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Superconductor-to-normal transition in finite nanowires

Gil Refael[1], Eugene Demler[2], Yuval Oreg[3]
[1]Dept. of Physics, California Institute of Technology, MC 114-36, Pasadena, CA 91125
[2]Dept. of Physics, Harvard University, 17 Oxford St., Cambridge MA, 02138
[3]Dept. of Condensed Matter, Weizmann Institute of Science, Rehovot, 76100, Israel

In this paper we discuss the interplay of quantum fluctuations and dissipation in uniform superconducting nanowires. We consider a phenomenological model with superconducting and normal components, and a finite equilibration rate between these two-fluids. We find that phase-slip dipoles proliferate in the wire, and decouple the two-fluids within its bulk. This implies that the the normal fluid only couples to the superconductor fluid through the leads at the edges of the wire, and the *local* dissipation is unimportant. Therefore, while long wires have a superconductor-metal transition tuned by local properties of the superconducting fluid, short wires have a transition when the *total* resistance is $R_{\rm total} = R_Q = h/4e^2$.

Quantum phase transitions have long been at the forefront of condensed matter theory. Especially interesting are systems of reduced dimensionality and size, where fluctuations are enhanced, and ordering is illusive, and far from being expalined by mean field theory. Such systems exhibit a surprising degree of universality; for instance, as observed in Refs. [1, 2], a mesoscopic Josephson junction shunted by a resistor R undergoes a (so-called Schmid) transition between a Coulomb-blockade (normal) and superconducting phase when the shunt resistor is $R = R_Q = h/4e^2 = 6.45k\Omega$ [3–5]. Fluctuations of the superconducting phase angle, i.e., phase-slips, induce this transition; they also control the onset of superconductivity in long thin wires and Josephson junction chains [6–10], where the competition between local charging energy, which creates phase-slips, and the superconducting stiffness tunes the transition [11–13].

Our focus is experiments on $Mo_{79}Ge_{21}$ (amorphous) nanowires as narrow as 5nm-15nm. Resistance vs. temperature curves showed a transition between superconducting (resistance decreasing upon cooling) and normal, or weakly insulating, (resistance non-decreasing upon cooling) behavior. A first set of measurements on wires of various diameters, and lengths $100nm < L < 200nm$, showed a remarkable result: a transition when the *total* resistance of the wire was $R_Q = h/4e^2 = 6.45k\Omega$ [14], as if the entire wire was a single shunted junction. But the coherence length of $MoGe$ is $\xi < 10nm$,

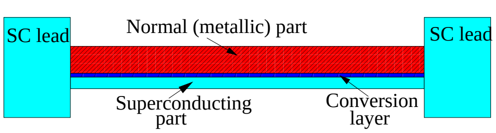

FIG. 1: A two-fluid model of a superconducting nanowire with a normal part, depicted as a separate region. In order for electrons to change from normal to superconducting, they need to pass through the conversion layer, which has a finite conductivity. Proliferated phase-slip dipoles inhibit two-fluid relaxation, and make the conversion layer insulating, which renders the normal part as an effective single shunt resistor.

thus the wire should differ dramatically from a single junction. Indeed, later experiments on longer wires, $200nm < L < 1000nm$, showed a weak transition that depended on the *resistance per length* or *cross-section* of the wires, i.e., on a local quantity, rather than the total resistance [15]. Later experiments [16], could neither prove nor disprove the global nature of the transition in the shorter wires.

