Superradiant instabilities in astrophysical systems

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CENTRA / IST,
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work in progress

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Outline

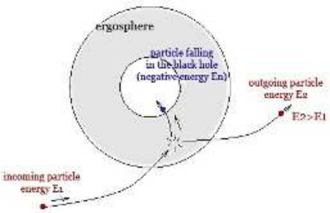
1. Motivation
2. Massive scalar fields
3. Massive vector fields
4. Conclusions

Motivation

Superradiance effect

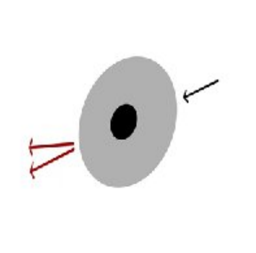
- Penrose process
  (Penrose '69, Christodoulou '70)
  - scattering of particles off Kerr BH

  $\Rightarrow$ reduction of BH mass



- superradiant scattering (Misner '72)
  - scattering of wave packet off Kerr BH
  - superradiance condition

$$\omega < m\Omega_H = m\frac{a}{2Mr_+}$$

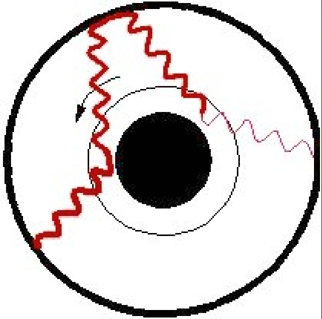


$\Rightarrow$ extraction of energy and angular momentum off BH
$\Rightarrow$ amplification of energy and angular momentum of wave packet

Superradiance instability

"black hole bomb" (Press & Teukolsky '72, Zeldovich '71)

- consider Kerr BH surrounded by mirror
- consider field with $\omega < m\Omega_H$
  $\Rightarrow$ superradiant scattering
- subsequequent amplification of superradiant modes

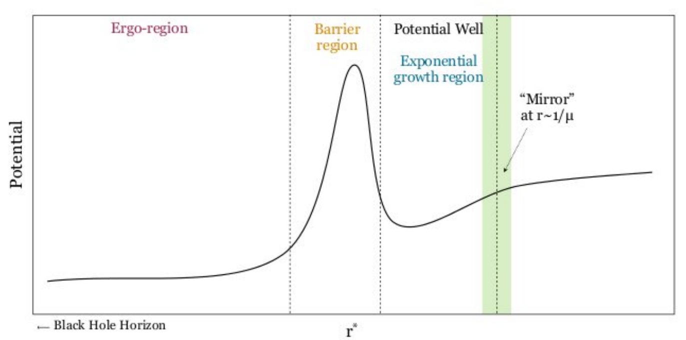


$\Rightarrow$ exponential growth of modes
$\Rightarrow$ instability due to superradiant scattering

Superradiance instability in physical systems

natural mirror provided by

- anti-de Sitter spacetimes
- massive fields with mass coupling $M\mu$
  (Damour et al. '76, Detweiler '80, Zouros & Eardley '79)



Arvanitaki & Dubovsky '11

Superradiance instability in physical systems

growth rate of massive scalar fields

- Detweiler '80: $M\mu << 1$

$$\tau \sim 24 \left(\frac{a}{M}\right)^{-1} (M\mu)^{-9} \left(\frac{GM}{c^3}\right)$$

- Zouros & Eardley '79 $M\mu >> 1$

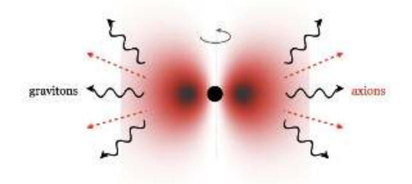
$$\tau \sim 10^7 \exp(1.84 M\mu) \left(\frac{GM}{c^3}\right)$$

- for astrophysical BHs and known particles: $M\mu \sim 10^{18}$
  $\Rightarrow$ insignificant for astrophysical systems?

Superradiance instability in physical systems

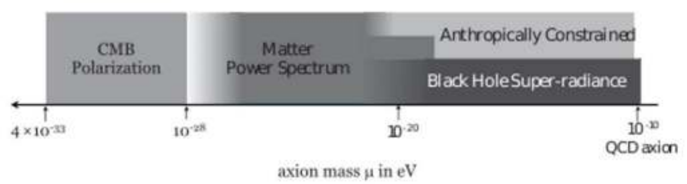
- most promising mass range: $M\mu \sim 1$
- ultralight bosons proposed by string theory compactifications: axions (Arvanitaki & Dubovsky '10)

- formation of bosonic bound states around astrophysical BHs
- gravitational wave emission
- "bosenova"-like particle bursts (see Yoshino's talk)



(Arvanitaki & Dubovsky '11)

Superradiance instability in physical systems



(Kodama & Yoshino '11)

