

---

# Superfluid Quantum Space and Evolution of the Universe

---

Valeriy I. Sbitnev and Marco Fedi

Additional information is available at the end of the chapter

<http://dx.doi.org/10.5772/68113>

---

## Abstract

We assume that dark energy and dark matter filling up the whole cosmic space behave as a special superfluid, here named “superfluid quantum space.” We analyze the relationship between intrinsic pressure of SQS (dark energy’s repulsive force) and gravity, described as an inflow of dark energy into massive particles, causing a negative pressure gradient around massive bodies. Since no superfluid has exact zero viscosity, we analyze the consequences of SQS’s viscosity on light propagation, and we show that a static Universe could be possible, by solving a modified Navier-Stokes equation. Indeed, Hubble’s law may actually refer to tired light, though described as energy loss due to SQS’s nonzero viscosity instead of Compton scattering, bypassing known historical problems concerning tired light. We see that SQS’s viscosity may also account for the Pioneer anomaly. Our evaluation gives a magnitude of the anomalous acceleration  $a_p = -H_\Lambda c = -8.785 \cdot 10^{-10} \text{ ms}^{-2}$ . Here,  $H_\Lambda$  is the Hubble parameter loaded by the cosmological constant  $\Lambda$ . Furthermore, the vorticity equation stemming from the modified Navier-Stokes equation gives a solution for flat profile of the orbital speed of spiral galaxies and discloses what one might call a breathing of galaxies due to energy exchange between the galactic vortex and dark energy.

**Keywords:** gravity, dark energy, Hubble’s law, tired light, Pioneer anomaly, flat profile

---

## 1. Introduction

A recent view of the evolution of the Universe suggests that it pre-existed the Big Bang. What we now observe seems, however, to be the result of such event. The Universe apparently continues to expand at an accelerated pace, as evidenced by the Doppler redshift of light coming from distant sources. To explain this accelerated expansion, scientists resort to dark energy. In addition, it turns out that spiral galaxies demonstrate a flat profile of orbital speeds.

---

Dark matter is used to explain this riddle [1]. Current evaluations of the presence of dark energy and dark matter in the cosmos say that the former is of about 69.1% and the latter about 25.9%. In total, they are about 95% of the whole energy matter in the Universe. The residue of 5% corresponds to baryon matter, which is the constituent material of all observed galaxies, stars, planets, etc. At present, space as a mere container of matter is therefore being revised. It is not an empty vessel: On the contrary, it may act as a quantum superfluid, named by us “superfluid quantum space” (SQS) [2]. It consists of dark energy and dark matter whose hydrodynamics generates perennially fluctuating particle-antiparticle pairs, which annihilate and newly arise, forming a dark fluid whose features are similar to a Bose-Einstein condensate [3–7]. Within this concept, the repulsive action of dark energy may be simply explained as the internal pressure of the SQS. It should be noted that there are scientists [8–12] who do not agree with a concept of Universe based on Big Bang, inflation and Doppler redshift. They explain its evolution without calling into play any “Deus ex machina” as cosmic inflation. On the contrary, they believe that light loses energy as a function of the traveled distance. We assert that this happens because of a nonzero viscosity of the SQS, in perfect agreement with the empirical Hubble’s law. This could be interpreted as a revised phenomenon of tired light, different from that proposed in 1929 by F. Zwicky. In effect, while Zwicky’s hypothesis based on light scattering [13] may be disproved, for example, by the absent blurring of distant cosmic objects, tired light due to SQS’s viscosity is a more robust concept, which seems not to conflict with the current observations. In addition, while a viscosity-related tired light would let us observe a Doppler-alike redshift, pressure phenomena of opposite sign, that is, repulsion caused by SQS’s internal pressure and gravity as an inflow of dark energy into massive particles [14], could balance and permit a not expanding Universe.

It is interesting to note the critical opinion of a greatest theorist of our time, of Roger Penrose. In his recently published book “Fashion, Faith, and Fantasy in the New Physics of the Universe” [15], he argues that most of the current imaginary ideas about the origins of the Universe could be not true. We agree with Penrose, being unsatisfied with the current mainstream. In this key, we speculate and analyze a different framework.

At the beginning, in Section 2, we present as much detail as possible on our idea of space as superfluid quantum space, including a short historical overview about the concept of ether, vacuum, and physical space. In Section 3, we introduce a general relativistic hydrodynamic equation, and we analyze the corresponding equation in non-relativistic limit, as a modified Navier-Stokes equation. Here, we discuss the issue of tired light, and we evaluate the Pioneer anomaly according to the nonzero viscosity of SQS. Section 4 deals with solutions of the vorticity equation derived from the modified Navier-Stokes equation. We obtain exact formulas for the flat profile of orbital speeds of spiral galaxies. Section 5 gives concluding remarks and a look on the overall issue of a superfluid Universe.

## 2. Space, vacuum and ether: toward a Superfluid Quantum Space

The issue concerning the concepts of space, time, motion and the existence, or not, of a real vacuum has accompanied the human knowledge all along [16]. The most distinct form of

representation about space and time has developed in the form of two dialectically opposite ideas, later known as the conceptions of Democritus-Newton and Aristotle-Leibniz. According to Democritus everything is formed of “atoms,” each of them is considered indivisible. Between atoms, we have empty space. Philosophical views of Sir Isaac Newton were focused on the idea that all material bodies move in Absolute Space and Absolute Time. Such a philosophy is extremely convenient in the analysis of motion based on Newton’s mechanics [17]. Huygens championed a different concept, according to which, the whole space is filled with a special substance, the ether [18]. In his view, each point in space was a virtual source of light waves. This implied the homogeneity of space, a feature which is important also in modern quantum field theory (QFT), where wave functions propagate along all available paths.

Exactly QFT has triggered the current concept of a not inert vacuum, seen as the scene of continuous, frantic physical events. What John Wheeler named quantum foam [19]. A sea of particle-antiparticle pairs which arise and annihilate according to Heisenberg’s principle of uncertainty, in a vacuum where energy can’t be always and surely zero. These pairs perform an endless dance by infinitely arising and annihilating. In Dirac’s opinion, the new theory of electrodynamics, which implies a vacuum filled with virtual particles, forces us to take into account the existence of an ether. In 1951, he stated [20]: “If one examines the question in the light of present-day knowledge, one finds that the aether is no longer ruled out by relativity, and good reasons can now be advanced for postulating an aether.” His new ether model was based on a stochastic covariant distribution of subquantum motion, which generates a vacuum dominated by fluctuations and randomness.

De Broglie stated that: “any particle, even isolated, has to be imagined as in continuous energetic contact with a hidden medium” [21]. The hydrodynamics of this medium could explain the outcome of the double slit experiment using electron beams, where the leptons interfere as waves, probably driven through the aether by pressure waves, generated by their motion, exactly as pressure waves forming the same patterns are involved in the case of sound propagating through a double slit. De Broglie-Bohm’s pilot-waves could be then explained as aether waves, which guide the electrons and show analogies with Faraday waves guiding a bouncing droplet along a surface of silicon oil [22]. Petroni and Vigier stated that: “one can deduce the De Broglie waves as real collective Markov processes on the top of Dirac’s aether” [23].

Robert Betts Laughlin, Nobel Laureate for the fractional quantum Hall effect, in his work [24], writes: “Studies with large particle accelerators have now led us to understand that space is more like a piece of window glass than ideal Newtonian emptiness. It is filled with “stuff” that is normally transparent but can be made visible by hitting it sufficiently hard to knock out a part. The modern concept of the vacuum of space, confirmed every day by experiment, is a relativistic ether. But we do not call it this because it is taboo.” Laughlin also tells us that this false vacuum can be treated with the laws of fluid dynamics: “About the time relativity was becoming accepted, studies of radioactivity began showing that the empty space had spectroscopic structure similar to that of ordinary quantum solids and fluids.”

In summary, it seems that any phenomenon occurring in quantum mechanics needs to interact with the vacuum, which consequently possesses a quantum physical structure, rather than

being real empty (zero-energy) space. Thus, a single unbound particle is always and anyway connected to its environment. We believe that this fact might also facilitate the explanation to quantum entanglement, in which quantum information would be transmitted from a particle to the other through, and thanks to, the quantum structure of space. Petroni and Vigier debate that: “the quantum potential associated with this ether’s modification, by the presence of EPR photon pairs, yields a relativistic causal action at a distance which interprets the superluminal correlations recently established by Aspect et al.” [23].

In our opinion, this is also the case of gravitational waves, for which the asserted space-time deformation could be actually interpreted as a negative pressure wave traveling through a superfluid quantum space (SQS) from the source up to a measuring point due to a mechanism that we call superfluid quantum gravity (SQG) [14], a quantum fluid dynamic explanation of gravity. In a few words, gravitational waves could be a hydrodynamic phenomenon in a SQS instead of a deformation of space. After all, it is unlikely that a deformation occurs in a non-solid substance. If space is not solid, we can then only observe fluid dynamic events, which can, indeed, fully replace and better justify any effect of SR and GR [14]. SQS also shares interesting analogies with Higgs field, being an ubiquitous fundamental scalar field with non-zero viscosity, which gives mass to particles. In our case, thanks to quantum fluid dynamic perturbations of the field, with formation of superfluid quantum vortices, akin to what happens in superfluid He-4.

