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Superfluidity of Λ-Hyperons Admixed in Neutron Star Cores[*]

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Λ-superfluidity in a hyperon core of neutron stars is studied with a realistic approach. It is found that Λ-superfluid exists in a restricted density region $\rho \simeq (\rho_t\text{–}\rho_d)$ with $\rho_t \simeq 2\rho_0$ (ρ_0 being the nuclear density) and $\rho_d \simeq (2.6\text{–}4.6)\rho_0$ depending on the pairing interaction and the hyperon core model. This restriction suggests that neutron stars compatible with hyperon cooling would be not so massive.

Introduction Study on the superfluidity of hyperons admixed in neutron star cores is of increasing interest not only with regard to many-body problems in hadronic matter including exotic components but also to the cooling scenario of neutron stars inferred from surface temperature observations. It has been suggested that some neutron stars are cooled much more rapidly than expected from the standard cooling scenario (i.e., modified URCA process; e.g., $n + n \to p + n + e^- + \bar{\nu}_e$, and the inverse process), and a more efficient cooling mechanism is needed.[2] In this connection, so-called "hyperon cooling",[3] associated with neutrino emission processes as of the β-decay type including hyperons ($\Lambda \to p + e^- + \bar{\nu}_e, \Sigma^- \to \Lambda + e^- + \bar{\nu}_e$, etc.), provides one of the rapid cooling mechanisms. The direct action of such rapid cooling, however, leads to a surface temperature much lower than that observed. This demands some suppression mechanism, most naturally hyperon superfluidity, to control the cooling rate. Therefore it becomes of special interest to investigate whether hyperon superfluidity is possible in the hyperon-mixed phase believed to exist in neutron star cores.

In this paper, as a typical example of hyperon superfluidity, we focus attention on Λ-hyperon pairing, since at present the uncertainty in the $\Lambda\Lambda$ interaction is relatively small due to the information from various experimental data (especially, the available data from double Λ hypernuclei) and theoretical studies. Very recently, Balberg and Barnea[4] studied this subject by using an effective $\Lambda\Lambda$ interaction $\tilde{V}_{\Lambda\Lambda}$ for $\Lambda\Lambda$ pairing interaction. However, their approach using an effective interaction for the pairing problem is not justified, as shown later. Moreover, the $\tilde{V}_{\Lambda\Lambda}$ they used[5] is based on the G-matrix calculations for two Λ particles immersed in symmetric nuclear matter, but such situation is basically different from the actual one of the hyperon-mixed phase composed of neutrons as a dominant component, protons, Λ-particles and other hyperons. In addition, the effective mass they used for Λ is not suitable.

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The aim of this paper is to present more realistic results of Λ superfluidity, carefully considering the following points: (i) We adopt the "bare" $\Lambda\Lambda$ potential $V_{\Lambda\Lambda}$ instead of the effective $\tilde{V}_{\Lambda\Lambda}$. (ii) We use the more realistic effective-mass parameter m^*_Λ, which is derived from the G-matrix calculation for neutron-plus-lambda matter with the lambda fraction Y_Λ, taken as a better approximation of the hyperon-mixed phase.

Outline of the approach Several works have undertaken on the hyperon mixing problem in neutron star matter. Hyperons become admixed at around twice the nuclear density ρ_0 ($\equiv 0.17$ nucleons/fm$^3 \simeq 2.8 \times 10^{14}$ g/cm^3), and mostly the population increases with increasing ρ. This aspect is, however, considerably model dependent. In Fig. 1, the fraction $Y_\Lambda(\equiv \rho_\Lambda/\rho)$ of Λ-particles of interest is shown for typical two cases, one from the conventional potential description approach (PD) of Pandharipande,[6] and the other from the relativistic mean-field (RMF) approach of Schaffner and Mishustin.[7] We see the difference between the two as a gradual (rapid) rise of Y_Λ for the former (the latter) approach and the largest Y_Λ of about 10% (25%) for the former (the latter). Here we consider these two cases for Y_Λ in order to see how the difference affects the Λ-superfluidity, and in particular its existence density region. The use of Y_Λ from RMA is intended only to make clear the influence of large Y_Λ and not for the purpose of discussing the pairing problem in the scheme of the RMF approach.

