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A very surprising development of the last two years is that not only supersymmetry but also supergravity have been accepted as basis for phenomenological models. In this talk, I would like to focus on a few theoretical aspects of supersymmetry and supergravity which might be relevant. for particle phenomenology in the near future. Thus $I$ have to apologize for not mentioning very fine work on the formal developments in extended supergravity $(\mathrm{N} \geq 2)$ and on the mathematical aspects of superspace.

First I would like to discuss nonlinear transformation laws of supersymmetry. They describe systems in which supersymmetry is spontaneously broken and all the masses of the SUSY-partners are very high (except the Goldstino mass) compared to the relevant energy scale. Nonlinear transformation laws are useful because they allow conciusions to be drawn in a very model-independent way.

Next, I am going to discuss some gneeral features of $N=1$ supergravity models coupled to gauge theories. On a very simple model I would like to explain the mechanism of a sliding scale. This mechan$i s m$, invented by $E$. Witten, 1 might allow us to construct a model where different scales (Weinberg-Salam scale, grand unified scale, SUSY breaking scale, gravitino mass) are all generated from one scale -in our case the Planck scale. That such ideas can be seriously discussed I consider to be one of the most exciting developments in supergravity.

After this discussion $I$ would like to mention $N=2$ gauge theories. Some of them are believed to be finite quantum field theories. It will be interesting to see if the finite models can be used for phenomenology. It would be one step in embedding phenomenological models in extended supersymmetric theories, a step which eventually has to be taken if we believe that a fundamental theory might be based on an extended supergravity theory.

Supersymmetry is a boson-fermion symmetry. All models are constructed in such a way that every known particle has a SUSY partner of opposite statistics. None of these partners have yet been observed. We believe that they are very heavy.

Supersymmetric couplings are usually of a type that an even number of SUSY partners enters a vertex ( $R$ invariance). If supersymmetry is spontaneously broken, there should be a fairly light Goldstino. It will be hard to discover this particle because it always changes a known particle to its heavy SUSY partner.


However, processes where this partner is virtual will contribute to low energy amplitudes. The general features of such processes are summarized in low energy theorems. Low energy theorems can be derived with the help of the nonlinear transforming AkulovVolkov field ${ }^{2}$ and the transformation law for matter fields. ${ }^{3}$

$$
\delta_{\xi_{0}} \lambda_{\alpha}=\xi_{\alpha}-i\left(\lambda \sigma^{m} \bar{\xi}-\xi \sigma^{m} \bar{\lambda}\right) \partial_{m} \lambda_{\alpha}
$$

$$
\delta_{\xi} \mathrm{A}=-i\left(\lambda \sigma_{\bar{\xi}}^{\mathrm{m}}-\xi \sigma^{\mathrm{m}} \bar{\lambda}\right) \partial_{\mathrm{m}} \mathrm{~A}
$$

This formalism has been developed by many authors. ${ }^{4}$ It has been brought into an especially workable form by E. Ivanoy, A. Korpustnikov, 5 and by S. Samuel and J. Wess. ${ }^{6}$ These authors have shown that any Lorentz invariant theory can be made supersymmetric by coupling in the Goldstino in a very well defined way. The resulting "low energy theorems" state that a pair of Goldstinos couples to the energy momentum tensor of the matter fields ${ }^{7}$ :

$$
\text { ik } \bar{\lambda} \gamma^{\nu} \partial^{\mu} \lambda T_{\mu \nu} .
$$

$k$ is an unknown constant.
The best way to look for the Goldstino is to look for events with missing energy. 0. Nachtmann, A. Reiter, and $M$. Wirbe $1^{8}$ have tried to place experimental bounds on $k$ by analyzing the events

$$
\begin{aligned}
& p+\bar{p} \rightarrow \text { one jet }+\lambda+\lambda+(\text { soft particles }) \\
& p+\bar{p}+\gamma+\lambda+\lambda+(\text { soft particles }) \\
& e^{+}+e^{-}+\gamma+\lambda+\lambda
\end{aligned}
$$

If they attribute the 5 single jet events and the one or two single photon events of UAI to these processes, they find that the ratio

$$
\sigma(p+\bar{p}+j e t) / \sigma(p+\bar{p}+\gamma)
$$

is in reasonable agreement with experiment. They find

$$
k^{-1 / 4} \sim 80 \mathrm{GeV}
$$

The process $e^{+} e^{-}+\gamma+\lambda+\lambda$ has a cross section that rises with the sixth power of $\sqrt{s}$ :

$$
\begin{aligned}
& \sigma\left(e^{+}+e^{-}+\gamma+\lambda+\lambda\right) \approx\left(\frac{\sqrt{s}}{36} 6 \mathrm{eV}\right)^{6} 10^{-2} \mathrm{pb} \\
& \text { cuts } 2 E_{j} / \sqrt{s} \geq 0.1 \quad 45^{\circ} \leq \delta_{j} \leq 135^{\circ}
\end{aligned}
$$

With the above value of $k$, one event should have been seen at DESY (JADE).

The UAI single jet events have also been attributed to gluino production by J. Ellis, H. Kowalski, and by E. Reyer and D.P. Roy. ${ }^{9}$ This would be consistent with a gluino mass of $20-40 \mathrm{GeV}$.