This suggests to even reconsider the pre-existence of a quantum space (as quantized dark energy) even before the Big Bang. By assuming that what we know to be the ubiquitous dark energy is a quantum superfluid, it could exactly correspond to our idea of SQS in a state of rest. Since dark energy still pervades the cosmos, corresponding to 69.1% of its energy, thinking that the Big Bang has rather been a perturbation event occurred in a previously quiet sea of dark energy, seems to be reasonable. From then on, cascade perturbations at Planck scale would have generated any existing particle as superfluid vortices or as pulses. Since no fluid or superfluid has real zero-viscosity, vortex-particles could attract the surrounding quanta, causing gravity as a fluid dynamic phenomenon. If the attracted space’s quanta were packed and re-emitted as virtual photons, stable particles could exist, and the link gravity-electromagnetism would be clear [2]. In such a view, quantum gravity is an apparent force which does not accelerate bodies by directly acting on them thanks to gravitons, they are rather dragged by the superfluid quantum space in which they are immersed that flows toward the site where greater absorption is exerted, i.e. toward the greater mass of a gravitational system, according to Newton’s law of universal gravitation. Cahill came to a similar conclusion in 2003, describing gravity as an inflow of quantum foam [25], though we consider more likely absorption of quantized dark energy.

Compared to QFT’s quantum vacuum, SQS would be at the lowest level (we believe that it is the very fundamental scalar field in nature) made up of dark energy’s quanta, whose hydrodynamic perturbation produces the continuous fluctuations which allow the formation and annihilation of particle-antiparticle pairs. In addition, Bohm and Vigier, moving from Dirac’s ether model, introduced in 1954 the idea of a sub-quantum medium, a hidden medium which all particles of the microphysical level constantly interact with [26]. The surface level of SQS, that is, the currently defined quantum foam or quantum vacuum, has to possess superfluid features as well and may act as a special Bose-Einstein condensate. The historical problem of vacuum’s infinite energy is solved by the infinite extent of the SQS.

As far as the Michelson-Morley test run in 1887 is concerned, we could wonder whether light interacts with the SQS, if it really exists, that is, if light interacts with dark energy. That test hypothesized a static ether and took into account Earth's motion. However, if we change the premise, by supposing that the Earth absorbs the ether, since massive particles absorb dark energy, we deal with a radial ether wind, independent of Earth's motion through the space, an ether wind which transports any object pointing toward the center of the Earth. In the hypothesis of fluid quantum gravity, this vertical ether wind exactly corresponds to the gravitational field [14]. This view would explain all the relativistic effects due to curved space-time, for example, the gravitational lensing and the Lense-Thirring precession. The correspondence between ether wind and gravitational field seems to be confirmed in a test run in 2009 by Martin Grusenick, who used a vertically placed Michelson's interferometer [27]. Maxwell's idea of an electromagnetic ether should be then revisited since, if a SQS exists, light could be a mechanical wave which propagates through an ether and its speed would merely correspond to the speed of sound through that specific fluid medium (i.e. of a pulse through dark energy) analogously to the case of sound through the air and for any other mechanical wave. In the case of light, this pulse would spin<sup>1</sup> and its velocity would arise from SQS's parameters such as density and compressibility [2, 14]. In short, a photon would be a spinning phonon through superfluid dark energy, whose mechanical interaction with dark energy's quanta would excite them, producing the photon's electromagnetic field. By starting from the formula which indicates the speed of a mechanical wave through a fluid,  $a = \sqrt{K/\rho}$ , in which  $K = V dP/dV$  is the bulk modulus, calculated by dividing the pressure increment,  $dP$ , by relative increment of the volume,  $dV/V$ , and  $\rho$  is the mass density and by putting  $\beta_S = 1/K$  as isentropic compressibility, we have  $a = 1/\sqrt{\beta_S \rho}$ . If we consider  $\beta_S = \beta_0$  as SQS's compressibility and  $\rho_0$  as its mass density, we get

$$a = \frac{1}{\sqrt{\beta_0 \rho_0}}, \tag{1}$$

expressing the speed of a photon as a phonon through the SQS, mathematically analogous to

$$c = \frac{1}{\sqrt{\epsilon_0 \mu_0}}, \tag{2}$$

as resulting from Maxwell's equations. The nonzero viscosity of the superfluid medium (SQS) would compel light to undergo redshift over very large distances: the more distant a galaxy the more stronger the observed redshift. This is fully compatible with Hubble's law, letting us doubt that an accelerated expansion of the Universe is really occurring.

### 3. Hydrodynamics of SQS

General relativity describes the Universe with a curved space-time metric due to presence of mass and energy. Observations show that the Universe, nevertheless, is flat at large distances

---

<sup>1</sup>Spinning sound waves have already been demonstrated [57], and thus, we can think of a photon as a spinning phonon through a superfluid medium.

and long times [28]. It means that the curvature tensor in the Einstein's field equations has to be omitted. In fact, as discussed below, we believe that what is supposed to be the curvature of space-time is rather a pressure force acting in a fluid, flat space, whose effect is compatible with that of general relativity's differential geometry. We come then to the general relativistic hydrodynamic equations [29, 30] containing the local conservation laws of the stress-energy tensor (the Bianchi identities) and of matter current density (the continuity equation) [5]:

$$\partial_\mu T^{\mu\nu} = 0, \quad (3)$$

$$\partial_\mu J^\mu = 0. \quad (4)$$

Here,  $\partial_\mu$  is the covariant derivative associated with the four-dimensional space-time metric  $\eta^{\mu\nu}$  having the signature  $(-+++)$ . The density current is given by  $J^\mu = \rho_m u^\mu$ , where  $u^\mu$  is the fluid 4-velocity and  $\rho_m$  is the rest-mass density in a locally inertial reference frame:

$$\rho_m = m\rho = m \frac{N_B}{\Delta V} = \frac{M}{\Delta V}. \quad (5)$$

Here,  $\rho$  is the density distribution of  $N$  particles within the unit volume  $\Delta V$ , where each of them has mass  $m$ . So,  $M = mN_B$  represents the bulk mass of the fluid occupying this volume.

The stress-energy tensor,  $T^{\mu\nu}$ , is expressed in units of pressure, whereas we need it in units of energy. Indeed, we further adopt the expression  $T^{\mu\nu}/\rho$  in order to have the possibility of getting the quantum potential  $Q = P_Q/\rho$ , where  $P_Q$  is the internal quantum pressure arising in SQS under influence of the external environment. We consider an incompressible, viscous fluid along with the gravitational potential  $\varphi$ . So, Eq. (3) reads [5]:

$$\partial_\mu \left( \frac{T^{\mu\nu}}{\rho} \right) = \partial_\mu \left( \frac{\varepsilon + p}{\rho} \gamma u^\mu u^\nu \right) + \partial^\nu Q - \partial^\nu \varphi + \partial_\mu \left( \mu(t)/\rho \right) \pi^{\mu\nu} = 0. \quad (6)$$

Here,  $\varepsilon$  and  $p$  are functions per unit volume. Divided by  $\rho$ , the sum  $\varepsilon + p$  has the dimension of energy. The term  $\mu(t)$  is the dynamic viscosity coefficient having the dimension of  $[\text{N s/m}^2]$ . Divided by  $\rho m$ , this function represents the kinetic viscosity coefficient  $\nu(t)$ , having the dimension of  $[\text{m}^2/\text{s}]$ . In our case, viscosity is a fluctuating-about-zero function of time. We suppose that its expectation vanishes in time but the variance is not zero. That is, we suppose the following average quantities:

$$\langle \mu(t) \rangle = \varepsilon \rightarrow 0_+, \langle \mu(t)\mu(0) \rangle > 0. \quad (7)$$

Here,  $0_+$  is an arbitrarily close-to-zero positive value, which describes the energy exchange with the zero-point energy of the SQS. The term  $\pi^{\mu\nu}$  reads:

$$\pi^{\mu\nu} = c(\partial^\mu u^\nu + \partial^\nu u^\mu) - c \frac{2}{3} \partial^\mu u_\mu \eta^{\mu\nu}. \quad (8)$$

In order to bring Eq. (6) to the relativistic Navier-Stokes equation, we shall repeat the computations of van Holten [31]. These calculations are reproduced also in Ref. [5].

