilon_1 > 0$, the singularity problem cannot appear in this situation. When $|s_1| < \varepsilon_1$, calculating the derivative of $x_{2d}$, we have the term $F_1(s_1) = \tau_{11} s_1 + 2 \tau_{12} |s_1| s_1$. It is apparent that this term will not turn to singular when $s = 0$ and $\dot{s} \neq 0$. However, the switching effect from $\text{sig}(s_1)^{\beta_1}$ to $\tau_{11} s_1 + \tau_{12} s_1^2 \text{sig}(s_1)$ may degrade the fixed-time convergence property. Thus, we should select a small enough $\varepsilon_1$ to maintain this convergence property and avoid the singular problem. Besides, the design of $\tau_{11}$ and $\tau_{11}$ can make $F_1(s_1)$ and its derivative continuous.

Step 1.2 (virtual control input for flight path angle): The sliding surface $s_2$ is selected as

$$s_2 = l_2 (z_2 - z_2(0) \exp(-\varphi_2 t)) \quad (70)$$

where $l_2 \geq 1, \varphi_2 > 0, z_2(0)$ is the initial error of $x_2$.

The time derivative of $s_2$ is

$$\dot{s}_2 = l_2 (g_{2s} x_3 + f_2 + d_2 - \dot{\sigma}_{21} + \varphi_2 z_2(0) \exp(-\varphi_2 t)) \quad (71)$$

The virtual control input $x_{3d}$ can be obtained as

$$x_{3d} = -\frac{1}{g_2} (f_2 + d_2 - \sigma_{22} + \varphi_2 z_2(0) \exp(-\varphi_2 t) + a_2 \text{sig}(s_2)^{\alpha_2} + b_2 F_2(s_2) + c_2 s_2 + g_1 s_1) \quad (72)$$

where $a_2, b_2$ and $c_2$ are positive constants, $\alpha_2 > 1, d_2$ is estimated by SSTDOB. $F_2(s_2)$ is constructed as

$$F_2(s_2) = \begin{cases} \text{sig}(s_2)^{\beta_2}, & |s_2| \geq \varepsilon_2 \\ \tau_{21} s_2 + \tau_{22} s_2^2 \text{sig}(s_2), & |s_2| < \varepsilon_2 \end{cases} \quad (73)$$

where $0 < \beta_2 < 1, \varepsilon_2$ is a small enough positive constant, $\tau_{21} = (2 - \beta_2) \varepsilon_2^{-1}, \tau_{22} = (\beta_2 - 1) \varepsilon_2^{-2}$. The FTD is designed as

$$\dot{\sigma}_{21} = \sigma_{21} - \lambda_{21} \text{sig}(\sigma_{21} - x_{2d})^{\alpha_2} + k_{21} \text{sig}(\sigma_{21} - x_{2d})^{\beta_2}$$

$$\dot{\sigma}_{22} = -\lambda_{22} \text{sig}(\sigma_{21} - x_{2d})^{\alpha_2} + k_{22} \text{sig}(\sigma_{21} - x_{2d})^{\beta_2} \quad (74)$$

where the selection conditions of $\lambda_{21}, \lambda_{22}, k_{21}$ and $k_{22}$ are same to Lemma 3, $\alpha_1 > 1, 0 < \beta_1 < 1, \alpha_2 = 2\alpha_3 - 1, \alpha_3 = 2\alpha_2 - 1, \beta_3 = 2\beta_2 - 1$. The virtual control is estimated by (74).

Similarly, define $z_3 = x_3 - \sigma_{31}, e_3 = \sigma_{31} - x_{3d}$. Then (71) is written as

$$\dot{s}_2 = l_2 (g_{2s} x_3 + e_3 + x_{3d}) + f_2 + d_2 - \dot{\sigma}_{21} + \varphi_2 z_2(0) \exp(-\varphi_2 t)$$

$$= l_2 (g_{2s} x_3 + g_2 e_3 + d_2 - a_2 \text{sig}(s_2)^{\alpha_2} - b_2 F_2(s_2) - c_2 s_2 - g_1 s_1) \quad (75)$$

where $d_2 = d_2 - d_2$.

Step 1.3 (virtual control input for pitching angle): The sliding surface $s_3$ and its time derivative $\dot{s}_3$ are expressed as follows

$$s_3 = l_3 (z_3 - z_3(0) \exp(-\varphi_3 t))$$

$$\dot{s}_3 = l_3 (x_4 - \sigma_{31} + \varphi_3 z_3(0) \exp(-\varphi_3 t)) \quad (76)$$

where $l_3 \geq 1, \varphi_3 > 0, z_3(0)$ is the initial error of $x_3$.

The virtual control $x_{4d}$ is designed as

$$x_{4d} = -\frac{1}{g_3} (\sigma_{32} + \varphi_3 z_3(0) \exp(-\varphi_3 t) + a_3 \text{sig}(s_3)^{\alpha_3} + b_3 F_3(s_3) + c_3 s_3 + g_2 s_2)$$

with

$$F_3(s_3) = \begin{cases} \text{sig}(s_3)^{\beta_3}, & |s_3| \geq \varepsilon_3 \\ \tau_{31} s_3 + \tau_{32} s_3^2 \text{sig}(s_3), & |s_3| < \varepsilon_3 \end{cases} \quad (77)$$
And the FTD is expressed as
\[
\dot{\sigma}_{41} = \alpha_4 \sigma_4 - \lambda_4 \text{sig}(\sigma_4 - x_{4d})^{\alpha_4} - \kappa_4 \text{sig}(\sigma_4 - x_{4d})^{\beta_4} \\
\dot{\sigma}_{42} = -\lambda_4 \text{sig}(\sigma_4 - x_{4d})^{\alpha_4} - \kappa_4 \text{sig}(\sigma_4 - x_{4d})^{\beta_4}
\] (79)

