

# Supernova pointing with neutrino detectors

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## ABSTRACT:

A future galactic SN can be located several hours before the optical explosion through the MeV-neutrino burst, exploiting the directionality of  $\nu$ - $e$ -scattering in a water Cherenkov detector. We study the statistical efficiency of different methods for extracting the SN direction and identify a simple approach that is nearly optimal, yet independent of the exact SN neutrino spectra. We use this method to quantify the increase in the pointing accuracy by the addition of gadolinium to water, which tags neutrons from the inverse beta decay background. We also study the dependence of the pointing accuracy on neutrino mixing scenarios and initial spectra. A TeV-neutrino burst is also expected to be emitted contemporaneously with the SN optical explosion, which may locate the SN to within a few tenths of a degree at a future km<sup>2</sup> high-energy neutrino telescope. If the SN is not seen in the electromagnetic spectrum, locating it in the sky through neutrinos is crucial for identifying the Earth matter effects on SN neutrino oscillations. These effects can be observed at a single detector through peaks in the Fourier transform of their “inverse energy” spectrum. The positions of these peaks are independent of the SN models and therefore the peaks can be used as a robust signature of the Earth matter effects, which in turn can distinguish between different neutrino mixing scenarios. We analyze the strengths and positions of these peaks as a function of the location of the SN in the sky and explore their features at a large scintillation detector as well as at a megaton water Cherenkov detector through Monte Carlo simulations.

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## 1. Introduction

The question “how well can one locate the SN in the sky by the neutrinos alone?” is important for two reasons. Firstly, the MeV-neutrino burst precedes the optical explosion by several hours so that an early warning can be issued to the astronomical community [3,

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†Based on Refs. [1, 2] in collaboration with A. Dighe, M. Kachelrieß, G. G. Raffelt and D. Semikoz.

4], specifying the direction to look for the explosion. Secondly, in the absence of any SN observation in the electromagnetic spectrum, a reasonably accurate location in the sky is crucial for determining the neutrino Earth-crossing path to various detectors since the Earth matter effects on SN neutrino oscillations may well hold the key to identifying the neutrino mass hierarchy [5, 6, 7, 8, 9, 10].

Nearly contemporaneously with the optical explosion an outburst of TeV neutrinos is expected due to pion production by protons accelerated in the SN shock [11]. Its detection would not only lead to an accurate measurement of the SN location, but also would be important as the first proof that SN remnants accelerate protons.

Although the optical signal can give the most accurate determination of the SN position in the sky, it could be possible that the SN is not seen in the entire electromagnetic spectrum. This can be the case if it is optically obscured, no suitable x- or  $\gamma$ -ray satellite operates, or simply the stellar collapse produces no explosion. In such a scenario, the best way to locate a SN by its core-collapse neutrinos is through the directionality of  $\nu e^- \rightarrow \nu e^-$  elastic scattering in a water Cherenkov detector such as Super-Kamiokande [12, 13, 14].

The pointing accuracy is strongly degraded by the inverse beta reactions  $\bar{\nu}_e p \rightarrow n e^+$  that are nearly isotropic and about 30–40 times more frequent than the directional scattering events. Recently it was proposed to add to the water a small amount of gadolinium, that would allow one to detect the neutrons and thus to tag the inverse beta reactions [15]. Evidently this would greatly improve the pointing.

Extracting information from “directional data” is a field in its own right [16, 17]. An efficient method is the “brute force” maximum likelihood estimate of the electron events. For a large number of events, the accuracy with this method in fact asymptotically approaches the minimum variance as given by the Rao-Cramér bound [18, 19]. However, for a small number of signal events,  $N_s \lesssim 200$ , the Rao-Cramér bound overestimates the pointing accuracy. On the other hand the likelihood method requires as input the functional form of the neutrino energy spectra that are only poorly known. Therefore we look for “parameter-free” methods that use only the information contained in the data and exploit the symmetry of the physical situation. We discuss the efficiency of two methods closely related to the harmonic analysis and find a simple iterative procedure making them nearly as efficient as the maximum likelihood approach. We use the most efficient method thus obtained for analyzing the simulated events at a detector. We also study the dependence of the pointing accuracy on the neutrino mixing parameters and the initial neutrino spectra.