- landscape of ultralight axions $\Rightarrow$ "string axiverse"
- bosonic fields with $M\mu \sim 10^{-22}$ as dark matter candidates
- small, primordial BHs with $M \sim 10^{-18} M_{\odot}$
- bosonic cloud around SMBHs ($M \sim 10^{9} M_{\odot}$) if $10^{-21} \leq M\mu \leq 10^{-16}$
  $\Rightarrow$ probe of photon mass (upper bound $\mu_{\gamma} \sim 10^{-18}$ (Nakamura et al.'10))

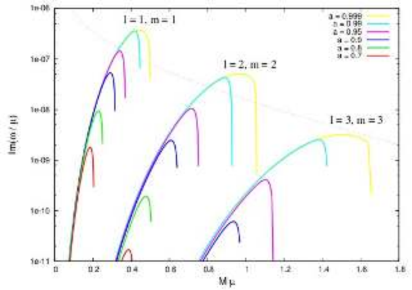
Massive scalar field

Massive scalar fields - recent results

- Klein-Gordon equation

$$(\Box - \mu^2)\psi = 0\,, \quad \text{with} \quad \psi = \exp(im\phi - i\omega t)S_{lm}(\theta)R_{lm}(r)$$

- bound states: maximum instability growth rate for $l = m = 1$, $a/M = 0.99$, $M\mu = 0.42$: $\frac{1}{\tau} \sim 1.5 \cdot 10^{-7} (\frac{GM}{c^3})^{-1}$ (Dolan '07)
- numerical results:
  - Strafuss & Khanna '05: $\frac{1}{\tau} \sim 2 \cdot 10^{-5} \frac{1}{M}$
  - Kodama & Yoshino '12: $\frac{1}{\tau} \sim 3.2 \cdot 10^{-7} \frac{1}{M}$



Dolan '07

Massive scalar fields - Code setup

goal: study time evolution of massive scalar field in Kerr background

- Kerr background in Kerr-Schild coordinates $\rightarrow$ excision of BH region
- Klein-Gordon equation $(\Box - \mu^2)\psi = 0$ as $3+1$ time evolution problem

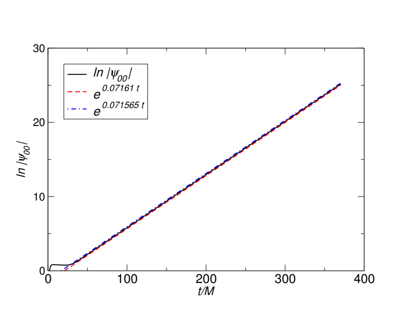
$$
\begin{aligned}
d_t\psi &= -\alpha\Pi \\
d_t\Pi &= -\alpha(D^i D_i\psi - \mu^2\psi - K\Pi) - D^i\alpha D_i\psi
\end{aligned}
$$

- initial data: gaussian wave packet
- 4^{th} finite differences in space, 4^{th} order Runge-Kutta time-integrator
- extraction of scalar field at fixed r_{ex}, mode decomposition

$$\psi_{lm}(t) = \int d\Omega\psi(t,\theta,\phi)Y^*_{lm}(\theta,\phi)$$

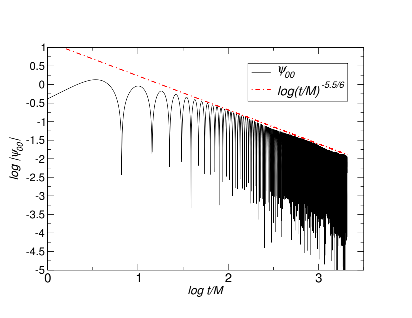
Massive scalar fields - Code tests I

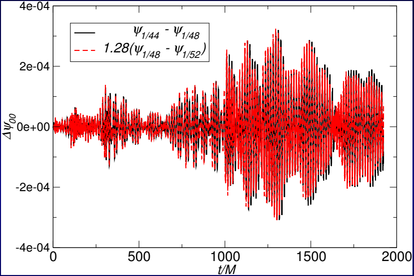
- consider *unphysical* scalar field mass $M\mu = -\frac{10}{r^4}$
  $\Rightarrow$ theoretical prediction:
  $\psi_{00} \sim \exp(\omega_I t)$ with $\omega_I = 0.071565$
- numerical result $\omega_I = 0.07161$
  $\Rightarrow$ agreement within 0.06%



