Supply Chain Coordination for False Failure Returns

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July 30, 2004

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ABSTRACT

False failure returns are products that are returned by consumers to retailers with no functional or cosmetic defect. The cost of a false failure return includes the processing actions of testing, refurbishing if necessary, repackaging, the loss in value during the time the product spends in the reverse supply chain (a time that can exceed several months for many firms), and the loss in revenue because the product is sold at a discounted price. This cost is significant, and is incurred primarily by manufacturer. Reducing false failure returns, however, requires effort primarily by the retailer, for example informing consumers about the exact product that best fits their needs. We address the problem of reducing false failure returns via supply chain coordination methods. Specifically, we propose a target rebate contract that pays the retailer a specific dollar amount per each unit of false failure returns below a target. This target rebate provides an incentive to the retailer to increase her effort, thus decreasing the number of false failures and (potentially) increasing net sales. We show that this contract is Pareto–improving in the majority of cases. Our results also indicate that the profit improvement to both parties, and the supply chain, is substantial.

1. Introduction

Product returns represent a growing financial concern for firms in the United States and the rest of the world, with recent estimates reaching $100 billion annually for the United States alone (Stock, Speh and Sheer 2002). Product returns are a result of two phenomena: consumer returns of products to the reseller during a 30, 60 or even 90 day return period, and product overstocks returned to the manufacturer by the reseller. *Consumer* product returns can occur at any time during the product life cycle, and are increasingly important to manufacturers. Hewlett-Packard (HP) recently discovered that the total costs of consumer product returns for North America exceeds 2% of total outbound revenue (Reiss 2003).

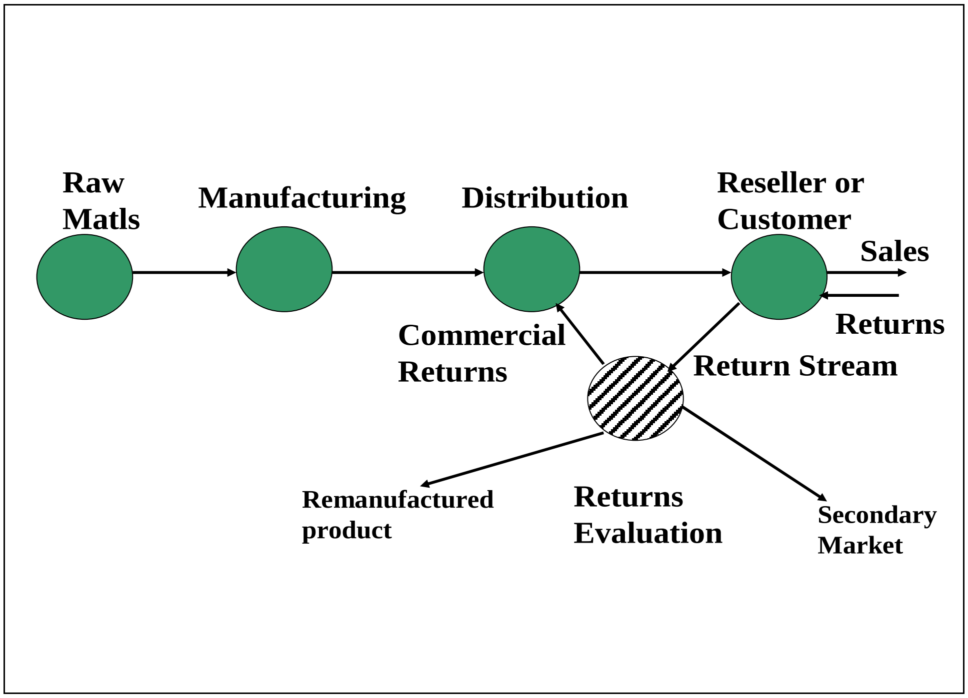
Product overstocks are the subject of a large body of research, but are not directly related to the problem of consumer product returns since overstocks are only returned at the end of the product lifecycle. Consumer product returns are driven by the 'consumer is king' attitude prevalent in the United States and supported by liberal product returns policies at most major resellers. Consumer product returns to the reseller are far less common in the European Union and the rest of the world, but many countries mandate some form of return period for Internet and catalog sales. The problems and costs of consumer product returns are projected to grow and many firms are just beginning to form teams to develop strategies and tactics to reduce the overall costs (Reiss 2003).

Consumer return rates range from 5-9 percent of sales for most resellers and up to 35 percent for fashion apparel (Toktay 2003). A percentage of these returns occur due to true product failure, however, a large percentage of returns have no verifiable functional defect. We refer to this class of consumer product returns as *false failure returns*; returns that have no functional or cosmetic defect. Managers cite a number of reasons why false failure returns occur, including: installation difficulties, product performance incompatibility with consumer preferences, and remorse (Kumar, Guide and Van Wassenhove 2002). For HP's inkjet printer group, false failure returns can account for up to 80% of their inkjet printer returns (Davey 2001). Since HP's total consumer product returns average slightly higher than 6% of sales, false failure returns average approximately 5% of sales. As of 1999, HP's inkjet printer division handled over 50,000 returns per month in North America (Davey 2001). The problem also persists outside of the high-tech industry. At Bosch Power Tools North America, false failure returns account for 2% of sales

(Valenta 2002). In the United Kingdom, manufacturers are seeing an increasing number of consumer returns to resellers disguised as 'defective' products (Helbig 2002). Because of the significant financial impact, manufacturers are interested in reducing false failure returns through improved relations and contracts with resellers.

When a manufacturer receives a false failure return, the product is routed through the firm's reverse supply chain involving several testing steps and repackaging before the product can become available for sale at a secondary market, typically at a price discount (see Figure 1). The cost of a false failure return includes the processing actions of testing, refurbishing if necessary, repackaging, the loss in value during the time the product spends in the reverse supply chain (a time that can exceed several months for many firms), and the loss in revenue because the product is sold at a discounted price. Thus, the cost of false failure returns is significant. The per–unit cost of a false failure return for computer manufacturers, including HP, is around 25% of the product's price.

Figure 1 - Reverse Supply Chain for Product Returns



In many settings, the retailer's effort is important in influencing the amount of false failure returns. A retailer may spend extra time with customers and listen to their needs before recommending a particular product. By doing so, customers have a higher probability of purchasing a product that matches their needs the first time. Retailers may also train their sales force so that the proper operating procedures of a product are clearly explained to customers upon purchase. After–sales support by the retailer may also reduce the number of false failure returns from customers who have trouble configuring the new product so that it performs as expected. HP, in the late 1990s, investigated the use of computer kiosks located in resellers to help a customer select the best HP printer for their needs in an effort to reduce false failure returns (Davey 2001). The project helped HP recognize that, to reduce the total business cost of returns, it is not sufficient to simply improve the efficiency of processing the returned units.

HP, along with most of their direct competitors, offers a customer returns policy to the retailer where the retailer receives full credit at the unit wholesale price. While the retailer may incur minimal processing and loss of goodwill costs, the manufacturer absorbs the majority of the false failure costs. In turn, the manufacturer receives the majority of the benefits from reducing false failure returns. The cost to reduce the number of false failure returns, however, may be incurred primarily by the retailer since she serves as the point of contact to the customer. Since the retailer incurs all effort–related costs but, essentially, no benefit from reducing the number of false failure returns, there is a need for contracts that coordinate the supply chain such that interests are aligned.