One of the important applications of an accurate SN pointing is the determination of the distance traveled by the neutrinos through the Earth before they reach the detector. When neutrinos pass through the Earth, their spectra may get modified due to the Earth matter effects. The presence or absence of these effects can distinguish between different neutrino mixing scenarios [20]. It is possible to ascertain the presence of these matter effects using the signal at a single detector. It has recently been pointed out [10] that the Earth matter effects on supernova neutrinos traversing the Earth mantle give rise to specific frequencies in the “inverse energy” spectrum of these neutrinos. These frequencies, which may be identified through the Fourier transform of the inverse energy spectrum, are independent of the initial neutrino fluxes and spectral shapes. Therefore, its identification

serves as a model independent signature of the Earth matter effects on SN neutrinos. We study the positions and strengths of these frequency components analytically in terms of the SN location.

Although it is difficult to isolate these frequencies individually from the background fluctuations from a single SN burst, we suggest a procedure that can identify the presence of these frequency components in a sizeable fraction of cases. We quantify the efficiency of this algorithm by simulating the SN neutrino signal at a large scintillation detector like LENA [21] and at a megaton water Cherenkov detector like Hyper-Kamiokande. Whereas the scintillation detector has the advantage of a much better energy resolution, this is compensated in part by the larger number of events in a megaton water Cherenkov detector.

We begin in Sec. 2 with a discussion of the statistical methodology for extracting information from directional data using a toy model. In Sec. 3 we study the realistic SN pointing accuracy of water Cherenkov detectors as a function of the neutron tagging efficiency, using realistic SN neutrino spectra. In Sec. 4 we turn to the pointing accuracy of high-energy neutrino telescopes. In Sec. 5, we discuss the positions and the strengths of the frequencies that characterize the “inverse-energy” spectra of the neutrinos crossing the Earth mantle as well as the core. In Sec. 6, we simulate the SN neutrino spectra at the detectors and study the features of the peaks with the background fluctuations averaged out. In Sec. 7, we introduce a method to identify the peaks in the presence of background fluctuations and make a quantitative estimation of the probability of peak identification as a function of the location of the SN in the sky. Sec. 8 is given over to conclusions.

## 2. Analyzing directional data

### 2.1 Pointing with maximum likelihood estimate

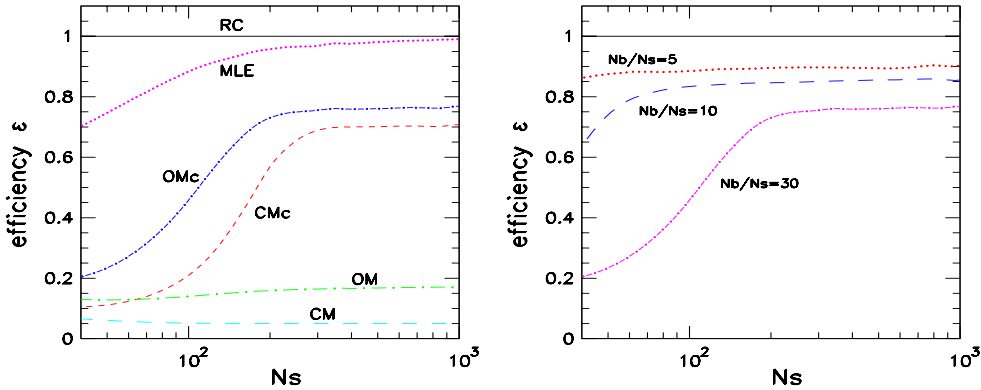
In order to analyze the statistical methodology of the directional data we will first consider the toy model introduced in Ref. [12], i.e. we imagine a directional signal that is distributed as a two-dimensional Gaussian on a sphere, together with an isotropic background. This would mimic the forward  $\nu$ - $e$  elastic scattering events and the nearly isotropic inverse beta decay background. The pdf on a sphere that represents  $N_s$  signal events distributed like a Gaussian around the direction  $(\vartheta_0, \phi_0)$  as well as the  $N_b$  isotropic background events is

$$f(\vartheta, \phi | \vartheta_0, \phi_0) d\vartheta d\phi = \frac{d\mu}{N_b + N_s} \left[ \frac{N_b}{4\pi} + \frac{N_s}{C} \exp\left(-\frac{\ell^2}{2\delta_s^2}\right) \right], \quad (2.1)$$

where  $\ell$  is the angular distance between the direction of an incoming neutrino and the experimentally measured direction of the Cherenkov cone, and  $\delta_s = 17^\circ$ .

