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Superhumps, resonances and accretion discs

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SUMMARY

The structure of accretion discs within binary systems is shown to be influenced by the excitation of resonances within the disc. Of particular importance is that near the 3:1 commensurability with the stars' orbit. This can be used to explain the superhump phenomenon of SU Ursae Majoris dwarf novae in superoutburst. This resonance can only appear for mass ratios which satisfy $M_2/M_1 \lesssim 0.25$–0.33: for larger mass ratios the available resonances are weaker and of the wrong form to produce the superhump phenomenon. The mass-transfer stream is shown to be an important contributor to the growth rate of the resonance.

1 INTRODUCTION

The dwarf novae known as SU Ursae Majoris (SU UMa) stars, characterized by their occasional very extended outbursts (superoutbursts), possess an important and intriguing photometric property: the superhump. This is a modulation of the optical light with a period a few per cent longer than the orbital period which always appears during the superoutburst. Numerical simulations carried out by one of us (Whitehurst 1988a) predict such a modulation provided that a steady-state disc exists in a system where the ratio $q=M_2/M_1$ of the secondary's and the primary's masses is small enough. The inviscid approximation to the critical value q_c of this ratio gave $q_c=0.25$.

In the following sections we will develop the arguments presented by Whitehurst (1988a) more fully by using more recent calculations. This allows us to show the physical origin of the superhumps are resonances near the 3:1 commensurability between material orbits in the disc and the motion of the secondary star. Furthermore, the value of the critical mass ratio q_c discussed above is found numerically to agree with that given earlier; however, to determine it exactly for a particular system the role of the viscosity must be taken into account, and in practice q_c lies in the range ~ 0.25–0.33.

2 RESONANCES

A crucial aspect of the superhump phenomenon is the resonant behaviour of the accretion disc in the potential-well of the secondary. The nature of this process is described briefly below. (Note that from here onwards the term 'particle' is used loosely to describe both numerical entities in a computer simulation and envisaged coherent gas flow in a real accretion disc.)

Resonance occurs in a disc when the frequency of radial motion of a particle orbit in the disc is commensurate with the angular frequency of the secondary star as seen by the particle. Hence, as the particle moves in and out around its orbit it will always 'see' the secondary in the same set of directions after each cycle. Now the epicyclic frequency is given by $\Omega-\omega$, where Ω is the particle's mean angular frequency around the primary and ω the apsidal precession frequency of its orbit, both measured in a non-rotating frame. Furthermore, the synodic frequency (which measures the relative motions of the particle and the secondary) is $\Omega-\Omega_{\rm orb}$, where $\Omega_{\rm orb}$ is the orbital frequency. Thus resonance requires

$$k(\Omega-\omega)=j(\Omega-\Omega_{\rm orb}), \qquad (1)$$

where j and k are positive integers.

An important aside is the observation that cases with $k>1$ only arise when orbital symmetry breaks down. This could be due to a small eccentricity of the binary orbit but here results from the random radial excursions of material in the disc caused by the viscosity.

The apsidal precession rate is zero for periodic (self-closing) orbits in the disc and in any case much smaller than Ω and $\Omega_{\rm orb}$ for aperiodic (rosette) orbits, so resonances appear close to commensurabilities of the form $j{:}j-k$ between particle and binary frequencies. The strength of a resonance is measured by its growth rate which goes as e^k, where e is the eccentricity of the particle orbit. Note that here we implicitly assume that the orbits can be characterized as Keplerian type ellipses; this is in fact a good approximation in almost all cases. Thus the strongest resonances arise at the $j{:}j-1$ commensurabilities, followed by $j{:}j-2$ and so on. This implies that near a given commensurability the orbits with the largest eccentricity compatible with stability will resonate with the secondary most strongly. Generally, these are aperiodic orbits with non-vanishing precession rates ω. (The aperiodic orbits near periodic orbits in phase space have higher eccentricities: this means that those orbits with the fastest growth rates will be aperiodic.)

It should be noted here that in an accretion disc truly periodic orbits will not be the norm; this is because material that follows such orbits conserves both energy and angular momentum; a disc consisting of such orbital streamlines cannot either evolve or even radiate heat! Any accretion disc we can see dissipates heat and therefore must be both (locally) evolving and contain aperiodically orbiting material.

Now because periodic orbits have the lowest degree of self-intersection they are energetically most favoured by the viscous process; the dissipation of energy within the disc acts to reduce the disc to concentric, periodic orbits centred around the primary. However, at some (probably small) level the resonances with the secondary will successfully oppose this trend, by acting as filters picking out some of the aperiodic orbits populated by the viscous perturbations. Note then that the tidal resonances are a viable source of the turbulent motions within the disc which provide the disc's viscosity.

The next obvious question is where in the disc we expect to find such resonant conditions. Now by assuming that even quite extreme aperiodic orbits do not differ greatly from Keplerian circles, we may measure the typical radii of resonant orbits near the general $j{:}k$ commensurability as

$$R_{jk}=(GM_1/\Omega_{jk}^2)^{1/3}. \tag{2}$$

Using Kepler's law the binary separation is

$$a=[G(M_1+M_2)/\Omega_{\rm orb}^2]^{1/3}, \tag{3}$$

so we find

$$R_{jk}/a=[(j-k)/j]^{2/3}(1+q)^{-1/3}. \tag{4}$$

To see what resonances are possible for a given mass ratio, we compare this with the disc's tidal radius $R_{\rm T}=0.9\,R_{\rm Roche}$, given by

$$R_{\rm T}/a=0.45/[0.6+q^{-2/3}\log_e(1+q^{-1/3})] \tag{5}$$

(Eggleton 1983), and ask when $R_{jk}<R_{\rm T}$.

It is readily apparent that the strongest ($k=1$) resonances can only occur for very small mass ratios: $j=2$, $k=1$ requires $q\lesssim 0.025$, with still smaller ratios required for larger j. The 2:1 resonance is therefore only possible in systems with mass ratios too extreme to be SU UMa dwarf novae but may be possible in some low-mass X-ray binaries.

