Progress of Theoretical Physics, Vol. 54, No. 5, November 1975

Supernova Explosion and Neutral Currents of Weak Interaction

Katsuhiko SATO

Research Institute for Fundamental Physics
Kyoto University, Kyoto

(Received May 12, 1975)

Diffusion of neutrinos in supernova core is investigated on the basis of the Weinberg theory of weak interaction. Escape rates of neutrinos and hydrodynamic collapse of the core are computed simultaneously on the assumption that supernova core is homogeneous (one-shell model). Computations are carried out for a number of sets of initial core temperatures, $T_i = 10^{9.5}$ and 10^{10}K and the Weinberg angles, $\sin^2\theta_W = 0.45$, 0.3 and 0. The initial core density and mass are fixed as $10^{9.5}$g/cm^3 and $1.26\,M_\odot$ (Chandrasekhar mass of iron star), respectively. We have obtained the following results.

1) The core becomes opaque to neutrinos before electron captures of nuclei proceed sufficiently, and neutrinos are confined in the core. Therefore neutralization of the nuclei is strongly suppressed by the Pauli principle of degenerate neutrino sea and by neutrino captures (the inverse process of electron captures), compared with the cases under the usual CVC theory.
2) The greater we assume the value of the Weinberg angle, the more neutrinos are confined and the collapse bounces at the lower density due to the pressure of confined leptons.
3) Features of bounce depend on the initial core temperature strongly compared with the cases under the usual CVC theory.
4) Under the neutral current theory, a supernova core will bounce by thermal pressure gradient instead of nuclear repulsive force.

§ 1. Introduction

According to the theory of stellar evolution, neutron stars are formed by gravitational collapse of the massive stellar cores at the end of evolution. Numerical experiment of such collapse was first worked out by Colgate and White.[1] They showed that a large amount of stellar mass is blown off by the neutrino deposition and a stellar core can avoid the general relativistic infinite collapse by this mass ejection. However, by refinements of neutrino processes and transfer, Arnett[2] and Wilson[3] showed that the less amount or nothing of stellar matter can be blown off; i.e., the mechanism of neutrino deposition is doubtful. If we stand on the basis of the usual CVC theory, it is marginal whether neutron stars are formed or not by such collapses.

However, recent experiments have established the existence of neutral currents of weak interaction.[4] It seems to change the features of collapse extensively. For example one of the processes detected by these experiments $\bar{\nu}_\mu + e^- \to \bar{\nu}_\mu' + e^{-\prime}$ obviously prevents the rapid cooling of shock front by emission of muon neutrinos which was thought to be most dominant cooling process under the usual CVC theory. Recently Wilson[5] worked out the numerical experiment on the basis of neutral current theory and showed that mass ejection is possible for $a_0=1$ but impossible for $a_0=0.45$, where a_0 is an adjustable constant of neutrino theory. This result indicates that even small improvements of equation of state, electron captures of nuclei and/or neutrino processes change the situation significantly. As he stressed, his result is preliminary and more careful treatments of neutrino interaction and equation of state are needed. Mazurek[6] and Schramm and Arnett[7] recently discussed the neutrino transfer and interaction more precisely.

In a previous paper,[8] we have shown that neutrinos are confined in the core because a stellar core becomes opaque to neutrinos before electron captures by nuclei (hereafter denoted simply as e-captures) proceed sufficiently. Confined neutrinos prevent e-captures, i.e., neutralization of nuclei, and vary the equation of state and the neutrino opacities. The purposes of the present paper are to discuss these problems more precisely and to demonstrate that the diffusion of neutrinos and the neutralization of nuclei should be computed simultaneously. On the contrary, in the previous computations of supernova explosion by Colgate and White,[10] Arnett[2] and Wilson,[3),5] mean molecular weight of electrons was simply assumed to be independent of evolution and diffusion of neutrinos, but it is not correct.

Nakazawa[9] and recently Kaminishi and Inomata[10] investigated the proceeding of e-captures and escape rate of neutrinos on the assumptoin that supernova core is homogeneous (one-shell model). In the present investigation, we compute the collapse of the core under the same one-shell model, but we take account of the reaction of neutrinos, electron captures and diffusion of neutrinos simultaneously. The other assumptions and basic equations of collapse are shown in § 2. In § 3, neutrino opacities under the Weinberg theory are discussed. Section 4 deals with nuclear statistical equilibrium at high densities, where interaction between free nucleons cannot be neglected. Numerical results are shown in § 5 and remarks are discussed in § 6.

§ 2. Basic equations and assumptions

One-shell model approximates the collapse as

$$\rho\frac{\partial^2 R}{\partial t^2}=\frac{p}{R}-f\frac{GM\rho}{R^2}, \tag{2·1}$$

where M, R, ρ and p are mass, radius, mass density and pressure of the core, respectively. The parameter f is adjusted to give a static equilibrium of initial model of the core, i.e., $p_i/R_i=fGM\rho_i/R_i^2$, where the subscript i denotes the initial value. Next we assume that constituents of the core matter are electrons, neutrinos, neutrons, protons and one species of nuclei (A, Z). The mass and charge number A, Z and mass fractions of the nuclei X_A, protons X_p and neutrons X_n are discussed in § 4. Conservation law of lepton number and transformation between electrons and neutrinos are described as

$$dy/dt = -dx/dt - y/\tau_d \tag{2·2}$$

and

$$dx/dt = R_{\nu n}X_n - (R_{eN} - R_{\nu N})X_A/A - R_{ep}X_p\,, \tag{2·3}$$

where

$$x \equiv n_e/n_B \quad \text{and} \quad y \equiv n_{\nu_e}/n_B\,.$$

In the above equations, n_e, n_{ν_e} and n_B are number densities of electrons, neutrinos and baryons, respectively. Reaction rates of neutrino capture $R_{\nu N}$ and e-capture R_{eN} of nuclei, e-capture of protons R_{ep} and neutrino capture of neutrons $R_{\nu n}$ are given in Appendix B. The time scale of neutrino diffusion τ_d is discussed in § 3. Reactions of protons and anti-neutrinos are neglected because their abundances are sufficiently small compared with those of electrons and neutrinos.