The maximum likelihood estimate (MLE) method is the most efficient way to extract information from statistical data. To estimate a lower bound on the uncertainty of the parameters extracted by the MLE method, the Rao-Cramér bound [18, 19], one commonly uses the Fisher information matrix [22],

$$F_{ij} \equiv \left\langle \frac{\partial^2 \ln L(\vartheta_0, \phi_0)}{\partial \Theta_i \partial \Theta_j} \right\rangle \Rightarrow (\Delta\vartheta)^2 \geq (\Delta\vartheta)_{\text{Fisher}}^2 \equiv 1/F, \quad (2.2)$$



**Figure 1:** Left: Efficiencies of different estimation methods described in the text for  $N_b/N_s = 30$ . RC corresponds to the Rao-Cramér minimum variance. Right: Efficiency of OMc with a  $40^\circ$  angular cut for different values of  $N_b/N_s$ .

where  $L(\vartheta_0, \phi_0)$  is the maximum likelihood function for the pdf of Eq. (2.1).

In the left panel of Fig. 1 we show the efficiency  $\varepsilon$  of the MLE in “Fisher units”,  $\varepsilon \equiv (\Delta\vartheta)_{\text{Fisher}}^2 / (\Delta\vartheta)^2$ , as a function of the number  $N_s$  of signal events while keeping the background-to-signal ratio  $N_b/N_s$  fixed at 30, which is the expected ratio of the inverse beta decay events to the elastic scattering events in a water Cherenkov detector. Though the MLE efficiency tends asymptotically to the Rao-Cramér bound for large values of  $N_s$ , this bound overestimates the MLE pointing accuracy by  $\sim 10\%$  for  $N_s \lesssim 200$ .

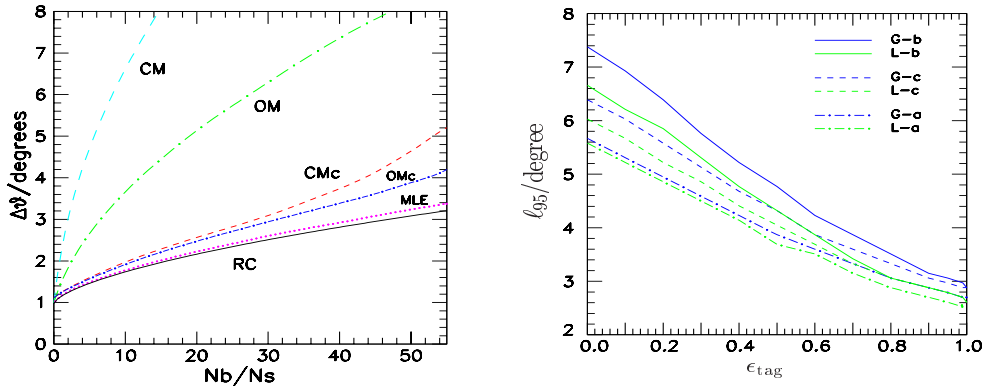
## 2.2 Efficiencies of parameter-free methods

The MLE is an optimal method to extract information from experimental data if the probability distribution function is known. This is not the case in our situation, where the exact forms of the neutrino spectra are needed and these are only poorly known. It is therefore worthwhile to look for other methods which may be less efficient, but which do not depend on the exact form of the pdf.

Let us consider two pointing methods that exploit the symmetries of our physical situation, but are independent of the exact details of the pdf: the “center of mass” (CM) method, and the “orientation matrix” (OM) method [16],

$$S_i \equiv \sum_{\alpha=1}^N x_i^{(\alpha)} \quad \text{and} \quad T_{ij} \equiv \sum_{\alpha=1}^N x_i^{(\alpha)} x_j^{(\alpha)}, \quad (2.3)$$

respectively, as described in Ref. [1]. Neither of these methods requires any prior knowledge of the neutrino spectra or cross sections. However, they involve some loss of information and hence will give larger pointing errors than the MLE. In order to quantify the efficiency of these methods we generate a data sample according to the pdf of Eq. (2.1) and show the respective pointing errors in the left panel of Fig. 2 as a function of  $N_b/N_s$  and for  $N_s = 300$ . Note that in the absence of neutron tagging this ratio is expected to be around 30–40. We observe that the error of MLE is almost the same as the Rao-Cramér (RC) bound. However, the errors of CM and OM are much larger.