In this paper, we describe nanowires using a two-fluid model, which assumes that Cooper pairs couple to a normal electron fluid, which provides local dissipation (Fig. 1). Remarkably, we find that *at sufficiently low temperature, the normal and superconducting fluids within a continuous nanowire decouple due to quantum phase fluctuations*, thus rendering the local dissipation unimportant. Therefore, the superconducting degrees of freedom can only couple to the dissipative normal fluid at the leads, on the edges of the wire, where they couple to its *total* normal-state resistance, $R_{\rm total}$. As a result, we show that indeed short wires may undergo a global dissipative Schmid transition tuned by the total wire resistance, when $R_{\rm total} = R_Q$. By short, we mean wires with length $L > \xi$, but shorter than both the thermal length, $L < \hbar c/T$ (with c the Mooij-Schön velocity, and T being the lowest temperature in the experiment)[17], and the 'quantum length' $A_C R_Q/\rho$, (with ρ the specific resistance, and A_C the largest cross-section area where quantum phase slips are not competely suppressed)[18]. After our work was completed, this result was verified in $MoGe$ wires with $50nm < L < 300nm$ [19]. Below we derive the two-fluid model, show how phase-slip dipoles decouple the normal and superconducting fluids, and apply to model to the case of a finite nanowire.

The hint of a Schmid transition, the long resistive tails seen in experiments, and the strong disorder of the $MoGe$ nanowires suggest the presence of local dissipation, which motivates the two-fluid model approach. We assume that charge can flow in the nanowires in two ways: as diffusive normal electrons with resistivity ρ - normal fluid - and as bosonic Cooper pairs - superfluid. The normal fluid stems from strong disorder and phase-fluctuations, which suppress the proximity effect and possibly give rise to normal regions and a finite density of states for single electrons at the Fermi level. The two fluids can have a different chemical potential, and can exchange charge with a finite, bare, relaxation time, $\tau_r = \Upsilon^{-1}$, in a bulk system (see Fig. 1, and Fig. 2b for a discrete model). This is related to the branch imbalance relaxation time [20, 21], first measured by Clarke [22] in Sn wires.

Before plunging to the analysis, note that earlier works on similiar models considered only the perfect normal-super fluid coupling, $\Upsilon = \infty$ case, and found a superconducing-metal transition tuned by the resistance per length [11–13, 23], as did Refs. [24, 25]. Alternative approaches assumed external dissipation coupled to the leads but not to the bulk of the wire [26], or discussed the onset of superconducting correlations and neglected phase fluctuations [27].

Indeed, in our model as well, sufficiently long but finite wires should exhibit a SC-normal crossover tuned by their cross-section area, which sets the bare fugacity of quantum phase slips [18], as well as their stiffness. But quantum fluctuations in the form of phase-slip dipoles, make the Cooper-pair to normal-electron conversion rate vanish at $T = 0$ in the bulk of the nanowire: $\Upsilon \to 0$. As claimed above, this leads to a true Schmid transition for short wires, which effecively become a short dissipationless superconducting wire, shunted through the leads by the total normal-state resistance R_T.

The crucial two-fluid decoupling is already evident in a simple two-junction system (Fig. 2a) [28, 29]. When $r = 0$ (i.e. vanishing conversion resistance), the two junctions in the system are independent in the d.c. limit. Phase slips - events where the phase across a Josephson junction tunnels by 2π - create a sudden voltage drop that opposes any supercurrent flowing, and thus induce dissipation. A Schmid transition occurs in each junction when $R_i = R_Q$ $(i = 1, 2)$. When $r > 0$, the two junctions become coupled, and phase-slips may form bound dipoles: simultaneous phase-slip and anti-phase-slip in the two junctions. Remarkably, dipoles do not destroy the coherence between the two leads, since they produce equal and opposite voltage drops. Nevertheless, as single phase slips block supercurrents across their Josephson junctions when they proliferate, dipoles block the normal-superfluid conversion channel: a conversion current $2i$ (Fig. 2a) flowing across r, with no lead-to-lead current, implies a current i on both junctions, but in opposite directions. i couples directly to the voltage drop of the dipoles; when proliferated, they block this current mode, and thus *decouple the normal and super fluids.* Phase slip dipoles proliferate roughly when $r > R_Q$. In this case a global Schmid transition takes place when $R_1 + R_2 = R_Q$.

