

Supersymmetric Rényi Entropy

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(U. Tokyo)

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[1306.2958] TN and I.Yaakov (Princeton)

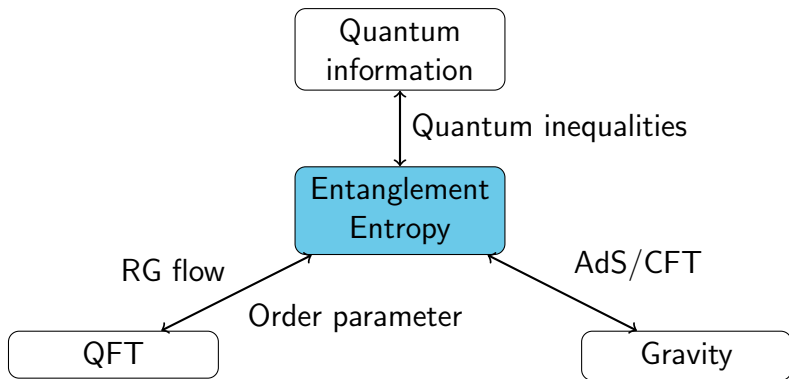
[1401.6764] TN

[1410.2206] N. Hama (Kyoto), TN and T. Ugajin (KITP)

Introduction

Entanglement
Entropy

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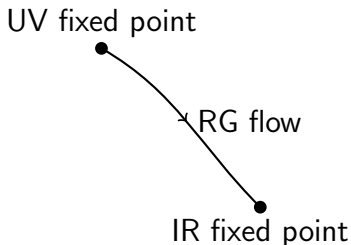


What's the role of entanglement entropy in QFT?

- ▶ Entanglement entropy as a measure of degrees of freedom
- ▶ Construct a monotonic function $c(\text{Energy})$ of the energy scale
 - ▶ Entropic c -theorem in two dimensions
 - ▶ F -theorem in three dimensions

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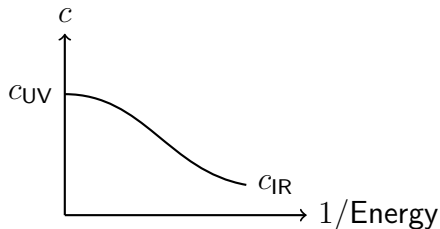
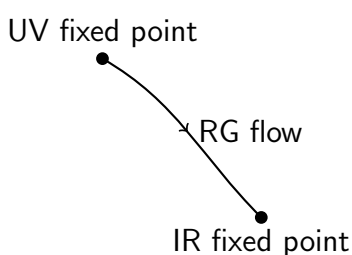
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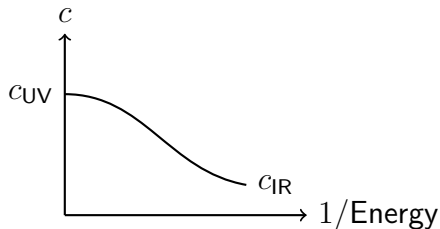
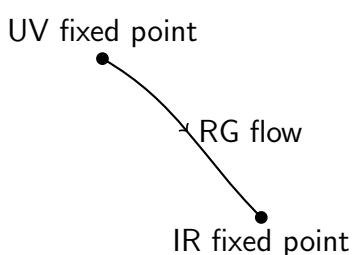
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Questions to be addressed

- ▶ If no, what is the most natural extension of them?
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Main question

Can we define entanglement (Rényi) entropies for SUSY gauge theories?