**Figure 2:** Left: pointing error  $\Delta\theta$  for different estimation methods for  $N_s = 300$ . Right: pointing accuracy  $\ell_{95}$  as a function of the neutron tagging efficiency  $\epsilon_{\text{tag}}$  for six cases corresponding to three neutrino mixing scenarios and two models for the initial neutrino spectra.

In order to increase the efficiency of CM and OM, we use the physical input that the signal is concentrated within a small region around the peak. Cutting off the events beyond a certain angular radius increases the signal to background ratio and gives a much better estimate of the incoming neutrino direction. We denote the CM and OM methods with this cutting procedure by CMc and OMc, respectively. Left panel of Fig. 2 shows how the pointing error decreases drastically with the cutting procedure. In particular the OMc method turns out to be more efficient than CMc in all the parameter ranges, henceforth we continue using only the OMc method for further estimations. Moreover, with neutron tagging, the value of  $N_b/N_s$  decreases, and that increases the efficiency of the OMc method, as can be inferred from the right panel of Fig. 1.

### 3. Supernova pointing accuracy of water Cherenkov detectors

We now apply the OMc method to a more realistic representation of the SN signal in a water Cherenkov detector.

In order to determine the pointing accuracy numerically we simulate a large ensemble of SN signals in a 32 kton water Cherenkov detector. To this end we assume that the SN is at a distance  $D = 10$  kpc and releases the neutron-star binding energy  $E_b = 3 \times 10^{53}$  erg in the form of neutrinos. Details of the assumed neutrino spectra and fluxes are given in Ref. [1]. The spread in the predicted neutrino spectra has been taken care of by using two models, a model from the Garching group (model G) [23] and a model from the Livermore group (model L) [24] as described in the same reference. We take into account the effects of neutrino flavor conversions by considering the three mixing scenarios, (a) normal mass hierarchy and  $\sin^2 \Theta_{13} \gtrsim 10^{-3}$ , (b) inverted mass hierarchy and  $\sin^2 \Theta_{13} \gtrsim 10^{-3}$ , and (c) any mass hierarchy and  $\sin^2 \Theta_{13} \lesssim 10^{-3}$ . As reaction channels we use elastic scattering on electrons  $\nu e^- \rightarrow \nu e^-$ , inverse beta decay  $\bar{\nu}_e p \rightarrow n e^+$ , and the charged-current reaction  $\nu_e + {}^{16}\text{O} \rightarrow X + e^-$ , while neglecting the other, subdominant reactions on oxygen.

If  $\ell$  is the angle between the actual and the estimated SN direction, one can define the opening angle  $\ell_\alpha$  for a given confidence level  $\alpha$  as the value of  $\ell$  for which the SN direction estimated by a fraction  $\alpha$  of all the experiments is contained within a cone of opening angle  $\ell$ . We show in Fig. 2 the opening angle for 95% C. L. for the six cases of neutrino parameters, for different efficiencies for neutron tagging. Qualitatively the pointing accuracy is governed by the background-to-signal ratio, being the most suitable scenario the one with the smallest ratio of  $\bar{\nu}_e$ - $\nu_x$  mixing to  $\nu_e$ - $\nu_x$  mixing, i.e. (L-a). One can see how for  $\varepsilon_{\text{tag}} = 0$ , at 95% C.L. the pointing accuracy is  $7.5^\circ$ , which improves to  $3^\circ$  for  $\varepsilon_{\text{tag}} = 1$ . For a larger detector like Hyper-Kamiokande, the desired accuracy can be calculated simply by rescaling according to the number of signal events. For a detector with 25 times the fiducial volume of Super-Kamiokande, the pointing accuracy is then expected to be  $2^\circ$  without gadolinium and  $0.6^\circ$  with  $\varepsilon_{\text{tag}} > 90\%$ .