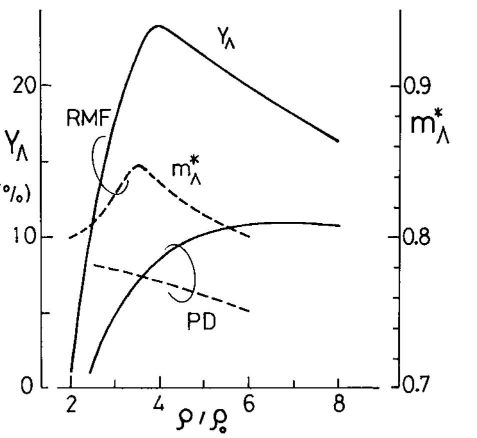

Fig. 1. Λ-fraction Y_Λ (solid lines) and Λ effective mass parameter m^*_Λ (dotted lines) as functions of the total baryon density ρ in units of the nuclear density ρ_0. PD (RMF) denotes the case of the potential description[6] (relativistic mean field[7]) approach.

Although Λ particles appear in the high-density region ($\rho \gtrsim \rho_t \simeq 2\rho_0$), while the density is relatively low because Y_Λ ((10–25)% at most) is not too large, therefore the Λ-Λ pairing responsible for the Λ superfluidity should be that of the $^1S_0$ pair state which is most attractive at low scattering energies. Thus we set up the problem to calculate the energy gap Δ by solving the gap equation of the $^1S_0$-type in a manner quite similar to that for the well-known nucleon $^1S_0$-pairing:[8]

$$\Delta(q) = -\frac{1}{\pi}\int_0^\infty q'^2 dq' \langle q' \mid V_{\Lambda\Lambda}(^1S_0) \mid q\rangle \Delta(q')/\sqrt{\tilde{\epsilon}(q')^2 + \Delta(q')^2}, \tag{1}$$

$$\tilde{\epsilon}(q) \equiv \epsilon(q) - \epsilon_F \simeq (q^2 - q_F^2)/2M^*_\Lambda, \tag{2}$$

$$\langle q' \mid V_{\Lambda\Lambda}(^1S_0) \mid q\rangle \equiv \int_0^\infty r^2 dr j_0(qr) V_{\Lambda\Lambda}(r; {}^1S_0) j_0(q'r). \tag{3}$$

Here $\Delta(q)$ denotes the energy gap function, $q_F = (3\pi^2\rho Y_\Lambda)^{1/3}$ is the Fermi momentum of Λ, and $\epsilon_F \equiv \hbar^2 q_F^2/2M_\Lambda$ is the Fermi energy, with M_Λ the lambda mass. Also, the effective-mass approximation is adopted for the lambda single-particle energy $\epsilon(q)$ with the effective-mass M^*_Λ. The $^1S_0$-gap equation (1) with the definitions in

Eqs. (2) and (3) are solved numerically when the $^1S_0$-pairing interaction $V_{\Lambda\Lambda}(r;{}^1S_0)$, the Λ-fraction Y_Λ and the effective-mass parameter $m^*_\Lambda \equiv M^*_\Lambda/M_\Lambda$ are given.

Generally, the energy gap depends sensitively on M^*_Λ, and so it is important to have a realistic m^*_Λ. In this work, we use m^*_Λ obtained as $m^*_\Lambda = (\hbar^2 q_F/M_\Lambda)/(\partial\epsilon(q)/\partial q)_{q_F}$ in terms of $\epsilon(q)$ from the G-matrix calculation with the Nijmegen-D potential for $\{n+\Lambda\}$ matter specified by Y_Λ. This m^*_Λ depends both on Y_Λ and ρ. It decreases with ρ when Y_Λ is fixed and increases with Y_Λ when ρ is fixed. For example, $m^*_\Lambda \simeq (0.80 \to 0.73)$ for $Y_\Lambda = 0.05$ and $\simeq (0.84 \to 0.77)$ for $Y_\Lambda = 0.15$, according to $\rho = (2 \to 6)\rho_0$. The ρ-dependence of m^*_Λ to take account of the Y_Λ-ρ relation from the two models mentioned above is inserted into Fig. 1. It is remarked that m^*_Λ is by far larger than the usual m^*_N in nucleon matter; e.g., $m^*_\Lambda(\text{PD}) \simeq (0.8 \to 0.75)$ for $\rho = (2 \to 6)\rho_0$ in contrast to $m^*_n = m^*_p \sim (0.65 \to 0.45)$ for symmetric nuclear matter and $m^*_n \sim (0.8 \to 0.6)$ for pure neutron matter, according to $\rho \simeq (1 \to 3)\rho_0$.[9] We note that m^*_Λ used in Ref. 4) is much different from ours; e.g., $m^*_\Lambda(\text{Balberg-Barnea}) \simeq (1.0 \to 0.8)$, whereas $m^*_\Lambda(\text{ours}) \simeq (0.8 \to 0.85)$ for the same parameters $Y_\Lambda = (0.06 \to 0.175)$ and $\rho = 2.5\rho_0$.