In (78), (78) and (79), \(a_4, b_4\) and \(c_4\) are positive constants, \(\alpha_4, \beta_4 > 0\), \(0 < \beta_3, \beta_4 < 1, \beta_3, e_4\) is a small enough positive constant, \(\tau_1 = (2 - \beta_3) \epsilon_{3}^{(\beta_{3}-1)}\), \(\tau_2 = (\beta_3 - 1) \epsilon_{3}^{(\beta_{3}-2)}\). The selection conditions of \(\lambda_4, \lambda_4, \kappa_4\) and \(\kappa_4\) are same to Lemma 3, \(\delta_4 = \gamma_4 - 1\), \(\beta_4 = 2\beta_4 - 1, \sigma_4\) is the virtual control estimated by (74).

With the definition of \(z_4 = x_4 - \sigma_{41}\) and \(e_4 = \sigma_4 - x_{4d}\), (77) becomes
\[
\dot{z}_3 = l_3 (g_3 (z_4 + e_4 + x_{4d}) - \delta_{41} - \varphi_z z_3 (0) \exp (-\varphi_z t)) \\
= l_3 (g_3 z_4 + g_3 e_4 - a_3 \text{sig}(\sigma_3)^{\alpha_3} - b_3 F_3 (\sigma_3) - c_3 z_3 \\
- g_2 z_2 + \sigma_{32} - \sigma_{31})
\] (80)

Step 1.4 (Actual Control Input for Pitching Rate Angle): The sliding surface \(s_4\) and its time derivative \(\dot{s}_4\) are designed as
\[
s_4 = l_4 (z_4 - z_4 (0) \exp (-\varphi_4 t))
\] (81)
\[
\dot{s}_4 = l_4 (g_4 u_1 + f_4 + d_4 - \delta_{41} - \varphi_z z_4 (0) \exp (-\varphi_z t))
\] (82)
where \(l_4 \geq 1, \varphi_4 > 0, z_4(0)\) is the initial error of \(x_4\).

Similar to the fore steps, the altitude subsystem actual controller \(u_1\) can be designed as
\[
u_1 = -\frac{1}{g_4} (f_4 + \delta_4 - \sigma_{42} + \varphi_z z_4 (0) \exp (-\varphi_z t)) \\
+ a_4 \text{sig}(\sigma_4)^{\alpha_4} + b_4 F_4 (\sigma_4) + c_4 z_4 + g_3 s_3)
\] (83)
where \(a_4, b_4\) and \(c_4\) are positive constants, \(\alpha_4 > 1, d_4\) is estimated by SSTDOB. \(F_4 (\sigma_4)\) is constructed as
\[
F_4 (\sigma_4) = \begin{cases} 
\text{sig}(\sigma_4)^{\beta_4}, & |\sigma_4| \geq e_4 \\
\tau_{41} s_4 + \tau_{42} s_4^2 \text{sign}(s_4), & |\sigma_4| < e_4
\end{cases}
\] (84)
where \(0 < \beta_4 < 1, e_4\) is a small enough positive constant, \(\tau_4 = (2 - \beta_4) \epsilon_{4}^{(\beta_{4}-1)}\), \(\tau_{42} = (\beta_4 - 1) \epsilon_{4}^{(\beta_{4}-2)}\).

Substituting (83) into (82), one gets
\[
\dot{s}_4 = l_4 (\delta_{44} - a_4 \text{sig}(\sigma_4)^{\alpha_4} - b_4 F_4 (\sigma_4) - c_4 s_4 \\
- g_3 s_3 + \sigma_{42} - \sigma_{41})
\] (85)
where \(\delta_{44} = d_4 - \delta_{44}\).

B. SSTDOB-FSMBC DESIGN FOR VELOCITY SUBSYSTEM

The design process of velocity subsystem is analogous to altitude subsystem.

Step 2.1 (virtual Control Input for Velocity): Define the velocity tracking error as \(z_5 = x_5 - x_{5d}\), the sliding surface \(s_5\) is designed as
\[
s_5 = l_5 (z_5 - z_5 (0) \exp (-\varphi_5 t))
\] (86)
where \(l_5 \geq 1, \varphi_5 > 0, z_5(0)\) is the initial error of \(x_5\). The time derivative of \(s_5\) is obtained as
\[
\dot{s}_5 = l_5 (g_5 x_6 + f_5 + d_5 - \dot{x}_{5d} + \varphi_5 z_5 (0) \exp (-\varphi_5 t))
\] (87)

The virtual control input \(x_{6d}\) is designed as
\[
x_{6d} = -\frac{1}{g_5} (f_5 + \dot{x}_{5d} - \dot{x}_{5d} + \varphi_5 z_5 (0) \exp (-\varphi_5 t)) \\
+ a_5 \text{sig}(\sigma_5)^{\alpha_5} + b_5 F_5 (\sigma_5) + c_5 s_5)
\] (88)

with
\[
F_5 (\sigma_5) = \begin{cases} 
\text{sig}(\sigma_5)^{\beta_5}, & |\sigma_5| \geq e_5 \\
\tau_{51} s_5 + \tau_{52} s_5^2 \text{sign}(s_5), & |\sigma_5| < e_5
\end{cases}
\] (89)