We shall further consider only the non-relativistic limit, since the orbital speeds of galaxies and of many intergalactic bodies are predominantly much lower than the speed of light [32]. The factor  $\gamma$  in Eq. (6) is a sign of relativistic/non-relativistic limit. When it tends to infinity, we have the relativistic limit. In this case, also the mass  $m$  tends to infinity. On the other hand, when  $\gamma$  converges to unit, it denotes a non-relativistic limit. In this case, the mass  $m$  becomes the rest mass. For the sake of simplicity, we further take into account only shear viscosity. Given the quantum and granular nature of the SQS, a dilatant behavior under a linear, great increase of shear stress would be plausible and that would help to explain the upper limit to the acceleration of a body in the Universe [14]: The more acceleration is supplied the much more resistance is encountered, following Lorentz factor. In the non-relativistic limit, the viscosity term can be cut up to  $(\mu(t)/\rho)\partial_i\partial^jcv^i \rightarrow (\mu(t)/\rho)\nabla^2\vec{v}$ . As a result, we come to the following non-relativistic modified Navier-Stokes equation:

$$\rho_m \frac{d\vec{v}}{dt} = -\rho_m \nabla\varphi + \rho\nabla\Sigma Q + \mu(t)\nabla^2\vec{v}. \tag{9}$$

Here,  $\Sigma Q$  calculates the contributions of the quantum potential within SQS. The gravitational potential,  $\varphi$ , is a function coming from a continuous mass distribution  $\rho_m$  [33]. Gravity by itself described as an inflow of SQS obeys Gauss's law for gravity (gravity as an incoming flux), which in differential form is  $(\nabla \cdot \vec{g}) = -4\pi G\rho_m$ . In our view, the classical gravitational potential  $\varphi = GM/r$  of the absorbing body, associated to the radial field in each point, that is,  $\vec{g} = -\nabla\varphi$ , can be interpreted [14] as a quantum potential expressed as the ratio of the pressure  $P_G$  of the incoming flux to mass density,  $\rho_m$ :

$$\varphi = -G \int_V \frac{\rho_m(\vec{r})}{r} dV = \frac{P_G}{\rho_m}. \tag{10}$$

Here,  $G$  is the Newtonian gravitational constant, and  $r$  is the distance from the volume element  $dV$  to a point in the field, and the integration is performed in the entire volume of the body, creating a field. We note that the gravitational constant in the rightmost part of Eq. (10) is absent, and we clearly look at gravity as a quantum phenomenon driven by the ratio pressure/density. The continuous mass distribution  $\rho_m(\vec{r})$  can be expressed using the Laplace operator,  $\Delta$ :

$$\rho_m(\vec{r}) = \frac{1}{4\pi G} \Delta\varphi. \tag{11}$$

We note that the mass density, in addition, submits to the continuity equation

$$\frac{\partial\rho_m}{\partial t} + (\nabla \cdot \vec{v})\rho_m = 0. \tag{12}$$

We can express from Eqs. (10) and (11) the gravitational pressure  $P_G$  as a function of the gravitational potential  $\varphi$ :

$$P_G = \frac{\varphi \cdot \Delta\varphi}{4\pi G} = \frac{\varphi^2}{4\pi G} ((\nabla \ln\varphi)^2 + \nabla^2 \ln\varphi). \quad (13)$$

We now see that in Eq. (9) two opposite quantum potentials act:

$$\rho_m \frac{d\vec{v}}{dt} = -\rho_m \nabla \left( \frac{P_G}{\rho_m} \right) + \rho \nabla \Sigma \left( \frac{P_Q}{\rho} \right) + \mu(t) \nabla^2 \vec{v} \quad (14)$$

The Navier-Stokes equation is written above in the modified form [34]. The modification is due to (a) presence of the quantum potentials  $Q = P_Q/\rho$  and  $Q_\varphi = M\varphi = MP_G/\rho_m = P_G/\rho$  ( $M$  is the mass of Universe, about  $10^{53}$  kg) and (b) existence of the dynamic viscosity coefficient  $\mu(t)$  that fluctuates about zero. In other words, we accept that there is an energy exchange between baryon matter and the SQS. The pair of Eqs. (12) and (14) represents a full set of equations, sufficient for describing the motion of baryon matter through SQS in the non-relativistic limit of the Euclidean geometry.

Referring to Eq. (14), we believe that baryon matter is reciprocally attracted due to the gravitational quantum potential  $Q_\varphi = P_G/\rho$  that we suppose justified by a hydrodynamic interaction occurring between SQS and baryon matter (attraction of dark energy's quanta toward vortex-particles, causing decrease of pressure and a consequent apparent attractive force [14]). On the contrary, the quantum potential  $Q = P_Q/\rho$  existing in SQS causes reciprocal repulsion of the baryon matter on large distance.

By omitting from consideration the viscous term in Eq. (14), we assume  $\mu = 0$  and we obtain Newton's second law describing variations of the acceleration  $\vec{a} = d\vec{v}/dt$  under the action of the two opposite quantum forces described above,  $\nabla Q_\varphi$  and  $\nabla \Sigma Q$ :

$$\begin{aligned} \rho_m \frac{d\vec{v}}{dt} &= -\rho_m \nabla \left( \frac{P_G}{\rho_m} \right) + \rho \nabla \Sigma \left( \frac{P_Q}{\rho} \right) = -\rho \nabla Q_\varphi + \rho \nabla \Sigma Q \\ &= -\nabla P_G + \nabla P_Q + \left( P_G \nabla \ln(\rho) - P_Q \nabla \ln(\rho) \right) = f_{QG}. \end{aligned} \quad (15)$$

We generally consider the volume of the whole visible Universe,  $\Delta V \rightarrow V$ , so the rest mass density  $\rho_m = mN_B/V = M/V$ , where  $M = mN_B$  is the total mass of the Universe (about  $10^{53}$  kg). Here,  $f_{QG}$  is the force density. It arises from the superposition of two forces within the considered volume, which are expressed through the gradient of the gravitational potential and that of the intrinsic quantum potential of SQS. They are represented by a negative pressure gradient around baryonic bodies,  $\nabla P_G$  (Superfluid Quantum Gravity) [14], and the quantum pressure gradient acting on SQS,  $\nabla P_Q$ .

The acceleration,  $\vec{a} = d\vec{v}/dt$ , vanishes if both potentials,  $Q_\varphi$  and  $Q$ , are uniformly distributed across the space. The uniformity of the potentials can be justified according to the reports of the Planck Observatory [28]. So that  $\mathcal{E} = -Q_\varphi/m + \Sigma Q/m$  reads

$$\mathcal{E} = G \int_V \frac{\rho_m(\vec{r})}{r} dV + N_D D^2 \left( \frac{\nabla^2 \rho_m}{\rho_m} - \frac{1}{2} \frac{(\nabla \rho_m)^2}{\rho_m^2} \right). \quad (16)$$



It should be constant at least within the visible Universe. The first term follows from Eq. (10) that is  $\varphi = P_G/\rho_m = Q_\varphi/M$  and the second term is the intrinsic quantum potential of SQS [5, 34] divided by mass. The integer multiplier  $N_D$  is equal to the sum of all the quantum potentials, which arise from the contribution of all dark energy and dark matter in SQS. This value is calculated from the fact that about 95% of mass-energy in the Universe accounts for this dark fluid, respectively, 69.1% dark energy and 25.9% dark matter. So, from here, we find  $N_D = 95/(100 - 95) \cdot N_B = 19 \cdot M/m$ . The number  $N_B = M/m$  follows from Eq. (5).

As for the mass density distribution  $\rho_m$  under the integral, we permit the existence of a static spherically symmetric Gaussian density of baryon matter

$$\rho_m(r) = \frac{M}{(\sigma\sqrt{2\pi})^3} \exp\left\{-\frac{r^2}{2\sigma^2}\right\}. \tag{17}$$

By accepting this result, Eq. (16) gives the following solution:

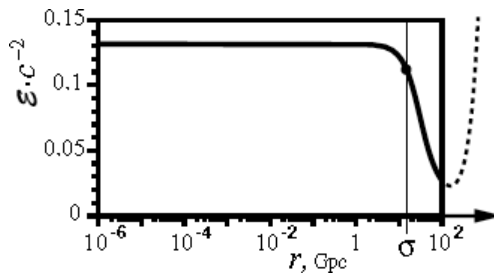
$$\varepsilon = \frac{GM}{r} \operatorname{erf}\left\{\frac{r}{\sigma\sqrt{2}}\right\} + N_D D^2 \left(\frac{r^2}{2\sigma^4} - \frac{1}{\sigma^2}\right) \tag{18}$$

The expression of  $\varepsilon$  reduced to dimensionless form by multiplying by  $c^{-2}$  ( $c$  is the speed of light) is shown as a function of  $r$  in **Figure 1**. We see that there is a *flat potential plateau* of baryon matter ranging in the radius of the visible Universe  $r < \sigma \approx 4.5 \cdot 10^{26} \text{ m} \approx 14.6 \text{ Gpc}$ . The negative pressure arising among the baryon bodies determines the attraction.

On the other hand, the repulsion is due to the quantum vacuum fluctuations in SQS. This repulsion is conditioned by the quantum potential  $Q$  represented by the second term In Eq. (18). In this case, the diffusion coefficient  $D$  reads

$$D = \frac{\hbar}{2m} \tag{19}$$

In the case of the proton mass,  $m$ , that is, about  $1.67 \cdot 10^{-27} \text{ kg}$ , we have  $D \approx 4.6 \cdot 10^{-8} \text{ m}^2 \cdot \text{s}^{-1}$ . The term  $N_D D^2$  in Eq. (18), however, reaches the enormous value of about  $10^{65} \text{ m}^4 \cdot \text{s}^{-2}$ .



**Figure 1.** Function  $\varepsilon$  reduced to dimensionless form by multiplying by  $c^{-2}$  ( $c$  is the speed of light) as a function of  $r$ , where  $\sigma$  is the radius of the visible Universe. A flat plateau ranging from 0 to about 10 Gpc tells us that the expansion of the Universe is almost absent. Dotted curve outside the chart shows divergence due to the quadratic term  $r^2/2\sigma^4$  in the quantum potential.

Therefore, the quantum potential outside the visible horizon gives divergence because of the quadratic term  $r^2/2\sigma^4$  in its representation. In **Figure 1**, this divergence is shown by a dotted curve.

