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Superposition of two squeezed vacuum states and interference effects

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The superposition of two squeezed vacuum states is analyzed by studying the photon-number probability distribution and the quadrature-phase-eigenstate marginal distributions. Interference fringes in the distributions are observed for some superposition states. The nonlinear oscillator generates a particular superposition of two squeezed vacuum states and the properties of this superposition state are discussed and contrasted with the other superposition states.

The generation of nonclassical states of light is an objective in quantum optics. In addition to providing a test for the quantum theory of light, nonclassical states can have practical applications. For example, the nonclassical squeezed state of light, in which the quantum fluctuations of one quadrature are reduced below the vacuum level, has potential applications in optical communications and gravitational wave detection.[1–4] Here we study nonclassical states which arise as the coherent superposition of two states. In particular, we investigate the superposition of two squeezed-vacuum states, which possesses interesting but simple interference properties.

The presence of interference fringes in phase space and marginal distributions has been studied for superposition states generated by the nonlinear oscillator.[5–8] In particular, the superposition of two squeezed-vacuum states has been investigated.[8] However, the nonlinear oscillator generates one particular superposition of two squeezed vacuum states. Here we discuss the general superposition of two squeezed-vacuum states by contrasting the photon-number and marginal distributions. We observe that interference in the distributions varies substantially for different superposition states.

In experiments the squeezed vacuum has been generated by using four-wave mixing interactions[9,10] and optical parametric oscillators.[11] The squeezed vacuum is defined to be

$$|z\rangle \equiv \exp[\tfrac{1}{2}(z\hat{a}^{+2}-z^*\hat{a}^{2})]|0\rangle \tag{1}$$

for $|0\rangle$ the vacuum state of the field. The in-phase and out-of-phase Hermitian quadrature operators are defined to be $\hat{a}_1$ and $\hat{a}_2$, respectively, where $\hat{a}=\hat{a}_1+i\hat{a}_2$, which satisfy the canonical commutation relation $[\hat{a}_1,\hat{a}_2]=\frac{1}{2}i$. Approximate measurements of the quadrature phase are performed by homodyne detectors[12] and the statistics of quadrature-phase measurements conform to the quadrature-phase eigenstate marginal distributions. The photon-number operator is $\hat{n}=\hat{a}^{\dagger}\hat{a}$, and the photon-counting statistics, obtained by a photon detector with a very high quantum efficiency, is approximated by the photon-number distribution.[13]

Let $\{|\alpha_\ell\rangle\}$ represent the eigenstates of the quadrature operators $\hat{a}_\ell$ with real eigenvalues $\{\alpha_\ell\}$. The marginal distributions for the squeezed vacuum with respect to the quadrature operators $\hat{a}_\ell$ are given by $|{}_\ell\langle\alpha_\ell|z\rangle|^2$, for $\ell=1,2$. The eigenstate representations for the squeezed vacuum, for $\ell=1,2$ are[1]

$${}_\ell\langle\alpha_\ell|z\rangle=(2\pi V_\ell)^{-1/4}\exp\{-\tfrac{1}{4}V_\ell^{-1}[1+(-1)^\ell i\sinh(2r)\sin\theta]\alpha_\ell^2-\tfrac{1}{2}i\delta_\ell\}\ , \tag{2}$$

where the z-dependent parameters V_ℓ and δ_ℓ are given below. For $z=re^{i\theta}$,

$$4V_1=e^{2r}\cos^2\frac{\theta}{2}+e^{-2r}\sin^2\frac{\theta}{2} \tag{3}$$

and $V_2(r)=V_1(-r)$. The phase-shift terms are

$$\delta_1=\tan^{-1}\left[\frac{\tanh r\sin\theta}{1+\tanh r\cos\theta}\right] \tag{4a}$$

and

$$\delta_2=\tan^{-1}[\sinh(2r)\sin\theta]-\delta_1\ . \tag{4b}$$

The marginal distributions are the Gaussian functions $|{}_\ell\langle\alpha_\ell|z\rangle|^2$ with means at zero and variances V_ℓ. For z real the product of the variances satisfies the Heisenberg minimum uncertainty equality. The phase dependence of the quantum fluctuations is a consequence of the photon correlations. Photons are created and annihilated in pairs. The number-state matrix elements for the squeezed vacuum are[1]

$$\langle 2n|z\rangle=\left[\begin{pmatrix}2n\\ n\end{pmatrix}\operatorname{sech}r\right]^{1/2}(-\tfrac{1}{2}e^{i\theta}\tanh r)^n \tag{5}$$

for the even number states and zero for the odd states. The nonzero elements of the photon-number distribution are therefore

$$|\langle 2n|z\rangle|^2=\begin{bmatrix}2n\\ n\end{bmatrix}(\tfrac{1}{2}\tanh r)^n \operatorname{sech} r\ . \qquad (6)$$