Another supersymmetry interpretation of these events would be via Wino production. This has been reported at this conference by $R$. Arnowitt. 10

If the nonlinear theory is coupled to supergravity we find that the transformation law of the Goldstino depends on the supergravity multiples

$$
\begin{aligned}
&\left(e_{m}^{\phi}, \psi_{m}^{\alpha}, b_{\alpha \dot{\alpha}}, M\right): \\
& \delta_{\xi_{\alpha}}^{\lambda_{\alpha}}=-\xi_{\alpha}(x)-\frac{1}{3} M^{*} \lambda^{2} \xi_{\alpha}-\frac{1}{6} \lambda^{2} b_{\alpha \dot{\alpha}} \bar{\xi}^{\alpha} \\
&+2 i\left(\lambda \sigma^{b} \bar{\xi}\right) \tilde{\sigma}_{b} \lambda_{\alpha}-2\left(\lambda \sigma^{b} \bar{\xi}\right)\left(\lambda \sigma^{c} \bar{\psi}_{b}\right) \tilde{\Delta}_{c} \lambda_{\alpha}
\end{aligned}
$$

The Goldstino field can be transformed away. It will furnish the missing degrees of freedom for a

The Goldstino field can be transformed away. It will furnish the missing degrees of freedom for a massive gravitino. This is the supersymmetric Higgs Kibble mechanism discovered by $S$. Deser and B. Zumino ${ }^{11}$ and by D.Z. Freedman and A. Das.

The nonlinear transformations provide a complete formalism describing the supergravity coupling. 12 It shows that almost all signs of supersynmetry can be hidden at arbitrary large energy. This caused S. Samuel to call it "secret supersymmetry."13

A generic indication of supersymmetry would be the existence of two Higgs fields ${ }^{4}$ in the standard Glashow-Weinberg-Salam model.

Let us now turn to more specific models. For rigid $N=1$ supersymmetry, they are formulated in terms of chiral superfields $S$ and vector superfields V . The most general supersymmetric and gauge invariant Lagrangian is ${ }^{15}$ :

$$
\begin{aligned}
L= & \int d^{2} \theta\left\{-\frac{1}{8} \bar{D}^{2} \Phi\left[S^{+} e^{V} S\right]+\frac{1}{4} W^{a} f_{a b}(S) W^{b}\right. \\
& +g(S)+h \cdot c \cdot\}
\end{aligned}
$$

Here we have used a superspace formulations which has been put on a more rigorous mathematical basis by $A$. Rogers. 16

For renormalizable models we have to demand that $g$, the superfield potential, be a polynomial of degree three and that

$$
\Phi\left[S^{+} e^{V} S\right]=S^{+} e^{V} S, f_{a b}(S)=\delta_{a b}
$$

There are also "nonstandard" formulations of supersymmetric matter multiplets. 17 Their coupling to gauge fields has recently been discussed by B.B. Deo and S.J. Gates, Jr. 18

There is a standard way ${ }^{19}$ to couple models with the above Lagrangian to minimal supergravity. We have to multiply the Lagrangian by the chiral density $\mathcal{E}$ and replace the chiral projection operator $\bar{D}^{2}$ by the covariant chiral projection operator ( $\mathcal{L}^{2}-8 R$ ):

$$
\begin{aligned}
L= & f d^{2} \theta 2 \mathcal{E}\left\{-\frac{1}{8}\left(b^{2}-3 R\right) \Phi\left[S^{+} e^{v} S\right]\right. \\
& \left.+\frac{1}{4} W^{a} E_{a b}(S) W^{b}+g(S)+h \cdot c \cdot\right\}
\end{aligned}
$$

This Lagrangian includes the supergravity action if $\Phi=-3+$ (field dependent terms). A constant in the superfield potential $g(S)$ contributes a supersymmetric cosmological term.

The Lagrangian can be decomposed into component fields. ${ }^{20}$ The dynamics is then governed by a particular combination of $\Phi$ and $g$, the Kähler potential ${ }^{21}$ :

$$
G(\bar{z}, z)=-3 \ln \left(-\frac{1}{3} \Phi(\bar{z}, z)\right)+\ln |g(z)|^{2}
$$

Here we have denoted the scalar components of $S$ by $z$. The Kähler potential defines the Köhler metric:

$$
g_{j}^{i}=\frac{\partial^{2} G}{\partial z_{i} \partial \bar{z}^{j}}
$$

From the metric a Ricci tensor $R^{i}$, can be constructed. The kinetic energy of the scalar fields depends on the Kähler metric:

$$
\mathrm{L}_{\mathrm{kin}}(\bar{z}, z)=e_{\mathrm{g}}{ }_{j}^{\mathrm{i}} \mathrm{a}_{\mathrm{m}} \bar{z}_{\mathrm{j}^{m} z_{i}}
$$

The potential energy of the scalar fields depends on the Kähler potential:

$$
\begin{aligned}
V= & e e^{G}\left\{3+g_{i}^{-1} j \frac{\partial G}{\partial z} \frac{\partial G}{\partial \bar{z}^{i}}\right\} \\
& ++\frac{1}{2} \text { e } \operatorname{Re} f_{a b}^{-1}\left(\frac{\partial G}{\partial z} T^{a} z\right)\left(\frac{\partial G}{\partial z} T^{b} z\right) .
\end{aligned}
$$

The last term is the generalization of the $D^{a}$ part of the potential energy to curved space.

We specify the model by assuming $f_{a b}=\delta_{a b}$ and further that the potential has a minimum such that
$\left\langle D^{a}\right\rangle=0$ and $\langle V\rangle=0$. Then we obtain the following mass relations:

$$
\left.\begin{array}{rl}
m_{3 / 2}= & e^{1 / 2\langle G\rangle} \\
\operatorname{StrM}^{2}= & 2 m_{3 / 2}^{2}\left\{(N+1)-R_{j}^{i} g_{i}^{-1} e^{g_{k}^{-1}} j^{j}\right. \\
& \frac{\partial G}{\partial z^{k}} \frac{\partial G}{\partial \bar{z}} e
\end{array}\right\} .
$$

$N$ is the number of scales multiplets and Str is the supertrace.