Massive scalar fields - Code tests II

massive scalar field $M\mu = 0.42$ in Schwarzschild background

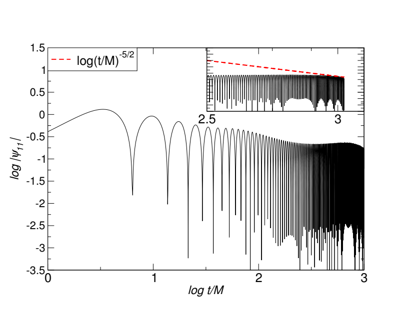


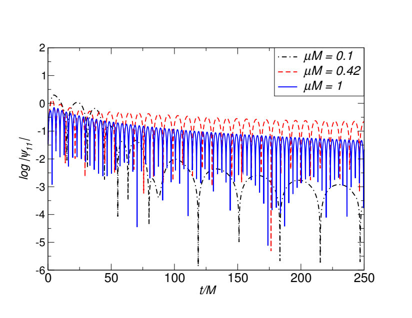


- late-time tail $\psi \sim t^{-5/6}$ (Koyama & Tomimatsu '02, Burko & Khanna '04)
- 2^{nd} order convergence
- discretization error:
  $\Delta\psi/\psi = 3.6\%$ @ $t \sim 1000M$,
  $\Delta\psi/\psi = 6.7\%$ @ $t \sim 1500M$

Massive scalar fields - Code tests II

massive scalar field in Schwarzschild background with $M\mu = 0.1, 0.42, 1$





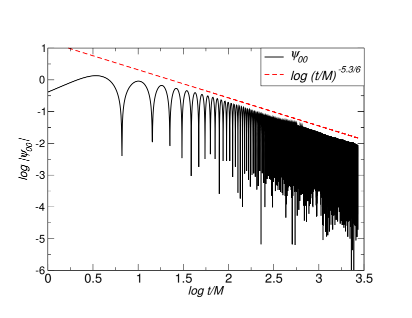
- tails in agreement with (Koyama & Tomimatsu '02, Burko & Khanna '04):

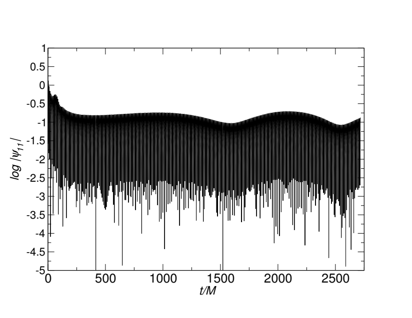
$$\begin{array}{ll} M\mu = 0.1 & \psi_{11} \sim t^{-l-3/2} \sin(\mu t) \\ M\mu = 0.42 & \psi_{11} \sim t^{-l-3/2} \sin(\mu t) \text{ @ } t \sim 1000M \\ M\mu = 1.0 & \psi_{11} \sim t^{-5/6} \end{array}$$

- slowest decay for $M\mu = 0.42$

Massive scalar fields in Kerr background

- evolution of scalar field with $M\mu = 0.42$ in Kerr background with $a/M = 0.99$
- animation of ψ along z-axis
- observation of quasi-resonant a state?





Massive vector fields

Massive vector fields

- massive hidden $U(1)$ vector fields from string theory compactification (e.g., Jaeckel & Ringwald '10)
- expected: superradiance effect stronger than in scalar field case
- rich phenomenology

- studied by Galt'sov et al '84, Konoplya '06, Konoplya et al '07, Herdeiro et al '11, *Rosa & Dolan '11*
- vector field eqs. in Kerr non-separable $\Rightarrow$ challenging problem

Massive vector fields in Schwarzschild background

Rosa & Dolan '11:

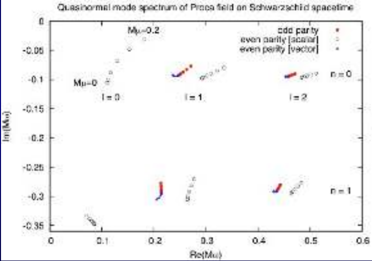
- Proca field equations

$$\nabla_\nu F^{\mu\nu} + \mu_A^2 A^\mu = 0 \quad F_\mu = \nabla_\mu A_\nu - \nabla_\nu A_\mu$$

- Lorenz condition has to be satisfied $\nabla_\mu A^\mu = 0$
  $\Rightarrow$ scalar mode gains physical meaning
- decomposition of A_μ in vector spherical harmonics $Z_\mu^{(i)lm}$
- continued fraction method and forward integration

Massive vector fields in Schwarzschild background

Rosa & Dolan '11: QNM spectrum

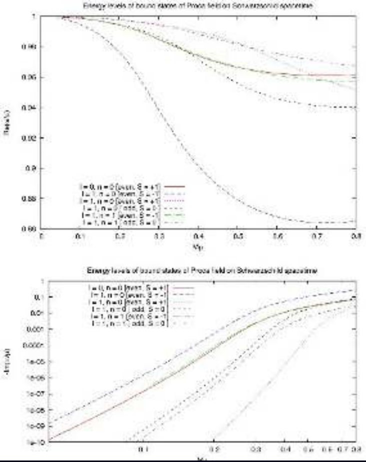


- for given l, n:
  2 even parity modes
  (scalar and vector field modes),
  1 odd parity mode (vector field mode)
- in electromagnetic limit ($M\mu_A \to 0$):
  - scalar mode = gauge mode
  - even and odd vector mode degenerate
- field mass - breaking of degeneracy
- distinct frequencies of even parity modes