A thermodynamic equation of the matter, in which the non-equilibrium beta processes[9),11),12)] and diffusion of neutrinos proceed, is described as

$$Tds/dt = -N_0\{(\mu_p + \mu_e - \mu_n - \mu_\nu)\,dx/dt - \mu_{\nu_e}\cdot y/\tau_d\} - \varepsilon_\nu\,, \tag{2·4}$$

where T, s and N_0 are temperature, specific entropy (including neutrino entropy) and the Avogadro number, respectively. The chemical potentials of neutron μ_n, protons μ_p, electrons μ_e and neutrinos μ_ν are discussed in § 4 and neutrino energy loss rate ε_ν in § 3 and Appendix A.

Simultaneous differential equations (2·1)~(2·4) are solved by the implicit difference method with the aid of the subsidiary equations.

§ 3. Neutrino opacities under the Weinberg theory

1) *Neutrino scattering with leptons*

The scattering cross section with lepton under the Weinberg theory was given by 't Hooft[13)] as

$$\sigma = \frac{1}{4}\sigma_{e\nu}^{\mathrm{CVC}}\left[(g_V + g_A)^2 + \frac{1}{3}(g_V - g_A)^2 + (g_A{}^2 - g_V{}^2)\frac{m_e c^2}{2E_\nu}\right]$$
$$\approx \frac{1}{3}(g_V{}^2 + g_A{}^2 + g_A g_V)\,\sigma_{e\nu}^{\mathrm{CVC}} \quad \text{for} \quad m_e c^2 \ll 2E_\nu\,, \tag{3·1}$$

where

$$\sigma_{e\nu}^{\mathrm{CVC}} = 1.7\times 10^{-44}\mathrm{cm}^2 (E_\nu/m_e c^2)^2\,. \tag{3·2}$$

In the above equations, E_ν is the incident neutrino energy, m_e mass of electron and $\sigma_{e\nu}^{\mathrm{CVC}}$ neutrino-electron scattering cross section under the usual CVC theory ($g_A = g_V = 1$). Recent experiments of neutral currents of CERN and FNAL[4)] seem to suggest $0.3 < \sin^2\theta_W \lesssim 0.45$, where θ_W is the Weinberg angle. In the present investigation computations are performed for the cases $\sin^2\theta_W = 0.45$, 0.3 and 0. Effective coupling constants of vector g_V and axial vector current g_A and the ratios of

Table I. The ratios of cross sections $\sigma/\sigma_{e\nu}^{\rm CVC}$.

	g_V	g_A	$a_0=0.9$	$a_0=0.6$	$a_0=0$
$\nu_e+e^-\to\nu_e+e^-$	$0.5+a_0$	0.5	0.97	0.67	0.25
$\bar\nu_e+e^-\to\bar\nu_e+e^-$	$0.5+a_0$	-0.5	0.50	0.30	0.083
$\nu_\mu+e^-\to\nu_\mu+e^-$	$-0.5+a_0$	-0.5	0.07	0.07	0.25
$\bar\nu_\mu+e^-\to\bar\nu_\mu+e^-$	$-0.5+a_0$	0.5	0.20	0.10	0.083

$\sigma/\sigma_{e\nu}^{\rm CVC}$ for various targets are shown in Table I.

The most important processes are scatterings by electrons and by electron type neutrinos because those number densities are much higher than those of the other leptons such as $\mu^\pm$, $\nu_\mu(\bar\nu_\mu)$, $\bar\nu_e$ and e^+. The opacity for electron type neutrinos is given by

$$\kappa_L=N_0\sigma_{e\nu}^{\rm CVC}[(a_0^2/3+a_0/2+1/4)x\cdot{\rm Min}\{1,\mu_e/E_\nu\}+\tfrac{1}{4}y\cdot{\rm Min}\{1,\mu_\nu/E_\nu\}], \quad (3\cdot3)$$

where

$$a_0=2\sin^2\theta_W, \quad (3\cdot4)$$

and function Min$\{A, B\}$ denotes the smaller values of A and B. The opacities of the other type neutrinos are easily obtained from the principle of symmetries between the leptons with the aid of Table I.

2) *Neutrino scattering and reactions with nuclei and nucleons*

Under the usual CVC theory scattering by electron is important only up to nucleus melting density, below which the cross section of the reaction $\nu+(N,Z)\to e^-+(N+1,Z-1)$ is two or more orders of magnitude smaller than that of scattering by electron. Above the nucleus melting density the reaction $\nu_e+n\to p+e^-$ is as effective as electron scattering, and the supernova core is opaque to neutrinos at the densities higher than about 10^{13}g/cm^3. The diffusion time is longer than freefall time for densities higher than $10^{13.5}$g/cm^3.

