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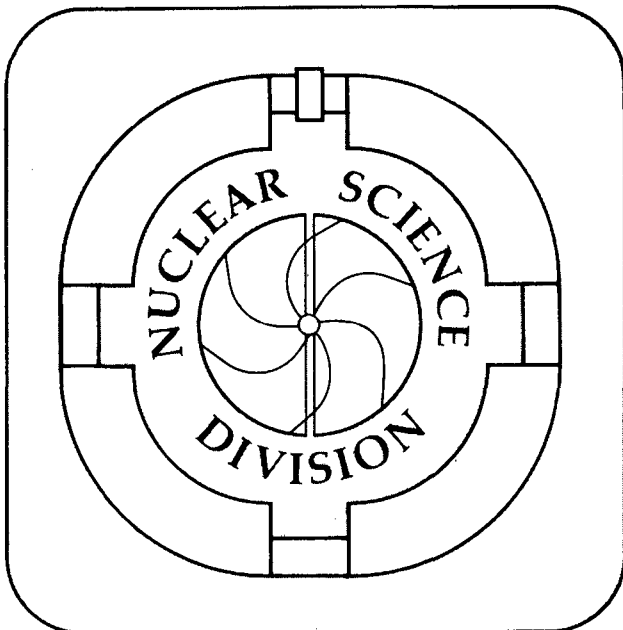
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SUPERLUMINAL TRANSFORMATIONS IN COMPLEX MINKOWSKI SPACES*

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Abstract

We calculate the mixing of real and imaginary components of space and time under the influence of superluminal boosts in the x-direction. A unique mixing is determined for this superluminal Lorentz Transformation when we consider the symmetry properties afforded by the inclusion of three temporal directions. Superluminal transformations in complex six-dimensional space exhibit unique tachyonic connections which have both remote and local space-time event connections.

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1. Introduction

It is well known that the superluminal Lorentz transformations (SLT), change real quantities into imaginary ones with their signs in order to preserve the magnitude of the line element in transforming between the moving frame (S' frame) and the laboratory frame (S frame) with superluminal relative velocity. For a superluminal boost in the x-direction, with velocity $v_x = \pm\infty$ along x-axis, the Lorentz transformations are⁽¹⁾

$$x' = \pm t, \quad y' = -iy, \quad z' = -iz, \quad t' = x, \quad (1.1)$$

where the upper sign corresponds to relative velocity, $v = +\infty$ and the lower sign corresponds to $v = -\infty$. Here x, y, z, t are real quantities forming a real four-dimensional Minkowski space. Similarly x', y', z', t' are also real quantities in the moving frame. The position vector and the time vector are separable quantities in both frames. However, the same superluminal transformations in complex Minkowski space exhibit the mixing of position and time vectors. This property of superluminal Lorentz transformations in complex Minkowski space has not been studied previously, and it is of interest to examine the behavior of faster than light particles (tachyons) in complex Minkowski space.^{2,3}

The purpose of the present paper is to show that superluminal transformations in the complex Minkowski space give an unique feature of the exchange of the real and imaginary parts of the space and time components which might exhibit new features of the tachyon type particles not previously postulated. We examine this point further in Section 3 and 4.

2. Complex Minkowski Spaces

The concept of complex Minkowski spaces is not a new one. It has been introduced by several authors in general relativity^{4,5} and in twistor theory.^{6,7} R. O. Hansen and E. T. Newman⁵ developed the properties of complex Minkowski space and explored the properties of this geometry in detail. Their formalism involves defining a complex Minkowski space which can be formed by coordinates labeled as

$$\tilde{z}^\mu = x_{\text{Re}}^\mu + ix_{\text{Im}}^\mu, \quad (2.1)$$

where \tilde{z} is a complex quantity while x_{Re} and x_{Im} are real quantities. Suffix Re and Im refer to the real and imaginary parts of the complex quantity \tilde{z} . The index μ runs over 0,1,2,3 representing 0 as time vector and 1,2,3, as spatial vectors. However, in the present paper we will use classical notation t,x,y,z to represent the four vectors of time and space.