For a possible explanation of the superhumps in SU MUa stars we must turn to the next strongest ($k=2$) resonances. The smallest radius is for $j=3$, and is inside $R_{\rm t}$ for $q\lesssim 0.33$. Larger values of j require ratios at least as extreme as for the 2:1 resonance. Since larger k values give weaker resonances, this shows that the strongest resonance possible in the discs of SU MUa stars is likely to be at $j=3$, $k=2$, and requires $q\lesssim 0.33$. Fig. 1 shows the tidal, 2:1 and 3:2 resonant radii as a function of mass ratio.

Once we have established the location of the resonances it is fair to question what the impact of this phenomenon is upon the disc. Since the 3:1 commensurability is the one we believe to be most significant we will dwell upon this resonance. By using the method of Hénon (1965) the stability of periodic orbits within a binary potential can be investigated. This is useful if we proceed as follows; first we assume that the disc is initially in an ideal state consisting of a concentric series of periodic (and therefore inviscid) orbits. Now the perturbations applied by the tidal forces can either

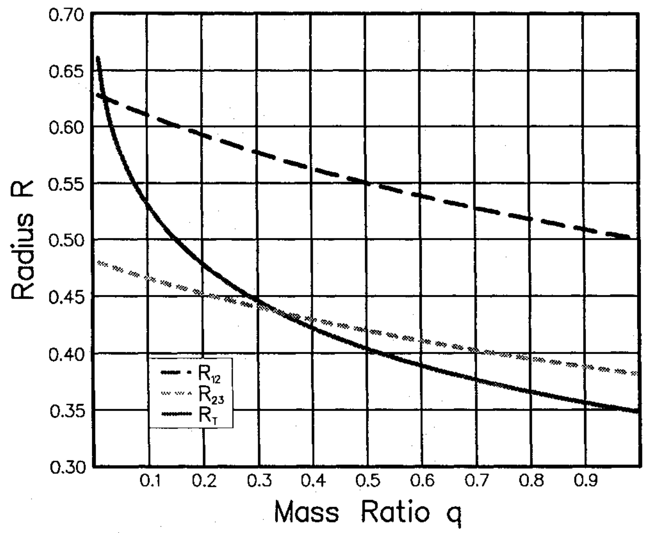

Figure 1. Plot showing the relationship between the resonant radii R_{21}, R_{32} and the tidal radius $R_{\rm T}$ with mass ratio q.

drive the orbit significantly away from the reference periodic orbit (an unstable or resonant orbit) or merely maintain some small degree of aperiodicity and hence turbulent viscosity (a stable orbit). We would therefore expect the resonant orbits to agree with the unstable orbits found by Hénon.

In repeating the calculations of Hénon our work is made much simpler by our only needing to consider simple, prograde orbits of the primary alone. The results of these calculations can be encapsulated nicely by the characteristic curves presented below in Fig. 2. Each curve demonstrates the size and stability of a single class of orbits for a given mass ratio. This is accomplished by plotting the orbits' intersections with the x-axis (which conventionally joins the stars' centres in the Roche potential) against the Jacobi constant, C, for each orbit. For the restricted three-body problem the Jacobi constant is the only constant of the motion and represents a modified energy integral. It is given by

$$C=(x+1)^2+y^2-\dot{x}^2\dot{y}^2+\frac{2\mu}{\sqrt{x^2+y^2}}+\frac{2(1-\mu)}{\sqrt{(x+1)^2+y^2}}, \tag{6}$$

where μ is the primary's mass fraction, $\mu=M_1/(M_1+M_2)$, and x and y are the coordinates of the considered point measured from the primary's centre. The knowledge that the orbits shown in Fig. 2 intersect the positive x-axis at right-angles along with the values of their Jacobi constants allow these orbits to be determined uniquely.

The abscissa shows the Jacobi constant for each orbit, the ordinate giving the distance of the orbit from the primary along the stars' line-of-centres. Note that negative ordinates lie between the stars. When a line is solid it represents sets of stable orbits, dotted lines showing unstable orbits. From these curves the size of an ideal accretion disc can be measured. The tidal radius corresponds to the maximum size the stable orbits achieve. This is reached at some small value of the Jacobi constant; as this constant decreases further stable orbits become smaller than this maximum. In an accretion disc these orbits will be naturally forbidden