And the corresponding FTD is designed as
\[
\dot{\sigma}_{61} = \alpha_6 \sigma_6 - \lambda_6 \text{sig}(\sigma_6 - x_{6d})^{\alpha_6} - \kappa_6 \text{sig}(\sigma_6 - x_{6d})^{\beta_6}
\] (90)

where \(a_6, b_5\) and \(c_5\) are positive constants, \(\alpha_6, \alpha_6 > 1, 0 < \beta_5, \beta_6 < 1, d_5\) is estimated by SSTDOB. \(\sigma_5\) is a small enough positive constant, \(\sigma_5 = (2 - \beta_6) \epsilon_{6}^{(\beta_{6}-1)}\), \(\tau_{62} = (\beta_6 - 1) \epsilon_{6}^{(\beta_{6}-2)}\). The selection conditions of \(\lambda_6, \lambda_6, \kappa_6\) and \(\kappa_6\) are also same to Lemma 3, \(\delta_6 = 2a_6 - 1, \beta_6 = 2\beta_6 - 1, \sigma_6\) is the virtual control estimated by (90).

Define \(z_6 = x_6 - \sigma_{61}, e_6 = \sigma_6 - x_{6d}\). (87) becomes
\[
\dot{s}_5 = l_5 (g_5 z_6 + e_6 + x_{6d}) + f_5 + d_5 - \dot{x}_{5d} \\
+ \varphi_5 z_5 (0) \exp (-\varphi_5 t))
\] (91)

where \(\delta_{5d} = d_5 - \dot{d}_5\).

Step 2.2 (virtual control input for the derivative of throttle setting): Similarly, the sliding surface \(s_6\) and its time derivative \(\dot{s}_6\) are expressed as
\[
s_6 = l_6 (z_6 - z_6 (0) \exp (-\varphi_6 t))
\] (92)
\[
\dot{s}_6 = l_6 (g_6 \dot{x}_7 - \delta_{61} + \varphi_6 z_6 (0) \exp (-\varphi_6 t))
\] (93)
where \(l_6 \geq 1, \varphi_6 > 0, z_6(0)\) is the initial error of \(x_6\).

The virtual control input \(x_{7d}\) is obtained as
\[
x_{7d} = -\frac{1}{g_6} (-\sigma_6 + \varphi_6 z_6 (0) \exp (-\varphi_6 t)) \\
+ a_6 \text{sig}(\sigma_6)^{\alpha_6} + b_6 F_6 (\sigma_6) + c_6 s_6 + g_5 s_5)
\] with
\[
F_6 (\sigma_6) = \begin{cases} 
\text{sig}(\sigma_6)^{\beta_6}, & |\sigma_6| \geq e_6 \\
\tau_{61} s_6 + \tau_{62} s_6^2 \text{sign}(s_6), & |\sigma_6| < e_6
\end{cases}
\] (94)

And the FTD is defined as
\[
\dot{\sigma}_{71} = \sigma_7 - \lambda_7 \text{sig}(\sigma_7 - x_{7d})^{\alpha_7} - \kappa_7 \text{sig}(\sigma_7 - x_{7d})^{\beta_7}
\] (95)
\[
\dot{\sigma}_{72} = -\lambda_7 \text{sig}(\sigma_7 - x_{7d})^{\alpha_7} - \kappa_7 \text{sig}(\sigma_7 - x_{7d})^{\beta_7}
\]
where \(a_6, b_6\) and \(c_6\) are positive constants, \(\alpha_6, \alpha_7 > 1, 0 < \beta_6, \beta_7 < 1, e_6\) is a small enough positive constant,
\( \tau_2 = (2 - \beta_1) \varepsilon_7^{-1}, \tau_2 = (\beta_2 - 1) \varepsilon_7^{-2}. \) The selection conditions of \( \lambda_1, \lambda_2, \kappa_1 \) and \( \kappa_2 \) are same to Lemma 3, \( \ddot{\sigma}_7 = 2 \sigma_7 - 1, \beta_1 = 2 \beta_2 - 1, \sigma_1 \) is the virtual control estimated by (95).

Define \( z_7 = x_7 - \sigma_7, \dot{z}_7 = \sigma_7 - x_7d, \) (93) becomes

\[
\dot{\sigma}_6 = l_6 \left( g_6 (z_7 + e_7 + x_7d) - \dot{\sigma}_6 + \psi_6 z_6 (0) \exp(-\psi_6 t) \right)
= l_6 (g_6 e_7 + g_6 e_7 - a_6 \text{sig}(s_6) \sigma_6 - b_6 F_0(s_6) - c_6 s_6
- g_5 s_5 + a_6 \dot{\sigma}_6 - \dot{\sigma}_6) \tag{96}
\]

Step 2.3 (Actual Control Input for the Throttle Setting): The sliding surface \( s_7 \) and its time derivative \( \dot{s}_7 \) are expressed as follows

\[
s_7 = l_7 (z_7 - \sigma_7 (0)) \exp(-\psi_7 t) \tag{97}
\]

\[
\dot{s}_7 = l_7 (g_7 u_2 + f_1 + \dot{d}_7 - \dot{\sigma}_7 + \psi_7 z_7 (0) \exp(-\psi_7 t)) \tag{98}
\]

where \( l_7 \geq 1, \psi_7 > 0, z_7 (0) \) is the initial error of \( x_7. \)