As for $V_{\Lambda\Lambda}(r;{}^1S_0)$, we consider two cases of the OBEP type. One case (referred to as "Ehime") has been proposed by the Ehime group,[10] based on a framework of nonet mesons and $SU(3)$ invariance, and has a soft repulsive core and a velocity-dependence of short-range interaction. The other (ND-Soft) is a Gaussian soft-core version of the Nijmegen-D hard core potential constructed by Yamamoto[11] so as to fit the t-matrix from the original Nijmegen-D potential. Both of these potentials reproduce the data of double Λ hypernuclei. Figure 2 compares these two potentials. The ND-Soft potential has a stronger short-range repulsion and a stronger intermediate-range attraction, compared with the Ehime potential. The use of two $V_{\Lambda\Lambda}(r;{}^1S_0)$ is expected to cover the present uncertainties of the $\Lambda\Lambda$ pairing interaction. In Fig. 2, the $^1S_0$ two-nucleon interaction for a well-known Reid-Soft-Core (RSC) potential is also shown for reference. Obviously, the occurence of Λ-superfluidity is less likely compared to the case of nucleon superfluidity, since the pairing attraction is weaker in the former. In view of the effective mass, however, larger ef-

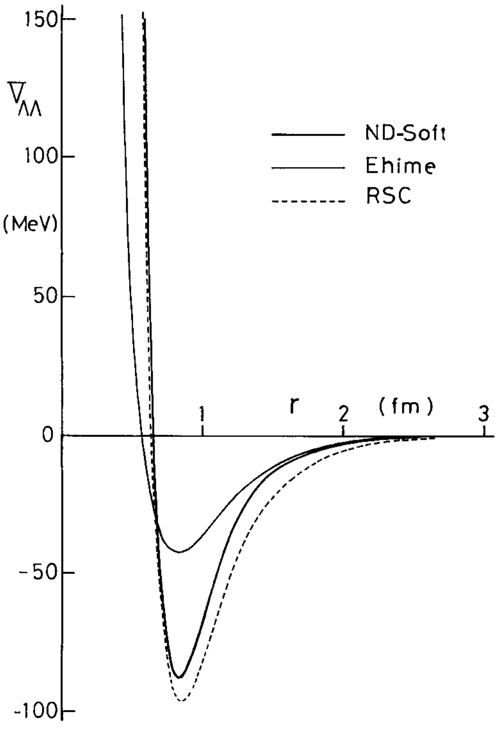

Fig. 2. Comparison of the $\Lambda\Lambda\,{}^1S_0$ potentials due to Yamamoto (ND-Soft) and that of the Ehime group (Ehime: $\epsilon_F = 17$ MeV). The $NN\,{}^1S_0$ interaction from the RSC potential is also shown for reference. Notation is described in the text.

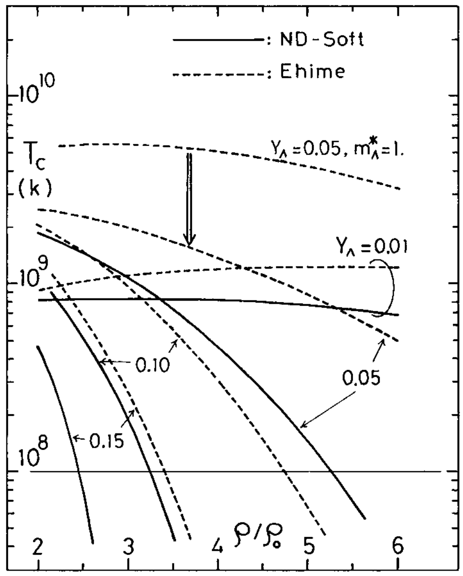

Fig. 3. Critical temperature T_c of Λ-superfluidity versus ρ for several values of Y_Λ and ρ-dependent m^*_Λ. The solid (dashed) lines correspond to the ND-Soft (Ehime) potential.