However, situations are very different if we take account of the neutral current interaction. Freedman[14] showed that coherent neutrino nucleus scattering has a very large cross section. In the limit $E_\nu\ll Mc^2$, where M is the rest mass of target nucleus, the total cross section is given as

$$\sigma=(3/4)(1.7\cdot10^{-44}\,{\rm cm}^2)a_0^2A^2\{1-(1-e^{-\alpha})/\alpha\}(\lambda_e/r_N)^2, \quad (3\cdot5)$$

where

$$\alpha=(2/3)(r_N/\lambda_e)^2(E_\nu/m_ec^2)^2. \quad (3\cdot6)$$

In the above equations, λ_e is the Compton wave length of electron, r_N and A are radius and mass number of target nucleus, respectively. Equation (3·5) is reduced to $\sigma=a_0^2\sigma_{e\nu}^{\rm CVC}A^2/4$ in the limit $\alpha\ll1$. If we have the target nucleus with $r_N\sim8$ fm ($A\sim100$), the condition $\alpha\ll1$ correspond to $E_\nu\ll25$ MeV. By this coherent scattering, opacity becomes about $A^2/4Z$ times greater than that of the usual CVC theory up to nucleus melting. As a result, the collapsing core becomes opaque before e-captures of nuclei proceed sufficiently.

Scattering cross sections with neutron and protons are easily derived from the results of Weinberg,[15] i.e.,

$$\sigma_p = \{1.08 + (0.5 - a_0)^2\} \sigma_{e\nu}^{\mathrm{CVC}}/8 \tag{3·7}$$

and

$$\sigma_n = 1.33 \sigma_{e\nu}^{\mathrm{CVC}}/8 . \tag{3·8}$$

The scattering opacity for electron type neutrinos is given as

$$\kappa_N{}^s = (N_0 \sigma_{e\nu}^{\mathrm{CVC}}/4) \cdot [(1 - \langle \cos\theta \rangle) \cdot A \cdot a_0{}^2 X_A + 0.5\{1.08 + (0.5 - a_0)^2\} X_p + 0.665 X_n]. \tag{3·9}$$

The cosine of mean scattering angle $\langle \cos\theta \rangle$ is 1/3 for coherent scattering in the limit $\alpha \ll 1$.

The absorption opacity of electron type neutrinos is given by

$$\begin{aligned}\kappa_N{}^a = N_0 \big[& X_n \sigma_{\nu n}/[1 + \exp\{(\mu_e - m_n c^2 + m_p c^2 - E_\nu)/kT)\}] \\ & + (X_A/A)\sigma_{\nu N}/[1 + \exp\{(\mu_e - m(N+1, Z)c^2 + m(N, Z+1)c^2 - E_\nu)/kT\}]\big] \\ & \times [1 + \exp\{(\mu_\nu - E_\nu)/kT\}],\end{aligned} \tag{3·10}$$

where $\sigma_{\nu n} = 2.29 \times 10^{-44}\,\mathrm{cm}^2\ (E_\nu/m_e c^2)^2$ and $m(N, Z)$ is the mass of nucleus.

The cross section of the reaction $\nu + n \rightarrow p + e^-$ is of course the same as that of the usual CVC theory because this is charged current interaction.

The neutrino capture cross section by nucleus $\sigma_{\nu N}$ is given by

$$\begin{aligned}\sigma_{\nu N}(E_\nu) = 2\pi^2 \ln 2 \cdot \langle \rho_L/ft \rangle \left(\frac{\hbar}{m_e c}\right)^2 \cdot m_e c \{(E_\nu + m(N+1, Z)c^2 \\ - m(N, Z+1)c^2)/m_e c^2\}^3/3 ,\end{aligned} \tag{3·11}$$

where $\langle \rho_L/ft \rangle$ is the strength function of beta process (see Appendix B).

3) *Emissivity of neutrinos for transparent cases*

For the case that "optical depth" of neutrinos is less than unity, energy loss rates are determined directly by emissivity instead of the rate of diffusion. Of course e-capture by nuclei and protons are most dominant emission process and, in this investigation, emissivity given by Nakazawa[9] (with some modifications[12]) is used. As for the pair, photo and plasma neutrino loss processes, the results of Dicus[16] is used.

§4. Nuclear statistical equilibrium of high density matter

Chemical equilibrium between neutrons, protons and one species of nuclei

(N, Z) are described as follows,

$$\mu_n^G(n_n, n_p, T) = \mu_n^A(N, Z), \tag{4·1}$$

$$\mu_p^G(n_n, n_p, T) = \mu_p^A(N, Z) \tag{4·2}$$

and

$$\mu_A(N, Z) = N\mu_n^G(n_n, n_p, T) + Z\mu_p^G(n_n, n_p, T). \tag{4·3}$$

The superscripts G and A denote gas and nucleus, respectively, and the others have their usual meanings. The chemical potentials of protons and neutrons in the gas are assumed simply as

$$\mu_n^G(n_n, n_p, T) = \mu_n^0(n_n, n_p) + \{\mu_n^{NFG}(n_n, T) - \mu_n^{NFG}(n_n, 0)\} \tag{4·4}$$

and

$$\mu_p^G(n_n, n_p, T) = \mu_p^0(n_n, n_p) + \{\mu_p^{NFG}(n_p, T) - \mu_p^{NFG}(n_p, 0)\}, \tag{4·5}$$

where μ_n^{NFG} and μ_p^{NFG} are chemical potentials of non-interacting Fermi gas which are calculated precisely by the Fermi-Dirac function (see, for example, Sugimoto et al.[17]). The chemical potentials of neutrons μ_n^0 and protons μ_p^0 in the cold nuclear matter are derived from the nuclear matter energy as discussed by Baym et al.[18] In the present investigation, nuclear energy per nucleon in units of MeV is assumed[18] as

$$W(k, \alpha) = -16.5 + 35.0(1 - k/1.43\ \mathrm{fm}^{-1})^2 + 33.0(k/1.43\ \mathrm{fm}^{-1})^2(1-2\alpha)^2, \tag{4·6}$$

where

$$k = \{1.5\pi^2(n_n + n_p)\}^{1/3} \text{ and } \alpha = n_p/(n_n + n_p).$$

It is interesting to note that the following simple relation is obtained by substituting Eq. (4·1) and (4·2) into Eq. (4·3).