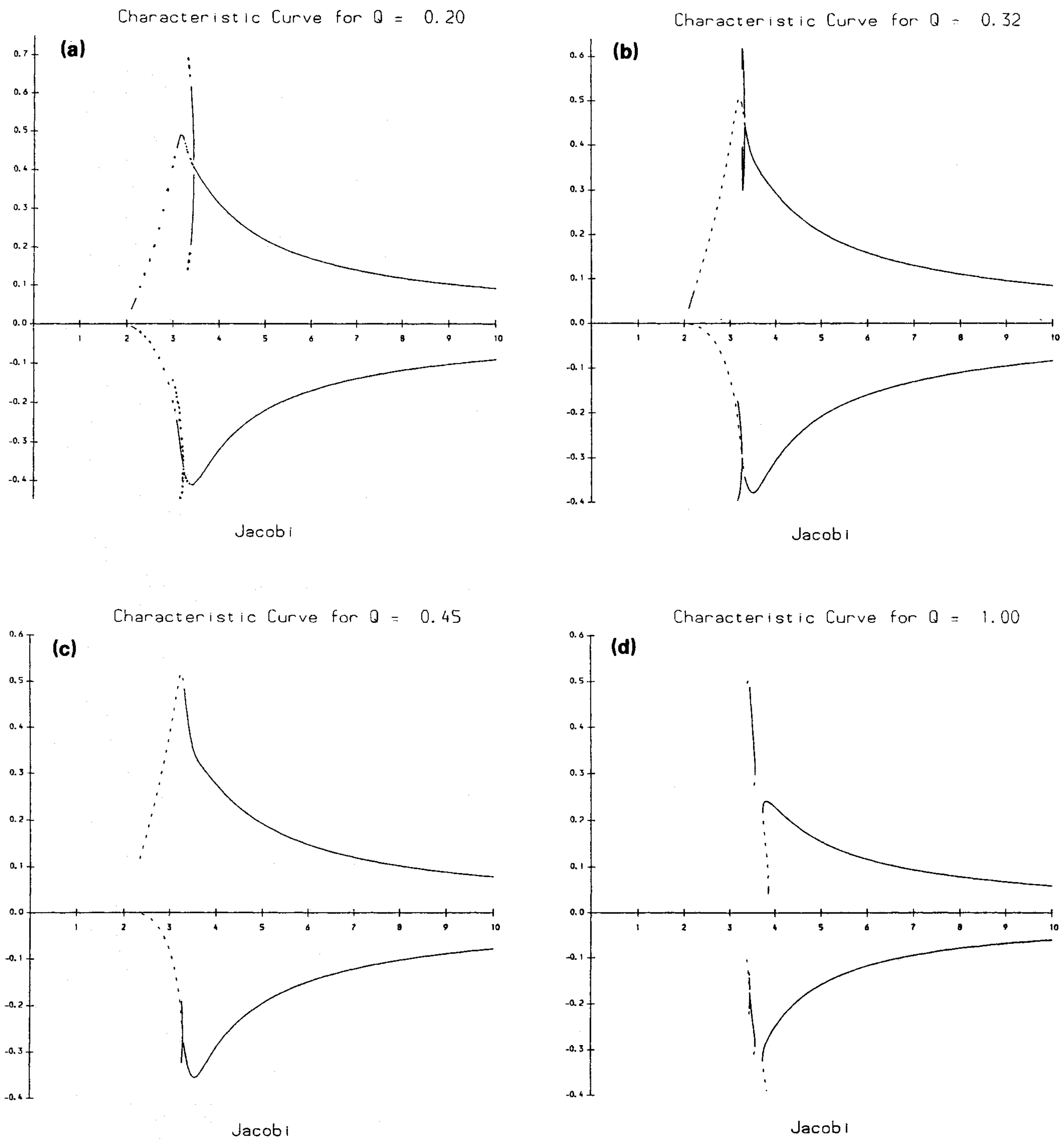

Figure 2. Characteristic curves for mass ratios $q = 0.2, 0.32, 0.45$ and 1.0.

because the inner orbits are already populated by material with higher values of the Jacobi constant.

Fig. 2 presents characteristic curves for a range of mass ratios. For mass ratios near unity there are two detached regions of stability (see Fig. 2d). Accretion discs in systems with this mass ratio will develop with orbits represented on the high-valued Jacobi constant branch of this curve. These discs will be simply tidally limited. At smaller mass ratios (Fig. 2c) these branches merge and between the stars an unstable region develops. This is marked by a bifurcation (period-doubling) of the stable orbit. This doubly periodic orbit cannot be achieved in an accretion disc however, because it lies well beyond the tidal radius of the disc (see Fig. 3c). These doubly periodic orbits themselves either reconnect to the singly periodic orbits or undergo further period-doublings until no stable periodic orbits exist. At

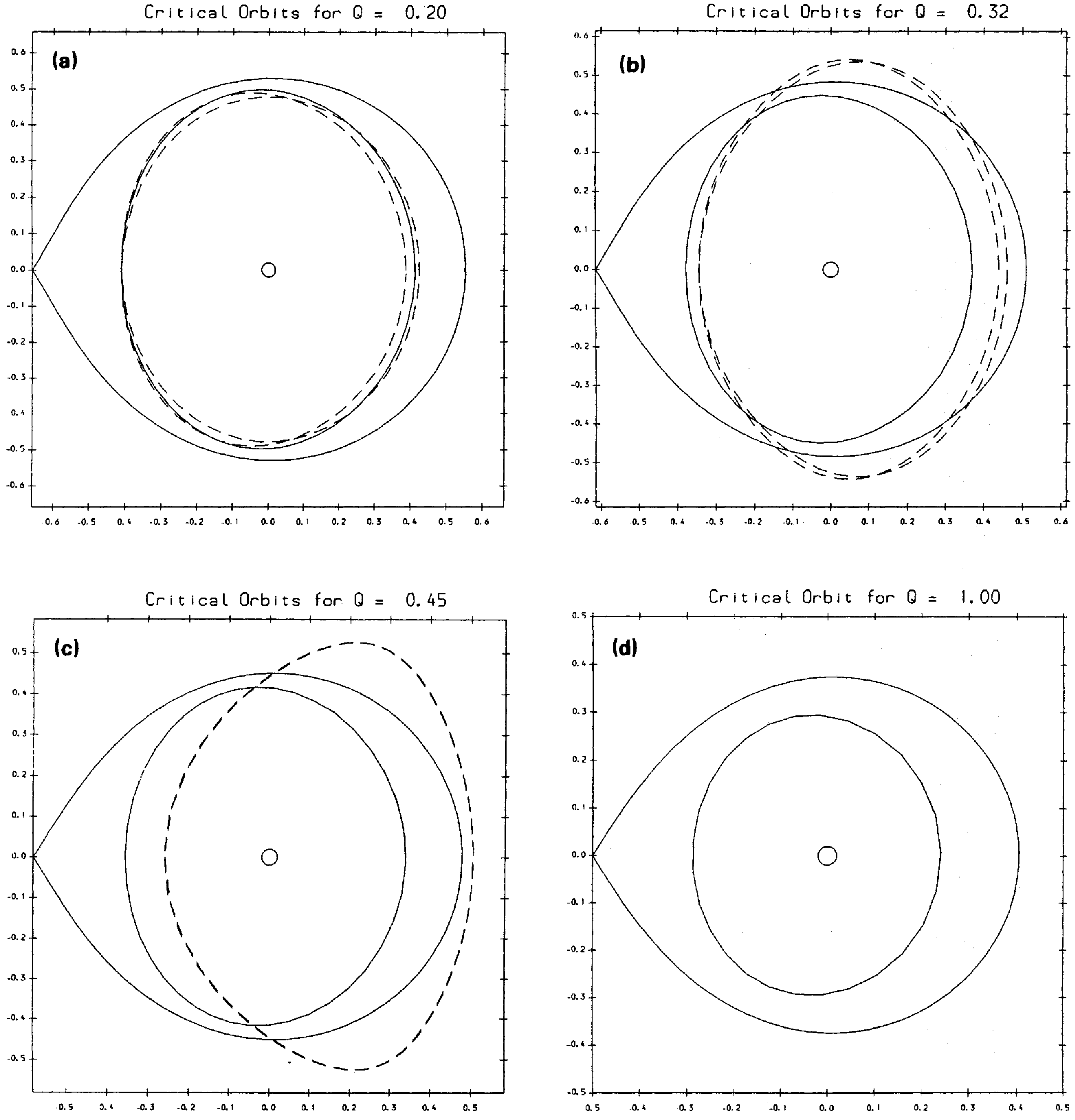

Figure 3. First stable doubly periodic orbits (dashed lines) and the limiting tidal orbit (solid line) for the mass ratios $q=0.2$, 0.32, 0.45 and 1.0.