We address the problem of reducing false failure returns via supply chain coordination methods. Specifically, we propose a *target rebate* contract that pays the retailer \$$u$ per each unit of false failure returns below a target T. This target rebate provides an incentive to the retailer to increase her effort of informing consumers about the exact product that best fits their needs, thus decreasing the number of false failures and (potentially) increasing net sales. We show that this contract is Pareto–improving in the majority of cases. Our results also indicate that the profit improvement to both parties, and the supply chain, is substantial. Additionally, we show that increased retailer's effort produces a magnitude of profit improvement per expected return of 239% and 118% (median values) for the supply chain and manufacturer respectively; where profit is defined as revenues minus the costs associated with false failures. This improvement, in the case of the retailer, depends heavily on the amount of uncertainty in the returns process.

This paper is organized as follows. In §2, we review the relevant literature from closed-loop supply chains and supply chain coordination. We define our model in §3, present and discuss a numerical study in §4. We discuss managerial implications of our findings and state our conclusions in §5.

2. Literature Review

Our research draws on two separate research streams; closed-loop supply chains and supply chain coordination contracts. In this section we provide an overview of recent work in each area and examine the implications for our research.

2.1. Closed-Loop Supply Chains

There is a growing body of literature on closed-loop supply chains, where both the forward and reverse flows of materials are considered. For example, a recent feature issue of *Interfaces* (**33**(6) 2003) focuses on the practice of closed-loop supply chains in a variety of industry settings. Guide and Van Wassenhove (2003) concentrate on the business aspects of developing and managing profitable closed-loop supply chains. They identify the common processes required by a closed-loop supply chain: product acquisition, reverse logistics, inspection, testing and disposition, remanufacturing, and selling and distribution. We use their framework of common processes in a closed–loop supply chain to classify the literature.

Product returns occur for a variety of reasons: end-of-life returns, end-of-use returns, consumer returns and overstocks. Our research focus is on consumer product returns, an area where little academic research presently exists; an exception is the recent work by Souza, Guide, Van Wassenhove and Blackburn (2004). They examine the impact of the value of time on reverse supply chain design for commercial product returns. They find that returns are often time sensitive and firms frequently lose much of the value remaining in their returned products by not making disposition decisions as quickly as possible. Their research, however, is focused on the appropriate reverse supply chain design, responsive or efficient, based on the rate of value decay. Our research is complimentary. Preventing false failure returns increases the revenue from lost sales while reducing the unnecessary expenses of product return, inspection, disposition and distribution for reuse.

Table 1 provides an overview of the more recent research addressing the common processes identified by Guide and Van Wassenhove (2003) to categorize the literature. There are obvious gaps in the research literature. The areas of testing and disposition, and distribution and selling of the remanufactured products have not been addressed at all from an academic perspective. The area of product acquisition has had a very limited amount of research. In particular, the use of contracts in a closed-loop supply chain context has received scant attention. Market creation has also been unexplored for recovered products. The operational aspects of remanufacturing have received the most attention and there are numerous publications dealing with production planning and control (PP&C), inventory control, and materials planning. Ironically, product recovery is often viewed as a narrowly focused technical operational problem without visibility at the corporate level. At the Hewlett Packard Company, customer returns were treated as a low–

level divisional problem until a thorough analysis showed that the total cost of product returns was equivalent to 2 percent of total outbound sales (Davey 2001). Unfortunately, academic research often tends to reinforce this limited view with its narrow focus on local optimization of operational issues.

More recent research efforts have started to consider remanufacturing from a more strategic perspective, examining the strategic interaction among the closed–loop supply chain players. Heese et al. (2003) consider the potential competitive advantage for a company actively engaging in product take-back via a game-theoretic model. Majumder and Groenevelt (2001) present a game-theoretic model of competition in remanufacturing. Their research suggests that incentives should be given to the original equipment manufacturer to increase the fraction of remanufacturable products available, or to decrease the costs of remanufacturing. Savaskan, Bhattacharya and Van Wassenhove (2004) develop a game-theoretic model that addresses the issues of channel choice and coordination of the channel. Debo, Toktay and Van Wassenhove (2003) investigate the joint pricing and production technology problem of a manufacturer that offers both new and remanufactured products. Ferrer and Ketzenberg (2004) consider the value of information in improving remanufacturing performance where there are long supplier lead times. General overviews of product recovery and remanufacturing are presented by Thierry et al. (1995), Fleischmann (2001), and Guide (2000). We also refer the reader to the book, edited by Guide and Van Wassenhove (2003), from the First Workshop on Business Aspects of Closed-Loop Supply Chains for a review and discussion of each of the areas in Table 1.

Table 1 - Recent research in closed loop supply chains processes

Product Acquisition	Guide and Van Wassenhove 2001	Guide et al. 2003		
Reverse Logistics	Fleischmann 2001	Fleischmann et al. 2001	Krikke et al. 1999	Krikke 1998
Inspection & Disposition	NA			
Remanufacturing	*Overview:* Nasr et al. 1998	*PP&C:* Ketzenberg, et al. 2003 Souza et al. 2002 Guide 2000 Guide and Srivastava 1998	*Inventory:* Mahadevan et al. 2003 Toktay et al. 2000 van der Laan et al. 1999 van der Laan 1997	*Materials planning:* Ferrer and Whybark 2001 Inderfurth and Jensen 1999
Distribution & Sales	NA			

Supply Chain Contracting

The pioneer work of Pasternack (1985) shows that if a manufacturer sells products to a retailer under demand uncertainty with a constant wholesale price per unit (higher than the manufacturer's cost but lower than the retail price) the retailer usually stocks less than the supply chain's optimal quantity. From a marketing perspective, Wood (2001) examines how returns policies affect consumer purchase probability and return rates. Wood shows that more lenient policies tend to increase product returns, but that the increase in sales is sufficient to create a positive net sales effect. Other research focuses on the problem of setting returns policy between a manufacturer and a reseller and the use of incentives to control the returns flow (Padmanabhan and Png 1997 1995, Pasternack 1985, Davis, Gerstner and Hagerty 1995, Tsay 2001). Choi, Li and Yan (2004) study the effect of an e-marketplace on returns policy in which internet auctions are used to recover value from the stream of product returns.

A considerable amount of research has been devoted to this problem with a focus on appropriate contracts—a transfer payment from the manufacturer to the retailer, which provides incentives for retailers to stock a level of inventory that is optimal for the entire supply chain. Cachon (2003) provides a thorough review on contracts. Contracts based on rebates are of particular importance to our research. Based on agency theory, the provision of incentives for managerial effort is necessary when effort is not directly observable. Taylor (2002) offers an application of channel rebates to induce forward supply chain coordination when demand is dependent on the sales effort of the retailer. We propose a similar type contract to induce the supply chain optimal amount of effort to reduce the number of false failure returns; we choose a rebate–type contract because the number of false failures per period is dependent on retailer effort *and* random; as a result retailer effort is unobservable by the manufacturer.

In the next section, we introduce a mathematical model of the false failure returns coordinating process, and suggest a target rebate contract to coordinate the supply chain.