Massive vector fields in Schwarzschild background

Rosa & Dolan '11: bound states

- in limit $M\mu_A \to 0$:
  hydrogenic spectrum
  $\omega_R \sim 1 - \frac{(M\mu_A)^2}{2N^2}$
- mode types: $S = 0, \pm 1$
- lowest energy mode: $l = 1, S = -1$
- power-law dependence
  $\omega_I \sim (M\mu_A)^\eta$
  with $\eta = 4l + 2S + 5$



Massive vector fields in Kerr - Code setup

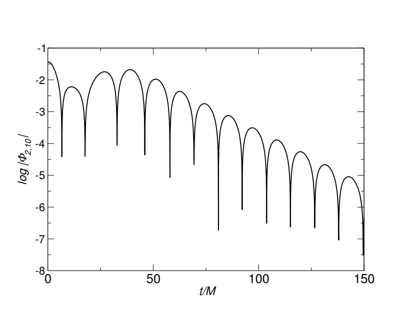
goal: study time evolution of Proca field in Kerr background
(work in progress)

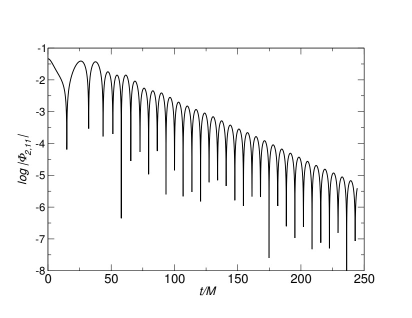
- Kerr background in Kerr-Schild coordinates $\rightarrow$ excision of BH region
- Proca equation $\nabla_\nu F^{\mu\nu} + \mu_A^2 = 0$
  Lorenz condition $\nabla_\mu A^\mu = 0$
- define $A_\mu = \mathcal{A}_\mu + n_\mu \varphi$, $E_\mu = F_{\mu\nu} n^\nu$
  $\Rightarrow$ formulation as $3+1$ time evolution problem
- initial data: gaussian wave packet
- 4^{th} finite differences in space, 4^{th} order Runge-Kutta time-integrator
- extraction of Newman-Penrose scalar Φ_2 at fixed r_{ex}, mode decomposition

$$\Phi_{2,lm}(t) = \int d\Omega \Phi_2(t,\theta,\phi)\, {}_{-1}Y^*_{lm}(\theta,\phi)$$

Massive vector fields in Kerr - Code test I

test massless vector field $\mu_A = 0$ in Kerr with $a/M = 0.99$



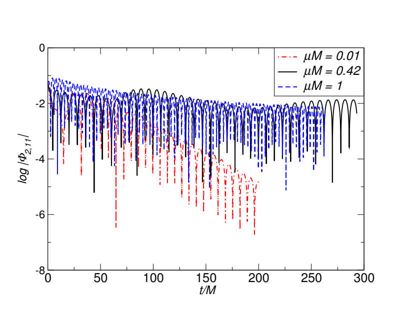


- QNM frequencies
  $\omega_{10} M = 0.277 - i\ (0.274 - i0.076)$
  $\omega_{11} M = 0.461 - i0.041\ (0.463 - i0.031)$
- agreement with theoretical prediction (Berti et al, '09)

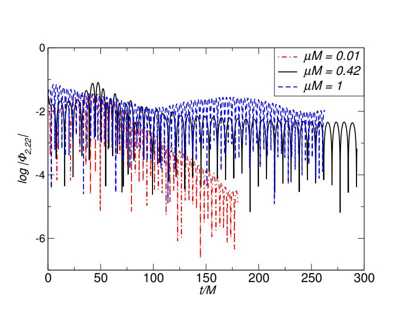
Massive vector fields in Kerr - Code test I

- vector field with $\mu_A = 0.01, 0.42, 1$ in Kerr with $a/M = 0.99$
- animation of $\mathcal{A}_z$ along z-axis

$l = m = 1$



$l = m = 2$



Conclusions

- massive fields in Kerr spacetimes exhibit extremely rich spectra
- evolution of scalar field wave packets
  - extensive code testing
  - resonant effect for $M\mu = 0.42$, $a = 0.99$?
- first evolutions of massive vector fields in Kerr background
  - low mass fields $M\mu_A = 0.01$ damped
  - quasi-resonant effect for $M\mu_A = 0.42$ and $M\mu_A = 1$?
  - still in its infantry $\Rightarrow$ more results to come soon

Thank you!

http://blackholes.ist.utl.pt