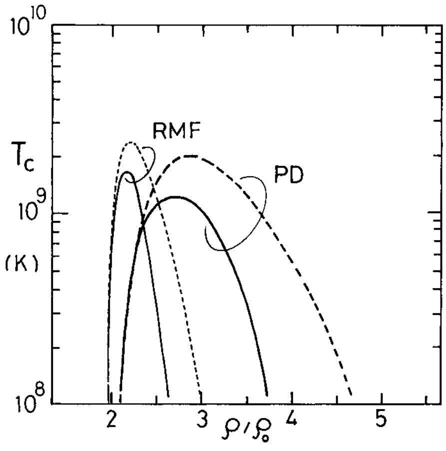

Fig. 4. T_c versus ρ taking account both of the Y_Λ-ρ and m^*_Λ-ρ relations in Fig. 1. The solid (dashed) lines correspond to the ND-Soft (Ehime) potential. PD and RMF specify the hyperon core model.

fective mass for Λ would act for a larger energy gap. In understanding the results, we note such counterbalancing features.

Results and discussion The energy gap $\Delta(\equiv \Delta(q_F))$ depends on ϵ_F and m^*_Λ, as well as the pairing interaction adopted. For convenience, we give numerical results in terms of the critical temperature T_c of Λ-superfluidity, which is related to Δ (in MeV) as $T_c \simeq 0.66\Delta \times 10^{10}$ K. The Λ-superfluid comes into existence when T_c exceeds $T_i \simeq 10^8$ K, the internal temperature of neutron stars. First we discuss the aspects of the ρ-dependence of T_c by keeping Y_Λ constant. The following points are noted from Fig. 3:

(i) The ρ-dependence of m^*_Λ makes T_c smaller than the simple case of $m^*_\Lambda = 1$, as shown by the arrow in the figure. This comes from the property that in general Δ is smaller for smaller effective mass and in the present case m^*_Λ decreases with ρ, for example, as $m^*_\Lambda = (0.804 \rightarrow 0.725)$ according to $\rho = (2 \rightarrow 6)\rho_0$ with $Y_\Lambda = 0.05$.

(ii) When Y_Λ is as large as (15–20)%, the Λ-superfluidity becomes less likely ($T_c \lesssim T_i$). This is because T_c has a peak value at around $\epsilon_F \simeq 15$ MeV and decreases with increasing ϵ_F (i.e., increase of $\rho_\Lambda = Y_\Lambda \rho$), due to the growth of the effects due to the short-range repulsion.

(iii) These aspects depend on the pairing interaction $V_{\Lambda\Lambda}$; on the higher density side, T_c from the ND-Soft potential are smaller than those from Ehime potential, leading to a narrower density region for Λ-superfluidity. This is due to the fact that the short-range repulsion is stronger for the former than for the latter.

Actually, the resulting T_c depends on the Y_Λ-ρ relation (namely, the hyperon

core model) through the Y_Λ- dependence of ϵ_F and m_Λ^*. The results obtained by taking account of the ρ-dependence of Y_Λ are shown in Fig. 4 for the two models:
(iv) The density region for the existence of Λ-superfluid is wider (narrower) for PD (RMF) models; $\rho \simeq (2\text{–}3.7)\rho_0((2\text{–}4.6)\rho_0)$ for PD, and $\rho \simeq (2\text{–}2.6)\rho_0((2\text{–}3.0)\rho_0)$ for RMF, corresponding to the use of ND-Soft (Ehime) potential. This is caused by a gradual (rapid) increase of Y_Λ and a maximum Y_Λ of about 10% (25%) for PD (RMF).