Finally, the actual control input \( u_2 \) is designed as

\[
u_2 = -\frac{1}{g_7} \left( f_1 + \dot{d}_7 - \sigma_7 + \psi_7 z_7 (0) \exp(-\psi_7 t) \right)
+ a_7 \text{sig}(s_7) \sigma_7^\alpha + b_7 F_7(s_7) + c_7 s_7 + g_6 s_6 \tag{99}
\]

where \( a_7, b_7 \) and \( c_7 \) are positive constants, \( \alpha_7 > 1, 0 < \beta_7 < 1, \) \( d_7 \) is estimated by SSTDOB. \( F_7(s) \) is constructed as

\[
F_7(s_7) = \left\{ \begin{array}{ll}
\text{sig}(s_7) \sigma_7^\alpha, & |s_7| \geq \varepsilon_7 \\
\tau_1 s_7 + \tau_2 s_7 \text{sign}(s_7), & |s_7| < \varepsilon_7
\end{array} \right. \tag{100}
\]

Substituting (99) into (98), we have

\[
\dot{s}_7 = l_7 (\dot{d}_7 - a_7 \text{sig}(s_7) \sigma_7^\alpha - b_7 F_7(s_7) - c_7 s_7 - g_6 s_6 + \sigma_7 - \dot{\sigma}_7) \tag{101}
\]

where \( \dot{d}_7 = d_7 - \dot{d}_7. \)

C. STABILITY ANALYSIS FOR SSTDOB-FSMC

In this section, the stability of the closed-loop system is analyzed by the Lyapunov theory.

Theorem 2: Consider the AHV system (16) and (17) with designed control inputs (66), (72), (78), (83), (88), (94), (99), and FTD (68), (74), (79), (90), (95) as well as SSTDOB (21). The closed-loop system is able to guarantee sliding surfaces and tracking errors converge to a small neighborhood of origin.

Proof: The Lyapunov function can be constructed as

\[
L = \sum_{i=1}^{7} \frac{1}{2} \dot{s}_i^2 \tag{102}
\]

Taking the time derivative of \( L \) yields

\[
\dot{L} = \sum_{i=1}^{7} \frac{1}{l_i} \dot{s}_i \dot{s}_i \tag{103}
\]

Substituting (69), (75), (80), (85), (91), (96) and (101) into (103), we obtain

\[
\dot{L} = -\sum_{i=1}^{7} a_i |s_i|^\alpha_i + \sum_{i=1}^{7} b_i |s_i|^\beta_i + \sum_{i=1}^{7} c_i s_i^2
+ \sum_{i=1, i \neq 4}^{6} s_i \ddot{d}_i + \sum_{i=1, i \neq 4}^{6} g_i s_i (z_{i+1} - s_{i+1})
+ \sum_{i=1, i \neq 4}^{6} g_i s_i e_{i+1} + \sum_{i=1, i \neq 4}^{6} s_i (s_{i+1, 2} - \dot{\sigma}_{i+1, 1}) \tag{104}
\]

Since \( \varepsilon_i \) is small enough, the switching effect caused by \( F_i(s_i) \) can be omitted. Thus, (104) can be reformed as

\[
\dot{L} = -\sum_{i=1}^{7} a_i |s_i|^\alpha_i + \sum_{i=1}^{7} b_i |s_i|^\beta_i + \sum_{i=1}^{7} c_i s_i^2
+ \sum_{i=1, i \neq 4}^{6} s_i \ddot{d}_i + \sum_{i=1, i \neq 4}^{6} g_i s_i (z_{i+1} - s_{i+1})
+ \sum_{i=1, i \neq 4}^{6} g_i s_i e_{i+1} + \sum_{i=1, i \neq 4}^{6} s_i (s_{i+1, 2} - \dot{\sigma}_{i+1, 1}) \tag{105}
\]

Based on the left side of inequalities in (63), one can get

\[
\dot{L} \leq -\sum_{i=1}^{7} a_i s_i^2 - \sum_{i=1}^{7} b_i |s_i|^\alpha_i + \sum_{i=1}^{7} c_i s_i^2
+ \sum_{i=1, i \neq 4}^{6} g_i s_i (\frac{1}{l_{i+1}} - 1) s_{i+1}
+ \sum_{i=1, i \neq 4}^{6} g_i s_i z_{i+1} (0) \exp(-\psi_{i+1} t)
+ \sum_{i=1, i \neq 4}^{6} g_i s_i e_{i+1} + \sum_{i=2, 4, 5, 7}^{6} s_i \ddot{d}_i
+ \sum_{i=1, i \neq 4}^{6} s_i (s_{i+1, 2} - \dot{\sigma}_{i+1, 1}) \tag{106}
\]
Since $\varphi_i > 0$, we have $|\exp(-\varphi_i t)| \leq 1$, and (106) becomes
\[
\dot{L} \leq -\sum_{i=1}^{7} a_i s_i^2 - \sum_{i=1}^{7} b_i |s_i|^{\varphi_i} + \sum_{i=1}^{7} c_i |s_i|^{\beta_i}
+ \sum_{i=1, i \neq 4}^{7} g_i s_i \left( \frac{1}{l_i} - 1 \right) s_{i+1} + \sum_{i=1, i \neq 4}^{7} g_i s_i \left( z_{i+1} - z_i \right)
+ \sum_{i=1, i \neq 4}^{7} g_i s_i e_i + \sum_{i=1, i \neq 4}^{7} s_i \bar{d}_i
+ \sum_{i=1, i \neq 4}^{7} s_i (\sigma_{i+1, 2} - \sigma_{i+1, 1})
\] (107)

According to (42), if $t > \max(t_i)$, $i = 2, 4, 5, 7$, $\bar{d}_i = 0$ are all satisfied. With Lemma 3 and its conclusion in Remark 6, we have $e_j = 0$, $j = 2, 3, 4, 6, 7$, and $\bar{d}_{i+1, 1} = \sigma_{i+1, 2}$ when $t > \max(t_i) + \max(t_{FTD})$. Thus, (107) becomes
\[
\dot{L} \leq -\sum_{i=1}^{7} a_i |s_i|^{\varphi_i} - \sum_{i=1}^{7} b_i |s_i|^{\beta_i} + \sum_{i=1}^{7} c_i s_i^2
+ \sum_{i=1, i \neq 4}^{7} g_i |s_i| \left( \frac{1}{l_i} - 1 \right) |s_i| |s_{i+1}|
+ \sum_{i=1, i \neq 4}^{7} g_i |s_i| |s_i| \eta_{i+1}
\] (108)