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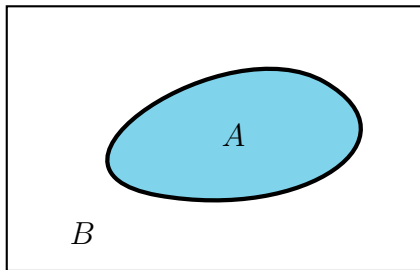
- 1 Review of entanglement entropy
- 2 Entanglement entropy and RG flow
 - C -theorem in 3d?
 - F -theorem
- 3 Supersymmetric Rényi entropy
 - 3d $\mathcal{N} = 2$ theories
 - Holographic supersymmetric Rényi entropy
 - 5d $\mathcal{N} = 1$ theories

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Definition of entanglement entropy

- ▶ Divide a system to A and $B = \bar{A}$: $\mathcal{H}_{tot} = \mathcal{H}_A \otimes \mathcal{H}_B$

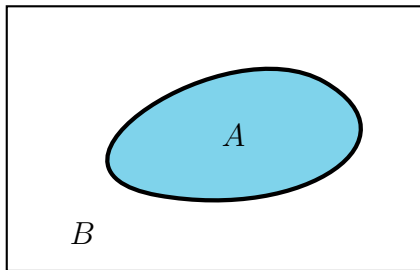


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- ▶ $|\Psi\rangle$: wave function of a ground state

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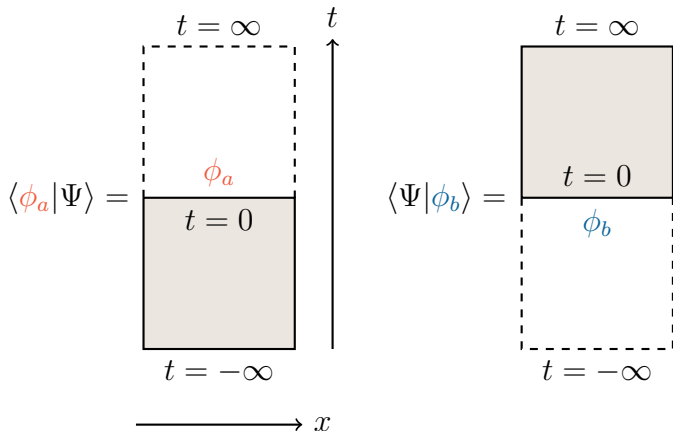
$$\rho_{tot} = \frac{1}{\langle \Psi | \Psi \rangle} |\Psi\rangle \langle \Psi|$$

- ▶ Reduced density matrix:

$$\rho_A = \text{tr}_B \rho_{tot} = \sum_i \langle \psi_B^i | \rho_{tot} | \psi_B^i \rangle$$

$\mathcal{H}_B = \{|\psi_B^1\rangle, |\psi_B^2\rangle, \dots\}$ orthonormal basis

Path integral representation of the wave function



States $|\phi_{a,b}\rangle$ are the boundary conditions at $t = 0$

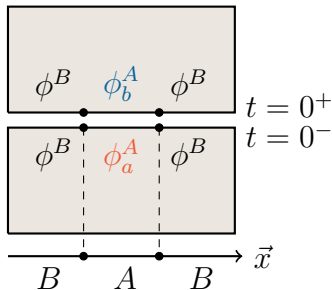
Replica trick and covering space

$$[\rho_A]_{ab} = \frac{1}{Z_1} \int [\mathcal{D}\phi^B(t=0, \vec{x} \in B)] (\langle \phi_a^A | \langle \phi^B |) |\Psi\rangle \langle \Psi| (| \phi_b^A \rangle | \phi^B \rangle) ,$$

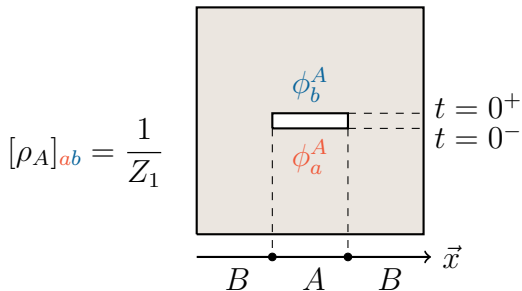
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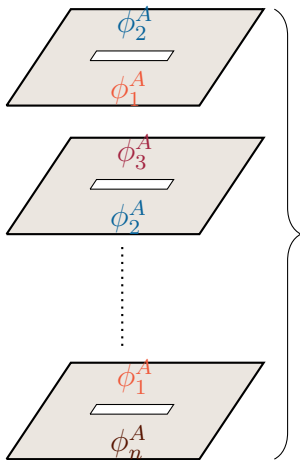