### 3.1. Viscosity of SQS: tired-light and the Pioneer anomaly

Let us return to the relativistic hydrodynamic equation [5] by considering the Klein-Gordon equation, loaded by the viscosity term. The kinetic energy of a relativistic particle, in this case, can be written as follows:

$$E = E_0 - 2mv(t)\frac{d\ln(\rho_m)}{dt} = E_0 - E_0\frac{2}{c^2}v(t)\frac{d\ln(\rho_m)}{dt}. \quad (20)$$

Here,  $v(t) = \mu(t)/\rho_m$  is the kinematic viscosity coefficient. Its dimension is  $\text{m}^2 \cdot \text{s}^{-1}$ . The second term here describes energy exchange with vacuum fluctuations during a particle's motion through SQS. Here, we took into account that  $E_0 = mc^2$ . By adopting  $E_0 = \hbar\omega_0$ , we can write a suitable wave function for a photon coming from a distant source:

$$\Psi(t) = \exp \left\{ -i\omega_0 t \left( 1 - \frac{2}{t} \int_0^t \frac{v(\tau)}{c^2} \frac{d\ln(\rho_m)}{dt} d\tau \right) \right\}. \quad (21)$$

We let the integral under the exponent be linked with the expanded Hubble parameter  $H_\Lambda$ , as follows:

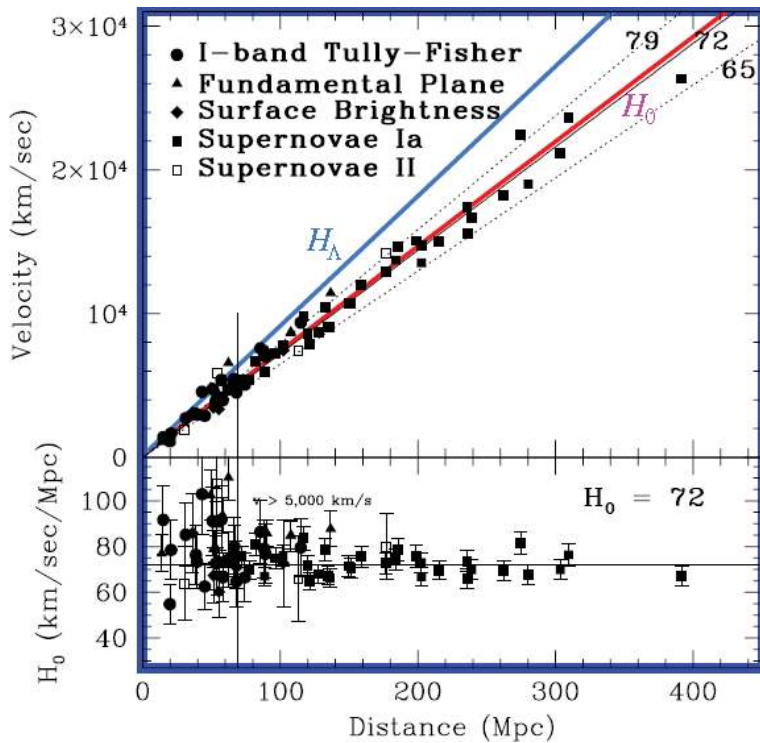
$$H_\Lambda = \frac{2}{t^2} \int_0^t \frac{v(\tau)}{c^2} \frac{d\ln(\rho_m)}{dt} d\tau = \frac{\dot{a}}{a} = \sqrt{\frac{8\pi G\rho_m}{3} - k\frac{c^2}{a^2} + \Lambda\frac{c^2}{3}}. \quad (22)$$

The rightmost terms under root in (22) result from the first Friedmann equation. Here,  $\Lambda$  is the cosmological constant (which refers to dark energy, i.e., to the SQS itself, being  $\Lambda = \kappa\rho_{sqs}$ , where  $\kappa = (8\pi G)/c^2$  is Einstein's constant),  $a$  the dimensionless scale factor, and  $k$  its Gaussian curvature. We further consider the case of a flat Universe,  $k = 0$ :

$$H_\Lambda = \sqrt{H_0^2 + \frac{\Lambda c^2}{3}}, \quad H_0 = \sqrt{\frac{8\pi G\rho_m}{3}}. \quad (23)$$

Being  $\Lambda$  omitted, the parameter  $H_\Lambda$  degenerates to  $H_0$ . We may evaluate  $H_0$  at the known critical density  $\rho_c = \rho_m \approx 10^{-26} \text{ kg}\cdot\text{m}^{-3}$  and knowing  $G$  [33]. We find  $H_0 \approx 2.36 \cdot 10^{-18} \text{ s}^{-1}$  in SI unit, while, in units adopted in astrophysics, it is about  $73 \text{ km} \cdot \text{Mpc}^{-1} \cdot \text{s}^{-1}$ .  $H_0$  fits well within the confidence interval estimated by Friedmann and others in [35] (see **Figure 2**).

We know that, to justify a ratio of the actual density to the critical density corresponding to a flat Universe, that is,  $\Omega = \rho_m/\rho_c = 1$ , we have to solve the flatness problem ( $|\Omega - 1| < 10^{-62}$  at the Planck era [36]). Its solution, as well known, is given by the theory of cosmic inflation, to



**Figure 2.** A Hubble diagram of distance versus velocity for secondary distance indicators, calibrated to Cepheids. Figure is taken from Ref. [35] and modified by adding the parameter  $H_\Lambda$ . Note that  $H_0$  belongs to the confidence interval, whereas  $H_\Lambda$  lies beyond it.

which many of the scientific community resort also to solve the magnetic-monopole problem and the homogeneity problem. In accordance with what above, we believe that no accelerating Universe exists, and consequently, the Doppler effect does not influence the observed redshift. We identify the cause of the redshift in the phenomenon of tired light, which in our case is due to the weak viscosity of SQS that leads to Eq. (21). The frequency  $\omega$  shifted with respect to the initial frequency  $\omega_0$  after the time  $t$  will be [37]:

$$\omega = \omega_0 e^{-H_0 t} \tag{24}$$

Cosmic inflation appears to us as a *deus ex machina*, which could actually hide the effect of viscosity on photons traveling through the SQS. As follows from Eq. (22), tired light occurs due to the existence of a tiny viscosity of SQS, in which photons are subject to by traveling through the cosmos. Fluctuations of the space-time metric (fluid dynamic fluctuations of dark energy, in our case) at the Planck scale [38] give a crucial contribution to the viscosity effect.

From Eq. (22), we can evaluate  $H_\Lambda \approx 2.93 \cdot 10^{-18} \text{s}^{-1}$  (about  $90 \text{ km} \cdot \text{Mpc}^{-1} \cdot \text{s}^{-1}$ ) at the adopted value of  $\Lambda = 10^{-52} \text{m}^{-2}$ . We observe that this parameter lies far outside the confidence interval,

marked in **Figure 2**. It means that the parameter  $H_\Lambda$ , most possibly, plays another role different from the Hubble constant  $H_0$ . Let us compute the acceleration of an object,  $\vec{a} = d\vec{v}/dt$  traveling through the Universe. It can be found from Eq. (9), by setting  $\nabla(m\varphi + \Sigma Q) = 0$ :

$$\vec{a} = v(t)\nabla^2\vec{v} = -v(t)\nabla(d\ln(\rho_m)/dt). \quad (25)$$

The term  $\nabla\vec{v} = -d\ln(\rho_m)/dt$  comes from the continuity Eq. (12). Now, by multiplying Eq. (22) by  $t^2/2$  and differentiating it with respect to  $t$ , we gain:

$$\frac{v(t)d\ln(\rho_m)}{c^2 dt} = \frac{1}{2} \frac{d}{dt} t^2 \sqrt{\frac{8\pi G\rho_m}{3} + \Lambda} \frac{c^2}{3} = tH_\Lambda + \frac{t^2}{4H_\Lambda} \left( \frac{8\pi G\rho_m}{3} \right) \frac{d\ln(\rho_m)}{dt}. \quad (26)$$

Then, by multiplying by  $c^2$  and by applying the operator  $\nabla$ , we get

$$a = -\frac{d}{d\ell} tc^2 H_\Lambda - \frac{d}{d\ell} \frac{t^2 c^2}{4H_\Lambda} (H_0^2) \frac{d\ln(\rho_m)}{dt}. \quad (27)$$

Here, the operator  $\nabla = d/d\ell$  calculates a gradient along the increment  $d\ell$ . Let us suppose that  $d\ell/dt$  represents an updated rate for the cosmic microwave background (CMB) fluctuations at the frequency  $\Omega_{\text{CMB}} = \omega_0$ , that is, the speed of light  $d\ell/dt = \lambda_{\text{CMB}}\Omega_{\text{CMB}} = c$ . By substituting  $c$  into Eq. (27) instead of  $d\ell/dt$ , we get:

$$a = -H_\Lambda c \left( 1 + \frac{H_0^2}{H_\Lambda^2} \cdot \left( \frac{1}{4} \frac{t}{\rho_m} \frac{d\rho_m}{dt} \right) \right). \quad (28)$$

We observe that the first term,  $H_\Lambda c$ , is equal to  $8.785 \cdot 10^{-10} \text{ m} \cdot \text{s}^{-2}$ . This indicates a good agreement with the acceleration  $a_p = (8.74 \pm 1.33) \cdot 10^{-10} \text{ m} \cdot \text{s}^{-2}$  which became known as the Pioneer anomaly [39–41]. From this, we have the fact that the term in second brackets vanishes, namely,  $d\rho_m/dt = 0$ , as follows from the continuity Eq. (12). It means that  $\rho_m = \rho_c = \text{const}$ . Eq. (28) suggests that the Pioneer anomaly is due to the presence of non-zero energy density (dark energy) of the vacuum, as reflected in the cosmological constant  $\Lambda \approx 10^{-52} \text{ m}^{-2}$  in metric units.