Using Young’s inequality and Assumption 3, we yield
\[
L \leq -\sum_{i=1}^{7} a_i (s_i^2)_{\varphi_i+1} - \sum_{i=1}^{7} b_i (s_i^2)_{\beta_i+1} - \sum_{i=1}^{7} c_i s_i^2
+ \sum_{i=1, i \neq 4}^{7} \bar{g}_i \left( \frac{1}{l_i} - 1 \right) s_i^2 + \sum_{i=1, i \neq 4}^{7} \bar{g}_i \left( s_i^2 + \eta_{i+1} \right)
\leq -\sum_{i=1}^{7} a_i (s_i^2)_{\varphi_i+1} - \sum_{i=1}^{7} b_i (s_i^2)_{\beta_i+1} - \sum_{i=1}^{7} c_i s_i^2
+ \sum_{i=1, i \neq 4}^{7} \bar{g}_i \left( \frac{1}{l_i} - 1 \right) \left( s_i^2 + s_{i+1}^2 \right)
+ \sum_{i=1, i \neq 4}^{7} \bar{g}_i s_i^2 + \sum_{i=1, i \neq 4}^{7} \bar{g}_i \eta_{i+1}^2
\] (109)

Merging all the same items of (109), we obtain
\[
\dot{L} \leq -\sum_{i=1}^{7} a_i l_i \left( \frac{s_i^2}{l_i} \right)^{\varphi_i+1} - \sum_{i=1}^{7} b_i l_i \left( \frac{s_i^2}{l_i} \right)^{\beta_i+1} - \sum_{i=1}^{7} c_i s_i^2
+ \sum_{i=1, i \neq 4}^{7} \bar{g}_i \left( \frac{1}{l_i} - 1 \right) \left( \frac{s_i^2}{l_i} + s_{i+1}^2 \right)
+ \sum_{i=1, i \neq 4}^{7} \bar{g}_i \left( \frac{s_i^2}{l_i} \right) + \sum_{i=1, i \neq 4}^{7} \bar{g}_i \left( \frac{\eta_{i+1}^2}{l_i} \right)
\]

Setting $A = \min(a_i l_i^{\frac{a_i+1}{2}})$ $(i = 1, 2, \cdots, 7)$, $B = \min(b_i l_i^{\frac{b_i+1}{2}})$, $K = \max(\bar{g}_i l_i^{\frac{\bar{g}_i+1}{2}})$, $K = \max(\bar{g}_i l_i^{\frac{\bar{g}_i+1}{2}})$, $\gamma_i = \max(\bar{g}_i l_i^{\frac{\bar{g}_i+1}{2}})$, $\sigma_i = \max(\bar{g}_i l_i^{\frac{\bar{g}_i+1}{2}})$, $\gamma_i = \max(\bar{g}_i l_i^{\frac{\bar{g}_i+1}{2}})$, $\sigma_i = \max(\bar{g}_i l_i^{\frac{\bar{g}_i+1}{2}})$, $\gamma_i = \max(\bar{g}_i l_i^{\frac{\bar{g}_i+1}{2}})$, $\sigma_i = \max(\bar{g}_i l_i^{\frac{\bar{g}_i+1}{2}})$.

Then (110) becomes
\[
\dot{L} \leq -A \sum_{i=1}^{7} \left( \frac{s_i^2}{l_i} \right)^{\varphi_i+1} - B \sum_{i=1}^{7} \left( \frac{s_i^2}{l_i} \right)^{\beta_i+1} + K \sum_{i=1}^{7} \left( \frac{s_i^2}{l_i} \right)^{\frac{\gamma_i+1}{2}} + \Gamma_0
\] (111)

By applying the inequality $x \leq x^p + x^q$ [38], where $x > 0, p > 1, 0 < q < 1$, one gets
\[
\dot{L} \leq -(A - K) \sum_{i=1}^{7} \left( \frac{s_i^2}{l_i} \right)^{\varphi_i+1} - (B - K) \sum_{i=1}^{7} \left( \frac{s_i^2}{l_i} \right)^{\beta_i+1} + \Gamma_0
\] (112)

Setting $a_i = r_1, b_i = r_2 (i = 1, 2, \ldots, 7)$, we have
\[
\dot{L} \leq -(A - K) \sum_{i=1}^{7} \left( \frac{s_i^2}{l_i} \right)^{\varphi_i+1} - (B - K) \sum_{i=1}^{7} \left( \frac{s_i^2}{l_i} \right)^{\beta_i+1} + \Gamma_0
\] (113)

where $\Gamma_0 = 2^{\gamma_i+1} \min(A, K), \Gamma_2 = 2^{\gamma_i+1} \min(B, K)$.

It is apparent that $A, B$ and $\Gamma_0$ are positive, $K > 0$ is satisfied by choosing appropriate parameters. Then we obtain $\Gamma_1 > 0, \Gamma_2 > 0, \gamma_i+1 > 1, 0 < \gamma_i+1 2 \gamma_i+1 < 1$. According to Lemma 1, the sliding surface will get close to a small region of zero in fixed time.