#### 4. High-energy neutrino telescopes

Nearly contemporaneously with the optical explosion an outburst of TeV neutrinos is expected due to pion production by protons accelerated in the SN shock [11]. Recently it was suggested that the high-energy neutrino signal would arrive just 12 hours after the SN explosion and would last for about one hour [11]. Following this calculation and assuming  $E_{\text{max}} = 1$  TeV we expect around 50 muon events in a  $\text{km}^2$  detector during 1 hour at a time about 12 hours after the SN explosion. In the case where the SN happens in a part of the sky that a given neutrino telescope sees through the Earth future  $\text{km}^2$  detectors like IceCube at the South Pole [25] can detect the high energy neutrinos with an angular resolution is around one degree for each event. Therefore, the pointing accuracy of a  $\text{km}^2$  detector is of the order of a few tenths of a degree. Even the existing smaller detectors, like AMANDA-II with an effective area of  $0.1 \text{ km}^2$  and angular resolution of  $2^\circ$  at TeV energies would be able to resolve the SN direction to better than  $1^\circ$ . If the high-energy SN neutrinos arrive “from above,” they are masked by the large background of atmospheric muons. For IceCube this background corresponds to nearly  $300 \text{ hour}^{-1} \text{ degree}^{-2}$  [26]. If we note that the angular resolution is about  $1^\circ$ , the expected signal of 100 events will be much larger than the background fluctuations in one pixel of the sky. Moreover, the expected SN neutrinos will have multi-TeV energies so that energy cuts will reduce the background and may allow one to detect the SN signal from the “bad” side of the sky.

#### 5. Earth matter effects on supernova neutrinos

The neutrino detectors, apart from a heavy-water detector like SNO, can give detailed spectral information only about the  $\bar{\nu}_e$  flux. We shall therefore concentrate on the  $\bar{\nu}_e$  spectrum in this paper. In the presence of flavor oscillations a  $\bar{\nu}_e$  detector actually observes the flux

$$F_e^D(E) = \bar{p}^D(E)F_e^0(E) + [1 - \bar{p}^D(E)] F_x^0(E), \quad (5.1)$$

where  $F_i^0$  and  $F_i^D$  stand for the initial and detected flux of  $\nu_i$  respectively, and  $\bar{p}^D(E)$  is the survival probability of a  $\bar{\nu}_e$  with energy  $E$  after propagation through the SN mantle and perhaps part of the Earth before reaching the detector.

In the absence of Earth effects, the dependence of the survival probability on  $E$  is very weak. A significant modification of  $\bar{p}^D$  due to the Earth effects takes place only in the cases (a) and (c). The identification of the Earth effects can then rule out the “null hypothesis” of an inverted hierarchy and  $|U_{e3}|^2 \gtrsim 10^{-3}$ . Let us consider then these scenarios (a) and (c). In these cases,  $\bar{\nu}_e$  produced in the SN core travel through the interstellar space and arrive at the Earth as  $\bar{\nu}_1$ . The oscillations inside the Earth are essentially  $\bar{\nu}_1$ – $\bar{\nu}_2$  oscillations [5] so that we need to solve a  $2 \times 2$  mixing problem.

When the antineutrinos pass through the Earth the survival probability  $\bar{p}^D$  may be written as an expansion in terms of  $\omega \approx 0.1$  in the form [2]

$$\bar{p}^D \approx \bar{A}_0 + \sum_{i=1}^{\alpha} \bar{A}_i \sin^2(\phi_i/2) + \mathcal{O}(\omega^2), \quad (5.2)$$

where  $\bar{A}_0 \equiv \cos^2 \vartheta_{12}$  and the coefficients  $\bar{A}_i \sim \mathcal{O}(\omega)$  depend only on the mixing parameters. When the neutrinos cross only the mantle there is a single frequency,  $\alpha = 1$ , located at  $\phi = \phi_m$ , whereas when they also cross the core three different frequencies can be clearly observed at  $\phi_1 = \phi_m/2$ ,  $\phi_2 = \phi_m/2 + \phi_c$  and  $\phi_3 = \phi_m + \phi_c$  [2]. These phases are connected to the neutrinos parameters through the expressions  $\phi_m \equiv 2\overline{\Delta m_{m,c}^2} L_{m,y}$  and  $\phi_c \equiv 2\overline{\Delta m_c^2} L_{c,y}$ , where  $\overline{\Delta m_{m,c}^2}$  is the mass squared difference between  $\bar{\nu}_1$  and  $\bar{\nu}_2$  inside the mantle(m) and the core(c), respectively, in units of  $10^{-5} \text{ eV}^2$ , and  $L_{m,c}$  is the distance traveled through the mantle and the core, in units of 1000 km. The “inverse energy” parameter is defined as  $y \equiv 12.5 \text{ MeV}/E_\nu$ .