$$\frac{\partial \ln m(N\cdot Z)}{\partial \ln N} + \frac{\partial \ln m(N, Z)}{\partial \ln Z} = 1 + \frac{kT}{m(N, Z)c^2} \ln\left\{\frac{n(N, Z)}{\omega(N, Z)}\left(\frac{2\pi^2\hbar^2}{m(N, Z)kT}\right)^{3/2}\right\}. \tag{4·7}$$

where $\omega(N, Z)$ is statistical weight of nucleus (N, Z). Obviously the second term on the right-hand side is negligibly small $(kT/m(N, Z)c^2 \ll 1)$. If we neglect this term, the above relation is reduced to the same relation as obtained for cold catalyzed matter.[19] In the present investigation, Bethe and Weizsäcker's mass formula is adopted for simplicity.

Using the conservation law of lepton and baryon numbers, we can easily calculate the mass and charge number of the most abundant nuclei and their abundances when mass density, temperature and mean molecular weight of electrons are given.

Total pressure of the matters p is calculated as the sum of pressures of radiation p_r, neutrinos p_ν electrons p_e, ions p_i and cold nuclear matter p_0.

Next we discuss the case in which chemical equilibrium of beta process is also established, i.e.,

$$\mu_n{}^G+\mu_\nu=\mu_p{}^G+\mu_{e^-}. \tag{4·8}$$

As shown in § 5, chemical equilibrium of beta process is roughly established where $\rho>10^{12.5}$g/cm^3. In Figs. 1 and 2, the loci of constant n_e/n_B are shown for the given ratios of n_L/n_B, where n_L is the lepton number density; $n_L=n_e+n_\nu$. It is clearly seen that neutralization of nuclei is strongly suppressed by neutrino degeneracy. Neutron drip curve on ρ-T diagram coincides almost with the locus of constant mass number $A=120$. This locus runs on the very high density side compared with the neutron drip density of cold catalyzed matter ($\rho_1=3\times10^{11}$ g/cm^3).

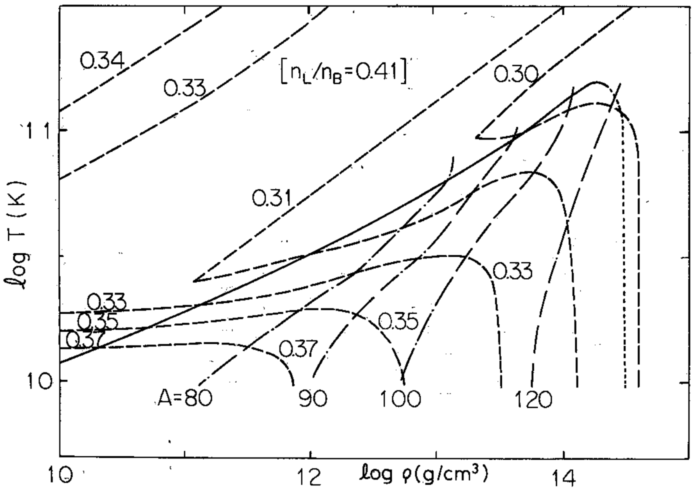

Fig. 1. The loci of constant value for the ratio n_e/n_B and mass number of nuclei under the chemical equilibrium. The ratio n_L/n_B is assumed as 0.41. The dashed curves represent the ratios n_e/n_B and the dash-dotted curves mass number A. The solid curve indicates that mass fraction of the nuclei X_A is 0.5.

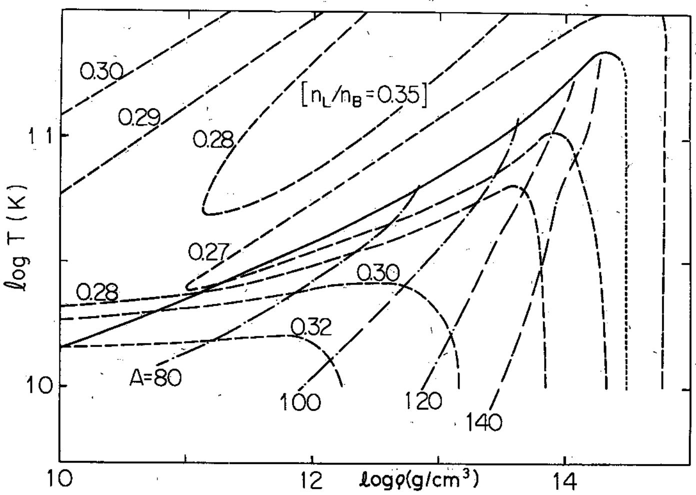

Fig. 2. The same as in Fig. 1, but for the case n_L/n_B $=0.35$.

§ 5. Numerical results and discussion

Collapse of the core and escape rate of neutrinos are computed for two cases of initial core temperatures $T_i = 10^{9.5}$ and 10^{10} K. The mass, initial density of the core and initial value of the ratio $n_e/n_B (\equiv x)$ are assumed as 1.26 $M_\odot$ (the Chandrasekhar mass of iron star), $10^{9.5}$ g/cm^3 and 0.4643, respectively. Previously, collapse computations,[1),2)] were carried out for more massive and high temperature cores, but recent computations of presupernova model[20)~23)] suggest that mass of the core including silicon shell is nearly equal to the Chandrasekhar mass and the temperature of the core is lower than that of previous computations.