Next, we generalize the normal-super fluids decoupling to wires, first using a discrete model (Fig. 1b), and

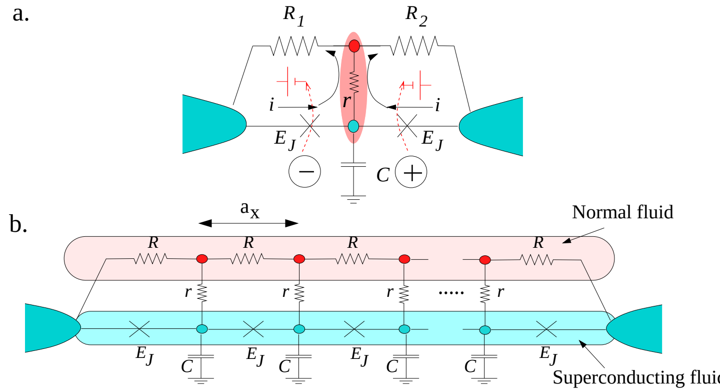

FIG. 2: (a) Single two-fluid grain (pink ellipse) between two leads: dipoles produce two opposing voltage spikes on the two junctions, which oppose super-to-normal conversion currents leaving the superconducting part of the grain (bottom circle in ellipse) and entering the normal part (top circle). When dipoles proliferate, the superfluid-normal conversion resistance (and time) effectively diverges, $r \to \infty$, and the normal and superfluid are completely decoupled. (b) We begin our study with a chain of two fluid grains, which is a discretized version of the nanowire in Fig. 1. Note that $1/r = \Upsilon a_x$ and $R = a_x \rho$.

then taking its continuum limit. Starting with an infinite chain of mesoscopic two-fluids grains (Fig. 1b) [30], the low-energy action for the chain is given in terms of a 2d gas of phase-slips, with interaction:

$$p_1 p_2 \left(K \log \frac{a_\tau}{\sqrt{x^2/c^2 + \tau^2}} + \alpha e^{-|x|/\lambda_Q} \log \frac{a_\tau}{|\tau|} \right). \quad (1)$$

$c = a_x\sqrt{E_J E_C}/\hbar$ is the Mooij-Schön velocity [31], and $a_\tau = a_x/c$. $p_i = \pm$ is the phase slip polarity. The first term is the usual isotropic interaction of a 1+1 XY model due to the plasmons in the Josephson junction array; $K = 2\pi\sqrt{E_J/E_C}$, $E_C = (2e)^2/C$. The second term is due to the dissipative interaction: $\alpha = max\{\frac{R_Q}{\sqrt{rR}}, \frac{R_Q}{R}\}$, and $\lambda_Q = max\{a_x\sqrt{r/R} = 1/\sqrt{\Upsilon\rho}, a_x\}$ is a new lengthscale that arises from the two-fluid finite relaxation time. As in the two-junction case, dipoles must be explicitly included in the low-energy description of this model [12, 13, 30]. We denote the fugacity of single phase slips as ζ, and the fugacity of a dipole with moment n as η_n. For completeness, we quote here the explicit field theory for the infinite chain:

$$\int \frac{d\omega dk}{(2\pi)^2} \left[\left(ck^2 + \frac{1}{c}\omega^2 \right) \frac{\theta^2_{(k,\omega)}}{4\pi K} + \frac{r}{4\pi R_Q}|\omega| \left(k^2 + \frac{R}{r} \right) \psi^2_{(k,\omega)} \right]$$
$$- \int d\tau \sum_i \left[\zeta \cos(\theta_i + \psi_i) + \eta_n \cos(\Delta_n \theta_i + \Delta_n \psi_i) \right] \quad (2)$$

with $c = a_x\sqrt{E_J E_C}/\hbar$, and θ, ψ mediating the plasmon and dissipative interactions, respectively. At high energies $\eta_n \sim \zeta^2$, and $\Delta_n f_i = f_{i+n} - f_i$. This is a representation dual to the SC phase representation, hence

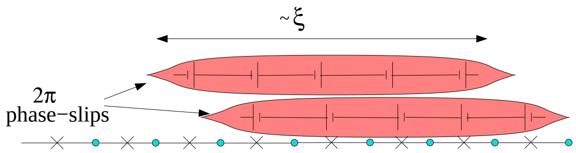

FIG. 3: To describe a continuous wire, we take the limit $a_x \to 0$ while keeping the coherence length and size of phase slip, ξ, fixed. This implies that phase slips are spread over ξ/a_x junctions, and can form dipoles with separation $x < \xi$. The voltage signs symbolize the voltage drop caused by a phase slip.