The energy dependence of  $\bar{p}^D$  introduces modulations in the energy spectrum of  $\bar{\nu}_e$ , which may be observed in the form of local peaks and valleys in the spectrum of the event rate  $\sigma F_\bar{e}^D$  plotted as a function of  $y$ . The modulations are equispaced, indicating the presence of a single dominating frequency. These modulations can be distinguished from random background fluctuations that have no fixed pattern by using the Fourier transform of the inverse energy spectrum [10].

The net  $\bar{\nu}_e$  flux at the detector may be written using Eqs. (5.1) and (5.2) in the form

$$F_\bar{e}^D \approx \sin^2 \vartheta_{12} F_\bar{x}^0 + \cos^2 \vartheta_{12} F_\bar{e}^0 + \Delta F^0 \sum_{i=1}^3 \bar{A}_i \sin^2(k_i y/2), \quad (5.3)$$

where  $k_i \equiv \phi_i/y$  are the dominating frequencies, and  $\Delta F^0 \equiv (F_\bar{e}^0 - F_\bar{x}^0)$  depends only on the primary neutrino spectra. The last term in Eq. (5.3) is the Earth oscillation term. The other terms as well as the coefficient  $\Delta F^0 \bar{A}_m$  are relatively slowly varying functions of  $y$ , and hence contain frequencies in  $y$  that are much smaller than  $k_m$ . The dominating frequencies  $k_i$  are the ones that appear in the modulation of the inverse-energy spectrum. These frequencies are completely independent of the primary neutrino spectra, and indeed can be determined to a good accuracy from the knowledge of the solar oscillation parameters, the Earth matter density, and the position of the SN in the sky.

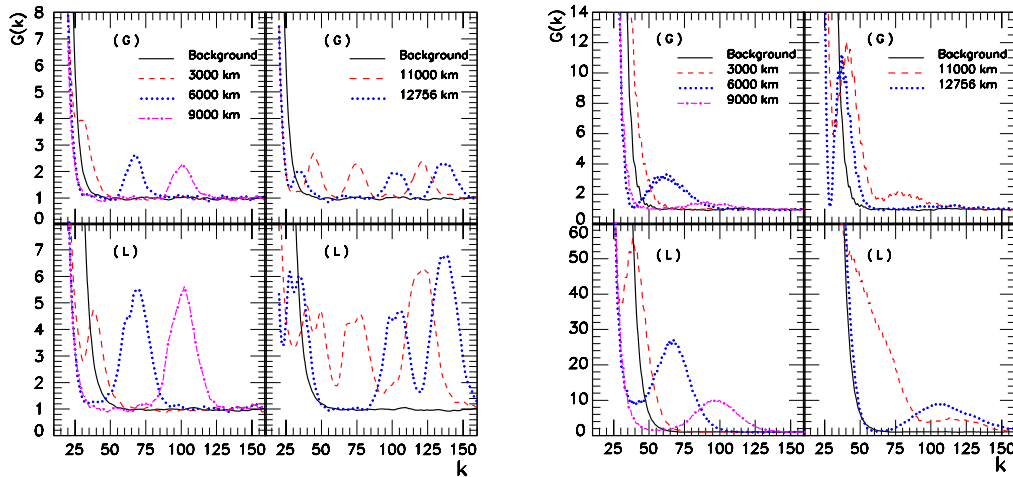
## 6. Peaks in the power spectrum of $\bar{\nu}_e$

We define the power spectrum of  $N$  detected events as

$$G(k) \equiv \frac{1}{N} \left| \sum_{i=1}^N e^{iky_i} \right|^2. \quad (6.1)$$

In the absence of Earth effect modulations,  $G(k)$  is expected to have an average value of one for  $k \gtrsim 40$  [10]. Earth effects introduce peaks in this power spectrum at specific frequencies, the identification of which correspond to the identification of the Earth effects.

To start with, we consider the power spectrum resulting from averaging 1000 SN simulations, assuming the SN is located at 10 kpc. This eliminates the fluctuations in the background, and illustrates the characteristics of the peaks in a clear manner. The power spectrum at a 32 kt scintillation detector for different distances traveled through the Earth is shown in the left panel of Fig. 3. The top panels use the Garching model whereas the bottom ones use the Livermore model. Only inverse beta decay events have been taken into account. The region  $k \lesssim 40$  is dominated by the “0-peak,” which is a



**Figure 3:** Averaged power spectra in the case of a large scintillation (left) and a megaton Water Cherenkov (right) detector for different SN models, Garching (G) and Livermore (L), and distances traveled through the Earth.