mass ratio $q=0.32$ another unstable regime develops opposite the primary (Fig. 2b). It is just beyond the tidal radius of the disc but may still be excited by the presence of viscosity allowing the accretion disc to grow slightly beyond its inviscid tidal radius. At mass ratios of $q=0.25$ or less (Fig. 2a shows an example for $q=0.2$) this unstable region lies within the tidal radius and so will always be excited in an accretion disc. Fig. 3 shows the tidal and limiting unstable orbits corresponding to Fig. 2. Note that the value of q, where the instability of singly periodic orbits to period-doubling, is exactly that expected from the analysis summarized by Fig. 1, namely $q \lesssim 0.33$.

3 SIMULATIONS

How the instabilities and resonances described above lead to the superhumps of SU UMa stars is best demonstrated with the help of a computer simulation. The simulation presented here is a simple two-dimensional model of an accretion disc in a binary potential with a mass ratio q of 0.12. It is calculated with th same methods as described by Whitehurst (1988b). One change is that the mass-transfer stream is not included, in order to remove any contribution it may make to any instability. Instead material is (artificially) inserted on a circular Keplerian orbit with the amount of angular

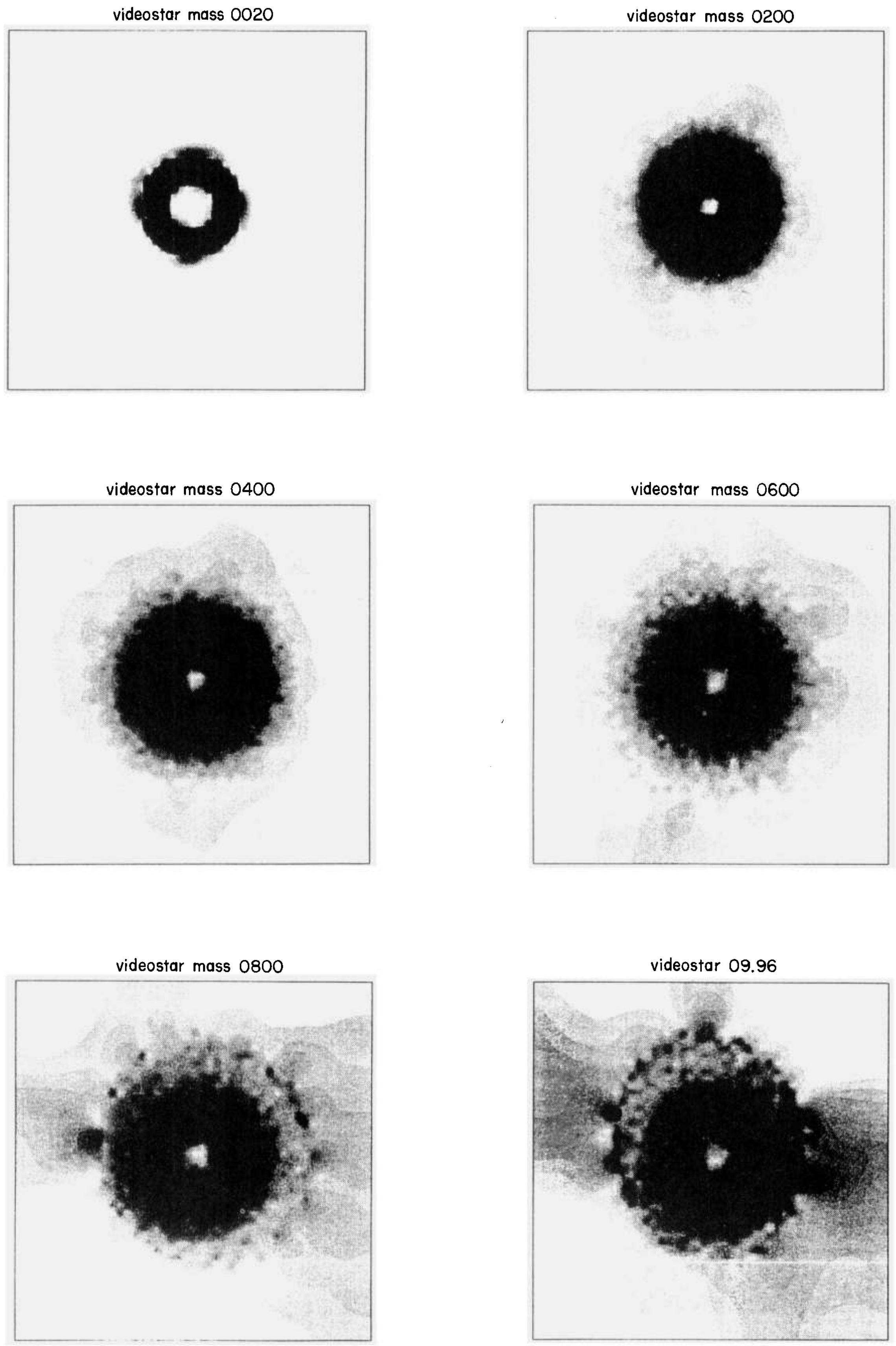

Figure 4. Evolution of the mass distribution in a disc simulation (simulation 'videostar') mass ratio $q = 0.12$, snapshots at intervals of approximately 20 orbital periods. (About 30 hr physical time.)

momentum it would have had at the inner Lagrangian point. (In other words at the circularization radius r_{circ}.) The disc forms as before however: initially the primary's Roche lobe is devoid of matter and a disc gradually forms as the material spreads under the influence of the disc's 'viscosity'.