In complex Minkowski space, these vectors are complex quantities defined as follows:

$$\tilde{t} = t_{\text{Re}} + it_{\text{Im}}$$

$$\tilde{x} = x_{\text{Re}} + ix_{\text{Im}} \quad (2.2)$$

$$\tilde{y} = y_{\text{Re}} + iy_{\text{Im}}$$

$$\tilde{z} = z_{\text{Re}} + iz_{\text{Im}}$$

Complex quantities as defined by eq. (2.1) or by eq. (2.2) give an eight dimensional space.⁸ A slice of this space gives four real dimensions of M^4 which will form a subspace in which the line element will be given by the real part of the complex quantities.⁴

It has also been suggested^{1,9,10} that in order to understand the appearance of imaginary quantities in the superluminal transformations, time should also be represented by a three dimensional vector, $\tilde{t}_x, \tilde{t}_y, \tilde{t}_z$, and that only its modulus $|t| = (t_x^2 + t_y^2 + t_z^2)^{1/2}$ has a direct physical meaning. This three dimensional time, introduced by P. Demers,⁹ is based on the symmetrical role of space and the time intervals in the relativistic physics. See Ref. (9) for details of the introduction of three dimensional time in relativistic physics. For the purpose of this present paper, we will consider time as a three dimensional complex quantity in order to analyze the superluminal Lorentz transformations in complex Minkowski space. We will define time as follows:

$$\tilde{t} = \tilde{t}_x \hat{x} + \tilde{t}_y \hat{y} + \tilde{t}_z \hat{z} \quad (2.3)$$

where

$$\tilde{t}_x = t_{x\text{Re}} + it_{x\text{Im}}$$

$$\tilde{t}_y = t_{y\text{Re}} + it_{y\text{Im}} \quad (2.4)$$

$$\tilde{t}_z = t_{z\text{Re}} + it_{z\text{Im}}$$

As before, suffix Re and Im refer to the real and imaginary part of the complex quantities. For example, both t_{Re} and t_{Im} are real quantities. Other formulations consider one complex temporal component or M^4 space but for symmetry considerations, we consider the six dimensional Minkowski space, M^6 . Consequences of superluminal, x direction boost give us a unique mixing of spatial and temporal real and imaginary quantities.

3. Superluminal Boost in Complex Minkowski Space

For a superluminal boost, $v_x = +\infty$, along the positive x direction we have the following transformations:

$$x'_{\text{Re}} + ix'_{\text{Im}} = t_{x\text{Re}} + it_{x\text{Im}} \quad (3.1)$$

$$y'_{\text{Re}} + iy'_{\text{Im}} = y_{\text{Im}} - iy_{\text{Re}}$$

$$z'_{\text{Re}} + iz'_{\text{Im}} = z_{\text{Im}} - iz_{\text{Re}}$$

$$t'_{x\text{Re}} + it'_{x\text{Im}} = x_{\text{Re}} + ix_{\text{Im}}$$

$$t'_{y\text{Re}} + it'_{y\text{Im}} = t_{y\text{Im}} - it_{y\text{Re}}$$

$$t'_{z\text{Re}} + it'_{z\text{Im}} = t_{z\text{Im}} - it_{z\text{Re}}$$

The transformations given in eq. (3.1) are the usual superluminal transformations which preserve the magnitude of line element but not the sign:

$$-x'^{\mu} x'^{\nu} = x^{\mu} x^{\nu} \quad (3.2)$$

In the direction orthogonal to the boost, the real part of the complex quantities transform to the imaginary part of the space and time components, and the imaginary parts of the complex quantities

transform to the real part of the space and time components. This is expected in superluminal Lorentz transformations. Now let us examine the effect of superluminal transformations on an event P located in the complex space and described by complex quantities.