3. Model

Consider a supply chain comprised of one manufacturer and one retailer. We present a single period model for ease of exposition, although the model can be easily extendable to multiple periods in a stationary setting because there is no dynamic linkage (e.g., beginning inventory) between periods. That is, if all parameters are stationary over an infinite horizon, and the firm maximizes average profit, then the optimal level of effort—our decision variable—is the same across periods. A period, in our context, can be thought of as a quarter. The retailer receives full credit for each false failure from the manufacturer, and false failure returns cost the manufacturer m per unit. As argued before, the cost m is comprised of the associated reverse logistics, refurbishing and remarketing costs, and the price difference between the refurbished product and a new product.

Let ρ, $\rho \geq 1$, denote the retailer's effort to reduce false failure returns. Effort may be thought of as the time the retailer spends working with the customer to choose the right product and demonstrating how to use it. The retailer's minimum effort is $\rho = 1$; it represents the retailer's optimal balance between the cost of effort and the goodwill cost (against the retailer) from customers that must return a product because it does not meet their expectations.

Given an effort ρ by the retailer, the number of false failure returns is $\beta X / \rho\theta$. Here, β is the expected number of false failure returns per period when the retailer exerts the minimum level of effort, $X \geq 0$ is a random variable that models the uncertainty in the returns process, and $\theta \equiv E\{X\}$. The random variable X may be thought of as the "white noise" in the process. If the mapping between the retailer's effort and the reduction of false failures was deterministic, the manufacturer could simply contract the supply chain optimal effort level. Due to the random component however, the retailer's effort is not directly observable. Let $F(x)$ and $f(x)$ denote the cumulative distribution function (cdf) and probability density function (pdf) of X respectively. We assume that F is strictly increasing, and differentiable. In this *multiplicative* model, the standard deviation of the number of false failure returns is proportional to the mean. We later show that our results do not significantly change if we assume an *additive* model of uncertainty, where the standard deviation of the number of false failure returns is *independent* of the mean.

The relationship between the retailer's effort and the reduction in the number of false failures is based on our conversations with HP. In practice, there is evidence of diminishing returns in the retailer's effort, just as there is in the more commonly studied sales effort. In our model, the reduction of false failure returns, $\beta X (1 - \frac{1}{\rho}) / \theta$, also has diminishing returns in the level of effort

ρ, and the number of false failure returns approaches zero as $\rho \to \infty$. The *expected* value of the reduction in false failure for any effort level ρ is $\beta(1-\frac{1}{\rho})$.

We assume that the retailer's cost of exerting effort is convex in her amount of effort, ρ. A simple illustrative case is $a\rho^2/2$, where $a/2 > 0$ can be interpreted as the cost associated with the minimum level of effort. Further, we assume the interesting case where $m\beta \geq a/2$, such that the total potential savings from reducing false failure returns are greater than the retailer's cost of exerting the minimum effort level.

The retailer's effort level may impact the overall net sales quantity. Souza , Guide, Van Wassenhove, and Blackburn (2004) argue that when the retailer spends more time with the customer to ensure that she gets the right product in the first place, then each return that is avoided by this effort contributes to *one unit* increase in *net* sales (that is, total sales net of returns). It is possible, however, that this unit increase in net sales may occur with a *competitive* product. There may also be situations where sales are not impacted by the effort. This may occur when the majority of the false failure returns are caused by the customer not receiving adequate instructions on how to operate the product, either before or after purchase.

We model the impact of effort on net sales by assuming that each false failure avoided by the retailer's effort results in a δ units increase in net sales for the product under consideration, where $0 \leq \delta \leq 1$. The case where $\delta = 1$ models the situation where the retailer's effort results in a unit increase in net sales for the product under consideration. The case $0 < \delta < 1$ models the situation where the retailer's effort may result in a unit increase in net sales for any of the competing products, including the product under consideration. Finally, if $\delta = 0$, then net sales are independent of the effort. Denote by p, w, and c, the product's retail price, wholesale price, and manufacturing cost, respectively, where we assume $p > w > c$ to focus on realistic and interesting cases.

We consider in our analysis only the costs and revenues impacted by the retailer's effort to reduce false failure returns. Specifically, we consider the manufacturer's savings from the reduction in false failure returns due to retailer's efforts, the retailer's cost of exerting that effort, and the increase in net sales as a result of the effort. As a result, the coordinated supply chain profit $\Pi(\rho)$ as a function of the retailer's effort is

$$\Pi(\rho) = \left(m + \delta(p-c)\right)\beta\left(1-\frac{1}{\rho}\right) - \frac{1}{2}a\rho^2. \tag{1}$$

The first term in (1) includes the savings in processing a reduced number of false failure returns plus the additional sales revenue as a result of the retailer exerting the effort. Recall that the expected value of the reduction in false failure for any effort level ρ is $\beta(1-\frac{1}{\rho})$. The second term includes the retailer's cost of effort. The additional profit (1) is concave in ρ, and is maximized at

$$\rho^C = \left[\frac{(m+\delta(p-c))\beta}{a}\right]^{1/3}, \tag{2}$$

where ρ^C denotes the optimal retailer effort in a coordinated supply chain. Note that (2) necessarily yields $\rho^C > 1$ (the coordinated supply chain exerts more than the minimum effort) because $m\beta \geq a/2$ by assumption.

In a non-coordinated supply chain, the retailer incurs all of the cost for reducing false failure returns but is only rewarded with part of the benefit. Thus, it seems unlikely that the optimal level of effort will be observed in the absence of some form of coordination contract. Specifically, the retailer's expected profit $\pi_R(\rho)$ is

$$\pi_R(\rho) = -\frac{a\rho^2}{2} + \delta(p-w)\beta\left(1-\frac{1}{\rho}\right). \tag{3}$$

In the absence of an incentive by the manufacturer, the retailer exerts the optimal effort level $\rho^D = \max\left\{\left(\delta(p-w)\beta/a\right)^{1/3}, 1\right\}$. Clearly, $\rho^C \geq \rho^D$ because $w > c$. It is easy to see that the manufacturer's profit in this decentralized case, given by (1) – (3), is

$$\pi_M(\rho^D) = \delta(w-c)\beta\left(1-\frac{1}{\rho^D}\right). \tag{4}$$

Since the manufacturer enjoys most of the benefits from reducing the number of false failure returns, the manufacturer must share some of these benefits with the retailer in order to achieve a coordinated solution. The manufacturer cannot mandate a given level of effort since effort is not observable due to the uncertainty component in the number of false failure returns. The manufacturer may contract on the number of false failure returns through the use of a target rebate. This is where the retailer receives a reward (a cash rebate) for every reduction in the number of false failures below a predetermined target level. A low number of false failure returns is a signal of good effort on the retailer's part to help the customer purchase the correct product and its operation. A target rebate creates an incentive for the retailer to exert effort in this respect.

To coordinate the supply chain we use a target rebate contract that pays the retailer *u* dollars for each unit of false failure returns below a target threshold level *T*. Sales target rebates have been used to coordinate supply chains where end-product demand is dependent on the retailer's sales effort (Taylor 2002). Here, the target rebate is used to solve the principle-agent problem that occurs in the reduction of false failure returns. HP has experimented with different forms of target rebate contracts to reduce the level of false failure returns (Reiss 2003). Little is known, however, about the value of these contracts and how they should be structured.