Here we comment that the use of an effective interaction for the pairing problem, as in Ref. 4), is not adequate. This is because the gap equation itself deals with the short-range correlation (s.r.c.), that is, the function $\Delta(q')/\sqrt{\tilde{\epsilon}^2(q')+\Delta^2(q')}$ together with the integration over q' plays the role of including both the s.r.c. and the pairing correlation.[8),12)] Therefore if we use $\tilde{V}_{\Lambda\Lambda}$ in place of $V_{\Lambda\Lambda}$, it leads to a "double counting" for the effects of s.r.c. In fact, the values of T_c from $\tilde{V}_{\Lambda\Lambda}$ are remarkably larger than those from $V_{\Lambda\Lambda}$, as shown in Fig. 5, where the parameter values $(Y_\Lambda, m_\Lambda^*, \rho = 2.5\rho_0)$ are taken the same as those used by Balberg-Barnea.[4)] This point has been also checked for the case of the $^1S_0$-gap of protons admixed in neutron star matter, by comparison between the gap Δ from the "bare" RSC potential $V_{\rm RSC}$ and $\tilde{\Delta}$ from the G-matrix effective potential $\tilde{V}_{\rm RSC}$. For instance, $\tilde{\Delta} \simeq 3.8$ MeV is larger by about a factor 7 than $\Delta \simeq 0.53$ MeV in the case with $\rho = 3\rho_0$, $Y_p = 0.05$ and $m_p^* = 0.7$. A way to remedy the defect of a too large gap for the effective interaction approach is to restrict the integration over q' near the Fermi surface, where the pairing correlation dominates. However, there arises a serious problem of how to quantitatively set the restriction region. Therefore, in order to unambiguously obtain an energy gap, we must solve Eq. (1) exactly. This is possible by suitably taking the starting values for the gap function in the iterative method,[12)] as we did in this work.

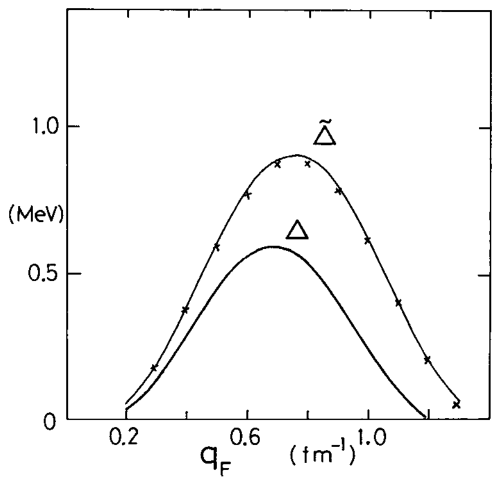

Fig. 5. Comparison of $^1S_0$ gap Δ from the "bare" pairing interaction $V_{\Lambda\Lambda}$ and that $\tilde{\Delta}$ from the effective one $\tilde{V}_{\Lambda\Lambda}$, for the same parameters $(Y_\Lambda, m_\Lambda^*, \rho = 2.5\rho_0)$ as used by Balberg-Barnea.[4)] Crosses indicate the results in Ref. 4) for $\tilde{V}_{\Lambda\Lambda}$. The abscissa corresponds to the Fermi momentum of Λ $q_F = (3\pi^2\rho Y_\Lambda)^{1/3}$.

Concluding remarks We have made a realistic study of Λ-superfluidity by closely considering the better selection of the pairing interaction and by taking into account the ρ- and Y_Λ-dependences of m_Λ^*. The main conclusions are as follows. Λ particles in the hyperon core of neutron stars can be in a superfluid state with critical temperature 10^{8-9} K. This originates mainly from a larger effective mass of Λ compensating for weaker pairing attraction, compared with the nucleon case. Superfluidity is realized as soon as Λ hyperons begin to be mixed at $\rho = \rho_t \simeq 2\rho_0$ with the sizable fraction ($Y_\Lambda \sim 1\%$), but it disappears at a higher density, where Y_Λ amounts to $\sim 10\%$ (15%), corresponding to PD (RMF). The density region for Λ-superfluidity to exist is found to be $\rho \simeq (\rho_t$–$\rho_d)$ with the disappearance density $\rho_d \simeq (2.6$–$4.6)\rho_o$ depending on the pairing interaction and the hyperon core model.

The existence of the restricted density region mentioned above implies that the effects of Λ-superfluidity on the cooling problem depend interestingly on the mass M of neutron stars. This is because neutron stars with larger M (hence higher central density ρ_c) would have a central region occupied by normal Λ-fluid when ρ_c exceeds ρ_d, lacking the suppresion mechanism for too rapid cooling. For example, if we consider neutron stars based on the Bethe-Johnson equation of state (BJ-1H)[13] and the superfluid region $(2$–$3.7)\rho_0$ from the PD plus ND-Soft case, neutron stars with $M \gtrsim 1.5M_\odot$ suffer this problem. This suggests that neutron stars rapidly cooled by hyperon cooling compatible with observation would be less massive stars, with $M \lesssim 1.5M_\odot$.

In this paper, discussion has concentrated on Λ-superfluidity. To make a full study of the hyperon cooling scenario, it is necessary to investigate the superfluidities of other hyperons, such as Σ^- and Ξ^-, admixed in neutron star cores. This remains as a future subject.

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