An event P in the six dimensional Minkowski space, M^6 , with a three dimensional temporal component can be described by:¹

$$P = (\underline{x}, \underline{y}, \underline{z}, \underline{it}_x, \underline{it}_y, \underline{it}_z) \quad , \quad (3.3)$$

$$= (x_{Re} + ix_{Im}, y_{Re} + iy_{Im}, z_{Re} + iz_{Im},$$

$$i(t_{xRe} + it_{xIm}), i(t_{yRe} + it_{yIm}), i(t_{zRe} + it_{zIm})) \quad .$$

In eq. (3.3) we have written each component in terms of its real and imaginary parts.

Event P can alternatively be represented¹ in three dimensional complex space c^3 ,

$$P = (\underline{x} + \underline{it}_x, \underline{y} + \underline{it}_y, \underline{z} + \underline{it}_z) \quad . \quad (3.4)$$

Referring to eqs. (2.2) and (2.4), we can rewrite the event P in the complex space c^3 , as

$$\begin{aligned}
P = & \left((x_{\text{Re}} - t_{x\text{Im}}) + i(x_{\text{Im}} + t_{x\text{Re}}), (y_{\text{Re}} - t_{y\text{Im}}) \right. \\
& \left. + i(y_{\text{Im}} + t_{y\text{Re}}), (z_{\text{Re}} - t_{z\text{Im}}) + i(z_{\text{Im}} + t_{z\text{Re}}) \right) \quad (3.5)
\end{aligned}$$

In eq. (3.4), the event, P is represented in a three dimensional complex space, c^3 . In eq. (3.5), event P is represented by complex quantities in c^3 space. As can be seen, the real part of the time vector is mixed with the imaginary part of the space vector and the imaginary part of the time vector is mixed with the real part of the space vector. See Fig. 1 as an example.

The superluminal transformation¹ in the M^6 space will effect the transition from a generic six vector $P = (\underline{x}, \underline{y}, \underline{z}, \underline{it}_x, \underline{it}_y, \underline{it}_z)$ to the corresponding six vector $P' = (\underline{t}'_x, \underline{t}'_y, \underline{t}'_z, \underline{ix}', \underline{iy}', \underline{iz}')$, where all six vectors are complex quantities given by eqs. (2.2) and (3.1).

In the complex space c^3 language, the superluminal transformation¹ corresponds to a transition from $P = (\underline{x} + \underline{it}_x, \underline{y} + \underline{it}_y, \underline{z} + \underline{it}_z)$ to

$$\begin{aligned}
P' &= (\underline{x}' + \underline{it}'_x, \underline{y}' + \underline{it}'_y, \underline{z}' + \underline{it}'_z) \\
&= (\underline{t}_x + \underline{ix}, \underline{t}_y - \underline{iy}, \underline{t}_z - \underline{iz}) \quad (3.6)
\end{aligned}$$

where the spatial and temporal vectors are complex quantities. Writing the real and imaginary parts of the complex quantities separately, we now get

$$\begin{aligned}
P' = & \left((t_{xRe} - x_{Im}) + i(t_{xIm} + x_{Re}), (t_{yRe} + y_{Im}) \right. \\
& \left. + i(t_{yIm} - y_{Re}), (t_{zRe} + z_{Im}) + i(t_{zIm} - z_{Re}) \right) \quad (3.7)
\end{aligned}$$

By comparison of eqs. (3.5) and (3.7), we see a superluminal transformation in the complex space, c^3 not only changes real quantities to imaginary ones, but also changes the mixing of real and imaginary parts of the spatial and temporal vectors. In Fig. 1, we represent an example of an M^2 space $(x, t_x + it_x)$ in which component mixing may lead to some interesting speculations about the connection of remote events.

In Fig. 1, we represent the relationship between the real part of the x vector component, and the real and imaginary time component t_x and it_x . In this Minkowski diagram, x dimensional spatial separations appear contiguous given the mix with complex time. An event P_1 ($x=0$) spatially separated from event P_2 ($x \neq 0$), appears as a point on the t_x axis such that P_1 coincides with P_2 in this M^2 space of $(x, t_{xRe} + it_{xIm})$. Real space, as a slice through complex space exhibits disconnected events that appear to be connected in the higher dimensional complex Minkowski space.⁸ In this example, we see that there are possible implications for remote connectedness by this unique mixing of real and imaginary components.