The convergence region $2\Omega$ and the convergence time $T_1$ are calculated by
\[
\Gamma_0 = \Gamma_1 \Omega^{\frac{\gamma_i+1}{2}} + \Gamma_2 \Omega^{\frac{\gamma_i+1}{2}}
\] (114)
\[
T_1 < \frac{1}{k} r_1 - 1 + \frac{1}{\Gamma_2 (2^{\gamma_i+1} 2 \gamma_i+1 1 - r_2)
\] (115)

Therefore, the whole convergence time $T$ evaluated by (42), (59) and (115) is derived as
\[
T < \max(t_i) + \max(t_{FTD}) + T_1
\] (116)
Besides, according to (114), the sliding surfaces $s_i$ 
(i = 1, 2, . . . , 7) will satisfy
\[ |s_i| = |l_i(z_i - z_i(0) \exp(-\varphi_i t))| \leq 2\Omega \quad (i = 1, 2, . . . , 7) \]  
(117)

Calculating the limit of following equation yields
\[ \lim_{t \to \infty} z_i(0) \exp(-\varphi_i t) = 0 \quad (i = 1, 2, . . . , 7) \]  
(118)
Then by substituting (118) into (117), the convergence region of tracking errors $z_i$ can be transformed into
\[ |z_i| \leq \frac{2\Omega}{l_i} \leq \Omega \quad (i = 1, 2, . . . , 7) \]  
(119)
where $\Omega = \frac{20}{\min(l_i)}$.

The proof of Theorem 2 is completed.

**Remark 8:** In the fore designing process, we combine BC strategy with tracking differentiator, sliding mode control, fixed-time control, as well as disturbance observer, in an attempt to make the composite controller more robust and effective. The explosion problem is solved by utilizing a fixed-time tracking differentiator and the disturbance observer is employed at each corresponding step to compensate the lumped disturbances. Compared with conventional sliding mode backstepping control, the sliding surfaces selected in each design steps can improve the convergence speed and modify the state trajectories right from the beginning by adding an exponential term. Besides, the convergence time of proposed FSMBC is independent of initial condition and the singularity problem in the derivative of virtual control is improved by introducing a switching logic.

**Remark 9:** From the viewpoint of practical application, the energy of each system is limited. Therefore, it is reasonable to claim the boundedness mentioned in Assumption 3-5.

**V. SIMULATION RESULTS**

In this section, several simulations are conducted to demonstrate the performance of proposed control scheme. The initial simulation conditions and model parameters are given in Table 1 and Table 2. Besides, Earth related constants are $\mu = 3.98842 \times 10^{14}$ N·m$^2$/kg and $R_E = 6.371 \times 10^6$m. The density of air can be defined as $\rho = \rho_0 \exp(-h/H_s)$ with $\rho_0 = 1.225$ kg/m$^3$ is the density of air at sea level, $H_s = 7200$m is the scale height.

**TABLE 1. Initial state of AHV system.**

<table>
<thead>
<tr>
<th>State</th>
<th>Value</th>
<th>State</th>
<th>Value</th>
<th>State</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$V$ (m/s)</td>
<td>4000</td>
<td>$\theta$ (°)</td>
<td>2.7203</td>
<td>$\rho$ (kg/m$^3$)</td>
<td>33000</td>
</tr>
</tbody>
</table>

As a representative case study, the desired altitude command $h_d$ and desired velocity command $V_d$ are generated by the following low-pass filters with their initial values at $h_d(0) = 33000$, $V_d(0) = 0$, $\dot{h}_d(0) = 0$, $\dot{V}_d(0) = 4000$, $\ddot{V}_d(0) = 0$

\[ h_d(s) = \frac{87.5}{(s + 0.05)^2}, \quad V_d(s) = \frac{40.5}{(s + 0.1)^2} \]

**TABLE 2. Model parameters of AHV system.**

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$m$ (kg)</td>
<td>$1.368 \times 10^6$</td>
</tr>
<tr>
<td>$J_z$ (kg·m$^2$)</td>
<td>$9.491 \times 10^6$</td>
</tr>
<tr>
<td>$\delta$ (m$^2$)</td>
<td>334</td>
</tr>
<tr>
<td>$l$ (m)</td>
<td>24.384</td>
</tr>
<tr>
<td>$e_z^1$</td>
<td>0.6203</td>
</tr>
<tr>
<td>$e_z^2$</td>
<td>$3.772 \times 10^{-3}$</td>
</tr>
<tr>
<td>$e_z^3$</td>
<td>$4.3378 \times 10^{-3}$</td>
</tr>
<tr>
<td>$c_0$</td>
<td>0.645</td>
</tr>
<tr>
<td>$c_0^1$</td>
<td>$0, \beta &lt; 1$</td>
</tr>
<tr>
<td>$c_0^2$</td>
<td>$0.0224, \beta \geq 1$</td>
</tr>
<tr>
<td>$c_0^3$</td>
<td>$0.02576, \beta &lt; 1$</td>
</tr>
<tr>
<td>$c_1$</td>
<td>$3.36 \times 10^{-3}, \beta \geq 1$</td>
</tr>
<tr>
<td>$m_0^1$</td>
<td>5.3261 \times 10^{-6}$</td>
</tr>
<tr>
<td>$m_0^2$</td>
<td>$7.417 \times 10^{-3}$</td>
</tr>
<tr>
<td>$m_0^3$</td>
<td>$-0.035$</td>
</tr>
<tr>
<td>$m_0^{\beta}$</td>
<td>$\frac{1}{2\pi} (-6.796\alpha^2 + 0.3015\alpha - 0.2289)$</td>
</tr>
<tr>
<td>$m_0^4$</td>
<td>0.0292</td>
</tr>
</tbody>
</table>