Fig. 4 shows a sequence of snapshots of the disc forming, and it is clear that the disc is asymmetric and ultimately forms a ring of material which collects at its outer edge, in other words a rim. The plots are logarithmic grey-scales of the density distribution within the disc, dark shades representing high density. (The logarithmic scale suppresses contrast so making low-density structure more apparent.) On all plots a square image covering the whole of the primary's Roche lobe is shown with neither the primary nor the secondary indicated. However, the primary is at the centre of the plotted area whilst the secondary lies on the x-axis away to the left; its surface would just meet the plotted area at the extreme left of the square (the inner Lagrangian point).

The rim that forms at the edge of the disc has its angular momentum removed by the tidal action of the secondary. Fig. 5 shows a sequence of snapshots one-fifth of an orbital period apart; the rim of the disc is clearly following a very non-circular orbit about the primary. By comparing snapshots of the disc exactly one-half of an orbit phase apart (Fig. 6), we can see that the disc's rim is following a doubly periodic orbit about the primary. When compared with Fig. 3 the motion is seen to be more extreme; this is because in Fig. 3 the dashed line indicates an orbit which is just doubly periodic. Resonances, as discussed earlier, favour more eccentric orbits such as those visible in Fig. 6. In Fig. 6 the underlying doubly periodic orbit crosses at approximately the 1, 5 and 9 o'clock positions. The outer track of the doubly periodic is on the right at time 91.5 and on the right at time 92.0. The orbit is not uniformly populated however, with only about one-half of the orbit containing a significant excess of material. This means that the form of the disc is modulated on the period of the doubly periodic orbit. This also implies that the dissipation within the disc is modulated on this period; this period and its corresponding variations can be identified with the superhump period and the superhump itself. Fig. 7 shows the relative dissipation within the disc as a logarithmic contour plot at the moments corresponding to the images of Fig. 5. The two arms of the doubly periodic orbit are evident in this figure.

It follows from the above that the behaviour of the superhump can be now understood, at least qualitatively. As the resonant-instability criterion is reached the perturbations applied by the secondary's tidal forces are strong enough to drive material away from the singly periodic orbits towards a family of doubly periodic orbits. However, the disc's material is unable to follow these orbits exactly or indeed to populate these orbits uniformly because of self-intersections. This self-collision occurs both because the inner loop of the doubly periodic orbits lies within the neighbouring singly periodic orbits and the doubly periodic crosses itself at three points. Dissipation forces any random irregularities along this orbital track to be amplified and soon one loop of the orbit must dominate; by a simple 'snow-plough' effect it will keep the other loop free of material. In this way the disc develops a rim of material which follows an approximately doubly period orbit around the primary. The interaction between this rim and the remainder of the disc causes a modulation in the disc's shape and light which is observed locally as the superhump.

It should be noted that this behaviour only occurs because the form of the viscosity used in the numerical simulations preserves the vector information in the flow. Numerical methods which use a scalar rule for transferring angular momentum will not reproduce this phenomenon. It is, however, important to note that this phenomenon is in no way a 'viscous instability'. As previously discussed by Whitehurst (1988a), the tidal properties alone are important in determining the superhump period (different viscosity parameterizations are found not to change the precession period and other properties of the superhump).

4 THE SUPERHUMP PERIOD

Definitive periods of the superhump can only be obtained from the simulations themselves. These are found to be always of the form $P_{SH}=P_{orb}+\Delta P$, where P_{SH} is the superhump period, P_{orb} the orbital period and $0<\Delta P\ll P_{orb}$. In practice ΔP is found to be dependent on the mass ratio q and to have a length a few per cent of that of the orbital period. This behaviour is largely to be expected, as described below.

It would seem at first that the superhump period would be simply the period of the doubly periodic orbits for each mass ratio. It is not quite that simple, however. First of all, for each mass ratio there is a family of doubly periodic orbits, each of which has a slightly different period. A more serious complication is that the orbits of material in the rim are not exactly equivalent to the doubly periodic orbits found by inviscid approximations. This is due to the influence of the material within the rim whose bulk resists the rim's inward excursion. This tends to increase the orbit's size and hence its period, and so complicates finding the superhump period by analytic means.

It is clear that the 3:1 commensurability must always occur near singly periodic orbits with a period of a half that of the orbit (in the corotating frame of reference). This means that the doubly periodic orbits will be always approximately equal to the orbital period of the system. Also, it is clear that, because of the influence of the disc, the orbit will always tend to be lengthened, so giving a period longer than one orbital period. Numerical simulations have always shown this superhump period to be longer than the orbital period; although there seems no compelling reason why a period slightly less than the orbital one cannot also occur in systems with very extreme mass ratios.

The parametric dependence of the superhump period on the orbital period can also be determined. When viewed in the reference frame, corotating with the binary, the resonantly driven doubly periodic orbits are seen to circulate twice. This implies that in the sidereal frame these (simply periodic) orbits are seen to circulate *three* times before closing on themselves, resulting in rosette orbits whose periods are 3 times those of the simply periodic orbits just inside the resonance, i.e. $6\pi/\Omega$. A similar result must hold for the aperiodic (precessing) orbits with non-zero ω which we know are the most likely to resonate in accretion discs. Since these have frequencies $\Omega=3\Omega_{orb}-2\omega$ just inside the resonance near the 3:1 commensurability (equation 1), the circulating orbits must have radial periods