Two obvious choices of Kähler manifolds have been extensively studied in the literature ${ }^{22}$ :

1) The "Köhler flat" manifold:

$$
g_{j}^{i}=\delta_{j}^{i}, \quad f_{a b}=\delta_{a b}
$$

From there we conclude:

$$
G=\bar{z} z+\ln |g(z)|^{2},
$$

$$
R_{i}^{j}=0
$$

$$
\Phi=-3 \exp \left[-\frac{1}{3} \bar{z} z\right]
$$

$$
\text { StrM }{ }^{2}=2(N+1) \mathrm{m}_{3 / 2}^{2}
$$

2) The "minimal" manifold:

$$
\Phi=-3+\bar{z} z, \quad f_{a b}=\delta_{a b} .
$$

From there we conclude:
$G=-3 \log \left(1-\frac{1}{3} \bar{z} z\right)+\ln |g(z)|^{2}$
$R_{i}{ }^{j}=-\frac{N+1}{3} g_{i}{ }^{j}$
StrM ${ }^{2}=0$
In this case, the mass relation is of particular importance. Quadratic divergences of the potential energy are, in general, proportional to StrM ${ }^{2}$.

The second case has 24 additional interesting features. It defines the noncompact version of $\mathrm{CP}(\mathrm{N})$, $U(N, 1) / U(N) \times U(1)$ as the underlying manifold. Because of its metric, this manifold is a complete space for $|z|^{2}<3$. It is transitive and the "boosts" in the direction $y$ are:

$$
y^{\prime}=\frac{y+\sqrt{3} a}{1+\frac{a^{*}}{\sqrt{3}} y} \quad z^{i}=\frac{z^{i} \sqrt{1-|a|^{2}}}{1+\frac{a^{*}}{\sqrt{3}} y}
$$

The Kähler metric is invariant under such "boosts." The superfield potential $g \notin z$ ) transforms:

$$
g\left(y^{\prime}, x^{\prime}\right)=\left[\frac{1+\frac{a^{*}}{\sqrt{3}} y^{\prime}}{\sqrt{1-|a|^{2}}}\right]^{3} g\left(y\left(y^{\prime}\right), z\left(z^{\prime}, y^{\prime}\right)\right)
$$

In general, this transforms a polynomial into a rational function. If, however, $g$ is a polynomial of degree three, it remains so under the transformation. This again characterized a minimal extension to curved space of a renormalizable model.

From the general form of the scalar potential, we see that $V$ can be identically zero even though $g(z)$ is different from zero. For the minimal manifold (case 2), we find that

$$
g(z)=\mu\left(1+\frac{z}{\sqrt{3}}\right)^{3}
$$

gives $V=0$. This potential does not determine the vev of the scalar field $z$ at the tree level. Its vev is a sliding parameter $\langle z\rangle / \sqrt{3}=$ a. In terms of this parameter, the gravitino mass slides as well:

$$
m_{3 / 2}=e^{1 / 2\langle G\rangle}=\mu\left[\frac{1+a}{\sqrt{1-|a|^{2}}}\right]^{3}
$$

Note that supersymmetry is also broken:

$$
\left\langle\frac{\partial g}{\partial z}\right\rangle=3 \mu(1+a)
$$

In general, we would like to have a scalar potential which has a local minimum, $\langle V\rangle=0$, and supersymmetry should be broken, $\left\langle\partial_{g} / \partial z\right\rangle \neq 0$. Dragon, Schmidt, and Ellwanger ${ }^{24}$ have examined the minimal case. They observe that the above boosts can be used to transform any nonvanishing vev to zero. An o'Raifeartaigh-like analysis lead them to conclude that the general form of a potential obeying the restrictions discussed above is

$$
\begin{aligned}
g(y, z)= & \mu\left(1+\frac{1}{\sqrt{3}} y\right)^{3}+O\left(z^{2}\right) \\
= & \mu\left(1+\frac{1}{\sqrt{3}} y\right)^{3}+\frac{1}{2}\left(1+\frac{y}{\sqrt{3}}\right) M Z^{2} \\
& +\frac{1}{2} N Z^{2}+\frac{1}{3} K z^{3}
\end{aligned}
$$

where $N$ and $M$ are mass matrices. Again, the vev of $y$ is a sliding parameter. Shifting away a nonvanishing vev of $y$ and taking the limit MPlanck $\rightarrow \infty$, Dragon et al. find that the field y decouples and that the surviving Yukawa interactions are determined by the superpotential:

$$
\mathrm{h}=\frac{1}{2}\left(\frac{1+\mathrm{a}}{\sqrt{1-|\mathrm{a}|^{2}}}\right) \mathrm{M}+\left(\frac{1}{\sqrt{1-|\mathrm{a}|^{2}}} \mathrm{~N}\right) \mathrm{Z}^{2}+\frac{1}{3} \mathrm{KZ}{ }^{3}
$$

The remaining scalar potential is given by:

$$
V=\sum_{i}\left|\frac{\partial h}{\partial z_{i}}\right|^{2}-\operatorname{Re}_{\mu}^{*} \frac{\left(1+a^{*}\right)^{2}}{1-|a|^{2}} N Z^{2}
$$

The additional part in $V$ shows explicit soft supersymmetry breaking, a generic feature of $N=1$ supergravity models. ihe gravitino mass slides

$$
m_{3 / 2}=\mu\left(\frac{1+a}{\sqrt{1-|a|^{2}}}\right)^{3}
$$

If a is close to minus one, $a=-1+\varepsilon$, SUSY breaking will be of the order $\varepsilon$. We also see that the factors

$$
\begin{aligned}
& \frac{1+a}{\sqrt{1-|a|^{2}}} M \sim \varepsilon^{1 / 2} \mathrm{M}, \quad \frac{1}{\sqrt{1-|a|^{2}}} \mathrm{~N} \approx \frac{1}{\sqrt{\varepsilon}} \mathrm{~N} \\
& \mu\left(\frac{1+\mathrm{a}}{\sqrt{1-|a|^{2}}}\right)^{3} \approx \varepsilon^{3 / 2} \mu
\end{aligned}
$$

exhibit different mass scales. The mass matrix $N$ has to have null directions to avoid the superheavy scale for the corresponding fields.