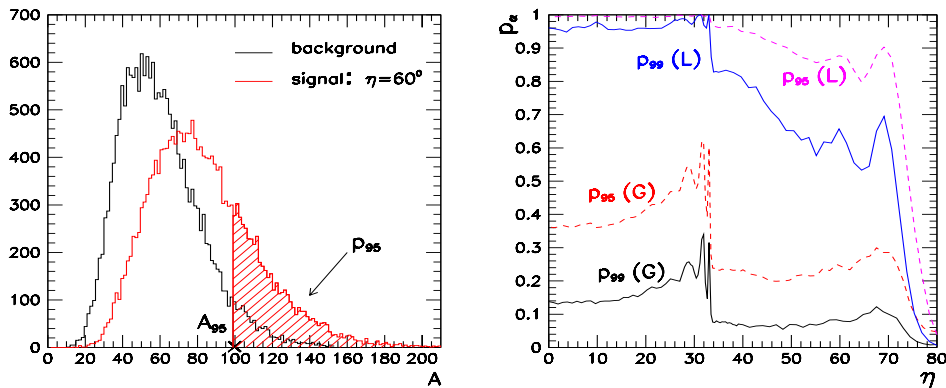
manifestation of the low frequency terms in Eq. (5.3). When the neutrinos traverse only the mantle, only one peak appears at the expected value of  $k_m$  that is proportional to the distance  $L_m$  traveled through the mantle. When the neutrinos travel also through the core, we observe three dominant peaks in each case, corresponding to  $k_1, k_2$  and  $k_3$  in Eq. (5.3), whose location depends on  $L_m$  and  $L_c$  as described in Section 5. The model independence of the peak positions may be confirmed by comparing the top and bottom panels of Fig. 3. The peaks obtained with the Livermore model are stronger as a result of the larger difference between the  $\bar{\nu}_e$  and  $\bar{\nu}_x$  spectra in that model, which increases the value



of  $\Delta F^0$  in Eq. (5.3). However, the positions of the peaks are the same as those obtained with the Garching model.

The energy resolution of a water Cherenkov detector is about a factor of six worse than that of a scintillation detector. This means that the energy spectrum is more “smeared out” and higher frequencies in the spectrum are more suppressed. This makes the peak identification more difficult, and even a detector of the size of Super-Kamiokande turns out not to be sufficient [10]. We show the power spectrum expected at a megaton water Cherenkov detector in the right panel of Fig. 3 for the two SN models considered here and for different locations of the SN.

## 7. Distinguishing the peaks from the background



**Figure 4:** Left: Area distribution of the background (black) and the signal (red) obtained for a 32 kton scintillation detector and Garching model for  $\eta = 60$ . Right: Comparison of  $p_{95}$  and  $p_{99}$  for the Garching and Livermore (L) models and the scintillation detector.

Although the analytic approximations seem to work well with the averaged power spectrum, in the real world the presence of fluctuations in the signal will spoil any naive theoretical peak. Therefore, in order to identify the peaks we need to introduce a prescription to carry out the analysis. Once we know the total distance traveled by the neutrinos through the Earth, we can calculate the position where the peak should lie. Then we consider the area around the position of the peak. Afterwards we compare the value of the measured area with the distribution of the area in the case of no Earth matter effects. Therefore, we perform a Monte Carlo analysis of the background case and calculate the exact distribution with which one can compare the actual area measured. We illustrate this in the left panel of Fig. 4. The confidence level of peak identification,  $\alpha\%$  C.L., may then be defined as the fraction of the area of the background distribution,  $A_\alpha$ , that is less than the actual area measured. In order to quantify the efficiency of the algorithm we simulate the area distribution for the “signal” using the neutrino mixing scenarios that allow Earth effects and compare it with the background distribution. The probability  $p_\alpha$  of peak identification at  $\alpha\%$  C.L. is the fraction of the area of the signal distribution above  $A_\alpha$ .

In the right panel of Fig. 4 we show  $p_{95}$  and  $p_{99}$  as a function of the nadir angle,  $\eta$ , in the case of a scintillation detector, for the two SN models considered. One can see that the presence of the core, i.e.  $\eta < 33^\circ$ , enhances the chances of detecting the Earth matter effects. As expected, the chances of peak identification are also higher when the primary spectra of  $\bar{\nu}_e$  and  $\bar{\nu}_x$  differ more.