The absence of odd-photon states in the distribution demonstrates the strong pair correlation between the photons.

The superposition of two squeezed-vacuum states $|re^{i\theta}\rangle$ and $|r'e^{i\theta'}\rangle$ is designated

$$|re^{i\theta},r'e^{i\theta'};e^{i\psi}\rangle=\mathcal{N}(|re^{i\theta}\rangle+e^{i\psi}|r'e^{i\theta'}\rangle) \qquad (7)$$

for $\mathcal{N}$, the normalization factor. Here we shall restrict our attention to the superposition of two squeezed-vacuum states where the magnitudes of squeezing are equal; i.e., $r=r'$. The distribution for the superposition state is equivalent to a sinusoidally modulated distribution for the squeezed vacuum:

$$|\langle 2n|re^{i\theta},re^{i\theta'};e^{i\psi}\rangle|^2 =4\mathcal{N}^2|\langle 2n|re^{i\theta}\rangle|^2\cos^2[\tfrac{1}{2}(\psi+n\theta'-n\theta)]\ , \qquad (8)$$

and all odd elements vanish. The interference between the two squeezed-vacuum states is evident in (8). The coefficient of the even state $|2n\rangle$ vanishes if $\Psi+n\theta'-n\theta$ is an odd multiple of π.

A maximal number of coefficients vanish for $\theta'-\theta$, a multiple of π, and $\psi=0$ or π. If $\psi=0$, then the coefficients of the states $|2n\rangle$ vanish for n odd; if $\psi=\pi$, then the coefficients of $|2n\rangle$ vanishes for n even. The photon-number distribution is shown in Fig. 1 for the state $|z,-z;1\rangle$. A minimal number of states vanish for $\theta'-\theta=\pi$ and $\psi=\pm\pi/2$. In this case the interference between the two squeezed-vacuum states is not manifested in the number distributions: the number distributions for the states $|z\rangle$ and $|z,-z;\pm i\rangle$ are identical. Photon-number distribution interference depends critically upon the coefficient phase ψ.

The marginal distributions are computed for the states $|r,-r;e^{i\psi}\rangle$ and $|ir,-ir;e^{i\psi}\rangle$. Both states are superpositions of two squeezed-vacuum states, squeezed in orthogonal directions in the phase-space picture. The states $|\pm r\rangle$ are minimum-uncertainty states with respect to the $\hat{a}_1$ and $\hat{a}_2$ quadratures and the states $|\pm ir\rangle$ are minimum-uncertainty states with respect to the quadrature operators $2^{-1/2}(\hat{a}_1\pm\hat{a}_2)$. For each case we obtain the marginal distributions for the $\hat{a}_1$ and $\hat{a}_2$ operators.

The marginal distributions for $|r,-r;e^{i\psi}\rangle$ are

$$|_{\ell}\langle\alpha_{\ell}|r,-r;e^{i\psi}\rangle|^2 =(\pi/2)^{1/2}\mathcal{N}^2\{e^{-r}\exp(-2e^{-2r}\alpha_{\ell}^2)+e^{r}\exp(-2e^{2r}\alpha_{\ell}^2) +2\exp[-2\cosh(2r)\alpha_{\ell}^2]\cos\psi\} \qquad (9)$$

for $\ell=1,2$. The distribution (9) represents the normalized sums of the Gaussian distributions for the two squeezed vacuum states $|\pm r\rangle$ and there is an added term due to interference. For $\psi=0$ and $\psi=\pi$, the effects of the interference term are maximized. For $\psi=\pm\pi/2$, interference is not present; that is, the constructive and destructive interference cancel. For the case that $\psi=\pi$, which corresponds to destructive interference, the distribution vanishes at

$$\alpha_i=\pm\left[\frac{1}{2}\frac{r}{\sinh(2r)}\right]^{1/2}, \qquad (10)$$

for $\ell=1,2$. At these values of α_{ℓ} (10) interference fringes are present. The marginal distributions for the states $|1,-1;\pm 1\rangle$ and $|1,-1;\pm i\rangle$ are shown in Fig. 2 and the distribution vanishes at $\alpha_{\ell}\approx 0.37$ as expected by substituting $r=1$ into (10).