First we discuss the case $T_i = 10^{9.5}$ K and $\sin^2\theta_W = 0.45$. A stellar core starts to collapse gradually which is triggered by e-captures of ^{56}Fe and protons . In early stages of the collapse, the temperature increases steeply as shown in Fig. 3. This is due to the entropy production by the non-equilibrium beta process as

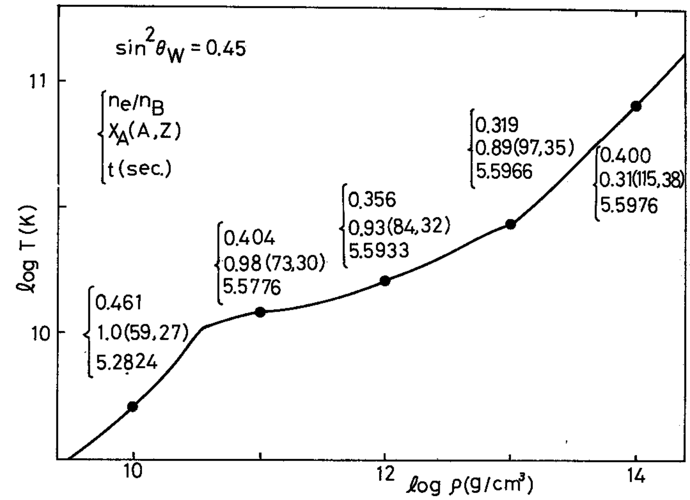

Fig. 3. The path on ρ-T diagram of collapsing core for the case $\sin^2\theta_W = 0.45$ and $T_i = 10^{9.5}$K. The ratio of n_e/n_B, mass fraction of nuclei, mass and charge number $X_A(A, Z)$ and time t are shown in figure.

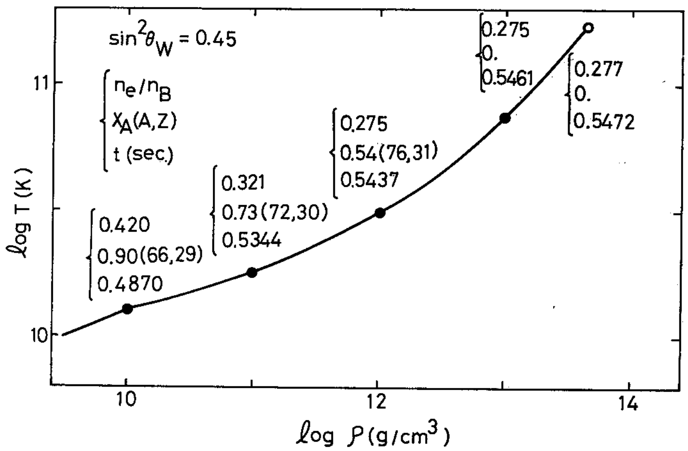

Fig. 4. The same as in Fig. 3, but for $T_i = 10^{10}$K. The open circle represents the bouncing point.

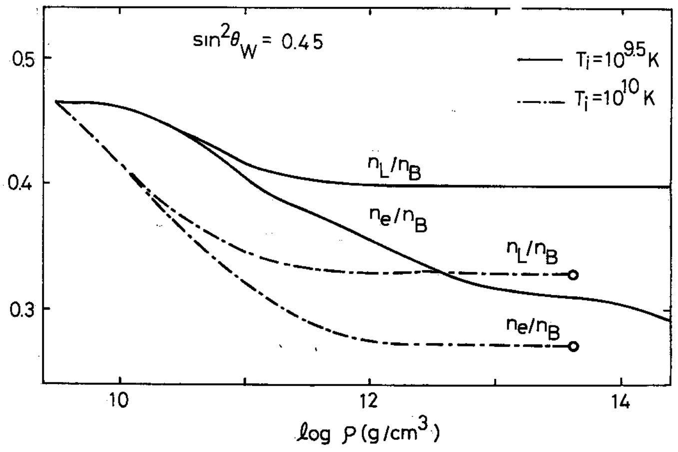

Fig. 5. Evolutions of the ratios of electrons to baryons n_e/n_B and of leptons to baryons n_L/n_B. The solid curves represent the case $T_i=10^{9.5}$K and the dash-dotted curves the case $T_i=10^{10}$ K.

discussed by Sugimoto,[11] Nakazawa[9] and Bisnovatyi-Kogan and Seidov.[24] At the densities lower than $10^{10.5}$ g/cm^3 neutrinos emitted by e-capture of protons and nuclei diffuse out before neutrinos are thermalized because $\sqrt{\kappa_L(E_\nu)\kappa(E_\nu)/3}\rho R<1$, where E_ν is the energy of the emitted neutrinos. (See Appendix A.)

On the other hand, for the case $T_i=10^{10}$ K (see Fig. 4), effect of entropy production by e-captures is minor. This result is the same as that of Nakazawa,[9] where neutrinos are assumed to escape freely from the core. As seen in Fig. 5, diffusion time of neutrinos becomes longer than the time scale of collapse at about 10^{12} g/cm^3, neutrinos are confined, and the ratio n_L/n_B is frozen up. Chemical equilibrium of beta process $\mu_n+\mu_{\nu_e}\approx\mu_p+\mu_{e^-}$ is almost established at this density and neutralization of nuclei is strongly suppressed. As the nuclei can neither drip neutrons nor melt, the coherent scattering is the dominant process for neutrino opacity even at high density. The opacity also increases with the rising of neutrino Fermi energy. These situation are illustrated in Fig. 6.