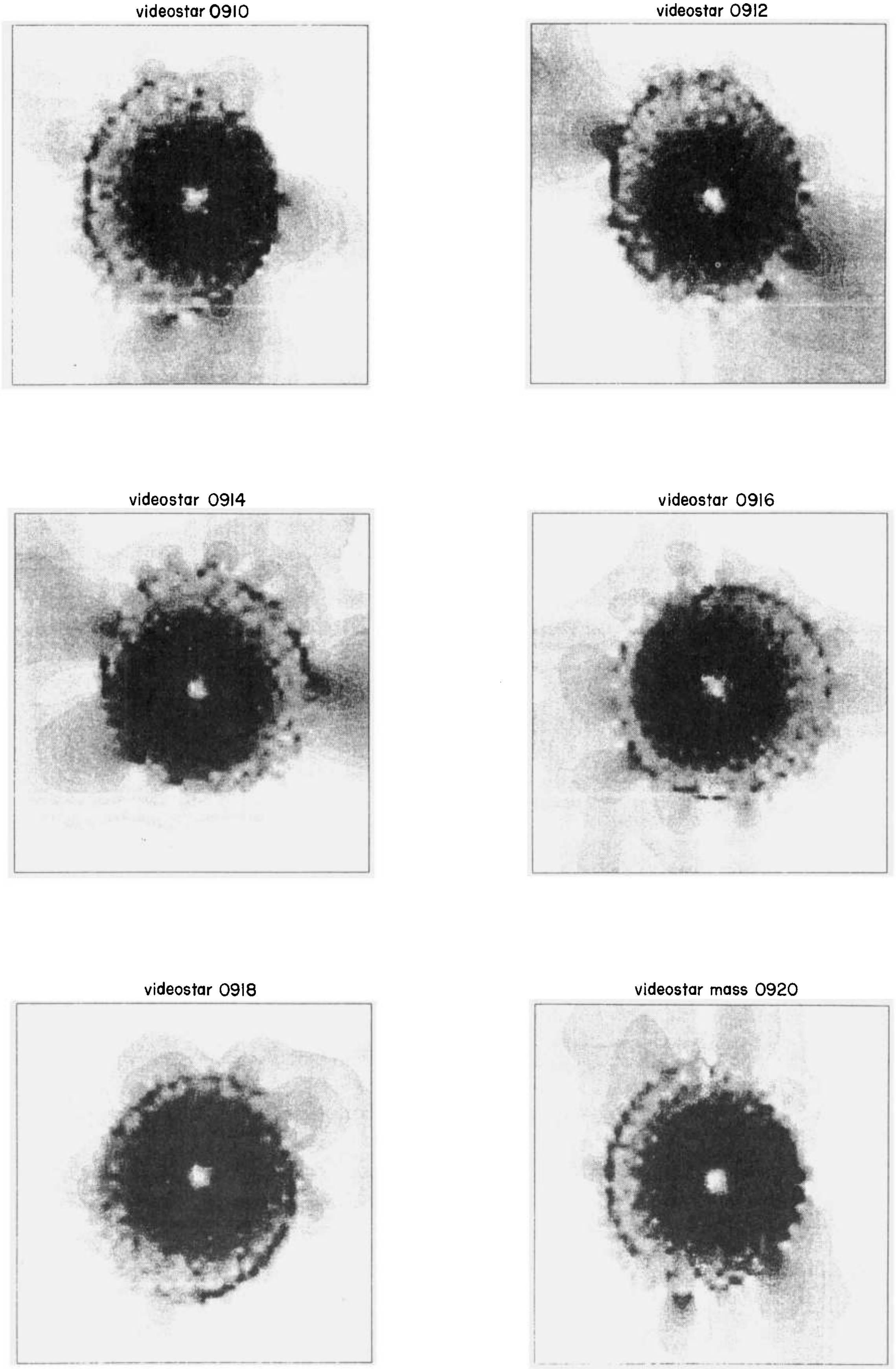

Figure 5. The resonantly excited disc with a rim following a doubly periodic orbital track. Snapshots are one-fifth of an orbital period apart.

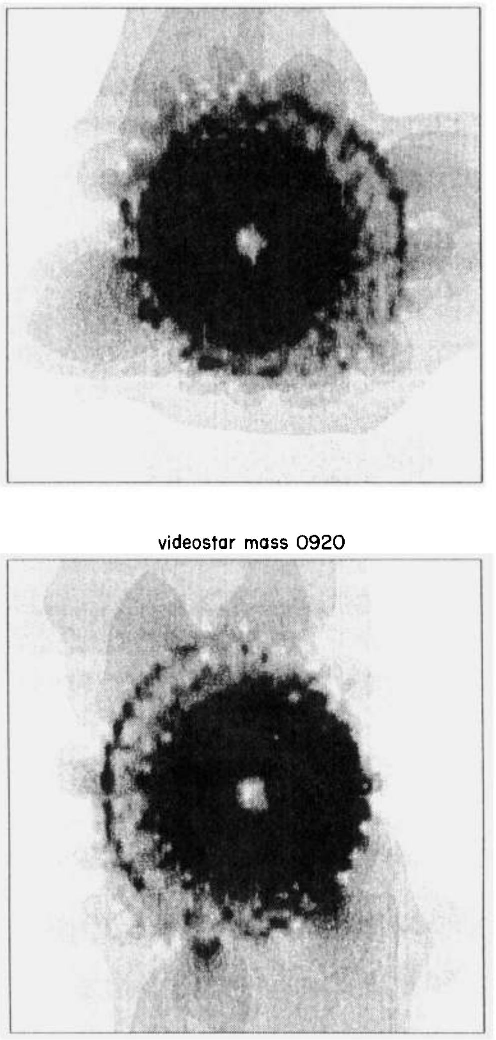

Figure 6. Opposite extremes of the doubly periodic orbit.

$$P_{SH} = 6\pi/\Omega \approx P_{orb}(1 + 2\omega/3\Omega_{orb}), \tag{7}$$

where $P_{orb} = 2\pi/\Omega_{orb}$ is the binary period and we have used the fact that $\omega \ll \Omega_{orb}$. Assuming the precession to be prograde ($\omega > 0$) gives that the radial period of the resonant orbits is slightly longer than P_{orb}. We cannot calculate by exactly how much without knowing exactly which precessing orbit resonates. Since, however, ω must vanish in the limit of vanishing q, we expect the fractional difference between P_{SH} and P_{orb} to be proportional to q. (If it were so simple it would be hardly necessary to construct two-dimensional simulations in the first place!) Note that the viscosity only plays a role in determining how fast material is transferred to the resonant orbits, i.e. in the growth rate of the resonance, and in the second-order effects determined by the relative density of the various orbits. The first-order effects of orbital size and period are entirely determined by the tidal influences of the secondary.

Despite the uncertainties we can still see that P_{SH} has precisely the properties of the observed superhump period. (And, for that matter, the properties of the simulations' 'superhump' period.) It is interesting to note that because the superhump *period* is independent of the disc's viscosity and density it will remain consistent from outburst to outburst. However, the superhump's growth rate and detailed shape should be dependent on the disc's properties, and so, in principle, may change in different outbursts. Furthermore we would expect the superhump's period to remain constant through a single superoutburst, but not necessarily assume the same to be true of its structure. All these expectations are borne out by observations of SU UMa systems.