In addition, constraints of the actuators and the external disturbances are listed as

\[ \left\{ \begin{array}{ll}
|\delta| \leq 30^\circ \\
0 \leq \beta_e \leq 1
\end{array} \right. \]

\[ d_2 (t) = \left\{ \begin{array}{ll}
0 & 0 \leq t < 100 \\
0.01 \sin(0.1 \pi t) & 100 \leq t < 325 \\
0.01 & 325 \leq t < 400
\end{array} \right. \]

\[ d_4 (t) = \left\{ \begin{array}{ll}
0 & 0 \leq t < 80 \\
0.005t - 0.4 & 80 \leq t < 100 \\
0.1 & 100 \leq t < 250 \\
0.1 \sin(0.1 \pi t) & 250 \leq t < 400
\end{array} \right. \]

\[ d_5 (t) = \left\{ \begin{array}{ll}
0 & 0 \leq t < 150 \\
0.5 \sin(0.02 \pi t) & 150 \leq t < 400
\end{array} \right. \]

\[ d_7 (t) = \left\{ \begin{array}{ll}
0 & 0 \leq t < 100 \\
0.001t - 0.1 & 100 \leq t < 200 \\
0.1 + 0.1 \sin(0.1 \pi t) & 200 \leq t < 400
\end{array} \right. \]

To verify the performance of the proposed controller (SSTDOB-FSMBC), the simulation results are divided into two groups as follows:

**Group 1:** Simulation on the AHV longitudinal model without aerodynamic uncertainties.

In this group, comparisons among SSTDOB-FSMBC, conventional super twisting disturbance observer based sliding mode control (CSTDOB-SMC) mentioned in Wang et al. [29] and a strictly-lower-convex-function based nonlinear disturbance observer based adaptive terminal sliding mode control (SDOB-ATSMC) designed in Wu et al. [26] are given to illustrate the robustness of proposed control approach.

Besides, the parameters of each control scheme are listed as follows.
SSTDOB-FSMBC:

\[ p_2 = p_4 = p_5 = p_7 = 2, \]
\[ k_{21} = 2, \quad k_{22} = 3, \quad k_{23} = 1.5, \quad k_{24} = 6, \]
\[ k_{41} = 1.5, \quad k_{42} = 3, \quad k_{43} = 1, \quad k_{44} = 6, \]
\[ k_{51} = 2, \quad k_{52} = 4, \quad k_{53} = 1, \quad k_{54} = 7, \]
\[ k_{71} = 2, \quad k_{72} = 3, \quad k_{73} = 1.7, \quad k_{74} = 7, \]
\[ \lambda_{j1} = 4, \quad \lambda_{j2} = 10, \quad \kappa_{j1} = 4, \quad \kappa_{j2} = 10, \quad j = 2, 3, 4, 6, 7, \]
\[ l_i = 1, \quad \varphi_i = 10, \quad \alpha_i = r_1 = 1.2, \quad \beta_i = r_2 = 0.8, \quad i = 1, \ldots, 7, \]
\[ a_1 = 1.1, \quad a_2 = 2, \quad a_3 = 2, \quad a_4 = 1, \quad a_5 = 2, \quad a_6 = 2, \quad a_7 = 1, \]
\[ b_1 = 0.01, \quad b_2 = 0.3, \quad b_3 = 0.5, \quad b_4 = 0.5, \quad b_5 = b_6 = b_7 = 0.9, \]
\[ c_1 = c_2 = c_3 = c_4 = c_5 = c_6 = c_7 = 0.1. \]

CSTDOB-SMC:

\[ \omega_{11} = 0.1, \quad \omega_{21} = 0.01, \quad \omega_{12} = 0.1, \quad \omega_{22} = 0.02, \]
\[ k_1 = 0.2, \quad k_2 = 0.3, \quad \gamma_1 = 0.95, \quad \gamma_2 = 0.95. \]

SDOBS-ATSMC:

\[ u = \left(\frac{5x^4}{6} + 5x^2 + \frac{5x^4_1}{6} + \frac{5x^4_2}{6} \right) + \frac{5x^4_3}{6} + \frac{5x^4_4}{6}, \]
\[ \Lambda = 60F_3, \quad c_1 = 1, \quad c_5 = 2, \]
\[ r_1 = 1, \quad r_2 = 0.7, \quad r_3 = 1.7, \quad r_4 = 0.65, \quad r_5 = 12, \quad r_6 = 6, \quad r_7 = 1, \]
\[ r_1 = r_2 = r_3 = r_4 = \frac{7}{9}, \quad r_5 = r_6 = r_7 = 0.95, \]
\[ \sigma_1 = 0.2, \quad \sigma_2 = 0.3, \quad \sigma_3 = 0.25, \quad \sigma_4 = 0.4, \]
\[ \sigma_5 = 0.3, \quad \sigma_6 = 0.4, \quad \sigma_7 = 0.7. \]

Group 2: Simulation on the AHV longitudinal model with aerodynamic uncertainties.

In this scenario, the simulations of proposed scheme considering external disturbances and aerodynamic uncertainties are performed. The control parameters of SSTDOB-FSMBC are same to that of Group 1. Here, the atmospheric force and moment coefficients can be rewritten as

\[ C_T = C_T^0(1 + \Delta_T)C_D = C_D^0(1 + \Delta_D) \]
\[ C_L = C_L^0(1 + \Delta_L)C_{mc} = C_L^0(1 + \Delta_M) \]

where \( C^0 \) and \( \Delta \) are the nominal value and fixed parameter uncertainty, respectively.