This should serve as a toy model to show how different mass scales can arise dynamically from one common scale for $M$, $N$, and $\mu$. The idea is that radiative corrections are going to fix $a=-1+\varepsilon, \varepsilon \ll 1$.

We have discussed the coupling of gauge theories to minimal supergravity. Besides the minimal supergravity multiplet ${ }^{25}$ ( 12 bosonic +12 fermionic components), there are several other multiplets known. These are the non-minimal multiplet $26(20+20)$; the new minimal $27(12+12)$, the flexible multiplet ${ }^{28}$ $(28+28)$, the new multiplet $29(16+16)$, and the reducible multiplet ${ }^{30}(24+24)$. All these multiplets can be obtained from the flexible multiplet. Gauge fixing in the flexible multiplet leads to the reducible multiplet. All others can be found in invariant subspaces of the reducible multiplet. However, all these multiplets have in common the graviton field $e_{m}$ and the gravitino field $\psi_{m}, \Psi_{m}$. They are different in their auxiliary field content. They all allow the construction of an invariant action and coupling to Yang-Mills theories.

All these theories can be constructed as geometrical theories in curved superspace. The superspace variable is $z^{M}=\left(x^{m}, \theta^{\mu}, \bar{\theta}_{\dot{\mu}}\right)$. The dynamical variables are the vielbein $E_{\mu}{ }^{A}$ and the connection $\phi_{M A}{ }^{B}$. Covariant derivatives can be defined:

$$
\mathcal{A}_{A} v^{B}=E_{A}^{M}\left\{\partial_{M} v^{B}+V_{\phi_{M C}}^{B}\right\}
$$

The commutator of two derivatives defines the curvature and the torsion:

$$
\left(\boldsymbol{D}_{\mathrm{C}}, \boldsymbol{\Delta}_{\mathrm{B}}\right] \mathrm{V}^{\mathrm{A}}=\mathrm{V}^{\mathrm{D}_{\mathrm{CBD}}}{ }^{\mathrm{A}}-\mathrm{T}_{C B} \mathrm{~F}_{\boldsymbol{D}_{\mathrm{F}}} \mathrm{~V}^{\mathrm{A}}
$$

The Jacobi-identities for the commutators lead to the Bianchi identities

$$
\oint\left\{R_{A B C}{ }^{\mathrm{D}}-A_{A} \mathrm{~T}_{\mathrm{BC}}{ }^{\mathrm{D}}-\mathrm{T}_{\mathrm{AB}} \mathrm{~F}_{\mathrm{FC}}{ }^{\mathrm{D}}\right\}=0
$$

$A B C$ :cyclic sum.
This formulation of supergravity theories is very similar to the formulation of general relativity in four dimensional space-time. There one puts $T=0$ as a constraint equation. This equation can be solved by expressing the connection (Christoffel symbol) in terms of the vielbein (metric tensor). Thus, the constraint equation $T=0$ allows us to eliminate the connection as an independent dynamical variable. The Bianchi identity $S_{a b c} R_{a b c}$, in this case, reduces the number of tensor components of $R_{a b c}{ }^{d}$.

In the case of supergravity, we have many variables. Thus, we could try to eliminate them by putting $T=0$. This, however, would eliminate all dynamical variables. 31

Tha analog to general relativity would be to ask for those constraints which allow us to solve for $\phi_{M A} B^{\text {in terms of the vielbein. The vielbein, how- }}$ ever, has several reducible components (under the Lorentz group). It is possible to eliminate the dimension $1 / 2$ component $E_{m}$ in terms of its dimension 0 , $\mathrm{E}_{\mathrm{m}}{ }^{a}, \mathrm{E}_{\mu}{ }^{\mathbf{0}}$, and dimension $-1 / 2$ component $\mathrm{E}_{\mathrm{L}}{ }^{\text {a }}$. The constraint equations by which thti can be achieve: are called natural constraints. 30 They are:

$$
\begin{aligned}
& T_{\alpha \beta}^{a}=T_{\dot{\alpha} \dot{\beta}}^{a}=0, \quad T_{\alpha \dot{\beta}}^{a}=2 i \sigma_{\alpha \dot{\beta}}^{a} \\
& T_{\gamma \dot{\alpha}}^{\dot{\beta}}=(n-1) \delta^{\dot{\beta}} \cdot{ }_{\dot{\alpha}}{ }_{\gamma} \\
& T_{\gamma \beta}^{\alpha}=(n+1)\left(\delta_{\beta}^{\alpha} T_{\gamma}+\delta_{\gamma}^{\alpha}{ }_{\gamma}{ }_{\beta}\right) \\
& T_{\gamma b}{ }^{a}=2 n \delta^{a}{ }_{b} T_{\gamma} \\
& T_{a b}{ }^{c}=0 .
\end{aligned}
$$