5 SUPERHUMPS AND SUPEROUTBURSTS

Recently Osaki (1989b) has argued that the normal outburst and superoutburst cycle of SU UMa stars is a result of a combination of the tidal instability mooted by Whitehurst (1988a), and the well-known thermal instability of accretion discs. In this model the normal outbursts are all too small to clear the disc of all its material and the disc grows until the critical size of the instability is reached during an outburst. At this point the disc becomes unstable which sustains a prolonged outburst which clears the disc of the mass accumulated since the previous superoutburst.

This mechanism is intriguing but has difficulties that are related to the growth rate of the superhump in the disc. Osaki's (1989a) analysis of the nature of the resonance (3:2) discovered by Whitehurst (1988a) and confirmed by Hirose & Osaki (1989) is correct and agrees with our identification but fails to consider the consequences of this resonance. As we have shown, the effect of the resonance is to excite the rim of the disc to follow a doubly periodic orbit. This excitation is not, however, instantaneous but proceeds at some rate.

As stated above, for perfectly symmetrical orbits near the 3:1 commensurability the growth rate of the disturbance is proportional to the square of both the eccentricity of the particle orbit, e, and that of the binary star e'. This gives a growth rate, g, of the form $g \propto e^2 e'^2$ (Greenberg 1984). However, in the simulations $e' = 0$, and the growth of the resonance is due to the presence of viscosity which breaks the symmetry of the disc's orbits. This, as argued above where we saw that perfectly periodic orbits cannot alone exist in accretion discs, must also be true in nature. With a typical viscous scalelength λ, the growth rate g is expressible as

$$g \propto \frac{e^2 \lambda^2}{a^2}, \tag{8}$$

where a is the semi-major axis of the binary orbit. (Fluctuations of size λ about an orbit with semi-major axis a mimic an eccentricity $e' \simeq \lambda/a$.) Since we are dealing here entirely with 'natural' units if we follow normal alpha-disc assumptions, in particular that turbulent cells have dimensions comparable to the disc's thickness so that $\lambda \simeq 0.01a$, we have that $g \lesssim 10^{-4}$, or that the natural growth time will be of the order of 1000 periods. This agrees with the numerical simulations where larger values of λ (0.05 and 0.025, respectively) gave growth-times of about 60 and 250 periods, respectively. Fig. 8(a) and (b) show the growth of the instability through the change in the disc's angular momentum; the rapid oscilla-

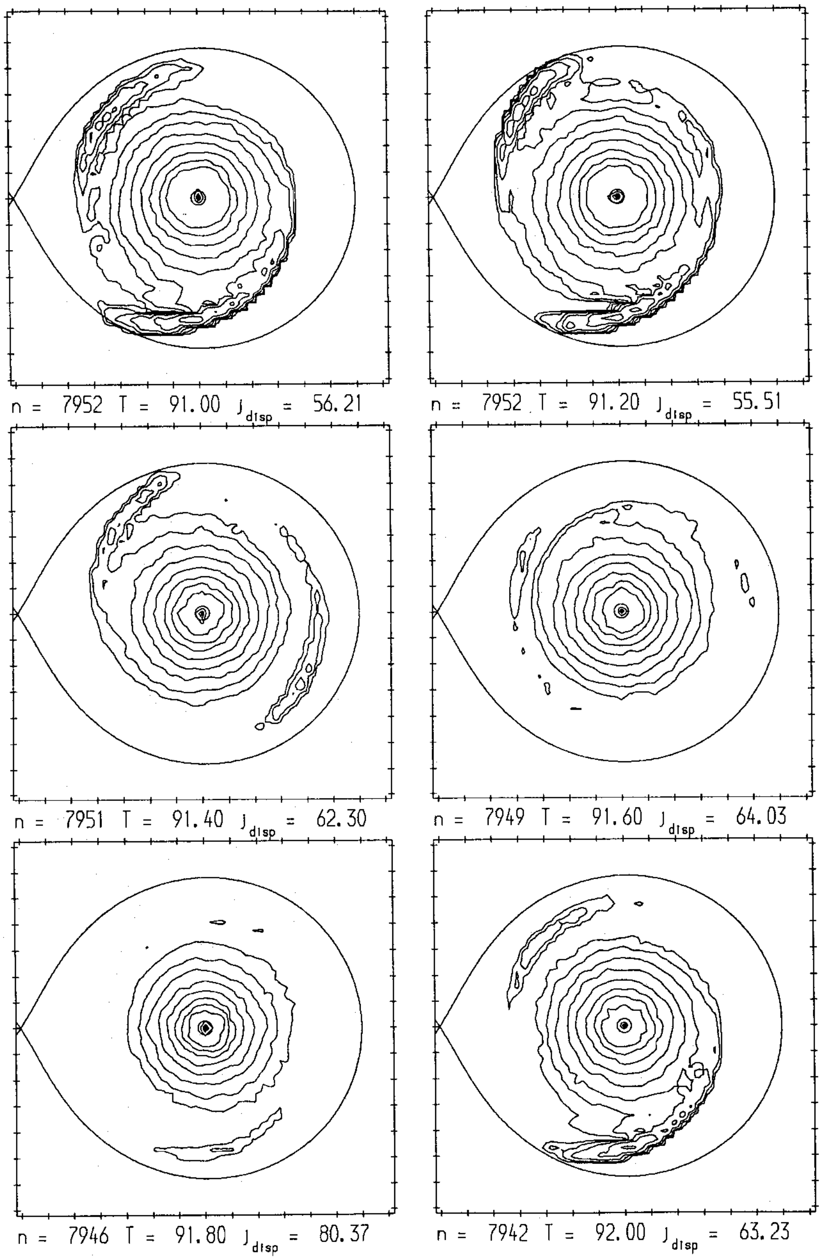

Figure 7. Viscous dissipation within the disc. Dissipation is concentrated at the disc's centre, but two arms which correspond to the tracks of the doubly periodic orbit are also visible.

tions are due to the varying phase of the disc's modulation at the instants the data points were calculated; the time-averaged value would remain negative and fairly constant.