The Hubble parameters,  $H_0$  and  $H_\Lambda$ , concern different manifestations of SQS. The first parameter is due to presence of the tiny non-zero, positive viscosity of the SQS, whereby light undergoes loss of energy (redshift) proportional to the traveled distance. The Hubble diagram in **Figure 2** shows in fact the relationship with distance expressed by our hypothesis, in which the role of the recessional velocity in causing the cosmological redshift has to be however substituted by that of energy dissipation. Since in our analysis (see Ch.2 and [14]) photons are phonons through the SQS, i.e. waves carrying a momentum, in agreement with the concept of photon, they lose energy while traveling huge distances, as no superfluid has perfectly zero viscosity.

As for the parameter  $H_\Lambda$ , it results from the trigonometric shear of the Hubble parameter  $H_0$  by adding the contribution of the cosmological constant  $\Lambda$ , see Eq. (23). The calculated acceleration (28), excellently close to the acceleration  $a_p$  of the Pioneer apparatus, is due to the contribution of the SQS (i.e., of dark energy and of its hydrodynamic perturbations) expressed by the

cosmological constant [42, 43]. Indeed, we know that the relationship between  $\Lambda$  and the energy density of free space is  $\Lambda = \kappa\rho_0$ , where  $\kappa = 8\pi Gc^{-2}$  is Einstein's constant and  $\rho_0$  is vacuum's (i.e., SQS's) energy density. Thus, by considering the SQS as a ubiquitous sea of quantized, perturbed dark energy, partially condensed as dark matter (25.9%), we see that its mass is  $m_{\text{darkenergy}} = \rho_0(t)V(t) = E_{\text{darkenergy}}c^{-2} = \beta_0\rho_0 E_{\text{darkenergy}}$ . Where  $V(t)$  is the volume of the Universe at an instant  $t$  (even in an expanding/shrinking Universe, we have  $\rho_0 V = \text{const}$ , so at any moment dark energy is neither created nor annihilated) and  $c^{-2} = \beta_0\rho_0$  from Eq. (1), where  $\beta_0$  and  $\rho_0$  are physical parameters of dark energy, responsible for a small non-zero, positive viscosity of free space and, consequently, for the investigated anomalous deceleration.

#### 4. Vorticity equation and solutions for orbital speeds of spiral galaxies

The modified Navier-Stokes Eqs. (9) and (14) can exhibit a manifestation of long-lived vortices in SQS. The last term in this equation is dissipative due to the presence of a weak viscosity of the medium fluctuating about zero. If the viscosity coefficient  $\mu$  is a fluctuating function of time, we can assume (see Eq. (7)) that (a) time-averaged, the viscosity coefficient vanishes; (b) its variance is not zero. Therefore, the viscosity coefficient is a function fluctuating about zero. We suppose that such fluctuations determine energy exchange between the existing baryon matter and the zero-point fluctuations of the superfluid physical vacuum [44].

Note first that the total derivative of  $\vec{v}$  with respect to  $t$  in the Navier-Stokes equation (9), rewritten through the partial derivatives reads:

$$\frac{d\vec{v}}{dt} = \frac{\partial\vec{v}}{\partial t} + (\vec{v} \cdot \nabla)\vec{v}. \tag{29}$$

Let us apply now the curl operator to the Navier-Stokes equation. We come to the equation for vorticity  $\vec{\omega} = [\nabla \times \vec{v}]$  [45]:

$$\frac{\partial\vec{\omega}}{\partial t} + (\vec{v} \cdot \nabla)\vec{\omega} = \nu(t)\nabla^2\vec{\omega}. \tag{30}$$

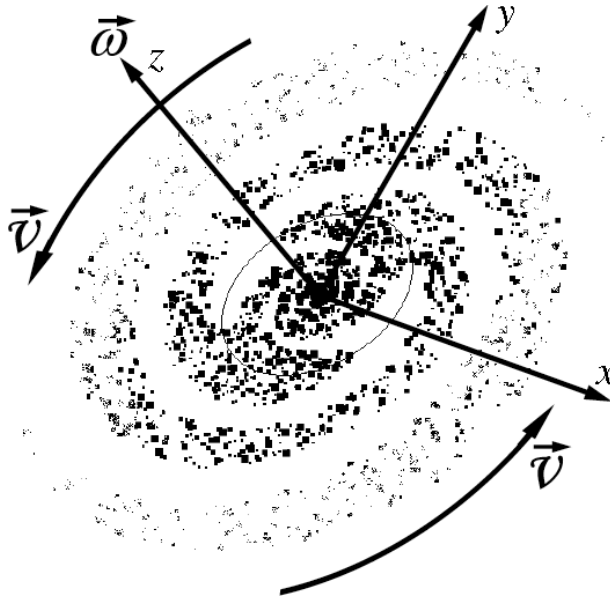
Here,  $\nu(t) = \mu(t)/\rho_m$  is the kinematic viscosity coefficient. The vector  $\vec{\omega}$  is directed along the rotation axis. In order to simplify this task, let us move to the coordinate system in which the rotation occurs in the plane  $(x, y)$  and the  $z$ -axis lies along the vorticity, **Figure 3**.

Under this transformation, the vorticity equation takes a particularly simple form:

$$\frac{\partial\omega}{\partial t} = \nu(t)\left(\frac{\partial^2\omega}{\partial r^2} + \frac{1}{r}\frac{\partial\omega}{\partial r}\right). \tag{31}$$

A general solution of this equation has the following view [5, 46]:

$$\omega(r, t) = \frac{\Gamma}{4\Sigma(\nu, t, \sigma)} \exp\left\{-\frac{r^2}{4\Sigma(\nu, t, \sigma)}\right\}, \tag{32}$$



**Figure 3.** A simulation of a rotating spiral galaxy: the orbital velocity  $\vec{v}$  lies in the plane  $(x, y)$ . The vorticity vector  $\vec{\omega}$  is oriented perpendicular to this plane.

$$v(r, t) = \frac{1}{r} \int_0^r \omega(r', t) r' dr' = \frac{\Gamma}{2r} \left( 1 - \exp \left\{ -\frac{r^2}{4\Sigma(v, t, \sigma)} \right\} \right). \tag{33}$$

The first function is vorticity; the second is the orbital speed. We do not mark the arrows above the letters  $v$  and  $\omega$  since the orbital velocity lies in the  $(x, y)$  plane and vorticity lies on  $z$ -axis. The denominator  $\Sigma(v, t, \sigma)$  in these formulas reads:

$$\Sigma(v, t, \sigma) = \int_0^t v(\tau) d\tau + \sigma^2. \tag{34}$$

Here,  $\sigma$  is an arbitrary constant such that the denominator is always positive.

Taking into account  $\langle v(t) \rangle = 0_+$ , see Eq. (7), we can see that the integral in Eq. (34) tends to zero and solutions of (32) and (33) in the limit of  $t \rightarrow \infty$  reduce to

$$\omega_{Gvc}(r, t) = \frac{\Gamma}{4\sigma^2} \exp \left\{ -\frac{r^2}{4\sigma^2} \right\}, \tag{35}$$

$$v_{Gvc}(r, t) = \frac{\Gamma}{2r} \left( 1 - \exp \left\{ -\frac{r^2}{4\sigma^2} \right\} \right). \tag{36}$$

That is, vorticity and angular speed are permanent in time. Here, the circulation  $\Gamma$  and the average radius  $\sigma$  are initially existing. The extra parameter  $\sigma$  comes from the Gaussian coherent vortex cloud [47]. The subscript  $G_{cvc}$  indicates the *Gaussian coherent vortex cloud*. The vortex cloud represents localized concentration of vorticity energy with a lifetime tending to infinity [48]. It does not significantly interact with any form of matter and exists in itself as long as possible.

**4.1. Flat profile of the orbital speed (evaluations)**

Solution (36) gives no flat profile. The function monotonically decreases with  $r \rightarrow \infty$ . This velocity is shown by curve 1 in **Figure 4**.