$\exp(i\psi_i + i\theta_i)$ is the operator that creates a phase slip on junction i.

It is useful to compare the relatively complicated interaction between phase slips in an infinite chain, with that of phase slips in a single Josephson junction. In a single junction the interaction is: $p_1 p_2 \frac{R_Q}{R} \log \frac{a_\tau}{|\tau|}$. The Schmid transition, which marks phase-slip proliferation, occurs when the gain in entropy due to separating a phase-slip from an anti phase slip, $S = \log(a_\tau T)$ equals the required interaction energy, $\frac{R_Q}{R}\log(a_\tau T)$. Employing the same argument for the two-fluid Josephson chain yields the approximate SC-normal phase boundary: $K + \frac{1}{2}\alpha \sim 4$. This transition is essentially the 1+1 Kosterlitz-Thouless-Berezinski (KTB) transition of the Josephson junction array in accordance with Ref. [11] (for a more refined analysis see Ref. [30]). But this argument, as well as the interactions in (1), ignores phase-slip dipoles. When dipoles are proliferated, the normal- and superfluids decouple, and the superconducting part of the wire exhibits a SC-normal KTB transition when $K \sim 4$. *Phase-slip dipoles*, we find, proliferate when:

$$\frac{2R_Q}{R}(1 - e^{-a_x/\lambda_Q}) < 1. \tag{3}$$

The left-hand side is the strength of dipole interaction, which consists of the self interaction of the slip and anti slip and also their mutual interaction. a_x is the distance between grains in the model.

In the continuum limit, *dipoles always proliferate* and cut off the superfluid-normal conversion. The continuum limit implies $a_x \to 0$; but this makes a Josephson junction (and therefore also a phase slip on a junction) shrink to length zero. But a phase slip occurring on physical nanowires has a characteristic length ξ (coherence length). To reconcile this we allow phase slips to smoothly spread over $\sim \xi/a_x$ junctions [30]. Technically, we transform the zeta term in Eq. (2) as $\cos(\psi+\phi)_{(x,\tau)} \to \cos\frac{\xi}{a_x}\sum_r f(r)(\psi+\phi)_{(x+r,\tau)}$, where $f(r)$ is a smooth, normalized, function centered around 0, with width ξ. We similarly treat the dipole η_n terms. The smearing reflects that in nanowires, phase slips can have an almost arbitrary overlap, $\Delta < \xi$ (Fig. 3), with other phase slips. The continuum generalization of Eq. (3) is that dipoles proliferate when: $\frac{R_Q}{R_\xi}\left(\frac{\Delta}{\max\{\lambda_Q,\xi\}}\right)^2 < 1$ (where $R_\xi = R_{\text{total}}\xi/L = \rho\xi$). Thus for any R_ξ, r there is a separation Δ_c below which dipoles proliferate. By incorporating the above analysis and the appropriate screening terms in Eq. (2) we see that the normal-superfluid conversion is cut off at temperatures (or frequencies) $T \sim \zeta_0^2 a_\tau$, where ζ_0 is the bare fugacity of phase slips, and $a_\tau^{-1} \sim (\xi/c)^{-1}$ is the UV cutoff in (2).