Whereas the presence of interference fringes in the $\hat{a}_{\ell}$ marginal distribution depends on ψ, the interference is evident in the $\hat{a}_{\ell}$ bases for the rotated superposition state $|ir,-ir;e^{i\psi}\rangle$ for all ψ. The marginal distributions for the superposition state $|ir,-ir;e^{i\psi}\rangle$ are normalized sinusoidal modulations of the marginal distributions for the state $|ir\rangle$:

$$|_{\ell}\langle\alpha_{\ell}|ir,-ir;e^{i\psi}\rangle|^2 =4\mathcal{N}^2|_{\ell}\langle\alpha_{\ell}|ir\rangle|^2\cos^2[\tfrac{1}{2}(\psi+\delta_{\ell}) +(-1)^{\ell}\tanh(2r)\alpha_{\ell}^2]\ , \qquad (11)$$

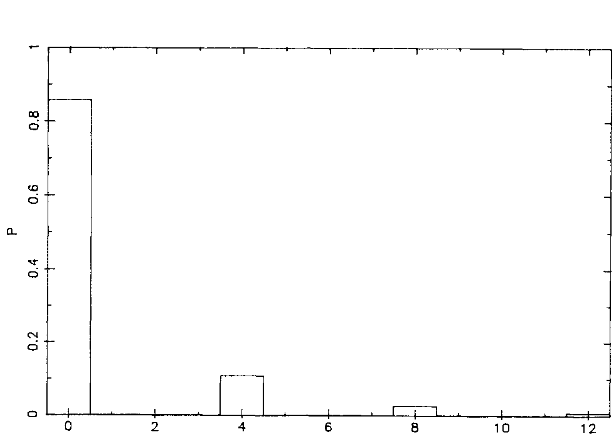

FIG. 1. Scaled photon-number distributions for the superposition state $|z,-z;1\rangle$ where $|z|=1$ and $0\le n\le 12$.

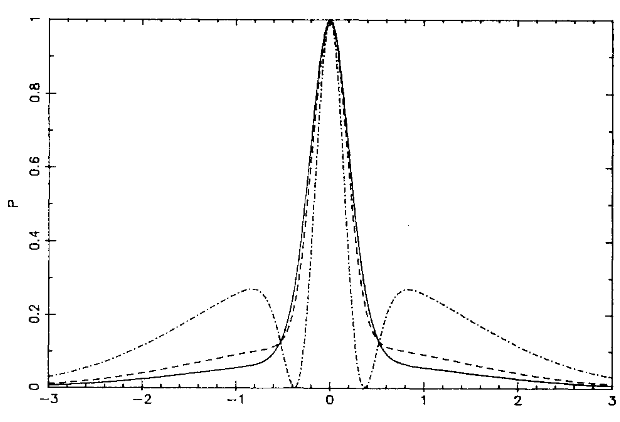

FIG. 2. Scaled marginal distributions for the superposition states $|1,-1;1\rangle$ (solid line), $|1,-1;\pm i\rangle$ (dashed line), and $|1,-1;-1\rangle$ (dotted-dashed line) in the $\hat{a}_{\ell}$ eigenbasis for $\ell=1,2$, and $-3\le\alpha_{\ell}\le 3$.

where $\ell=1$ and 2, and δ_ℓ are given by Eqs. (4) for $\theta=\pi/2$. The marginal distributions for the state $|ir\rangle$ are

$$|_\ell\langle\alpha_\ell|ir\rangle|^2=\left[\frac{2}{\pi}\operatorname{sech}(2r)\right]^{1/2}\exp[-2\operatorname{sech}(2r)\alpha_\ell^2] \quad (12)$$

for $\ell=1,2$. Interference effects are evident in expression (11). The phase ψ is responsible for a phase shift in the modulation of (11). Plots of the marginal distributions for $|i,-i;-1\rangle$ are shown in Fig. 3.