A. SIMULATION ANALYSIS OF SSTDOB-FSMBC WITHOUT AERODYNAMIC UNCERTAINTIES

Figures 3 and 4 show the tracking curves of altitude \( h \) and velocity \( V \), respectively. It can be observed that all these controllers show satisfactory performances when the disturbances are not involved in the first 100 seconds. In the presence of external disturbances, the proposed SSTDOB-FSMBC performs well in tracking of altitude and velocity command, whereas the CSTDOB-SMC and SDOB-ATSMC exhibit worse because of their weaker disturbance rejection. Besides, Figures 3 and 4 also reveal that SSTDOB-FSMBC has faster convergence rate without overshoot, and other methods will produce slow convergence with undesired large overshoot, which suggests that the proposed SSTDOB-FSMBC is more effective than CSTDOB-SMC and SDOB-ATSMC.

Figures 5 and 6 represent the control inputs of elevator deflection angle \( \delta_e \) and demand of throttle setting \( \beta_t \), respectively. As shown in the simulation results, the control inputs \( \delta_e \) and \( \beta_t \) vary in their constraints during the whole process.
The estimation results of proposed SSTDOB are depicted in Figures 7-10. Since the sign function will directly appear in the CSTDOB, serious chattering may exist when the observation errors vary in the neighbor of zero, which would cause a variety of troubles in practical systems. As for the SSTDOB, the possible sign function can be hidden in the integral items, then the outputs of SSTDOB will become continuous and the chattering phenomenon is well eliminated. Therefore, the curves of SSTDOB are much smoother than that of CSTDOB. The outputs of SDOB are also smoother than CSTDOB because of the low-convex-function designed in SDOB. Howbeit, we can see that the estimation errors of SDOB are larger than that of SSTDOB. Besides, it is shown in Figure 10 that CSTDOB exhibits poor observation performance for disturbances with faster change rates. This result will further explain the vibrations of CSTDOB-SMC’s tracking curves in Figures 3 and 4. The above analysis also indicates that SSTDOB do has a better disturbance observation ability than CSTDOB and SDOB. Thus, this SSTDOB can enhance the robustness of proposed controller.

B. SIMULATION ANALYSIS OF SSTDOB-FSMBC WITH AERODYNAMIC UNCERTAINTIES

In order to examine the control performance of proposed SSTDOB-FSMBC under aerodynamic uncertainties, we select the uncertainties as follows.
Case 1: $\Delta_T = 20\%$, $\Delta_D = 20\%$, $\Delta_L = 20\%$, $\Delta_M = 20\%$.
Case 2: $\Delta_T = -20\%$, $\Delta_D = -20\%$, $\Delta_L = -20\%$, $\Delta_M = -20\%$.

The tracking curves of altitude $h$ and velocity $V$ are shown in Figures 13 and 14. The simulation results demonstrate that proposed control scheme can still present rapid convergence rate and excellent tracking precision in the presence of external disturbances and parameter uncertainties.

Figure 15 displays the curves of control inputs elevator deflection angle $\delta_e$. Figure 16 shows the demand of throttle setting $\beta_c$. It can be seen that when Case 1 and Case 2 are introduced respectively, the control inputs $\delta_e$ and $\beta_c$ still satisfy the constraints. Therefore, we can draw a conclusion.
that the proposed controller possesses fast convergence rate and excellent anti-interference ability.

VI. CONCLUSION AND FUTURE WORK

In this paper, a robust composite control scheme SSTDOB-FSMBC is developed to achieve the attitude and velocity tracking of AHV. The SSTDOB is firstly designed to estimate the disturbances exhibited in AHV, which can effectively enhance the anti-interference performance of control system. Moreover, the “explosion of complexity” inherent in conventional BC is avoided by utilizing a fixed-time tracking differentiator. In order to improve the singularity problem caused by the derivative of virtual control in FSMBC, we introduce a switching logic. Then, the stability of closed-loop system is proven by Lyapunov method. Simulation results illustrate the effectiveness of proposed SSTDOB-FSMBC. In this paper, we just consider the amplitude limit of control inputs, which is relatively simple. Future work will focus on the AHV’s fixed-time control problem with input saturations and full state constraints.

REFERENCES


YUNJIE WU was born in 1969. She received the Ph.D. degree in navigation guidance and control from Beihang University, Beijing, China, in 2006. She is currently a Professor with the School of Automation Science and Electrical Engineering, Beihang University. Her research interests include system simulation, intelligent control, servo control, aircraft guidance, and control technology.

FEI MA was born in 1993. He received the B.S. degree from the College of Automation Engineering, Nanjing University of Aeronautics and Astronautics, Nanjing, China, in 2016. He is currently pursuing the Ph.D. degree with the School of Automation Science and Electrical Engineering, Beihang University, Beijing, China. His research interests include flight control, missile guidance, and control.

YUEYANG HUA was born in 1989. He received the Ph.D. degree from Beihang University, in 2019. He is currently an Engineer with the China Academy of Launch Vehicle Technology, Beijing, China. His research interests include sliding-mode control, game theory, robust control, and multiagent system control.

XIAODONG LIU was born in 1987. He received the Ph.D. degree in navigation guidance and control from Beihang University, in 2014. He is currently an Engineer with the Beijing Aerospace Automatic Control Institute, Beijing, China. His research interests include aircraft guidance and control, servo control systems, and so on.

GUOFEI LI was born in 1992. He received the B.S. and M.S. degrees from Lanzhou Jiaotong University, Lanzhou, China, in 2013 and 2016, respectively. He is currently pursuing the Ph.D. degree with the School of Automation Science and Electrical Engineering, Beihang University, Beijing, China. His research interests include nonlinear control, servo system control, cooperative control, and cooperative guidance.