Thus a finite continuous wire is effectively described by a chain of Josephson junctions, shunted by the *global resistance* in the chain. Phase slips now exhibit an interaction due to the plasma waves in the chain, $K \log \frac{a_\tau}{\sqrt{x^2/c^2+\tau^2}}$, and also due to the dissipation through the normal resistance, which is couple through the leads: $\frac{R_Q}{R_{\text{total}}}\log\frac{a_\tau}{|\tau|}$. Naively, a transition will now occur when:

$$K + \frac{R_Q}{R_{\text{total}}} \sim 4 \tag{4}$$

(see Fig. 4a). Less naive considerations show that the KTB transition, tuned by K is a cross-over for lengths $L < c/\hbar T$, and an even stronger effect may appear due to the bare fugacity of phase slips being exponentially suppressed with K. The total resistance, however, still drives a Schmid transition when $R_{\text{total}} = R_Q$.

Our conclusions could be easily related to the nanowire experiments [14–16, 32]. In long wires, we expect a SC-Normal crossover tuned by stiffness (as in [11, 23]), but in short wires, we expect a Schmid transition tuned by the total normal-part resistance. In Fig. 4b we recast the diagram of Fig. 4a for the *MoGe* nanowire experiments, plotting $L/R_{\text{total}} \propto A$ vs. L, with L the length of the wire, and A its cross section area. The diagonal line marks $R_{\text{total}} = R_Q$. Above it we expect $T = 0$ superconductivity. The horizontal line marks the SC-normal cross-over in longer wires. This line most probably arises from the exponential dependence of the bare quantum-phase-slip fugacity on thickness[18], but may also be associated with a KTB transition at $K \sim 4$, or a fermionic T_C suppression mechanism, which also depends on R_ξ/R_Q [25]. After completing the analysis described here, Bezryadin and coworkers measured a large number of short samples with $L < 150nm$. These show near perfect fit with our prediction of a universal transition at $R_{\text{total}} = R_Q$ for shorter wires [16, 32] [37].

The application of our simple theory to the nanowire experiments requires several caveats. (1) It is natural to associate the resistance of the nanowire devices at temperatures just below the SC transition of the leads, with the total normal-part nanowire resistance, R_{total}. It is unclear, however, how this resistance is related to the normal-state resistance of the nanowires at temperatures above the bulk critical temperatures for $MoGe$. (2) In addition, the origin and precise nature of normal electrons in the wires is unknown. Possibly, phase fluctuations or the strong disorder stifle the proximity