Old studies of the supernova explosion[2),3)] gave evolutionary paths on the density-temperature diagram, which run through the high temperature side compared with our results. This might look contradictory, because the CVC theory (old studies) gives a neutrino energy loss greater than the Weinberg theory (ours). However, the following two points in our computation seem to yield the reduction of temperature, which overweigh an increase of temperature due to the different

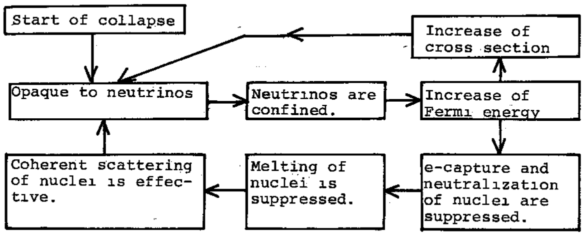

Fig. 6. Schema of mutual influences of beta processes and diffusion of neutrinos.

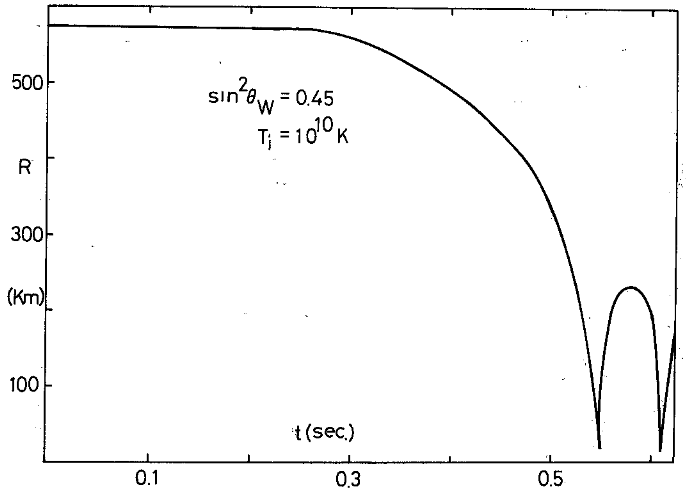

Fig. 7. Radius versus time history of the core for the case $\sin^2\theta_W=0.45$ and $T_i=10^{10}$K.

theory of neutrino. One is the refined treatment of thermodynamic quantities, i.e., entropy fitting method (see § 2). In particular, in this method, the effects of the nucleus melting and the change in the mean molecular weight are computed easily, but very precisely. The other is the simplified treatment of dynamics as described in § 2, by which we have avoided artificial entropy production. In one of the old studies,[4)] the complicated numerical hydrodynamics have lead to spurious production even at the center of the core (see Fig. 7 in Ref. 5)), where no shock wave appeared, and where there was no physical process of entropy production. Usually, the method of artificial viscosity of von Neumann is used to reproduce the shock wave in the numerical computation of hydrodynamics. However, it should be noticed that in the computation of rapid collapse of the stars or gaseous clouds, this method produces the entropy spuriously at the center.

As shown in Fig. 4, the supernova core bounces at $10^{13.6}$ g/cm³ for the case of initial temperature $T_i=10^{10}$K. However, it did not bounce for the case $T_i=10^{9.5}$K up to $10^{14.3}$ g/cm³ where our computations are stopped. This result obviously comes from the difference of the core temperatures: For the former case ion pressure is greater than 55% of total pressure at the bouncing point, while it is less than 10% at $10^{14.3}$ g/cm³ for the latter case.

As shown in Table II, the supernova core bounces at densities below 10^{14} g/cm³ for either value of the Weinberg angle for $T_i=10^{10}$ K. Therefore, under the neutral current interaction theory, difference of initial temperature (entropy) strongly affects the features of explosion. Under the usual charged current interaction theory, on the contrary, temperatures of the core converge to $10^{11.2}$ K in spite of the difference of initial temperatures because of rapid cooling by the muon type neutrinos. Table II also shows that if we assume the smaller value for the Weinberg angle, the bouncing point shifts to higher density. This result comes from the fact that the pressure (especially lepton pressure) is lower because the number of confined leptons is smaller for the case of the smaller angle.

Under the presence of neutral current interaction, the supernova core will bounce by thermal pressure gradient, while pressure gradient due to nuclear repulsive force makes the core bounce under the usual CVC theory.

Table II. Physical conditions at the bouncing point. Initial core temperature is assumed as 10^{10}K. Nuclei are completely melted at this density.

$\sin^2\theta_W$	0.45	0.3	0
$\log\rho(g/\mathrm{cm}^3)$	13.6	13.7	14.0
$\log T$(K)	11.2	11.3	11.5
n_e/n_B	0.277	0.260	0.196
n_L/n_B	0.330	0.307	0.224
μ_ν(MeV)	76.5	79.6	86.2
p_l/p	0.55	0.59	0.67
time (sec)	0.5472	0.5221	0.5161

Table III. Physical conditions at $10^{14.3}g/\mathrm{cm}^3$. Initial core temperature is assumed as $10^{9.5}$K.

$\sin^2\theta_W$	0.45	0.3	0
$\log T$(K)	11.1	11.1	11.2
(A, Z)	(136,41)	(141,42)	(257,57)
X_A	0.855	0.855	0.724
n_e/n_B	0.295	0.286	0.235
n_L/n_B	0.399	0.387	0.294
μ_ν(MeV)	165.	157.0	140.
$(p_e+p_\nu)/p$	0.92	0.92	0.77
time (sec)	5.5978	5.5791	5.5310

§ 6. Remarks

In order to investigate the amount of mass ejection and to distinguish whether the stellar core becomes neutron star or black hole, precise hydrodynamic computation is necessary. However, the results of the present paper suggest that mass ejection from the core is very likely under the presence of neutral current interaction.