A superposition of two squeezed vacuum states is generated by the nonlinear oscillator. The Hamiltonian is[5-8]

$$\hat{H}=\hbar\omega\hat{a}^\dagger\hat{a}+\hbar\lambda(\hat{a}^\dagger\hat{a})^2 \ . \quad (13)$$

In the interaction picture an initial squeezed vacuum state evolves according to the expression

$$|z(t)\rangle=\exp[-i\lambda t(\hat{a}^\dagger\hat{a})^2]|z\rangle \ . \quad (14)$$

The superposition states $e^{i\pi/4}|z,-z;-i\rangle$ and $e^{-i\pi/4}|z,-z;i\rangle$ are obtained for $t=\pi/2\lambda$ and $t=3\pi/2\lambda$, respectively.

The superposition states which are generated by the nonlinear oscillator produce photon statistics which are identical to the squeezed-vacuum-photon statistics. Photon-counting techniques cannot be used to distinguish between the states. Furthermore the $\hat{a}_1$ and $\hat{a}_2$ marginal distribution statistics are identical for the squeezed vacuum $|r\rangle$ and the superposition states $|r,-r;\pm i\rangle$. In order to distinguish between the states by employing quadrature phase measurements, the local oscillator phase is varied. Interference fringes arise for the superposition state but not for the squeezed vacuum. However, the superposition states $|r,-r;\pm 1\rangle$ display photon-number-distribution interference and can be distinguished from the squeezed vacuum by photon-counting experiments. Also the $\hat{a}_1$ and $\hat{a}_2$ quadrature-phase statistics for the superposition states $|r,-r;\pm 1\rangle$ exhibit interference and fringes are present for the state $|r,-r;-1\rangle$. Interference effects in the photon-counting and the quadrature-phase statistics are critically dependent on ψ. The relation between phase-space interference and photon-number interference has been analyzed for the highly squeezed coherent state with nonzero amplitude[14,15] but the techniques are not easily applied to the study of squeezed-vacuum superposition states.

Whereas the superposition states $|z,-z;\pm i\rangle$ can be generated from the vacuum by a nonlinear oscillator interaction, we are not aware of a technique to generate arbitrary superposition states $|z,-z;e^{i\psi}\rangle$. Nevertheless, the presence of interference in the number and marginal distribution is interesting and the special nature of the states $|z,-z;\pm i\rangle$ has been noted. In practice, the interference fringes will be degraded. The statistical distribution of photon-counting measurements for the superposition state are given by the photon-number probability distribution for an ideal photon detector. The photon-number distribution fringes can be observed with an ideal photon detection scheme. However, the fringes decay very rapidly as the detector efficiency is reduced.[15] In practice, the number distribution fringes may be difficult to observe. The quadrature-phase observables are approximately measured by a homodyne detection scheme.[12] In the limit of intense coherent local oscillators the quadrature phase is accurately measured. A homodyne detector provides a method for measuring the marginal distribution interference fringes. However, a photon detector with nonunit quantum efficiency will partially suppress the marginal distribution fringes.[5-8]

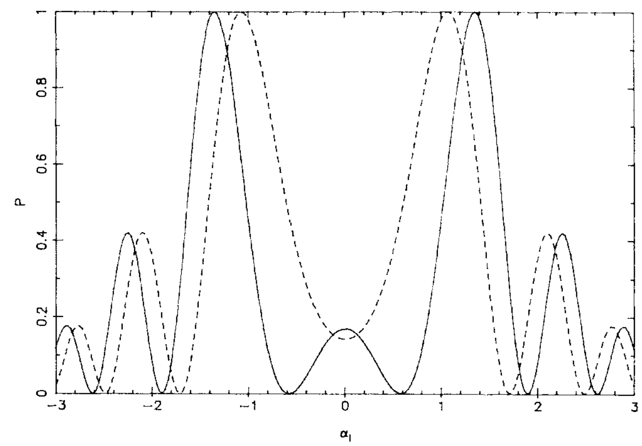

FIG. 3. Scaled marginal distribution for the superposition state $|i,-i;-1\rangle$ in the $\hat{a}_1$ eigenbasis (solid line) and the $\hat{a}_2$ eigenbasis (dashed line), where $-3\le\alpha_\ell\le 3$.

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