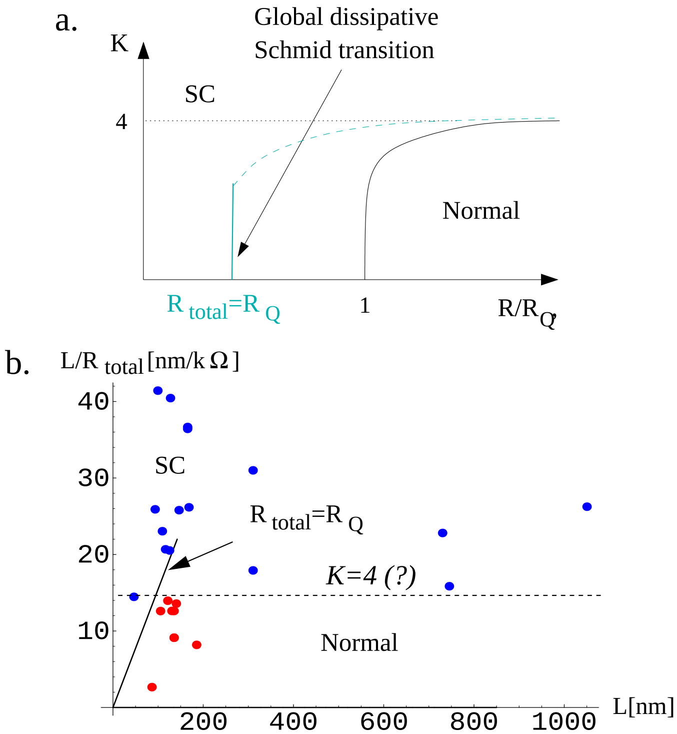

FIG. 4: (a) Phase diagram of the two-fluid chain and wire. The black line describes (roughly) the SC-normal boundary of an infinite two-fluid chain (Fig. 2b), with $r < R_Q$; both $K = 2\pi\sqrt{E_J/E_C}$ and R are local quantities. When $r > R_Q$, normal-superfluid conversion is cut off, the local dissipation becomes unimportant, and only the horizontal line applies. In finite continuous wires (grey line), the transition takes a dissipative nature when the wire is short, and occurs when $R_{\text{total}} = R_Q$; longer wires have a local crossover - dashed line - tuned by the superconducting stiffness $\sim K$. (b) Phase diagram for $MoGe$ nanowires of Refs. [14–16]. The y-axis, $L/R_{\text{total}} \propto A/\rho_{MoGe}$ is proportional to the surface area, which is proportional to K. The x-axis is the length. The blue dots are insulating samples, whereas the red dots are superconducting. The horizontal dashed line marks the long-wires cross over, while the diagonal black line marks the transition line $R_{total} = R_Q$.

effect and give rise to a normal part. Particularly, if the normal-superconductor relaxation rate is indeed suppressed, each phase slip gives rise to a long-lived population of quasi-particles, as is the case in Ref. [33] where dissipation at phase-slip centers is investigated. (3) The resistance vs. temperature curves measured by Bezryadin on the superconducting side show sharp exponential, activated-like, decay of the resistance, contrary to a naive quantum phase-slip theory, where an algebraic dependence of the resistance on temperature is expected. Similarly, the wires remaining normal show a weakly insulating behavior, with a charge-gap that corresponds to the Coulomb-blockade of the leads [32]. These observations do not contradict the possibility of a Schmid transition, and can be understood by also considering the effective dissipation produced by a finite density of phase slips, which is too large to justify the perturbative analysis pursued here. Such considerations appeared to describe the intermediate-coupling regime of the two-junction system [29]. Using the results presented here regarding the Schmid transition, but adding a finite density of phase-slips, we successfully explained the sharp temperature dependence of short wires in Refs. [34, 35].

The main result of this paper is the divergence of the normal-superfluid relaxation time, $\tau_r = \Upsilon^{-1}$, in continuous uniform nanowires due to quantum fluctuations. Apart from the direct application of our theory to the $MoGe$ nanowire experiments, this effect could be directly investigated in meso and nanoscopic systems where quantum fluctuations are apparent at relatively high temperatures. Some early experiments in this direction on nanostructures not uniform enough, but with quantum fluctuations are described in Ref. [36]. In future work we hope to address the issues of the origin and nature of the normal-part in nanowires, and its interplay with phase-slip density, and the diverging relaxation time.

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[1] J. S. Penttilä, Ü. Parts, P. J. Hakonen, M. A. Paalanen, and E. B. Sonin, Phys. Rev. Lett. **82**, 1004 (1999).
[2] M. Watanabe and D. B. Haviland, cond-mat/0301340.
[3] A. Schmid, Phys. Rev. Lett. **51**, 1506 (1983).
[4] S. Chakravarty, Phys. Rev. Lett. **49**, 681 (1982).
[5] S. A. Bulgadaev, JETP Lett. **39**, 315 (1984).
[6] P. Xiong, A. V. Herzog, and R. C. Dynes, Phys. Rev. Lett. **78**, 927 (1997).
[7] P. Delsing, C. D. Chen, D. B. Haviland, T. Bergsten, and T. Claeston (1997).
[8] Y. Takahide, R. Yagi, A. Kanda, Y. Ootuka, and S. Kobayashi, Phys. Rev. Lett. **85**, 1974 (2000).
[9] F. Altomare, A. M. Chang, M. R. Melloch, Y. Hong, and C. W. Tu, Phys. Rev. Lett. **97**, 017001 (2006).
[10] M. Zgirski, K. P. Riikonen, V. Touboltsev, and K. Y. Arutyunov, Phys. Rev. B **77**, 054508 (2008).
[11] A. D. Zaikin, D. S. Golubev, A. van Otterlo, and G. T. Zimányi, Phys. Rev. Lett. **78**, 1552 (1997).
[12] S. E. Korshunov, Sov. Phys. JETP **68**, 609 (1989).
[13] P. A. Bobbert, R. Fazio, and G. Schön, Phys. Rev. B **45**, 2294 (1992).
[14] A. Bezryadin, C. N. Lau, and M. Tinkham, Nature (London) **404**, 971 (2000).
[15] C. N. Lau, N. Markovic, M. Bockrath, A. Bezryadin, and M. Tinkham, Phys. Rev. Lett. **87**, 217003 (2001).
[16] A. T. Bollinger, A. Rogachev, M. Remeika, and A. Bezryadin, Phys. Rev. B **69**, 180503 (2004).