Replica trick and covering space



Replica trick and covering space

$$\mathrm{tr}_A \rho_A^n = \frac{1}{(Z_1)^n}$$



n copies $\equiv Z_n$

$$= \frac{Z_n}{(Z_1)^n}$$

Replica trick and covering space

Entanglement entropy

$$S_A = -(\partial_n - 1) \log Z_n \Big|_{n=1}$$

All we need to know is the partition function Z_n on the n -fold cover $\mathcal{M}_n!$

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One parameter extension ($S_A = \lim_{n \rightarrow 1} S_n$)

n -th Rényi entropy

$$S_n = \frac{1}{1-n} \log \frac{Z_n}{(Z_1)^n}$$

Properties of entanglement entropy

For a pure ground state

$$S_A = S_{\bar{A}}$$

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The area law divergence in QFT_d

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Strong subadditivity [Lieb-Ruskai 73]

$$S_{A \cup B \cup C} + S_B \leq S_{A \cup B} + S_{B \cup C}$$

$$S_A + S_C \leq S_{A \cup B} + S_{B \cup C}$$

for any three disjoint regions A , B and C

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C-theorem in 3d?

Thermal c-theorem

$$F_{\text{Therm}} \sim c_{\text{Therm}} T^3$$

C_T -theorem [Petkou 94]

$$C_T|_{\text{UV}} \geq C_T|_{\text{IR}} , \quad \langle T_{\mu\nu}(x) T_{\rho\sigma}(0) \rangle_{\text{CFT}} = C_T \frac{I_{\mu\nu,\rho\sigma}(x)}{x^6}$$

F -theorem [Jafferis-Klebanov-Pufu-Safdi 11, Myers-Sinha 10]

$$F_{\text{UV}}(\mathbb{S}^3) \geq F_{\text{IR}}(\mathbb{S}^3) , \quad F = -\log Z(\mathbb{S}^3)$$

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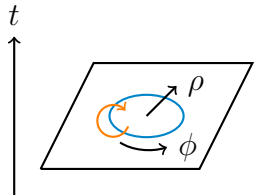
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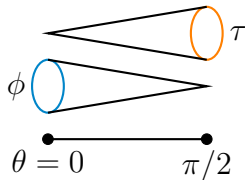
Conformal map



\mathbb{R}^3

$$ds^2 = dt^2 + d\rho^2 + \rho^2 d\phi^2$$

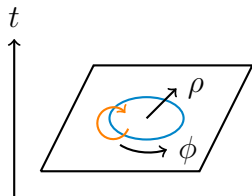
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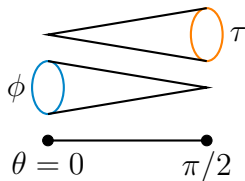
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\mathbb{S}_n^3 : n -fold cover of \mathbb{S}^3

EE in CFT_3 and F -theorem

The Rényi entropy of a disc for CFT

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Taking $n \rightarrow 1$ limit

For CFT_3 [Casini-Huerta-Myers 11]

$$S_1(R) = \log Z[\mathbb{S}^3] = -F(\mathbb{S}^3)$$

up to a UV divergence

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Proof of the F -theorem by using entanglement entropy?