A solution of the Bianchi identities, subject to these constraints, shows that the curvature and the torsion can be expressed in terms of a few tensor fields:

$$
\mathrm{R}, \mathrm{G}_{\alpha \dot{\alpha}}, \mathrm{T}_{\alpha}, \mathrm{W}_{\alpha \beta \gamma}
$$

These tensor fields are still constraint, as for example,

$$
\boldsymbol{D}_{\beta}{ }_{\alpha}+\boldsymbol{D}_{\alpha} \mathrm{T}_{\beta}=0 .
$$

If we solve this constraint through a prepotential, $T_{\beta}=\mathcal{A}_{\beta} \psi$, we obtain the flexible multiplet $(28+28)$ from the natural constraints. This solution allows for a gauge transformation $\psi+\psi+\Lambda, \theta_{\alpha} \Lambda=0$. If we fix the gauge we obtain the reducible multiplet $(24+24)$. This multiplet can be further reduced in different ways:

1) $T_{\alpha}=0, R \neq 0$ : minimal multiplet $(12+12)$
2) $n \neq 0, \quad R=0, T_{\alpha} \neq 0$ : non-minimal multiplet $(20+20)$
3) $\mathrm{n}=0, \mathrm{R}=0, \mathrm{~T}_{\alpha} \neq 0$ : These constraints are invariant under a super Weyl transformation. Choosing a particular gauge leads to the new minimal multiplet ( $12+12$ ),

In all the cases, where $n \neq 0$, we obtain the action as the covariant volume element $L=E$. For $\mathrm{n}=0$, this volume element is zero. However, also in this case there is a superspace action:

$$
\begin{aligned}
& L=E\left(\frac{T^{\alpha} T_{\alpha}}{S}+\frac{\bar{T}_{\dot{\alpha}}^{\bar{\alpha}^{\dot{\alpha}}}}{\bar{S}}\right) \\
& \bar{S}=\mathcal{O}_{\alpha}-T^{\alpha} T_{\alpha} .
\end{aligned}
$$

The coupling to matter field ${ }^{2}$ proceeds in the same way as in the minimal case. ${ }^{32}$ There is a chiral density . It is, hwoever, singular in $\bar{S}$ (see above). The chiral projectin operator exists as well:

$$
\bar{\Delta}=\overline{\mathcal{A}}^{2}-3(n+1) \bar{T} \cdot \bar{\alpha}_{\dot{\alpha}}^{\dot{\alpha}}-2(n+1) \bar{S}
$$

The matter coupling in curved superspace has the following Lagrangian:

$$
\begin{aligned}
\mathscr{L}= & \int d^{2} \theta \mathcal{L}\left\{\bar{\Delta} \Phi\left[S^{+} e^{v} S\right]\right. \\
& +\operatorname{Tr}\left(\bar{\Delta} e^{-\bar{\Omega} / 2} e^{-v} \boldsymbol{A}_{\alpha} e^{v}\right)\left(\bar{\Delta} e^{-\bar{\Omega} / 2} e^{-v} \boldsymbol{B}^{\beta} e^{v}\right) \\
& +g(S)+h \cdot c \cdot\} \\
\Omega= & 2(3 n+1) \frac{\overline{\mathrm{T}} \overline{\mathrm{~T}}^{\dot{\alpha}} \dot{\bar{\alpha}}}{\bar{S}} .
\end{aligned}
$$

For the case $n=0$, the matter coupling has to be $R$ invariant. For $n \neq 0$, non $R$ invariant theories are possible. In general, they will be singular in $S$. It can be shown, however, that $R$ invariant theories are not singular ${ }^{3}$ and that their dynamics is again governed by a Kähler potential.

Another very interesting development was the discovery of $N=2$ ultraviolet finite gauge theories. ${ }^{34}$ It is well known that supersymmetry softens the divergencies of a theory ${ }^{35}$; that it completely removes all divergencies is a remarkable result.

The theories in question are constructed in terms of $N=2$ vector multiplets and hypermultiplets. The helicity states of these multiplets are best shown in the following figures:

$$
\mathrm{N}=1 \text { : V vector }
$$



Here we have indicated how the two supersymmetries connect the states. It shows that the $\mathrm{N}=2$ vector multiplet decomposes into an $N=1$ vector multiplet $V$ and an $N=1$ chiral multiplet $\phi$. The states $\lambda$ and $\psi$ are connected by an $\mathrm{SU}(2)$, an authomorphism group of $\mathrm{N}=2$ supersymmetry.

The hypermultiplet has a similar decomposition:


The Yang-Mills potentials $\mathrm{v}^{\mathrm{m}}$ have to be in the adjoint representation of the gauge group G'. This now applies to the full $\mathrm{N}=2$ vector multiplets.

The "usual" matter fields might be thought of as being members of the $N=1$ chiral superfield $x$. This determines the group representation of the hypermultiplet.
$\mathrm{N}=1$ supersymmetry has given a SUSY partner to each known particle. $N=2$ supersymmetry adds mirror gauge and matter multiplets $\phi$ and $\mathrm{Y}^{+}$. All the $\mathrm{N}=2$ gauge theoires are vector-like theories. Their Lagrangian can be written in terms of the above mentioned $N=1$ superfields:

$$
\begin{aligned}
L= & \frac{1}{8 g^{2}} \int d^{2} \theta W^{\alpha} W_{\alpha}+i \sqrt{2} g \int d^{2} \theta Y \phi X+h . c . \\
& +2 \int d^{2} \theta d^{2} \bar{\theta}\left[\operatorname{Tr} \phi^{+} e^{2 g V_{\phi}} e^{-2 g V}\right. \\
& \left.+X^{+} e^{2 g V_{X}}+Y^{+} e^{-2 g V^{T}} Y\right] \\
& + \text { gauge fixing }+ \text { ghosts. }
\end{aligned}
$$

The gauge couplings and the Yukawa couplings all depend on the single coutling constant g. Considered as an $N=1$ gauge theo-y the coupling $\int d^{2} \theta y \phi x$ would have been independent. It is an F-type coupling for which the $N=1$ non-renormalizaton theorem applies.