Under a target rebate contract, the retailer's profit is

$$\pi_R(\rho \mid T,u) = uE_X\left\{\left[T-\frac{\beta X}{\rho\theta}\right]^+\right\} - \frac{a\rho^2}{2} + \delta(p-w)\beta\left(1-\frac{1}{\rho}\right), \tag{5}$$

and the manufacturer's profit is given by subtracting (5) from (1), or

$$\pi_M(\rho \mid T,u) = \left(m+\delta(w-c)\right)\beta\left(1-\frac{1}{\rho}\right) - uE_X\left\{\left[T-\frac{\beta X}{\rho\theta}\right]^+\right\}. \tag{6}$$

We find the values of u and T that coordinate this supply chain by equating to (2) the value of ρ that maximizes (5). To do so, we first show that (5) is concave in ρ through the following proposition.

Proposition 1: The retailer's profit under a target rebate contract is concave in ρ.

Proof: By assumption, $X \geq 0$ and $\rho \geq 1$ so $T - \dfrac{\beta X}{\rho\theta}$ is concave in ρ for any given value of X. The function $\left[T - \dfrac{\beta X}{\rho\theta}\right]^+$ is constant at zero for any given value of X and $\rho < \dfrac{\beta X}{T\theta}$, thus the entire function is concave in ρ. Since concavity is maintained under expectation, the first term of (5) is concave. It is easy to check that the last two terms of (5) are concave in ρ and the sum of a set of concave functions is also concave, therefore (5) is concave in ρ. □

To calculate the supply chain coordinating contract values, it becomes necessary to assume specific distributions for X. Since there is no data available, we need to make some reasonable choices in selecting specific distributions to use. We demonstrate the uniform and normal distribution cases below; the uniform distribution, which has a constant coefficient of variation, allows us to have closed–form solutions; the normal distribution case covers cases with varying degrees of uncertainty (coefficient of variation) in the returns process.

Uniform Distribution for X

Consider that X ~ Uniform (0, 2θ). Then, $E_X\left\{\left(T - \frac{\beta X}{\rho\theta}\right)^+\right\} = u\int_0^{\frac{T\rho\theta}{\beta}}\left(T - \frac{\beta x}{\rho\theta}\right)\cdot\frac{1}{2\theta}\,dx = T^2 u\rho/4\beta$.

Substituting this expression in (5) yields

$$\pi_R(\rho\,|\,T,u) = \frac{T^2 u\rho}{4\beta} - \frac{a\rho^2}{2} + \delta(p-w)\beta\left(1 - \frac{1}{\rho}\right). \tag{7}$$

The value ρ^* that maximizes (7) must satisfy the first–order conditions:

$$T^2 u(\rho^*)^2 - 4a\beta(\rho^*)^3 + 4\delta(p-w)\beta^2 = 0. \tag{8}$$

Substituting ρ^* with (2), we find an expression that relates u and T, so that the supply chain is coordinated by the contract:

$$T = \frac{2\beta^{2/3} a^{1/3} \left(m + \delta(w-c)\right)^{1/2}}{u^{1/2} \left(m + \delta(p-c)\right)^{1/3}}. \tag{9}$$

For a fixed value of *u*, the corresponding value of *T* is obtained from (9). The value of *u* (and *T*) has to satisfy $T\rho\theta/\beta \le 2\theta$, that is, the upper limit in the integral $E_X\{(T - \frac{\beta X}{\rho\theta})^+\}$ cannot exceed the upper limit of the uniform distribution, 2θ. Substituting *T* from (9) and ρ from (2), yields $u \ge m + \delta(w-c)$. This is interesting since it shows that the value of *u* that coordinates this contract is larger than the cost of a return to the manufacturer. Still, the manufacturer can be better off with a target rebate contract as Proposition 2 shows below.

Proposition 2: Suppose *X* is uniformly distributed. A (*u*, *T*) contract that coordinates the supply chain always makes the retailer better off, and makes the manufacturer better off if $\rho^C \ge 2 + \delta(w-c)/m$.

Proof: See Appendix.

Normal Distribution for X

Now, consider $X \sim N(\theta, \sigma^2)$, and assume that σ is small enough so that negative values of *X* can be ignored. After some algebra, (5) becomes:

$$\pi_R(\rho \mid T, u) = u\left\{T - \frac{\beta}{\rho}\left[1 - \frac{\sigma}{\theta} L\left(\frac{T\rho/\beta - 1}{\sigma/\theta}\right)\right]\right\} - \frac{a\rho^2}{2} + \delta(p-w)\beta\left(1 - \frac{1}{\rho}\right), \tag{10}$$

where $L(\cdot)$ is the standard normal loss function. The first–order condition with respect to ρ results in an implicit equation that can be solved for *T*, given a value of *u* or vice-versa:

$$\Phi\left(\frac{T\rho^C/\beta - 1}{\sigma/\theta}\right) - \frac{\sigma}{\theta}\phi\left(\frac{T\rho^C/\beta - 1}{\sigma/\theta}\right) = \frac{\left(m + \delta(w-c)\right)}{u}, \tag{11}$$

where, as before, ρ^C is given by (2), and $\phi(\cdot)$ and $\Phi(\cdot)$ are the standard normal pdf and cdf respectively. Given that $\Phi(\cdot)$ and $\phi(\cdot)$ are necessarily less than one, then a necessary condition for (11) to have a solution is that the right–hand side of (11) is less than one, which implies

$u \geq m + \delta(w - c)$, just like in the uniform distribution case. Further, if we define $z = \left(T\rho^C / \beta - 1\right) \Big/ \left(\sigma / \theta\right)$, and (arbitrarily) set $u = k\left(m + \delta(w - c)\right)$, where k > 1 is a constant, we can rewrite (11) as

$$\Phi(z) - \frac{\sigma}{\theta}\phi(z) = k \,. \tag{12}$$

Denoting the solution of (12) by z^*, then $T = \frac{\beta}{\rho}\left(\frac{\sigma}{\theta}z^* + 1\right)$. Note that z^* only depends on the coefficient of variation $\frac{\sigma}{\theta}$ and the constant *k*, and thus the design of such a contract is relatively straightforward. Unlike the uniform distribution case, however, the retailer is not always better off under a normal distribution for *X*. As (10) indicates, and we will see numerically in Section 4, the retailer's profit is very much dependent on the amount of uncertainty $\frac{\sigma}{\theta}$ in the returns process *X*—under reasonable values for costs and parameter values, the retailer is better off with a (*u*, *T*) contract at moderate to high levels of uncertainty $\frac{\sigma}{\theta}$. In short, a higher degree of uncertainty implies a higher expected number of returns below the target *T*, *ceteris paribus*, and consequently a higher profit for the retailer.

To illustrate how the target rebate contract works for both the uniform and normal distribution cases, we provide the following numerical example.

Example 1: Consider m = 10, β = 20, a = 10, δ = 0 (sales independent of effort), and $\frac{\sigma}{\theta} = 1/\sqrt{3}$ (so that the uniform and normal distribution cases have the same coefficient of variation, $1/\sqrt{3}$). The optimal level of effort, given by (2), is ρ^C = 2.714. The coordinated supply chain's profit at the optimal level of effort (1) is $\Pi(\rho^C) =$ \$ 89.75. To coordinate the supply chain, different combinations of *u* and *T* are possible. A necessary condition for *u*, under both uniform and normal distributions, is $u \geq m + \delta(w - c) = m$ = 10. We consider five combinations of *u* and *T*, for both cases; and present the results in Table 2. Note that the distribution of profits does not depend on the contract parameters for the uniform distribution case. This is not an obvious result, and is a consequence of the unique shape of the uniform distribution—the term

$E_X\{(T-\frac{\beta X}{\rho\theta})^+\}$, the only term dependent of (u, T) in the retailer's profit function (7), is a linear function of ρ when X is uniformly distributed. However, a higher value of u (resulting in a lower T) results in increasing profits for the manufacturer and decreasing profits for the retailer for the normal distribution case.