In this investigation, we adopted the Weinberg theory. However, there are some other theories which include the neutral current interaction. For example, baryon current theory by Sakurai[25] predicts the coherent scattering cross section 15 times greater than that of the Weinberg theory with $\sin^2\theta_W=0.3$. However, qualitative consequences of the present investigation will not be changed.

Acknowledgements

The author would like to thank Professor C. Hayashi for continuous encouragement and valuable discussions. He also thanks Dr. K. Nakazawa for critical discussions and Professor D. Sugimoto for critical reading of the manuscript and helpful comments. He is grateful to Soryushi Shogakukai for financial aid. Numerical computations were carried out by FACOM 230-75 at the Data Processing Center of Kyoto University.

Appendix A

——Diffusion of degenerate neutrinos——

The transfer equation of neutrinos is essentially the same as that of radiation[26] and is previously discussed by Schwartz[27] and Imshenik and Nadezhin.[28] We shall discuss the case that neutrinos are in thermal equilibrium with matter (Eq. (A·4)) but not necessarity in chemical equilibrium (Eq. (A·6)). In the supernova core thermal equilibrium is established faster than chemical equilibrium because rates of scatterings are greater than those of reactions. Energy flux $\mathscr{F}_3$ and particle flux $\mathscr{F}_2$ are described as

$$\mathscr{F}_n = -(n+1)\frac{7}{16}\frac{4ac}{3}\frac{T^n}{k^{3-n}}\left(\frac{30}{7\pi^4}\right)\left\{\frac{1}{\kappa_n{}^T\rho}F_n(\psi)\frac{dT}{dr}+\frac{1}{\kappa_n{}^\psi\rho}\frac{T}{(n+1)}F_n'(\psi)\frac{d\psi}{dr}\right\}, \tag{A·1}$$

where k is the Boltzmann constant, $F_n(\psi)$ and $F_n'(\psi)$ are Fermi Dirac function[29] and its derivative, respectively, and $\psi \equiv \mu_\nu/kT$. The generalized Rosseland means $\kappa_n{}^T$ and $\kappa_n{}^\psi$ are given by

$$\kappa_n{}^T = \int(\partial I_n/\partial T)_\psi d\nu \Big/ \int(1/\kappa(h\nu))\,(\partial I_n/\partial T)_\psi d\nu \tag{A·2}$$

and

$$\kappa_n{}^\psi = \int(\partial I_n/\partial\psi)_T d\nu \Big/ \int(1/\kappa(h\nu))\,(\partial I_n/\partial\psi)_T d\nu\,, \tag{A·3}$$

where

$$I_n = (h\nu)^{n-2}(\nu^2/c^2)/\{e^{(h\nu-\mu_\nu)kT}+1\} \tag{A·4}$$

and

$$\kappa(h\nu) = \kappa^s(h\nu) + \kappa^a(h\nu)\{1+\exp(\mu_\nu'-h\nu)/kT\}. \tag{A·5}$$

In the above equations, κ^s and κ^a are opacities due to scattering and absorption, respectively. The chemical potential μ_ν' is not of neutrinos, i.e., $\mu_\nu' \neq \mu_\nu$ but

$$\mu_\nu' = \mu_p + \mu_e - \mu_n\,. \tag{A·6}$$

Diffusion time of particle $\tau_d{}^{(1)}$ and energy loss rate by diffusion per unit mass ε_ν are derived from Eq. (A·1) as

$$1/\tau_d{}^{(1)} \equiv -(dy/dt)^d/y = c/R^2\left\{3\left\langle\frac{d\ln T}{d\ln r}\right\rangle\frac{1}{\kappa_2{}^T\rho}+\left\langle\frac{d\psi}{d\ln r}\right\rangle\frac{F_2'(\psi)}{F_2(\psi)}\frac{1}{\kappa_2{}^\psi\rho}\right\} \tag{A·7}$$

and

$$\varepsilon_\nu = N_0(\mathcal{F}_3/\mathcal{F}_2)y/\tau_d^{(1)} \cdot \tag{A·8}$$

In this investigation, we simply assume that mean derivatives $\langle d\ln T/d\ln r\rangle = 1$ and $\langle d\psi/d\ln r\rangle = r$, and $\psi \gg 1$. In this limit, $\kappa_n^T = \kappa_n^\psi = \kappa(\mu_\nu)$ and $\mathcal{F}_3/\mathcal{F}_2 = \mu_\nu$; therefore, Eqs. (A·8) and (A·9) are reduced to simpler ones.

Strictly speaking, energy spectrum of neutrinos are not Fermi distribution, especially for low energy neutrinos, because mean free path of low energy neutrinos is greater than the radius of the core as discussed by Schramm and Arnett.[7] Therefore we add the terms which describe the diffusion from the bottom of the Fermi distribution.

$$1/\tau_d = 1/\tau_d^{(1)} + 1/\tau_d^{(2)} \tag{A·9}$$

and

$$\varepsilon_\nu = N_0 y(\mu_\nu/\tau_d^{(1)} + h\nu_B/\tau_d^{(2)}), \tag{A·10}$$

where

$$\tau_d^{(1)} = \kappa(\mu_\nu)\rho R^2/6c \tag{A·11}$$

and

$$\tau_d^{(2)} = \kappa(h\nu_B)(\mu_\nu/h\nu_B)^3 \rho R^2/3c\,. \tag{A·12}$$

The energy at the bottom of neutrino spectrum $h\nu_B$ is determined by the equation $\sqrt{\kappa(h\nu_B)\kappa_L(h\nu_B)/3}\cdot\rho R = 1$, where κ_L is the opacity due to lepton scattering (§ 3). Neutrinos are thermalized mainly by lepton scattering because the rate of energy exchange is greater compared with the nucleon or nucleus scatterings. Therefore neutrinos which have the energy lower than $h\nu_B$ diffuse out before thermalization (the Fermi distribution). However, the window of the bottom of neutrino energy spectrum is never so large as to destroy the neutrino degeneracy. Equation (A·12) gives the upper limit of diffusion rate from this window.