In the direction of the superluminal boost, one would expect real space and imaginary time components to be interchanged as given by eq. (1.1). But if space and time are complex quantities, then we see

from eq. (3.5) and (3.7) (referring only to x components), that due to the exchange of space and time coordinates in the direction of the superluminal boost, the quantity $x_{\text{Re}} - t_{x\text{Im}}$ changes to $t_{x\text{Re}} - x_{\text{Im}}$. By the application of the superluminal x-direction boost, we exchange the real and imaginary components of space and time in the x-direction. This implies that, for event P in the S frame, there is the mixing of real part of space with imaginary part of time while for the same event in S' frame, the imaginary part of space is mixed with real part of time. This complex exchange and mixing of space and time component is a unique feature which is different from the superluminal transformations in a real Minkowski space.

A similar behavior of space and time component is also present in the direction orthogonal to the direction of the superluminal boost, for example, $y_{\text{Re}} - t_{y\text{Im}}$ becomes $t_{y\text{Re}} + y_{\text{Im}}$. In this type of transformation, the total length of the components also changes because the modulus is further changed by a reversed algebraic summation. Similar analysis can be made for the z components in eqs. (3.5) and (3.7) for orthogonal directions to the boost. In Table I, we summarize the properties of the real and imaginary components of the complex Minkowski metric under the transformation of an x-directional superluminal boost. We compare these components for the S and S' frames of reference. Also the "modulus" for the two systems is compared.

This unique exchange and mixing of components affects the properties of tachyons in real and complex Minkowski spaces. In M^4 space, a free tachyon appears to be a "time point" extended in

space, whereas a free tachyon in c^3 space will be an imaginary "space point" which is extended in imaginary time. Again referring to our analogy in Figure 1, we see that the origin at event P_1 , is a real time point. The separation between P_1 and P_2 extends in real space. It also extends from P_1 to P_3 in imaginary time.

In ref. 8, we examine the relationship between macroscopic interconnectedness inherent in the structure of complex Minkowski spaces and microscopic interconnectedness. The J. F. Clauser and W. A. Horne test of the statistical predictions of quantum mechanics implies nonlocality based on the predictions of Bell's theorem.^{12,13} Further examination of the principle of nonlocality and its relationship to causality, as suggested in ref. 8 is of interest.

4. Conclusion

We have examined the effects on the components of the metric in a complex Minkowski space of a superluminal boost. A superluminal boost in the x direction in the real Minkowski space transforms space into time components, time into space components, and in the orthogonal direction, also transforms real quantities into imaginary quantities. But the same superluminal boost in the complex Minkowski space exchanges space and time components and exchanges real and imaginary parts of the complex quantities irrespective of the direction of the superluminal boost. This exchange of real and imaginary components is unique to this superluminal transformation in the complex Minkowski space. The properties of tachyonic signals in this space gives a microscopic remote connectedness.

A free tachyon in "real dimensioned space" appears to be a "time point" extended in space while the same free tachyon in the "complex dimensioned space" will look like a point "fixed" in the imaginary part of space and the real part of time, and which is "extended" in the real part of the space and the imaginary part of time. The physical validity and interpretation of the above statement will be examined in detail in order to apply it to physical systems. The structure and properties of this tachyonic signal may have implications for macroscopic remote connection of events in a complex Minkowski space that appear to be space - time separated in a real Minkowski space.

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Table 1. Properties of the Complex Minkowski Metric under a superluminal boost, $v_x = \infty$ in the x-direction carrying P into P'.