Note that the delays in the onset of the superhump in the above simulations would correspond to growth times of about 2 and 10 d for the two simulations. The lower value corresponds to a very high viscosity limit of $\alpha \simeq 5$, so the suggested value of $\lambda \simeq 0.01$ which corresponds to $\alpha \simeq 1$ a waiting time of about a month would be required. These waiting times are very much longer than the entire duration of normal outbursts and most superoutbursts in SU UMa stars. Therefore, it is not surprising that no superhumps are observed in normal outbursts – the disc has nowhere near enough time to trigger the instability. (Normal outbursts being of 2 to 3 d in duration are therefore about an order of magnitude too short.) At first sight it seems that we have replaced one problem with another; superhumps cannot occur in normal outbursts, but why do they happen at all? Surely superoutbursts are too short as well?

Both in the simulations discussed above and the foregoing analysis no accretion stream was present. This is important because it acts as a source of strong perturbations on the outer disc. In the simulations described by Whitehurst (1988a) it allows the disc to become unstable much faster by giving an extra perturbation effectively much stronger than that provided by the viscous terms alone. (Growth times of a

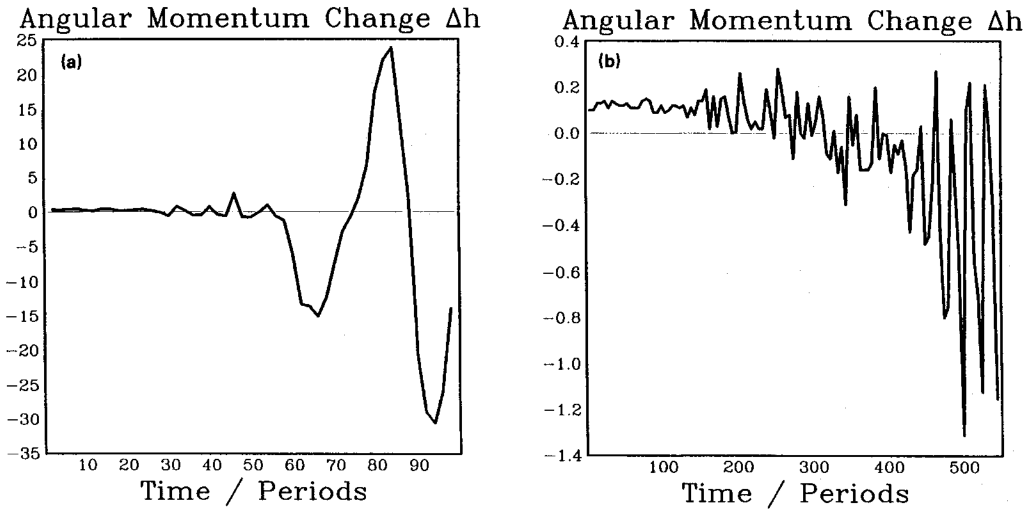

Figure 8. Growth of the resonant disc demonstrated by the angular momentum change in the disc. (The change is caused by the disturbance of the secondary's mass; it grows as the disc becomes doubly periodic in its outer regions.)

few dozen periods were observed in these simulations with $\lambda \simeq 0.03$, consistent with the observations of real SU UMa dwarf novae.)

The reason why the stream has such a dramatic effect is revealed by the simple argument that follows. If we assume that the stream is in fact feeding the superoutburst then the mass and momentum flux of the stream will balance approximately the loss of mass and momentum during a superoutburst. This means that in pseudo-steady state the stream will have a momentum flux equivalent to that of the disc's rim (if we assume all this material is deposited locally). Then the stream will have the same perturbing effect as a large turbulent eddy within the disc. However, since the size of this 'eddy' is not of order λ, but instead a sizeable fraction of the orbital separation, a, the local growth rates will be an order of magnitude larger.

It therefore seems most unlikely that the superhump is entirely a product of the disc, as envisaged by Osaki (1989b). It seems instead that superhumps are linked to superoutbursts by a common causative action; namely enhanced mass transfer from the secondary star. This has the effect both of sustaining the disc throughout the superoutburst and of accelerating the growth of the superhump within the accretion disc. The lack of an enhanced mass-transfer stream during normal outbursts is the natural corollary of this conclusion. This implies that normal outbursts are indeed driven by viscous instabilities, and superoutbursts are sustained by mass-transfer instabilities, perhaps similar to the form proposed by Bath (1974).

Another (possible) role for the stream is that it establishes an asymmetry between the otherwise equivalent loops of the doubly periodic orbit. This would mean that the superhump would be forced to be a single-humped feature per superhump period and not double humped. Without the stream there is no obvious means to prevent the superhump from becoming double humped. This is indeed observed in simulations allowed to run for long periods of time (when no stream is present) a second hump becoming visible at the counter-phase to the original superhump.

6 CONCLUSIONS

While not all the details of the superhump and superoutburst phenomena of SU UMa stars are yet understood, it seems we can now itemize their broad characteristics. In particular the arguments given above show compellingly that superhumps are a consequence of a tidal resonance with commensurability near 3:1 in the accretion discs of SU UMa systems. The resonance is fed by perturbations from both the stream and the viscous turbulence within the disc. Because the stream is potentially a much larger perturber, and superhumps are restricted to the relatively energetic superoutbursts, it seems likely that superoutbursts are fed by enhanced mass transfer. This argument is a major point against the superoutburst–superhump model of Osaki (1989b).

Furthermore, a mass ratio condition exists which limits superhumps to systems with mass ratios, q, satisfying $q \lesssim 0.33$, which agrees well with the distribution of SU UMa stars almost entirely below the period gap.

Future work should concentrate on the effects of a more complete gas dynamical treatment of the accretion disc. Although the simulations discussed here do include conservation of mass, momentum and energy that are equivalent to the well-known 'Smoothed Particle Hydrodynamics' (SPH) techniques (e.g. Monaghan 1985), they do not allow for the extra degrees of freedom available to a gas through heating and cooling. These terms probably have the effect of reducing the growth rate of the instability. However, calculations made using SPH or similar techniques may be misleading; this is because they introduce a scalar numerical viscosity which will tend to destroy the genuine momentum transfer of the disc by non-conservation of *local* angular momentum.

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