Let us begin to search for a solution of Eq. (31) by perturbing the solution (32) through a function  $g(r)$  not equal to one, that is,  $\omega(r, t) \cdot g(r)$ . When we substitute this function into Eq. (31), we get:

$$g \frac{\partial \omega}{\partial t} = g v(t) \left( \frac{\partial^2 \omega}{\partial r^2} + \frac{1}{r} \frac{\partial \omega}{\partial r} \right) + v(t) \left( \omega \frac{\partial^2 g}{\partial r^2} + \left( 2 \frac{\partial \omega}{\partial r} + \frac{1}{r} \omega \right) \frac{\partial g}{\partial r} \right). \tag{37}$$

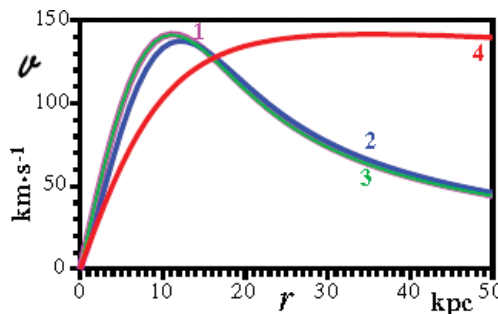
Here, we obtain two independent differential equations. The first one is for the function  $\omega(r, t)$ . We return to the same solution (32). While the second equation for the function  $g(r)$  becomes equal to zero. In this case, we introduce an auxiliary function  $\varphi = \partial g / \partial r$  for which this equation takes the form:

$$\frac{\partial \varphi}{\partial r} + \left( \frac{2}{\omega} \frac{\partial \omega}{\partial r} + \frac{1}{r} \right) \varphi = \frac{\partial \varphi}{\partial r} + \left( -\frac{r}{\Sigma} + \frac{1}{r} \right) \varphi = 0. \tag{38}$$

In the second part instead of  $(2/\omega) \cdot \partial \omega / \partial r$ , we put its solution  $-r/\Sigma$ . For the sake of simplicity, we write  $\Sigma$  instead of  $\Sigma(v, t, \sigma)$ . The function  $g(r)$  stemming from the solution of Eq. (38) reads:

$$g(r) = \int_0^r \frac{1}{\xi} \exp \left\{ \frac{\xi^2}{2\Sigma} \right\} d\xi. \tag{39}$$

Next, we find the orbital speed



**Figure 4.** (1, 2, 3) are monotonically decreasing profiles with  $r \rightarrow \infty$ ; 4 is an example of flat profile for large  $r$ , but when  $r$  tends to infinity, the curve vanishes.

$$v'(r, t) = \frac{1}{r} \int_0^r \omega(r', t) \cdot g(r') \cdot r' dr'. \tag{40}$$

This speed is shown as curve 2 in **Figure 4**. We have to observe, however, that the weight function  $g(r)$  can be approximated by the continued fraction [49]

$$E_1(x) = \frac{e^{-x}}{x + \frac{1}{1 + \frac{1}{x + \frac{2}{1 + \frac{2}{x + \frac{3}{1 + \frac{3}{x + \frac{4}{1 + \dots}}}}}}}} \tag{41}$$

In this way, we find an approximated function of the orbital speed

$$v''(r, t) = \frac{1}{r} \int_0^r \omega(r', t) \cdot E_1\left(\frac{r'}{2\Sigma}\right) \cdot r' dr'. \tag{42}$$

This speed is shown as curve 3 in **Figure 4**. One can see that all curves, 1, 2, and 3, accurate to the scaling, show good accordance with each other.

As for the curve 4 in this figure, it follows from the function

$$v'''(r, t) = \frac{\Gamma}{r} \sum_{n=1}^N \frac{1}{4\sigma_n^2} \int_0^r E_1\left(\left(\frac{r'}{2\sigma_n}\right)^2\right) \cdot r' dr'. \tag{43}$$

This function is drawn with linear growth of  $\sigma_n$  when  $n$  goes on,  $\sigma_n = 10 \cdot n$ . For  $n$  large enough, it shows a good outcome for the flat profile at  $r \gg 1$ .

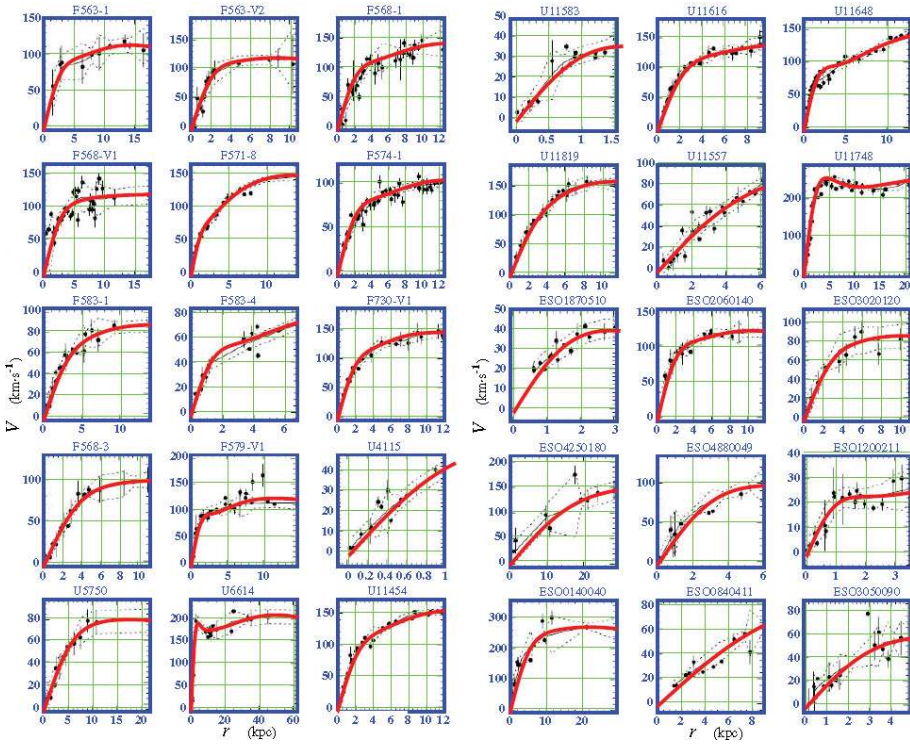
#### 4.2. Flat profile of the orbital speed (a general case)

A rich gallery of galactic rotation curves showing output on a flat profile is presented in [50].

These flat profiles of the orbital speeds are here rearranged, and they are shown in **Figure 5**. The curves draw approximations of these profiles.

Equation (43) gives a hint for getting flat profiles of orbital speeds, which are typical for spiral galaxies. In this section, we present formulas which show the formation of flat profiles evolving in time. First, we hypothesize that the above-mentioned Gaussian coherent vortex clouds (see Eqs. (35) and (36)) have a long-term memory, and they can therefore manifest themselves as dark matter. As shown in **Figure 4** by the curve 4, the clouds can support flat profiles for a long time through their superposition (see Eq. (43)). For the sake of demonstration, let us set [46]





**Figure 5.** Families F#, U#, and ESO# of the flat profiles of the orbital speeds taken from [50]. The curves approximate these profiles marked by the black points.

$$v(t) = v \frac{e^{i\Omega t} + e^{-i\Omega t}}{2} = v \cos(\Omega t). \quad (44)$$

In this case  $\Sigma(v, t, \sigma) = (v/\Omega) \sin(\Omega t) + \sigma^2 = (v/\Omega) (\sin(\Omega t) + \zeta)$ ,  $\sigma^2 = (v/\Omega) \cdot \zeta$  and  $\zeta > 1$ . Note first that the Gaussian coherent vortex clouds show self-similarity.

From this view, let us assume that the fluctuating viscosity reads as follows [5]:

$$v_n(t) = \frac{c^2}{\Omega_n} \cos(\Omega_n t). \quad (45)$$

The kinetic viscosity coefficient  $c^2/\Omega_n$  has dimension  $[m^2 \cdot s^{-1}]$ . Here,  $c$  is the speed of light, and  $\Omega_n$  is the angular frequency of a vacuum oscillation. That is, there is a periodic exchange of energy  $\hbar\Omega_n = n^{-1}$  with SQS. The energy tends to zero at  $n$  going to infinity, whereas the viscosity goes to infinity. It can mean that SQS acquires a high viscosity on very small frequencies of the vortex energy exchange. Note, however, that  $\langle v_n(t) \rangle = 0_+$  for any  $n$ .

Let us compute the flat profile for the orbital speed of a spiral galaxy guided by the rule formulated above. To see its formation, we perform computations of sets collected from modes

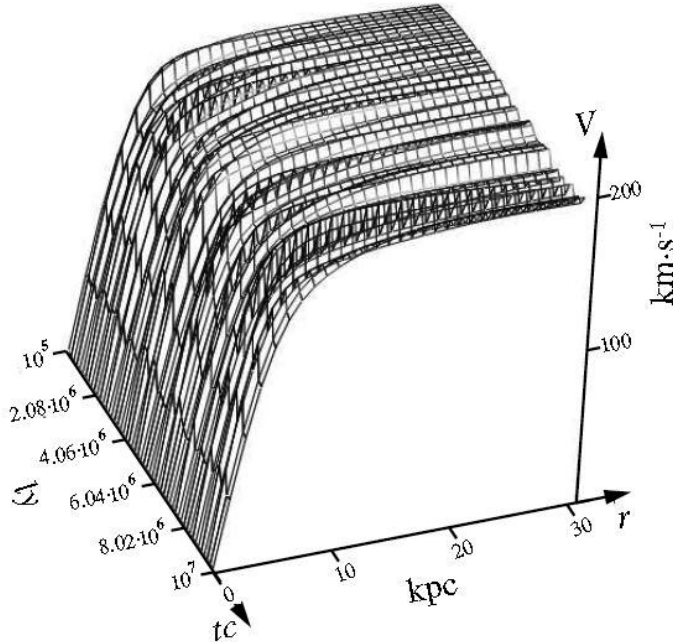
(45),  $n = 1, 2, \dots, N$ . Let us substitute the expression (45) into the integral (34). After computing it, we get the following view of the denominator  $\Sigma(v, t, \sigma)$ :

$$\Sigma_n(t) = \frac{c^2}{\Omega_n^2} \left( \sin(\Omega_n t) + \zeta \right). \tag{46}$$

Since  $\Omega_n = n^{-1}$ , the coefficient  $\Sigma_n = c^2/\Omega_n^2$  tends to infinity as  $\Omega_n$  goes to zero while  $n$  increases. From here, it follows that the expression  $1 - \exp\{-r^2/4\Sigma_n\}$  in Eq. (33) reaches 1 the more slowly with increasing  $r$ , the larger is  $\Sigma_n$ . As a result, the set of coefficients  $\Sigma_n$  for  $n = 1, 2, \dots$  can give output to the flat profile of the orbital speed. Let us, therefore, compute a sum of possible orbital speeds of galaxies for all entangled modes for which baryon matter is allowed to exchange energy with the SQS. Our statistical sum reads as follows:

$$V(r, t) = \frac{\Gamma}{2r} \sum_{n=1}^N \left( 1 - \exp\left\{ -\frac{r^2}{4\Sigma_n(t)} \right\} \right). \tag{47}$$

The orbital speed  $V(r, t)$  versus  $r$  and  $t$  is shown in **Figure 6**. Here, for the evaluated calculations, we used  $\Gamma = 3 \cdot 10^{25} \text{m}^2 \cdot \text{s}^{-1}$  and the angular frequency  $\Omega_n$  ranges from  $10^{-11} \text{s}^{-1}$  to  $1.667 \cdot 10^{-13} \text{s}^{-1}$  as  $n$  runs from 1 to 60. The angular frequencies are extremely small, while the wavelengths,  $\lambda_n = c/\Omega_n$ , are in the range from 0.97 to 58.3 kpc. These oscillating modes cover areas from the galactic core up to the size of the galaxy itself.