In addition to the $N=2$ supersymmetry the above Lagrangian is also $R$ invariant and invariant under the previously mentioned $\operatorname{SU}(2)$ group.

Such a Lagrangian can give rise to an ultraviolet. finite field theory. This statement is based on the following arguments:

1) In the $N=2$ superspace background field method, there are no counterterms above one loop for any $\mathrm{N}=2$ model which can be written in terms of $\mathrm{N}=2$ superfields.
2) By the same argument one concludes that the wavefunction renormalizations $Z_{x}=Z_{y}=1$.
3) The $N=1$ non-renormalization theorem then tells us that

$$
\begin{aligned}
& Z_{g} z_{\phi}^{1 / 2}=1 \\
& \text { 4) } B(g)=\frac{1}{16 \pi^{2}}\left\{2 C_{2}(G)-2 T(R)\right\} g^{3}
\end{aligned}
$$

Here, $C_{2}(G)$ is the quadratic Casmir for the adjoint representation and $T(R) \delta^{a b}=T r T^{a} T^{b}$.

It is possible to have $\beta(g)=0$ by choosing for the matter fields the proper representation 36 This e.g, is always the case for $N=4$ theories. ${ }^{36}$ There the matter representation is the adjoint representation when looked upon as an $N=2$ theory.

The statement that such theories are ultraviolet finite has been checked and verified by explicit two loop calculations. ${ }^{37}$ However, there is no known supersymetric and gauge invariant regularization.

All explicit calculations have been done in the dimensional reduction scheme (red). ${ }^{38}$ This scheme is supersymmetric but it is known to have difficulties with unitarity above the two loop level. ${ }^{39}$ Hever, it was argued that this should not be the case up to two loops.

On these grounds, R. van Damme and G. 't Hooft ${ }^{40}$ have questioned the results obtained with dimensional reduction* They have considered the $\mathrm{N}=2$ gauge theory as a regular gauge theory, depending on four independent coupling constants. They use the regular dimensional regularization scheme (minimal subtraction
ruin) and then tried to compare their results with the others. The renormalized parameters should be finitely related

$$
g_{i}^{\text {red }}=g_{i}^{\operatorname{mn}}+\alpha_{i}(g)
$$

if both schemes are to make sense. There are two ways to relate the results: compare the the b-functions or compare renormalization invariant charges. For R, van Damme and G. 't Hooft the two ways of comparing the theories gave a different redefinition of the coupling constants. They concluded that the results obtained by dimensional reduction (the supersymetric results) have to be inconsistent.

A similar analysis was then done by L.V. Avdeev, D.I. Kaxakov, and O.V. Tarasov ${ }^{41}$ and by D. Maison, 42 They do not find any discrepancy and conclude that there is (at present) no contradiction to the existence of gauge invariant and supersymmetric $N=2$ theories. This result is also supported by recent work of R. Oehme, K. Sibold, and W. Zimmermann. ${ }^{43}$ They show that there is a consistency condition if a theory with several coupling constants (four constants if the $N=2$ theory is considered as a gauge theory only) is reduced to a theory with fewer constraints (one constant for the $\mathrm{N}=2$ theory).

$$
\beta_{i}\left(g_{i}(g)\right)=\beta(g) \frac{d g_{i}(g)}{d g}
$$

This consistency condition has a solution in the case of interest and it reproduces the results obtained with the dimensional reduction scheme. This becomes even more convincing when we start from the two parameter $N=1$ gauge theory related to our $N=2$ theory. O. Pigeut and K. Sibold ${ }^{44}$ have shown in previous work that $N=1$ supersymmetry and gauge invariance are consistent to all orders in perturbation theory. Again, the consistency condition relating the two parameter theory to the one parameter theory has a solution which agrees with the previous results. A gauge invariant and supersymmetric renormalization does not require a regularizaton scheme having this property. There is a good chance that there are $N=2$ supersymmetric and finite gauge theories.

There remains the infrared problem. P.S. Howe and P.C. West ${ }^{37}$ have conjectured that models which are ultraviolet finite at one loop are also infrared finite at two loops. 0. Pigicet and K. Sibold ${ }^{45}$ in a very recent paper have shown that in supersymmetric gauge theories off-shell-Green's function do exist such that Green's functions of gauge invariant operators are supersymmetric. They conclude that the offshell infrared problem is a gauge artifact.

What are the phenomenological possibilities of $N=2$ finite models? ${ }^{46}$ To think about it is only possible because there are "soft" supersymmetry breaking operators. They preserve the finiteness of the theory and they allow us to introduce mass terms and explicit breaking of the chiral symmetries. Potentials can be obtained which, at the tree level, give large masses to mirror fermions. It is, however, very difficult (impossible?) to construct realistic models. The finiteness condition is very restrictive on the group representations. It is very difficult to see how several families can be accommodated. Moreover, it is difficult to obtain a generation dependence of quark and lepton masses from an $N=2$ finite GUT theory because the only fundamental parameters of the theory are gauge couplings. The gauge
group might have to be enlarged to include a family group as well.