Table 2: Different combinations of *u* and *T* that coordinate the supply chain in Example 1

	Uniform Distr. for *X*			Normal Distr. For *X*		
u	***T***	π_R	π_M	***T***	π_R	π_M
11	14.05	36.84	52.64	14.79	45.54	43.94
12	13.45	36.84	52.64	13.33	36.59	52.89
13	12.92	36.84	52.64	12.43	32.09	57.39
14	12.45	36.84	52.64	11.76	29.36	60.12
15	12.03	36.84	52.64	11.24	27.56	61.92

Additive Uncertainty Case

In the additive uncertainty case, the number of false failure returns is $(\beta+X)/\rho$, where $E\{X\}=0$ and we now allow X to take negative values as long as $X\geq-\beta$; X has the same interpretation—noise in the returns process—as in the multiplicative uncertainty case studied before. Note that the variance of the number of false failure returns is now independent of the mean. The expected supply chain profit and optimal level of effort are still given by (1) and (2). The retailer's expected profit is a slight modification of (5):

$$\pi_R(\rho\,|\,T,u)=uE_X\left\{\left[T-\frac{\beta+X}{\rho}\right]^+\right\}-\frac{a\rho^2}{2}+\delta(p-w)\beta\left(1-\frac{1}{\rho}\right). \tag{13}$$

If X ~ Uniform $(-\beta,\beta)$, the results are identical to the multiplicative uncertainty case. This is a reasonable comparison since the lower bound of the distribution should be set to $-\beta$ so that the lower bound on the number of false failure returns is zero, and the upper bound should be set to β so that $E\{X\}=0$.

If X ~ Normal (0, γ^2), then the retailer's profit is

$$\pi_R(\rho\,|\,T,u) = u\left\{T - \frac{\beta}{\rho}\left[1 - \frac{\gamma}{\beta}L\left(\frac{T\rho/\beta - 1}{\gamma/\beta}\right)\right] + \right\} - \frac{a\rho^2}{2} + \delta(p-w)\beta\left(1 - \frac{1}{\rho}\right), \tag{14}$$

and the (u, T) contract parameters that coordinate the supply chain satisfy:

$$\Phi\left(\frac{T\rho^* - \beta}{\gamma}\right) - \frac{\gamma}{\beta}\phi\left(\frac{T\rho^* - \beta}{\gamma}\right) = \frac{m + \delta(w-c)}{u}. \tag{15}$$

We note that (14) is identical to (10) if $\gamma = \beta\frac{\sigma}{\theta}$. Thus, we conclude that our modeling approach is robust to the choice of uncertainty modeling, multiplicative or additive, in the number of false failure returns.

3.1. Incentives to Game Traditional Sales Contracts

Up to this point, we have assumed that all false failure returns are unintentional on the retailer's part and are due primarily to the retailer's lack of effort in helping customers select the right product and teaching them how to operate it. In this section, we explore whether there are incentives for the retailer to intentionally encourage false failure returns (a practice we term "gaming"). For example, HP offers a sales-based coordination contract with its retailers where any unsold product may be returned to HP for a partial refund at the end of the product's last selling season. For our purposes, we define the last selling season as the time when the demand rate for a particular product decreases to a point where the retailer no longer wishes to replenish it. This happens when an older model is being phased out to make room for its replacement version. This contract, termed a buy-back contract in the literature, is shown by Pasternack (1985) to coordinate the channel, provided that appropriate values for wholesale and buy-back prices are selected, by eliminating the double-marginalization effect.

If the retailer is offered a full refund for a false-failure return (the current practice), then the retailer has an incentive to push a slow–selling product (during its last selling season) to consumers, fully knowing that the product does not fully meet the consumer's needs and thus has a high probability of being returned. We are, in our example, ignoring the consumer's loss of goodwill to the retailer for selling the wrong product. Still, the retailer receives a higher payment from false-failure returns than from returning unsold product under the partial buy-back contract. This savings may dominate the retailer's loss of goodwill cost and cause her to intentionally act in ways that increase the number of false-failure returns. Thus, a relevant question is: Does our proposed target rebate contract encourage or discourage the retailer to game her sales-based contract?

A large number of manufacturer-retailer sales-based contracts are used in practice and/or have appeared in the literature. For a detailed review and description of these contracts, the reader is referred to Cachon (2003). We examine several of the most popular contracts for the sole purpose of determining where incentives for gaming exist at the end of the selling season. We then show how our proposed target rebate contract for false failure returns helps mitigate these gaming incentives. In all of our analysis, we assume the more interesting case of voluntary compliance. This is where the manufacturer delivers the amount, not to exceed the retailer's order, that maximizes his profit given the terms of the contract. Let g_m represent the loss of goodwill to the manufacturer and g_r represent the loss of goodwill to the retailer if the customer purchases a product that does not meet their needs. Note that this is a different penalty cost than the more traditional loss of goodwill due to unsatisfied demand. Let v represent the per unit salvage value of any unsold product for the retailer at the end of the product's last selling season.

We begin with the simplest contract, the wholesale price-only contract, where the manufacturer charges the supplier a fixed wholesale price for each unit purchased. In general, this contract does not coordinate the supply chain, but it is easy to monitor and still common in practice. In the absence of a target rebate contract, the incentive to game occurs when the retailer is left with unsold inventory at the end of the last selling season and the retailer's salvage value is less than the net benefit from a false failure return (the wholesale price minus the loss of goodwill to the retailer), i.e. $v \le w - g_r$. Our target rebate contract changes this condition to

$$v \le w - g_r - u \;\text{ if }\; T < \frac{\beta}{\rho^C}, \tag{16}$$

which reduces the incentive for gaming.

Now, consider buy–back and revenue sharing contracts. Cachon (2003) shows that the two are equivalent under most simple settings. We therefore restrict our discussion to the buy–back contract. In a buy–back contract, the manufacturer sells the product to the retailer at the per unit wholesale price w, but pays the retailer b, $b \le w$, for each unsold unit at the end of the last selling season. Without a target rebate contract, the retailer has an incentive to game if $b + v \le w - g_r$. Thus, a buy–back contract ensures a smaller risk of gaming than does a wholesale price-only contract. The effect of the target rebate is similar to (16) but includes the buy–back price on the left–hand side. Therefore, the target rebate also reduces the incentive for gaming with a buy-back contract.

Next, consider the quantity flexibility contract. In this contract, the retailer receives a full refund from the manufacturer on any leftover inventory at the end of the selling season that is below a predetermined target level λq, where q is the retailer's order quantity over the product's last selling season and $\lambda \in [0,1]$ is a contract parameter. Let $Y \in [0, \Omega]$ be a random variable representing demand over the last selling season, with pdf $f_Y(y)$. The condition for gaming under a quantity flexibility contract with a target rebate may be written as

$$v\int_{\lambda q}^{\Omega} f_Y(y)dy \le w - g_r - u \;\text{ if }\; T < \frac{\beta}{\rho^C}.$$

Since the integral is less than one by definition, the quantity flexibility contract ensures a smaller risk to gaming than a wholesale price-only contract and the target rebate reduces this risk even more.