[17] The upper bound marks the crossover length where finite spatial-size corrections to the thermodynamic behavior occur. To get a sense for this scale we note that $L_{CO} = \hbar c/T \approx \xi\sqrt{N_\perp}\Delta_0/T$, where Δ_0 is the BCS gap, and $N_\perp$ is the number of transverse channels.
[18] The fugacity of quantum phase slips is $\zeta \sim e^{-cR_Q/R_\xi} = e^{-cAR_Q/\rho\xi}$, with c of order 1, $R_\xi = R_{\rm total}\xi/L$ and A the cross-section area [23]). In wires too thick, $A > A_C$, no quantum phase slips would be observed. A_C depends on the prefactor of the exponent in ζ, which could be quite large.
[19] A. T. Bollinger, R. C. Dinsmore III, A. Rogachev, and A. Bezryadin, arXiv.org:0707.4532 (2007).
[20] T. J. Rieger, D. J. Scalapino, and J. E. Mercereau, Phys. Rev. B **27**, 1787 (1971).
[21] J. Clarke, U. Eckern, A. Schmid, G. Schn, and M. Tinkham, Phys. Rev. B **20**, 3933 (1979).
[22] J. Clarke, Phys. Rev. Lett. **28**, 1363 (1972).
[23] D. S. Golubev and A. D. Zaikin, Phys. Rev. B **64**, 014504 (2001).
[24] Y. Oreg and E. Demler, in *Electronic Correlations: From meso- to nano-physics, Proceedings of the XXXVI Rencontres de Moriond*, edited by T. Martin, G. Montambaux, and J. T. T. Van (2001).
[25] Y. Oreg and A. M. Finkel'stein, Phys. Rev. Lett. **84**, 191 (1999).
[26] H. P. Büchler, V. B. Geshkenbein, and G. Blatter, Phys. Rev. Lett. **92**, 067077 (2004).
[27] S. Sachdev, P. Werner, and M. Troyer, Phys. Rev. Lett. **92**, 237003 (2004).
[28] G. Refael, E. Demler, Y. Oreg, and D. S. Fisher, Phys. Rev. B **68**, 214515 (2003).
[29] P. Werner, G. Refael, and M. Troyer, J. Stat. Mech. p. P12003 (2005).
[30] G. Refael, E. Demler, Y. Oreg, and D. S. Fisher, Phys. Rev. B **75**, 014522 (2007).
[31] J. E. Mooij and G. Schön, Phys. Rev. Lett. **55**, 114 (1985).
[32] A. Bollinger, A. Rogachev, and A. Bezryadin, Europhys. Lett. **76**, 505 (2006).
[33] G. J. Dolan and L. D. Jackel, Phys. Rev. Lett. **39**, 1628 (1977).
[34] D. Meidan, Y. Oreg, and G. Refael, Phys. Rev. Lett. **98**, 187001 (2007).
[35] D. Meidan, Y. Oreg, G. Refael, and R. A. Smith (2007).
[36] K. Y. Arutyunov, D. A. Presnov, S. V. Lotkhov, A. B. Pavolotski, and L. Rinderer, Phys. Rev. B **59**, 6487 (1999).
[37] Note that Büchler et al. [26], considered a SC-Metal transition tuned by the resistance in the measuring circuit - this results in similar physical behavior as our model.