**Figure 6.** Orbital speed  $V$  is a function of the radius  $r$  from the galactic center (in kiloparsec) and time  $t$  (in light years). Variations of the orbital speed in time are evident.

**Figure 6** shows that the orbital speed experiences small fluctuations in time, resembling the breathing of the galaxy. This trembling of galaxies within the  $1/f$  spectrum is caused by the exchange of vortex energy with the SQS on the ultra-low frequencies  $\Omega_n$ .

De Broglie wavelength,  $\lambda_n = c/\Omega_n$ , by changing in the range from about 20 kpc to 2 Mpc covers all galactic scales. One can evaluate the mass of axion-like particles [51],

$$m = \frac{\hbar\Omega_n}{c^2}. \quad (48)$$

It ranges from about  $10^{-62}$  kg to  $5 \cdot 10^{-66}$  kg. They are in the range shown in Ref. [52]. These particles may correspond to dark energy's quanta and be responsible for exchange phenomena among baryon objects in the frequency range from  $\Omega_n = 10^{-11} \text{s}^{-1}$  to  $\Omega_n \approx 5 \cdot 10^{-15} \text{s}^{-1}$ . We note that the frequency  $\Omega_c$  is  $2.2 \cdot 10^{-18} \text{s}^{-1}$ . We obtain  $c/\Omega_c \approx 1.36 \cdot 10^{26} \text{m}$ , which is close enough to the Compton wavelength evaluated for the visible Universe in [53]. This corresponds to the radius of the Hubble sphere  $r_{HS} = c/H_0$ , which is about  $4 \cdot 10^3 \text{Mpc}$  (at the Hubble constant  $H_0 = 73 \text{ km} \cdot (\text{s} \cdot \text{Mpc})^{-1}$ ). On these cosmological scales, we can evaluate the mass of a graviton (or more likely of a quantum of dark energy, since we don't need gravitons in superfluid quantum gravity [2, 14]), by resorting to a wavelength that is commensurable with the radius of the Universe stated above. An extreme mass of the axion-like particle for the observable Universe is  $m_g = \hbar\Omega/c^2 \approx 2.6 \cdot 10^{-69}$  kg. This value finds a good agreement with the evaluation that comes from the holographic screen model to be the boundary of the visible Universe [52]. This evaluation is also in agreement with the graviton mass given in Ref. [54], here interpreted as a quantum of dark energy. We finally add that ultra-light dark matter particles produced in the vacuum have been predicted in Ref. [10].

We can continue the calculation of the orbital speed (44) up to the point  $\Omega_c = 2.2 \cdot 10^{-18} \text{s}^{-1}$ . This would allow us to affirm that the observable Universe rotates about some center with an orbital speed, which has a flat profile through enormous distances. Excepting a central region where the orbital speed grows from zero to the maximal value corresponding to the profile level. This rotation possibly takes place around the richest Super Cluster in the Sloan Great Wall, SCI-126, and especially around its core, resembling a very rich filament [55].

## 5. Conclusion

We have shown that the fluid dynamics of SQS could explain the astrophysical observations without resorting to far-fetched auxiliary concepts, such as cosmic inflation and accelerated expansion.

In general, the fluid dynamics of SQS is described by the conservation equations of energy, momentum, orbital momentum, etc. In the non-relativistic limit, these equations are reduced to the modified Navier-Stokes equation and to the continuity equation of mass density. The modification leads to the emergence of a quantum potential,  $Q(t)$ , and reduces the viscosity coefficient,  $\mu(t)$ , to a weak term fluctuating about zero,  $\langle \mu(t) \rangle = 0_+$ ,  $\langle \mu(t)\mu(0) \rangle > 0$ . Because

of that, this term acquires an absolutely different physical meaning. Firstly, an active exchange of energy between cosmological structures and SQS takes place and extends their lifetime.

We applied the modified Navier-Stokes equation to describe a balance within the visible Universe between the gravitational potential,  $\varphi$ , expressed as the quantum potential  $Q_\varphi = P_G/\rho_m$  and the intrinsic quantum potential,  $Q$ , of SQS. Outside this range, strong repelling forces act (see dotted curve in **Figure 1**), probably due to osmotic expansion of dark energy in a really empty space. Figuratively speaking, baryon matter in the Universe is similar to a hydrophobic droplet floating in a hydrophilic medium filling the vast space. However, there is a difference between a “droplet model” and the Universe, since the latter consists of numerous clumps of baryonic matter separated by vast voids. These baryonic clumps are concentrated on vortex filaments that permeate the whole Universe and form an intricate cosmic web [56] with galaxies strung on these filaments.

Since  $\langle\mu(t)\rangle = 0_+$  (it differs from zero to a tiny value), light coming from distant stars shows a frequency shift due to a loss of energy when traveling through the SQS. We therefore introduce an updated concept of tired light without resorting to Compton scattering and overcoming in this way the known objections to the classical concept of tired light.

The Pioneer anomaly has a lot in common with the revised tired light effect. The same loss of energy due to motion through the SQS most likely led to a deceleration of the space apparatus. An essential contribution to the deceleration comes from a non-zero small correction of the Hubble parameter thanks to the cosmological constant, which refers to dark energy, i.e., to SQS itself). This correction gives a value of the negative acceleration of the cosmic apparatus  $a = -H_\Lambda c = -c\sqrt{H_0^2 + \Lambda c^2/3} \approx -8.785 \cdot 10^{-10} \text{ m} \cdot \text{s}^{-2}$ , which acceptably falls within the measured anomalous acceleration of the Pioneer probes 10 and 11.

Eventually, the considered superfluid dark medium is capable of explaining the flat profile of the orbital speed of spiral galaxies, due to their interactions with the SQS. We can observe flat profile solutions by putting (46) as denominator in Eqs. (30) and (31), with  $\sigma_0 > 1$ , where the set of coefficients  $\Sigma_n$  for  $n = 1, 2, \dots$  shows the flat profile.

## Author details

Valeriy I. Sbitnev<sup>1,2\*</sup> and Marco Fedi<sup>3</sup>

\*Address all correspondence to: valeriy.sbitnev@gmail.com

1 Petersburg B. P. Konstantinov Nuclear Physics Institute, NRC Kurchatov Institute, Gatchina, Russia

2 Department of Electrical Engineering and Computer Sciences, University of California, Berkeley, CA, USA

3 Ministero dell’Istruzione, dell’Università e della Ricerca (MIUR), Italy

## References

- [1] Rubin VC. A brief history of dark matter. in Livio Mn editor *The Dark Universe: Matter, Energy and Gravity*. Cambridge. Cambridge University Press. 2004, pp. 1–13.
- [2] Fedi M. Gravity as a fluid dynamic phenomenon in a superfluid quantum space. *Fluid Quantum Gravity and Relativity*. 2016. URL: <https://hal.archives-ouvertes.fr/hal-01248015>
- [3] Ardey A. Dark fluid: A complex scalar field to unify dark energy and dark matter. *Phys. Rev. D*, 2006; **74**: 043516. doi:10.1103/PhysRevD.74.043516
- [4] Hajdukovic DS. Quantum vacuum and dark matter. *Astrophys. Space Sci.* 2011; **337**: 9–14. doi:10.1007/s10509-011-0938-9
- [5] Sbitnev VI. Hydrodynamics of the physical vacuum: dark matter is an illusion. *Mod. Phys. Lett. A*. 2015; **30**: 1550184. doi:10.1142/S0217732315501849
- [6] Albareti FD, Cembranos JAR, Maroto AL. Vacuum energy as dark matter. *Phys. Rev. D*. 2014; **90**: 123509. doi:10.1103/PhysRevD.90.123509
- [7] Das S, Bhaduri RK. Dark matter and dark energy from Bose-Einstein condensate. *Class. Quant. Grav.* 2015; **32**: 105003. doi:10.1088/0264-9381/32/10/105003
- [8] Baum L, Frampton PH. Entropy of Contracting Universe in Cyclic Cosmology. *Mod. Phys. Lett. A*. 2008; **23**: 33-36. doi:10.1142/S0217732308026170
- [9] Gurzadyan VG, Penrose R. Concentric circles in WMAP data may provide evidence of violent pre-Big-Bang activity. 2010. URL: <http://arxiv.org/abs/1011.3706>
- [10] Chefranov SG, Novikov EA. Hydrodynamic vacuum sources of dark matter self-generation in an accelerating universe without a Big Bang. *JETP*. 2010; **111**: 731–743.
- [11] Crawford DF. Observational evidence favors a static universe. 2014. URL: <http://arxiv.org/abs/1009.0953>
- [12] López-Corredoira M, Melia F, Lusso E, Risaliti G. Cosmological test with the QSO Hubble diagram. *Int. J. Mod. Phys. D* 2016; **25**: 1650060. doi:10.1142/S0218271816500607
- [13] Zwicky F. On the Red Shift of Spectral Lines through Interstellar Space. *PNAS*. 1929; **15**: 773–779. doi:10.1073/pnas.15.10.773
- [14] Fedi M. Quantum gravity as dark energy inflow, 2016, URL: <https://hal.archives-ouvertes.fr/hal-01423134>
- [15] Penrose R. *Fashion, Faith, and Fantasy in the New Physics of the Universe*. Princeton and Oxford. Princeton University Press. 2016.
- [16] *Stanford Encyclopedia of Philosophy*. Absolute and Relational Theories of Space and Motion. 2015. URL: <http://plato.stanford.edu/entries/spacetime-theories/>
- [17] Motte A. *Axioms or Laws of Motion* (translation of Newton's *Principia* (1687)). Published by Daniel Adee, 45 Liberty str., N. Y. 1846.