The forth contract type is the sales rebate contract. This contract is similar to the target rebate contract discussed in this paper, except that it offers a reward for each unit *sold* above a predetermined target threshold. We must be careful here in how we define a sale: Is the retailer rewarded for sales regardless of whether or not the items are eventually returned, or for net–sales where the number of false failure returns are subtracted out at the end? The exact contractual definition results in very different incentives for gaming. If a sale is defined as the former, a clear incentive exists (above even the wholesale price only contract) when the retailer is above her target threshold and receives an additional bonus for each product sold. If a sale is defined as the latter, the use of a sales rebate contract should not add any additional incentive for gaming. Unfortunately, sales rebate contracts do not coordinate the supply chain in isolation and are thus often used in combination with other coordination contracts when sales are dependent on the sales effort of the retailer. Thus, the incentive for gaming may still exist with these contracts but can be reduced by specifying net–sales in the conditions of the sales rebate contract.

Finally, consider the quantity discount contract where the wholesale price is a decreasing function of the order quantity. Since this contract induces the retailer to order more by lowering the wholesale price instead of repurchasing unsold units, the condition for gaming is the same as the wholesale price–only contract except that the single price w is replaced by the average wholesale price paid, $\bar{w}$, in (16). Since coordination requires that $\bar{w} < w$ then a quantity discount contract ensures a smaller risk for gaming than a wholesale price-only contract. A target rebate contract reduces the probability of gaming here as well, just as it does for the wholesale price-only contract.

Summarizing, our analysis reveals that traditional sales-based contracts designed to coordinate the forward supply chain may induce gaming at the end of a product's last selling season. This is because, for most contracts, the retailer's unsold inventory commands a lower per unit revenue than the wholesale price, but she receives the full wholesale price credit for each false failure return. Therefore, retailers have an incentive to push products during the last selling season to customers, knowing that these products have a high probability of being returned. We argue, however, that our target rebate contract, designed to coordinate false failure returns in the reverse chain, reduces these gaming incentives when they exist. Further, we believe that the low volumes typically found during the last selling season combined with the side benefits of our target rebate contract implies that there is no need to further design or alter existing contracts to eliminate the gaming incentives.

4. Numerical Analysis

In this section, we perform a numerical analysis that has several objectives:1) to analyze the influence of the distribution of the uncertainty in the returns process (*X*) on our results; 2) to quantify the benefits of coordination (how better off is each party in the supply chain by adopting our target rebate contract?), and 3) to identify how the benefits of adopting our contract vary with different parameter values. We use the multiplicative uncertainty case in our analysis, but our results hold for the additive uncertainty case.

The base numbers are grounded on product prices and returns volume for HP deskjet printers, and the other model parameters are varied over a wide range. We set p = \$200 (median price across different inkjet models), $m = 0.25p$ = \$50, and a 50% gross margin, which implies c = \$100. We vary the remaining parameters according to the full–factorial experimental design shown in Table 3, which we justify as follows. We use the two distributions (uniform and normal) for the uncertainty in the returns process studied theoretically. We consider three values for the coefficient of variation (when *X* is normally distributed) σ/θ, from 0.15 to 0.45 (we do not consider values for σ/θ higher than 0.45 because then there is a significant probability of negative values for *X*). We use five values for the cost of effort per expected return, a/β, from

\$0.10 to \$5. We use six values for the parameter δ, which measures the impact of effort on net sales, from the theoretical minimum of zero to the theoretical maximum of one. Finally, we consider five values for the wholesale price as a fraction of the product's price, w / p, from 0.55 to 0.95. The experimental design of Table 3 yields 600 cases, 150 for the uniform distribution and 450 for the normal distribution.

Table 3: Experimental Design

Parameter	***Symbol***	***Levels***
Distribution of *X*	-	Uniform, Normal
Coefficient of variation for *X*	σ / θ	0.15, 0.30, 0.45
Cost of effort per expected return	a / β	\$ 0.1, 0.5, 1.0, 2.5, 5.0
Influence of effort on sales	δ	0.0, 0.2, 0.4, 0.6, 0.8, 1.0
Wholesale/retail price ratio	w / p	0.55, 0.65, 0.75, 0.85, 0.95

Our main performance measure is the profit improvement per expected return (when $\rho = 1$) for each party or the supply chain as a result of adopting the target rebate contract. For example, for the retailer this is defined as $\Delta_R = \left(\pi_R\left(\rho^C \mid T,u\right) - \pi_R\left(\rho^D\right)\right) / \beta$, or, in relative terms, $\%\Delta_R = 100\%\Delta_R / \left|\pi_R\left(\rho^D\right) / \beta\right|$. A similar comparison is done for the manufacturer and the supply chain. We report relative and absolute values of profit improvement here because *both* are important in providing a complete picture of the contract benefits, as we argue below. If ρ^D = 1, then $\pi_R(\rho^D)$ = 0, and consequently $\%\Delta_R$ is infinite. As a result we do not include the cases where ρ^D = 1 in our statistics for $\%\Delta_R$ (they necessarily occur when $\delta = 0$; they also occur in a few cases when $\delta > 0$), although we do for the absolute change Δ_R. Detailed results are available upon request.

The statistics for *relative* profit improvement $\%\Delta$ for the retailer, manufacturer, and supply chain are reported in Table 4, tabulated by the distribution of *X*: uniform, normal over all cases, and normal for each value of the coefficient of variation of *X*, σ/θ. We do not report the maximum values for $\%\Delta$ because they occur in cases where the non-coordinated supply chain has such a small positive profit that any improvement has a huge relative impact and relative numbers can be misleading. A similar phenomenon occurs in Table 4—the extremely small minimum values of $\%\Delta_R$ in the normal distribution case (first row) occurs when $\pi_R(\rho^D)$ has a small absolute value. Again, this points out to the need for also reporting *absolute* values of profit improvement, which we do below.

Table 4: Statistics for Profit Improvement $\%\Delta_R$ Across Experimental Design

Statistic	**Party**	**Distribution of *X***				
		Uniform	Normal Overall	Normal $\sigma/\theta = 0.15$	Normal $\sigma/\theta = 0.30$	Normal $\sigma/\theta = 0.45$
Minimum	Retailer	6%	-2312%	-2312%	3%	5%
	Manufacturer	33%	52%	77%	72%	52%
	Supply chain	53%	53%	53%	53%	53%
25^{th} perc.	Retailer	45%	5%	-16%	19%	36%
	Manufacturer	105%	138%	169%	141%	116%
	Supply chain	85%	85%	85%	85%	85%
Median	Retailer	118%	28%	2%	46%	94%
	Manufacturer	179%	258%	296%	263%	216%
	Supply chain	139%	139%	139%	139%	139%
75^{th} perc.	Retailer	473%	115%	6%	158%	372%
	Manufacturer	453%	580%	621%	578%	512%
	Supply chain	293%	293%	293%	293%	293%