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## References

1. E. Witten, Phys. Lett. 105 B (1981) 267.
2. V.P. Akulov and D.V. Volkov, JETP Lett. 16 (1972) 438; Phys. Lett. 46 B (1973) 109.
3. T. Umeatsu and Z. Zachos, Nucl. Phys. B 201 (1982) 250.
4. D.V. Volkov and V. Soraka, JETP Lett. 18 (1973) 312; A.I. Pashnev, Theor. Math. Phys. 20 (1974) 725; B. Zumino, Nuc1. Phys. B 127 (1977) 189; K. Shima, Phys. Rev. D 15 (1977) 2165; V.I. Ogievetsky and E. Sokatchev, JETP Lett. 23 (1976) 58; M. Rocek, Phys. Rev. Lett. 41 (1978) 451; V. Lindström and M. Rocek, Phys. Rev. D 19 (1979) 2300.
5. E. Ivanov and A. Kapustnikov, J. Phys. A 11 (1978) 2375; J. Phys. 68 (1982) 167.
6. S. Samuel and J. Wess, Nucl. Phys. B 221 (1983) 153.
7. J. Wess, Quantum Theory of Particle and Fields, Birthday volume dedicated to Jan Zopuozeniski, B. Jancewicz, L. Lukierski, World Scientific, p. 223.
8. O. Nachtmann, A. Reiter, and M. Wirbel, Single Jet and Single Photon Production in Proton Antiproton Collisions and $e^{+}-e^{-}$Annihilation in a Supersymetric Model, HD--TEP-84-11, Heidelberg, FRG.
9. J. Ellis and H. Kowalski, Gluino Signatures at the pp Collider, Ref. TH. 3843, CERN; E. Reya and D.P. Roy, Have Gluinos Been Observed at the pp Collider? DO-TH 84/11, Dortmund, FRG.
10. A.H. Chamseddine, P. Natn, and R. Arncwitt, Experimental Signals for Supersymmetric Decays of the $1 /$ and $Z$ Bosons, HVTB-83/AO40, NVB 2588, Boston, Mass.
11. S. Deser and B. Zumino, Phys. Rev. Lett. 38 (1977) 1433; D.Z. Freedman and A. Das, Nucl. Phys. B 120 (1977) 221.
12. S. Samuel and J. Wess, Nuc1. Phys. B 226 (1983) 289; E.A. Ivanov and A.A. Kapustnikov, On a Mode Independent Description of Spontaneously Broken $N=1$ Supergravity in Superspace, Dubna 1984.
13. S. Samuel and J. Wess, Nucl. Phys. B 233 (1984) 488.
14. P. Fayet, Gauge Bosons/Higgs Boson Unification, LPTENS 83/25-June 1983.
15. J. Wess and J. Bagger, Supersymmetry and Supergravity, Princeton Series in Physics, Princeton University Press (1983).
16. A. Rogers, Consistent Superspace, March(84) Preprint King's college, London.
17. S.J. Gates and W. Siegel, Nucl. Phys. B 187 (1981) 389.
18. B.B. Deo and S.J. Gates, Comments on Non-minimal $N=1$ Scalar Multiplets, to be published in Nucl. Phys.
19. J. Wess and B. Zumino, Phys. Lett. 66 B (1977) 361; Phys. Lett. 74 B (1978) 51.
20. J. Bagger and E. Witten, Phys. Lett. 115 B (1982) 202; J. Bagger, Nucl. Phys. B 211 (1983) 302.
21. E. Cremmer, B. Julia, J. Scherk, P. van Nieuwenhuizen, S. Ferrara, and L. Girardello, Phys. Lett. 79 B (1978) 23; Nucl. Phys. 147 B (1979) 105.
22. E. Cremmer, S. Ferrara, L. Girardello, and A. van Proeyen, Phys. Lett. 116 B (1982) 231; Nucl. Phys. 212 B (1983) 413; S. Weinberg, Phys. Rev. Lett. 48 (1982) 1776; A. Chamseddine, R. Arnowitt, and P. Nath, Phys. Rev. Lett. 49 (1982) 970; C. Kounnas, D.V. NAnopoulos, M. Srednicki, and
M. Quiros, Phys. Lett. 127 B (1983) 82; J. Ellis, D.V. Nanopoulos and K. Tamvaski, Phys. Lett. 121 B (1983) 123; L. Ibanez, Phys. Lett. 118 B (1982) 73; L. Ibanez and C. Lopez, Phys. Lett. 126 B (1983) 54; H.P. Nilles, Phys. Lett. 115 B (1982) 1973; L. Alvarez-Gaumé, J. Polchinski, and M. Wise, Nucl. Phys. B 221 (1983) 495; R. Barbieri, S. Ferrara, and C. Savoy, Phys. Lett. 119 B (1982) 343; H.P. Nilles, M. Srednicki, and D. Wyler, Phys. Lett. 124 B (1983) 337; J. Ellis, J. Hagelin D.V. Nanopoulos, and K. Tamvakis, Phys. Lett. 125 B (1983) 275; U. Ellwanger, Phys. Lett. 124 B (1983) 34; C. Kounnas, A.B. Lahanas, D.V. Nanopoulos, and M. Quiros, Phys. Lett. 132 B (1983) 95; B.A. Ovrut and P.J. Steinhardt, Phys. Lett. 133 B (1983) 161; B.A. Ovrut and S. Raby, Phys. Lett. 134 B (1984) 51; Phys. Lett. 121 B (1983) 381.