One of the features of this algorithm is its robustness. However in some cases it turns out to be very conservative. In this sense we have checked that for some particular cases it is possible to enhance the efficiency of the method by optimizing the area integration around the peaks.

## 8. Summary and Conclusions

The MeV neutrinos from the cooling phase of a SN will arrive at the Earth several hours before the optical explosion. These neutrinos will not only give an early warning of the advent of a SN explosion, but they can also be used to determine the location of the SN in the sky, so that the optical telescopes may concentrate on a small area for the observation.

In a water Cherenkov detector like Super-Kamiokande, the  $\nu$ - $e$  scattering events are forward peaked and thus can be used for the pointing. The main background comes from the inverse beta decay reactions  $\bar{\nu}_e p \rightarrow n e^+$ , which is nearly isotropic and has a strength more than 30 times that of the electron scattering “signal” reaction.

We have observed that even the most efficient method, the maximum likelihood estimate, can reach the minimum variance bound only for a large number of signal events. More importantly, the maximum likelihood method (MLE) needs as an input the exact form of the fit function, which is not available due to our currently poor knowledge of the neutrino spectra. Instead, we explore some parameter-free methods that only use the data and exploit the symmetries inherent in the physical situation, and therefore give a model independent estimation of the pointing accuracy while sacrificing some information from the data. We find that a method that uses the “orientation matrix” with an appropriate angular cut (OMc) is almost as efficient as the MLE.

One may add gadolinium to Super-Kamiokande in order to tag neutrons and therefore reduce the background due to inverse beta decay events. With a simulation of a water Cherenkov detector like Super-Kamiokande, we determine the pointing accuracy obtained from a SN at 10 kpc. The accuracy has a weak dependence on the neutrino mixing scenarios and the initial neutrino spectra. The OMc method gives the pointing accuracy of  $7.5^\circ$  at 95% C.L. without neutron tagging, which improves to  $3.2^\circ$  at 95% tagging efficiency. Beyond this, the pointing accuracy saturates due to the presence of the oxygen events and the limited angular resolution of the detector.

The SN shock wave may produce a TeV neutrino burst that arrives at the Earth within a day of the initial MeV neutrino signal. This can give about 100 events with  $E > 1$  TeV at a  $\text{km}^2$  detector like IceCube. Since the angular resolution of this detector is as good as  $1^\circ$ , the SN may be located to an accuracy of a few tenths of a degree.

When neutrinos coming from a core-collapse supernova pass through the Earth before arriving at the detector, the spectra may get modified due to the Earth matter effects.

The presence or absence of these effects can distinguish between different neutrino mixing scenarios. We have seen that these Earth matter effects on supernova neutrinos can be identified at a single detector through peaks in the Fourier transform of their “inverse energy” spectrum.

We have performed an analytical study of the positions and the strengths of these frequencies for different neutrino trajectories through the Earth. In the case that the SN neutrinos only traverse the mantle a single peak shows up in the power spectrum, whereas they travel through both the mantle and the core as many as three distinct frequencies can be clearly observed in the inverse energy spectrum, what leads to an easier identification of the Earth matter effects.

In order to illustrate the qualitative features of the present analysis we have considered the power spectrum resulting from averaging 1000 SN simulations for different SN models and different detector capabilities. In particular we have assumed a 32 kton scintillation detector and a megaton water Cherenkov detector. We have shown how the energy resolution turns out to be crucial in detecting the modulation introduced in the neutrino spectra by the Earth matter effects. We have observed that the strength of the peaks is larger in those SN models with bigger differences between  $\bar{\nu}_e$  and  $\bar{\nu}_\mu$  spectra. However, their position is model independent. Therefore their identification serves as a clear signature of the Earth matter effects on SN neutrinos, which in turn can help to discard the neutrino mass scheme with inverted mass hierarchy and  $\sin^2 \Theta_{13} \gtrsim 10^{-3}$ .

We have introduced a simple algorithm to identify the peaks in the presence of background fluctuations, based on the integration of the area around the expected position of the peak. By comparing the area distribution without and with the spectral modulations induced by the Earth matter effects we have analyzed the statistical significance of the result. As expected the presence of the core as well as a larger difference in the initial spectra enhance the probability of identifying the Earth effects.

## Acknowledgments

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