Table 4 reveals that *relative* profit improvements are substantial, the median values for $\%\Delta_M$ are 179% and 258%, and median values for $\%\Delta_R$ are 118% and 28%, for the uniform and normal distribution cases, respectively. The median value for supply chain profit improvement is 239%, which does not depend on the distribution of *X*. We caution that our analysis is restricted only to profit per false failure return as impacted by the retailer's effort, not *total* profit (forward + reverse supply chain) for each party. To illustrate this, false failure returns are about 5% of total sales. Both parties are better off ($\%\Delta > 0$) in *all* uniform distribution cases. For the normal distribution case, the manufacturer is always better off and the retailer is better off in the majority of our test cases, with the exception occurring when the coefficient of variation of *X* is low ($\sigma/\theta = 0.15$). The retailer's (manufacturer's) improvement increases (decreases) as σ/θ

increases, for example, the median value of $\%\Delta$ is 2%, 46% and 94% (296%, 263%, and 216%) for σ/θ = 0.15, 0.30, and 0.45, respectively. The higher the level of uncertainty σ/θ in the returns process, the higher the *expected* number of returns below the target *T*, ceteris paribus, as (10) shows. Because the retailer is not penalized when returns are above the target, but is paid (by the manufacturer) \$*u* per return below the target, the retailer's (manufacturer's) profit increases (decreases) as uncertainty in the returns process increases, *ceteris paribus*.

Because the coefficient of variation for the uniform distribution is $1/\sqrt{3}$ = 0.58, we expect $\%\Delta$ to be the most similar between the uniform and normal distribution cases when σ/θ = 0.45; this is indeed observed—compare the "Uniform" column with the three right "Normal" columns in Table 4. Further the % difference in the value of the contract parameter *T* between the uniform and normal distribution cases (not reported in Table 4) is 36%, 23%, and 8% for σ/θ = 0.15, 0.30 and 0.45, respectively (regardless of other experimental factors), where across all cases we select the contract parameter $u = 1.1\left(m + \delta(w - c)\right)$. Overall the statistics in Table 4 show remarkable improvements with the use of a target rebate contract.

Next we study how *absolute* profit improvement Δ is related to the factors in our experimental design. For each factor, we average Δ across *all* experiments for each factor level. We focus on the normal distribution results and note that the results are similar for the uniform distribution, except that the magnitude of improvement is higher for the retailer in the uniform case (since we average across all coefficients of variation). The results are shown in Figure 2 for factors σ/θ (left) and a/β (right), and in Figure 3 for factors δ (left) and w/p (right). We note that the average profit improvement for the supply chain is equal to the sum of improvements across the retailer and the manufacturer. We have already commented on the effect of σ/θ on Δ, shown in Figure 2 (left). Figure 2 (right) shows that both the supply chain and the manufacturer's profit improvement decrease with the retailer's normalized effort cost a/β. This is because a higher effort cost induces a lower coordinated optimal effort level ρ^C, making profits closer to the non-coordinated case for both parties; as a consequence benefits of coordination decreases. In turn, this results in increasing benefits of coordination for the retailer.

Figure 2: Average Profit Improvement ($) per Expected Return as a Function of the Coefficient of Variation for X (left) and the Retailer's Effort Cost per Expected Return (right)

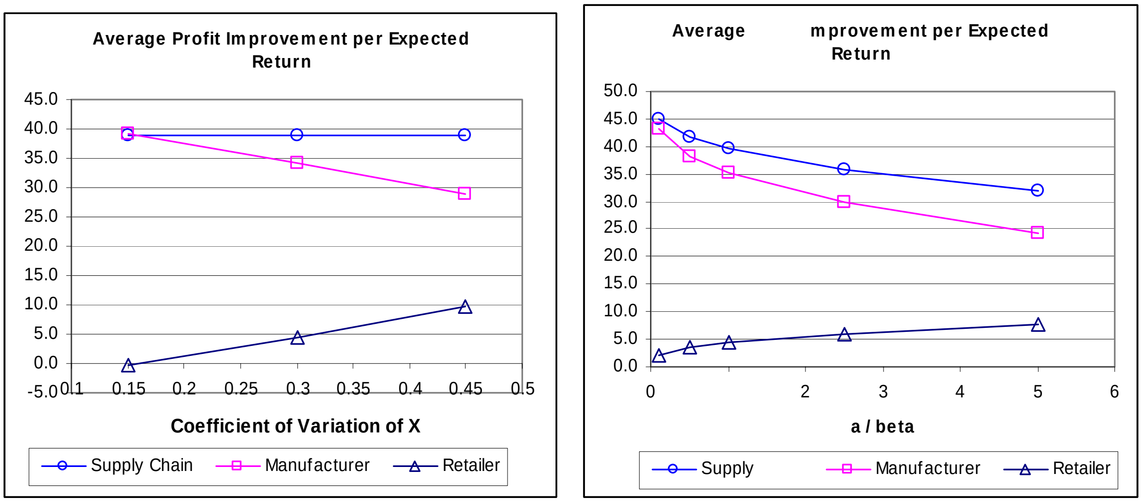
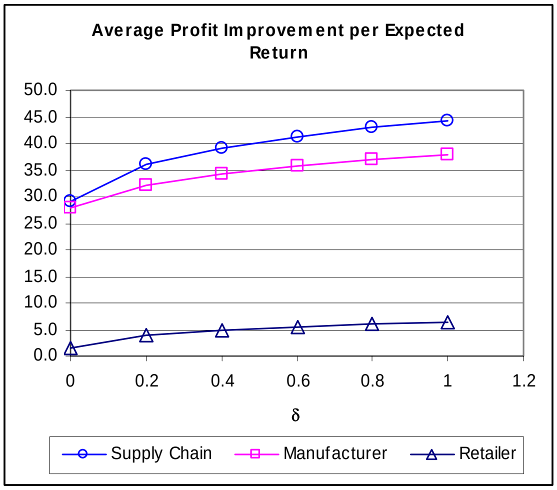
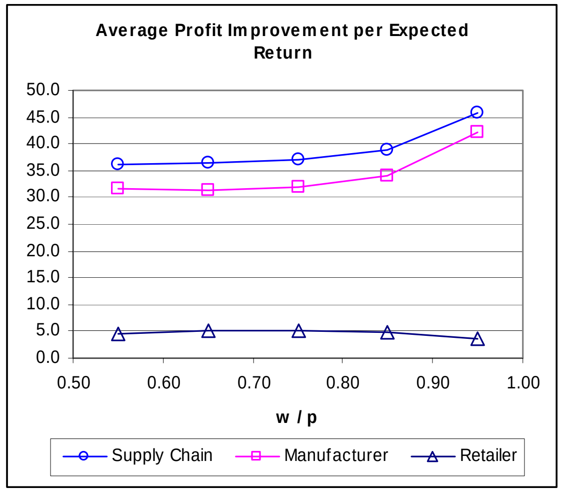


Figure 3 (left) shows that all parties' profit improvement increase with δ; this is expected because a higher δ implies a higher positive impact of effort on net sales. Finally, Figure 3 (right) shows that the retailer's profit *improvement* is fairly insensitive to the wholesale price w/p, except for very low or very high values. We note that the retailer's *profit* is clearly decreasing in the wholesale price since its profit margin is smaller. This somewhat surprising result can be explained by the reasoning that most of the retailer's profit is derived from the contractual agreements at high values of w/p. For the manufacturer, a higher wholesale price means a higher profit margin in each additional net sale generated as a result of the retailer's effort. In sum, our numerical results indicate that a target rebate contract results in significant improvements in the reverse supply chain profits.