Appendix B

——*Reaction rates of electron and neutrino captures*——

In the present investigation, reaction rates are calculated on the assumption that electrons and neutrinos are strongly degenerate. Reaction rates of electron and neutrino captures are described as

$$\begin{bmatrix} R_{eN} \\ R_{\nu N} \end{bmatrix} = \ln 2\langle \rho_L/ft\rangle m_e c^2 \cdot \begin{bmatrix} 2K(X,Y,W)\cdot(1-S) \\ K(Y,X,-W)\cdot(1-1/S) \end{bmatrix} \quad \text{for} \quad \begin{bmatrix} X > Y+W \\ X < Y+W \end{bmatrix}, \tag{B·1}$$

where

$$K(x,y,z)\equiv 1/3(x^3-z^3)\{(x-z)^3-y^3\}-3/4z^2\{(x-z)^4-y^4\}$$
$$-3/5z\{(x-z)^5-y^5\}-1/6\{(x-z)^6-y^6\} \tag{B·2}$$

and

$$S=\exp\{(\mu_n+\mu_\nu-\mu_p-\mu_e)/kT\}. \tag{B·3}$$

In the above equations, W, X and Y are threshold energy of e-capture of nucleus and the chemical potential, of electrons and neutrinos in unit of m_ec^2, respectively. The terms S and $1/S$ represent the contributions of the inverse processes which are derived from the principle of detailed balance. Strength function of beta process $\langle\rho_L/ft\rangle$ is simply assumed as $10^{-5.5}\,\mathrm{MeV}^{-1}\,\mathrm{sec}^{-1}$.[30]

References

1) S. A. Colgate and R. H. White, Astrophys. J. **144** (1967), 626.
2) W. D. Arnett, Can. J. Phys. **45** (1967), 1621.
3) J. R. Wilson, Astrophys. J. **163** (1971), 209.
4) D. C. Cundy, Report on International Conference of High Energy Physics, London, 1974 July.
5) J. R. Wilson, Phys. Rev. Letters **32** (1974), 849.
6) T. J. Mazurek, Nature **252** (1974), 287.
7) D. N. Schramm and W. D. Arnett, Phys. Rev. Letters **34** (1975), 113, Astrophys. J. **198** (1975), 628.
8) K. Sato, Prog. Theor. Phys. **53** (1975), 595.
9) K. Nakazawa, Prog. Theor. Phys. **49** (1973), 1932.
10) K. Kaminishi and M. Inomata, Phys. Rep. Kumamoto Univ. **1** (1974), 21.
11) D. Sugimoto, Astrophys. J. **161** (1970), 1069.
12) K. Sato, Prog. Theor. Phys. **51** (1974), 726.
13) G. 't Hooft, Phys. Letters **37B** (1971), 195.
14) D. Z. Freedman, Phys. Rev. **D9** (1974), 1389.
15) S. Weinberg, Phys. Rev. **D5** (1972), 1412.
16) D. A. Dicus, Phys. Rev. **D6** (1972), 941.
17) D. Sugimoto, K. Nakazawa and C. Hayashi, Scientific Papers of College of General Education, University of Tokyo **22** (1972), 145.
18) G. Baym, H. A. Bethe and C. Pethick, Nucl. Phys. **A175** (1971), 225.
19) K. Sato, Soryusiron Kenkyu **40** (1969), B6 (In Japanese).
H. A. Bethe, G. Börner and K. Sato, Astron. and Astrophys. **7** (1970), 279.
20) S. Ikeuchi, K. Nakazawa, T. Murai, R. Hoshi and C. Hayashi, Prog. Theor. Phys. **46** (1971), 1713; **48** (1972), 1870.
21) Z. Barkat, Y. Reiss and G. Rakavy, Astrophys. J. **193** (1974), L21.
22) D. Sugimoto and K. Nomoto, "Late Stages of Stellar Evolution", *Proc. IAU Symp.* No. 66, ed. Taylor (Reidel Publ., Dordrecht, 1974), p. 105.
23) W. D. Arnett, Astrophys. J. **195** (1975), 727.
24) G. S. Bisnovatyi-Kogan and Z. F. Seidov, Soviet Astron.-AJ 14 (1970), 113.
25) J. J. Sakurai and L. F. Urrutia, Phys. Rev. **D11** (1975), 159.
26) H. Y. Chiu, *Stellar Physics*, (Blaisdell, New York, 1967).
27) R. A. Schwartz, Ann. of Phys. **43** (1967), 42.
28) V. S. Imshennik and D. K. Nadezhin, Soviet Phys.-JETP **36** (1973), 821.
29) A. W. Guess, *Advances Astron. Astrophys.*, ed., Z. Kopal (Academic Press, New York and London) **4** (1966), 153.
30) M. Yamada, *Proceedings of International Conference on the Properties of Nuclei far from the Region of Beta-Stability* (Leysin, 1970).