Renormalized entanglement entropy

Two requirements for the proof of the F -theorem

- ▶ Interpolating function between F_{UV} and F_{IR}
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- ▶ Proof of monotonicity [Casini-Huerta 12]

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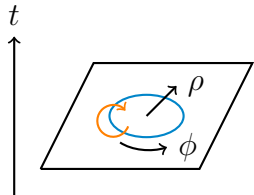
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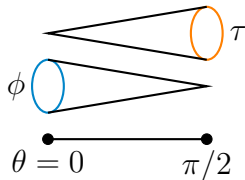
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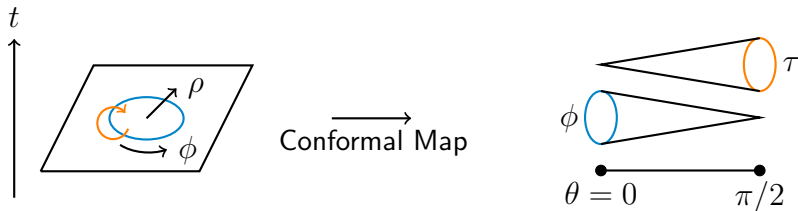
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[Klebanov-Pufu-Sachdev-Safdi 11]
- ▶ For SUSY gauge theories, $Z[\mathbb{S}^3]$ ($n = 1$) can be obtained by
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SUSY on singular space

- ▶ SUSY is **broken** on the singular space \mathbb{S}_n^3

$$ds^2 = d\theta^2 + n^2 \sin^2 \theta d\tau^2 + \cos^2 \theta d\phi^2, \quad \tau \sim \tau + 2\pi$$

- ▶ To recover SUSY, turn on the **R -symmetry b.g. gauge field**

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SUSY partition function on \mathbb{S}_n^3

$$Z^{\text{susy}}[\mathbb{S}_n^3] = Z^{\text{susy}}[\mathbb{S}_b^3]$$

\mathbb{S}_b^3 : squashed three-sphere with squashing parameter $b = \sqrt{n}$

Some properties

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Expansion around $n = 1$

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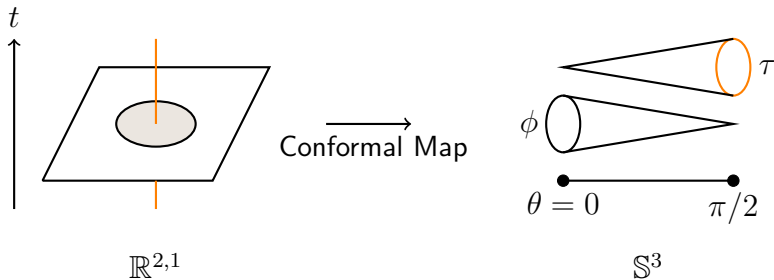
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Large- N limit

$$S_n^{\text{susy}} = \frac{3n + 1}{4n} S_1$$

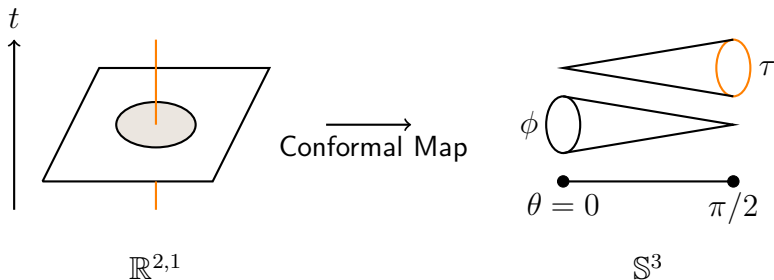
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The variation of RE by the loop

$$S_{W,n} = \frac{1}{1-n} (n \log |\langle W \rangle_1| - \log |\langle W \rangle_n|)$$

Wilson loop in the large- N [TN 14]

- ▶ 1/6-BPS Wilson loop in ABJM on S_n^3

$$\log \langle W \rangle_n = \frac{\pi(n+1)}{2} \sqrt{2\lambda} + O(\log N)$$

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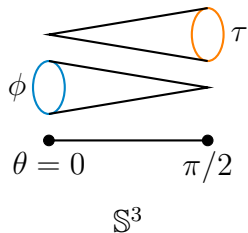
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- ▶ The variation does not depend on the Rényi parameter n !