23. B. Ovrut and J. Wess, Phys. Lett. 112 B (1982) 347 ; E. Cremmer, S. Ferrara, C. Kounnas, and D.V. Nanopoulos, Phys. Lett. 133 B (1983) 61; R. Barbieri, S. Ferrara, D.V. Nanopoulos, and K.S. Stelle, Phys. Lett. 113 B (1983) 219; S. Ferrara and A. van Proeyen, LPTENS 84/01, Paris 1983; Ngee-Poug Chang, S. Ouvry, and Xizeng Wu, Phys. Rev. Lett. 5i (1983) 327; S. Ouvry, Phys. Lett. 136 B (1984) 165; CCNY-HEP 84/21, New York 1984; K.T. Mahanthappa and G.M. Stoebler, Imperial TP/ 83-84/47.
24. N. Dragon, M.G. Schmidt, and U. Ellwanger, CERN Ref. TH 3915, CERN (1984); Xizeng Wu, CTP 1164 , Cambridge, Mass, 1984 (submitted to Phys. Rev.).
25. S. Ferrara and P. van Nieuwenhuizen, Phys. Lett. 74 B (1978) 333; K.S. Stelle and P.C. West, Phys. Lett. 74 B (1978) 330.
26. P. Breitenlohner, Nucl. Phys. B 124 (1977) 500; W. Siegel and J. Gates, Nucl. Phys. B 147 (1979) 77.
27. V. Akular, D. Volkov, and V. Soroka, Theor. Math. Phys. 31 (1977) 12; M.F. Sohnius and P.C. West.s Phys. Lett. 105 B (1981) 353; S.J. Gates, M. Rocek, and W. Siegel, Nucl. Phys. B 198 (1982) 113.
28. A. Galperin, V. Ogievetsky, and E. Sokatchev, JINR preprint E2-83--589 Dubna 1983.
29. A.S. Galperin, V.I. Ogievetsky, and E.S. Sokatchev, J. Phys. A: Math. Gen. 15 (1982) 3785; G. Girardi, R. Grimm, M. Muller, and J. Wess, Lapp-TH-113, Annecy-le-.-Viense 1984.
30. G. Girardi, R. Grimm, M. Muller, and J. Wess, LAPP-TH-102, Annecy-le-Viense 1984.
31. N. Dragon, Z. Phys. C 2 (1979) 29.
32. A. Galperin, V. Ogievetsky, and E. Sokatchev, Dubna preprint E2-83-589 (1983); R. Grimm, M. Müller, and I. Wess, CERN Ref. Th. 3881, CERN (1984).
33. S. Ferrara, I. Girardello, T. Gugo, and A. von Proeyen, Nucl. Phys. B 223 (1983) 191.
34. P. Howe, K. Stelle, and P. West, Phys. Lett. 124 B (1983) 55; P.C. West, Talk at La Jolla Workshop, January 1983, Talk at Shelter Island III, June 1983; S. Rajpoot, J. Taylor, and M. Zaini, Phys. Lett. 127 B (1983).
35. J. Wess, B. Zumino, Phys. Lett. 49 B (1974) 52; J. Iliopoulos and B. Zumino, Nucl. Phys. B 76 (1974) 310; S. Ferrara, J. Iliopoulos, and B. Zumino, Nucl. Phys. B 77 (1974) 41.
36. D.R.T. Jones, Phys. Lett. 72 B (1977) 199; E. Poggio and H. Pendleton, Phys. Lett. 72 B (1977) 200; 0. Tarasov, A. Vladmirov, and A. Yu, Phys. Lett. 93 B (1980) 429; M.T. Grisaru, M. Rocek, and W. Siegel, Phys. Rev. Lett. 45 (1980) 1063; W.E. Caswell and O. Zanon, Nucl. Phys. B 182 (1981) 125; M. Grisaru and W. Siegel, Nucl. Phys B 201 (1982) 292; S. Mandelstam, Proceedings of the XXI International Conference on High Energy Physics, Paris, 1982; K. Stelle, Proceedings of the XXI International Conference on High Energy Physics, Paris, 1982.
37. P.S. Howe and P.C. West, The Two Loop B-Function in Models with Extended Rapid Supersymmetry, Imparial College preprint, London 1984 (to be published in Nucl. Phys.).
38. W. Siege1, Phys. Lett. 84 B (1979) 193
39. V. Elias, . McKeon, S.B. Phillips, and R.B. Mann, Phys. Lett. 133 B (1983) 83.
40. R. van Damme and G. 't Hooft, Univ. Utrecht preprint, corrected version, 1984.
41. L.V. Avdeev, D.I. Karakov, and O.V. Tarasov, Dubna preprint E2-84-479 (1984).
42. D. Maison, MPI-PAE/PTU 48/84 (revised) Munich, July (1984).
43. R. Oehme and W. Zimmermann, MPI-PAE/PTh 60/84, Munich, May (1984); R. Oehme, K. Sibold, and W. Zimmermann, MPI-PAE/PTh 36/84, Munich, May (1984).
44. O. Piguet and K. Sibold, UGVA-DPT 1984/02-416, Geneva University, UGVA-DPT 1984/04-423; MPI-PAE/ PTh 21/84; Nucl. Phys. B 196 (1982) 428,447; Nucl. Phys. B 197 (1982) 257.
45. O. Piguet and K. Sibold, MPI-PAE/PTh 57/84, Murich, July (1984).
46. F. del Aguila, M. Dugan, B. Grinstein, L. Hall, G.G. Ross, and P. West, HUTP-84/A001, Cambridge, Mass. (1984).
47. Ian Jack, Impexial/TP 83-81/34 March (1984) P. Howe, A. Parkes and P. West Dimensional Regularization and Supersymmetry, Math. Dept. King's College, London.

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While the written version of this talk was prepared, $R$. van Damme and G. t'Hooft have withdrawn their cxiticism. Two other papers that clarified the situation have come to my knowledge (47.).