	<u>Event, P (eq. (3.5))</u>	<u>Event, P' (eq. 3.7)</u>	<u>Algebraic Summing in Modulus of term</u>
<u>X components</u>			
Real Parts	$x_{Re} - t_{xIm}$	$t_{xRe} - x_{Im}$	+ - to + - Same
Imaginary Part	$x_{Im} + t_{xRe}$	$t_{xIm} + x_{Re}$	+ + to + + Same
<u>Y Components</u>			
Real Parts	$y_{Re} - t_{yIm}$	$t_{yRe} + y_{Im}$	+ - to + + Change
Imaginary Parts	$y_{Im} + t_{yRe}$	$t_{yIm} - y_{Re}$	+ + to + - Change
<u>Z components</u>			
Real Part	$z_{Re} - t_{zIm}$	$t_{zRe} + z_{Im}$	+ to + + Change
Imaginary Parts	$z_{Im} + t_{zRe}$	$t_{zIm} - z_{Re}$	+ + to + - Change

Figure 1. We illustrate an example in which a real space-like separation of events P_1 and P_2 appears to be contiguous by the introduction of the complex time, $t_{xRe} + it_{xIM}$ such that from the point of view of event P_3 , $(x_2(P_2) - x_1(P_1))$ appears to be zero.

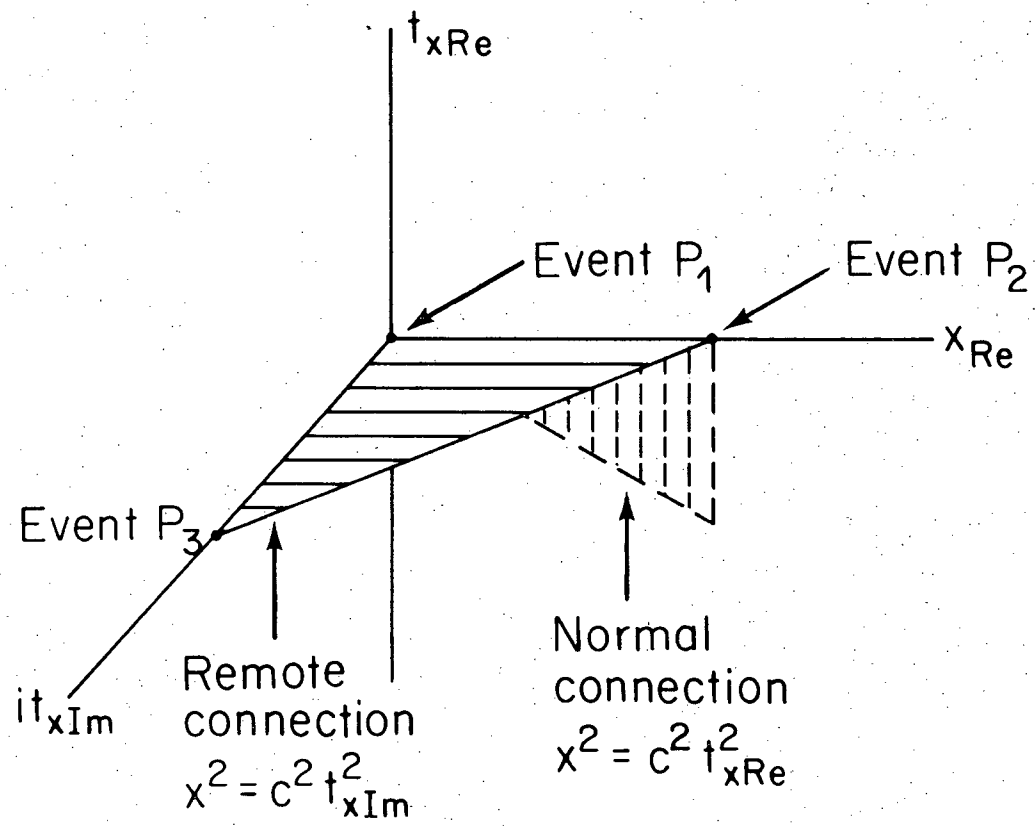


Fig. 1

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