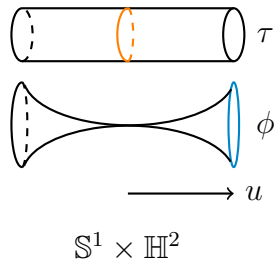
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Further conformal map



Conformal Map



$$ds^2 = d\theta^2 + \sin^2 \theta d\tau^2 + \cos^2 \theta d\phi^2$$

$$ds^2 = d\tau^2 + du^2 + \sinh^2 u d\phi^2$$

The n -fold cover has $\tau \sim \tau + 2\pi n$

The dual gravity solution

- ▶ The 1/2-BPS $U(1)$ charged topological AdS_4 black hole in $\mathcal{N} = 2$ gauged SUGRA

$$ds^2 = f(r)d\tau^2 + \frac{dr^2}{f(r)} + r^2 ds_{\mathbb{H}^2}^2$$

$$A = Q \left(\frac{1}{r} - \frac{1}{r_H} \right) d\tau$$

The horizon is at $r = r_H$ where $f(r_H) = 0$

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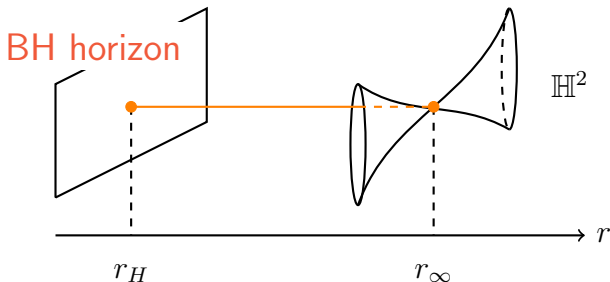
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Agrees with the **large- N result!** [Huang-Rey-Zhou 14, TN 14]

Adding holographic Wilson loop

- ▶ The fundamental string dual to the Wilson loop



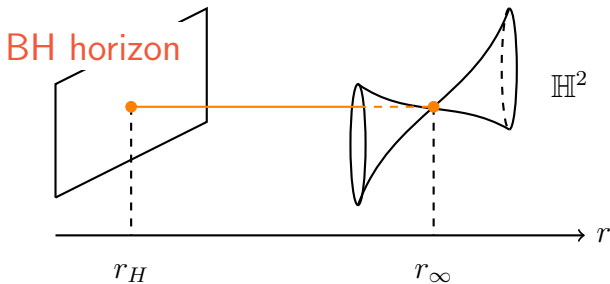
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n -fold cover of S^5 [Hama-TN-Ugajin 14]

- ▶ Consider $\mathcal{N} = 1$ five-dimensional theories on S_n^5

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The localized partition function

$$Z^{\text{susy}}[\mathbb{S}_n^5] = Z^{\text{susy}}[\mathbb{S}_{(\omega_1, \omega_2, \omega_3)}^5 = (1/n, 1, 1)]$$

$\mathbb{S}_{(\omega_1, \omega_2, \omega_3)}^5$: squashed sphere

5d SRE in the large- N limit

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With a BPS Wilson loop

$$S_{W,n}^{\text{susy}} = \sqrt{\frac{2}{8 - N_f}} \pi N^{1/2}$$

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Summary

- ▶ SUSY does not manifest itself in the Rényi entropy
- ▶ A new observable, **supersymmetric Rényi entropy**, is introduced
- ▶ The **holographic duals** of the **supersymmetric Rényi entropies** are given by **the BPS charged topological AdS black holes**

Future direction

- ▶ Can SRE be defined in other dimensions? (4d SRE by [Huang-Zhou 14], [Crossley-Dyer-Sonner 14])
- ▶ Boundary SRE or squashed SRE?
- ▶ Entangling surface as a surface operator? [Drukker-Okuda-Passerini 14]