- [18] Huygens C. *Treatise on light*. London. Macmillan and Co. Ltd. 1912.
- [19] Wheeler JA, Ford K. *Geons, Black Holes, and Quantum Foam*. Norton & Co. 1995.
- [20] Dirac PAM. Is there an Aether? *Nature*. 1951; **168**: 906–907.
- [21] De Broglie L. Interpretation of quantum mechanics by the double solution theory. *Annales de la Fondation Louis de Broglie*. 1987; **12**: 1–22.
- [22] Couder Y, Protière S, Fort E, Boudaoud A. Dynamical phenomena—walking and orbiting droplets. *Nature*. 2005; **473**: 208. doi:10.1038/437208a
- [23] Petroni NC, Vigier JP. Dirac’s aether in relativistic quantum mechanics. *Foundations of Physics*. 1983; **13**: 253–286.
- [24] Laughlin RB. *A Different Universe: Reinventing Physics from the Bottom Down*, N. Y.: Basic Books, 2005.
- [25] Cahill RT. Gravity as quantum foam in-flow. 2004. URL: <https://arxiv.org/abs/physics/0307003>
- [26] Bohm D, Vigier JP. Model of the causal interpretation of quantum theory in terms of a fluid with irregular fluctuations. *Phys. Rev*. 1954; **96**: 208–216. doi:10.1103/PhysRev.96.208
- [27] Grusenick M. Extended Michelson-Morley Interferometer experiment. 2009. URL: <https://www.youtube.com/watch?v=7T0d7o8X2-E>
- [28] Planck Collaboration: Ade PAR, Aghanim N, et al. (260 co-authors). Planck 2015 results. XIII. Cosmological parameters. 2015. URL: <http://arxiv.org/abs/1502.01589>
- [29] Font JA. Numerical Hydrodynamics and Magnetohydrodynamics in General Relativity. *Living Rev. Relat*. 2008; **11**: 1–131. doi:10.12942/lrr-2008-7
- [30] Romatschke P. New Developments in Relativistic Viscous Hydrodynamics. *Int. J. Mod. Phys. E*. 2010; **19**: 1–53. doi:10.1142/S0218301310014613
- [31] Van Holten JW. *Relativistic fluid dynamics*. NIKHEF. Amsterdam NL. 2006. Available from: <http://www.nikhef.nl/~t32/relhyd.pdf>
- [32] Parkhomov AG. Dark matter: its role in cosmo-terrestrial interactions. *Consciousness and Physical Reality*. 1998; **3**: 24–35.
- [33] Ryden B. *Introduction to Cosmology*. The Ohio State University. 2006.
- [34] Sbitnev VI. Hydrodynamics of the physical vacuum: I. Scalar quantum sector. *Foundations of Physics*. 2016; **46**: 606–619. doi:10.1007/s10701-015-9980-8
- [35] Freedman WL, et al. (14 co-authors). Final Results from the Hubble Space Telescope Key Project to Measure the Hubble Constant. *Astrophys. J*. 2001; **553**: 47–72. doi:10.1086/320638
- [36] Spergel DN, et al. (21 co-authors). Wilkinson Microwave Anisotropy Probe (WMAP) Three Year Results: Implications for Cosmology. *Astrophys. J. Suppl*. 2007; **170**: 337–408. doi:10.1086/513700

- [37] Khaidarov KA. Galilean Interpretation of the Hubble Constant. GALILEAN ELECTRO-DYNAMICS. 2005; **16**: 103–105.
- [38] Crowell LB. Quantum fluctuations of spacetime. Singapore. World Scientific. Co. Pte. Ltd. 2005.
- [39] Anderson JD, Laing PA, Lau EL, Liu AS, Nieto MM, Turyshev SG. Indication from Pioneer 10/11, Galileo, and Ulysses Data, of an Apparent Anomalous, Weak, Long-Range Acceleration. Phys. Rev. Lett. 1998; **81**: 2858–2861. doi:10.1103/PhysRevLett.81.2858
- [40] Page GL, Dixon DS, Wallin JF. Can minor planets be used to assess gravity in the outer solar system? Astrophys. J. 2006; **642**: 606–614. doi:10.1086/500796
- [41] Turyshev SG, Toth VT, Kinsella G, Lee S-Ch, Lok SM, Ellis J. Support for the thermal origin of the Pioneer anomaly. Phys. Rev. Lett. 2012; **108**: 241101. doi:10.1103/PhysRevLett.108.241101
- [42] Mukhopadhyay U. et al. Generalized Model for  $\Lambda$ -Dark Energy. URL: arXiv:0802.1032, Int. J. Mod. Phys. 2009; D18: 389–396. doi:10.1142/S021827180901456X
- [43] Zhao H. An Uneven Vacuum Energy Fluid as  $\Lambda$ , Dark Matter, MOND and Lens. 2008. URL: arXiv:0802.1775, Mod. Phys. Lett. A23:555–568, 2008. doi:10.1142/S021773230802656X
- [44] Sbitnev VI. Dark matter is a manifestation of the vacuum Bose-Einstein condensate. 2016. URL: <http://arxiv.org/abs/1601.04536>
- [45] Sbitnev VI. Hydrodynamics of the physical vacuum: II. Vorticity dynamics. Found. of Physics. 2016; 1–15. URL: <http://rdcu.be/kdon>. doi: 10.1007/s10701-015-9985-3
- [46] Sbitnev VI. Physical vacuum is a special superfluid medium. In: Pahlavani MR, editor. Selected Topics in Applications of Quantum Mechanics. Rijeka: InTech; 2015. pp. 345–373. doi:10.5772/59040
- [47] Kevlahan NK-R, Farge M. Vorticity filaments in two-dimensional turbulence: creation, stability and effect. J. Fluid Mech. 1997; **346**: 49–76. doi:10.1017/S0022112097006113
- [48] Provenzale A, Babiano A, Bracco A, Pasquero C, Weiss JB. Coherent vortices and tracer transport. Lect. Notes Phys. 2008; **744**: 101–116. doi:10.1007/978-3-540-75215-8\_5
- [49] Tseng P-H, Lee T-C. Numerical evaluation of exponential integral: Theis well function approximation. J. Hydrol. 1998; **205**: 38–51. doi:10.1016/S0022-1694(97)00134-0
- [50] De Blok WJG, McGaugh SS, Rubin VC. High-resolution rotation curves of low surface brightness galaxies. II. Mass models. Astron. J. 2001; **122**: 2396–2427. doi:10.1086/323450
- [51] Mielczarek J, Stachowiak T, Szydlowski M. Vortex in axion condensate as a dark matter halo. Int. J. Mod. Phys. D. 2010; **19**: 1843–1855. doi:10.1142/S0218271810018037
- [52] Mureika JR, Mann RB. Does entropic gravity bound the masses of the photon and graviton? Mod. Phys. Lett. A. 2011; **26**: 171–181. doi:10.1142/S0217732311034840
- [53] Goldhaber AS, Nieto MM. Photon and graviton mass limits. Rev. Mod. Phys. 2010; **82**: 939–979. doi:10.1103/RevModPhys.82.939

- [54] Triple S. A simplified treatment of gravitational interaction on galactic scales. JKAS. 2013; **46**: 41–47. doi:10.5303/JKAS.2013.46.1.41
- [55] Einasto E, et al. (10 co-authors). The Sloan Great Wall. Morphology and galaxy content. *Astrophys. J.* 2011; **736**: 25 p. doi:10.1088/0004-637X/736/1/51
- [56] Laigle C, Pichon C, et al. (10 co-authors). Swirling around filaments: are large-scale structure vortices spinning up dark haloes? *MNRAS*. 2013; **446**: 2744–2759. doi:10.1093/mnras/stu2289
- [57] Santillán AO, Volke-Sepúlveda K. A demonstration of rotating sound waves in free space and the transfer of their angular momentum to matter. *Am. J. Phys.* 2009; **77**: 209–215. doi:10.1119/1.3056580