Figure 3: Average Profit Improvement ($) per Expected Return as a Function of the δ (left) and Wholesale Price (right)





5. Conclusion

In this paper we propose a target rebate contract to coordinate false failure returns. This contract stipulates a payment of $*u* for each false failure below a target *T*. We propose a simple, but insightful model, which reveals the substantial benefits that can be derived by enticing retailers to exert a higher level of effort to decrease the number of false failure returns. This higher level of effort has two benefits for the supply chain. First, it reduces the overall processing cost for false failures, which is significant considering observed false failure cost averaging 25% of the product price. Second, a higher level of effort may result in a higher level of net sales (sales that are not returned).

We show that our contract has a number of attractive properties, including Pareto–improvement under most common circumstances. That is, our numerical study shows that Pareto–improvement always occurs for moderate to high levels of uncertainty in the returns process. This is because as uncertainty increases, *ceteris paribus*, the retailer's profit increases due to an increase in the expected number of false failures below the contract target *T*. In our numerical study, Pareto improvement occurs in all cases where the returns process follows a uniform distribution; if the returns process follows a normal distribution, then Pareto improvement occurs for *all* cases when the coefficient of variation is at least 0.3. We find that the magnitude of profit (associated with false failures) improvement per expected return as a result of the increased retailer's effort is very significant, with a median value of 239% and 118% for the supply chain and manufacturer respectively. For the retailer, the profit improvement per expected return increases with the uncertainty in the returns process.

Our research is a first step in understanding how a target rebate contract provides incentives to the retailer to improve her effort in the sales process targeted at reducing the number of false failure returns. We use a stylized model that summarizes the retailer's effort by a single variable, a common approach in the current body of contract research. We also discuss the implications of using quantity-based coordination contracts on the false failure return problem, and offer guidance on which ones provide the least incentive to game. One possibility for future research is to generalize our results to other settings. Another is to monitor the implementation of such a contract in practice, and compare the improvements against the theory. In any event, our research indicates that significant savings may be obtained by reducing the number of false failure returns.

Appendix – Proof of Proposition 2

Proposition 2: Suppose *X* is uniformly distributed. A (*u*, *T*) contract that coordinates the supply chain always makes the retailer better off, and makes the manufacturer better off if $\rho^C \geq 2 + \delta(w-c)/m$.

Proof: First, consider the manufacturer. The manufacturer is better off with the (*u*, *T*) contract if $\pi_M\left(\rho^C \mid T,u\right) \geq \pi_M\left(\rho^D\right)$. The manufacturer's profit (6) can be rewritten as

$$\pi_M(\rho^C \mid T,u) = \left(m + \delta(w-c)\right)\beta\left(\frac{\rho^C - 1}{\rho^C}\right) - \frac{T^2 u \rho^C}{4\beta}. \tag{18}$$

We can rewrite (8) as $T^2u(\rho^C)^2 = 4a\beta(\rho^C)^3 - 4\delta(p-w)\beta^2$, and from (2), $(\rho^C)^3 = \left(m+\delta(p-c)\right)\beta / a$; thus $T^2u(\rho^C)^2 = 4\left(m+\delta(p-c)\right)\beta^2 - 4\delta(p-w)\beta^2$ = $4\beta^2\left(m+\delta(w-c)\right)$. Substituting this in (18) yields:

$$\pi_M(\rho^C \mid T,u) = \left(m+\delta(w-c)\right)\beta\left(\frac{\rho^C-1}{\rho^C}\right) - \frac{\beta\left(m+\delta(w-c)\right)}{\rho^C} = \left(m+\delta(w-c)\right)\beta\left(\frac{\rho^C-2}{\rho^C}\right)$$

Again, we need $\pi_R\left(\rho^C \mid T,u\right) \geq \pi_R\left(\rho^D\right)$, where $\pi_M(\rho^D)$ is given by (4). We know that $\rho^C \geq \rho^D$, and thus $\pi_M(\rho^C) \geq \pi_M(\rho^D)$. Consequently, it is enough to show conditions under which $\pi_M(\rho^C \mid T,u) \geq \pi_M(\rho^C)$. Simple algebra shows that this is true if $\rho^C \geq 2 + \delta(w-c)/m$.

Now, we consider the retailer. Define $\Delta_R = \pi_R\left(\rho^C \mid T,u\right) - \pi_R\left(\rho^D\right)$. For the retailer to be better off, we need $\Delta_R \geq 0$. From (8), $T^2u/4\beta = a\rho^C - \delta\beta(p-w)/(\rho^C)^2$, which can be substituted into (7), yielding

$$\pi_R(\rho \mid T,u) = \frac{a(\rho^C)^2}{2} - \frac{\delta(p-w)\beta}{\rho^C} + \delta(p-w)\beta = \frac{a(\rho^C)^3 - 4\delta\beta(p-w)}{2\rho^C} + \delta(p-w)\beta \qquad (19)$$

Now, from (2), $(\rho^C)^3 = \left(m+\delta(p-c)\right)\beta / a$, which can be substituted in (19), yielding

$$\pi_R(\rho \mid T,u) = \frac{\beta\left[m+\delta(4w-3p-c)\right]}{2\rho^C} + \delta\beta(p-w). \qquad (20)$$

In the decentralized case, note that (3) can be rewritten as

$$\pi_R(\rho^D) = \frac{-a(\rho^D)^3 - 2\delta\beta(p-w)}{2\rho^D} + \delta(p-w)\beta. \qquad (21)$$

But $\rho^D = \max\left\{\left(\delta(p-w)\beta/a\right)^{1/3}, 1\right\}$. First, consider the case where ρ^D > 1; in this case $\rho^D = \left(\delta(p-w)\beta/a\right)^{1/3}$, which implies $a(\rho^D)^3 = \delta\beta(p-w)$; substituting this in (21) yields

$$\pi_R(\rho^D) = -\frac{3\delta\beta(p-w)}{2\rho^D} + \delta(p-w)\beta. \qquad (22)$$

Taking the difference (20) – (22) yields

$$\Delta_R = \pi_R(\rho^C \mid T,u) - \pi_R(\rho^D) = \frac{\beta\left[m+\delta(4w-3p-c)\right]}{2\rho^C} + \frac{3\delta\beta(p-w)}{2\rho^D} \overbrace{\geq}^{\rho^C \geq \rho^D}$$

$$\frac{\beta\left[m+\delta(4w-3p-c)\right]}{2\rho^C} + \frac{3\delta\beta(p-w)}{2\rho^C} = \frac{\beta\left[m+\delta(w-c)\right]}{2\rho^C} \geq 0. \tag{23}$$

The other case to consider is if $\rho^D = 1$; in this case $\pi_R(\rho^D) = -a/2$. Thus, from (20),

$$\Delta_R = \pi_R(\rho^C \mid T,u) - \pi_R(\rho^D) = \frac{\beta\left[m+\delta(4w-3p-c)\right]}{2\rho^C} + \delta\beta(p-w) + \frac{a}{2}. \tag{24}$$

If the first term on the right–hand side of (24) is positive, then $\Delta_R \geq 0$ because the remainder two terms of (24) are positive. If, on the other hand, the first term on the right–hand side of (24) is negative, then we can write

$$\Delta_R = \frac{\beta\left[m+\delta(4w-3p-c)\right]}{2\rho^C} + \delta\beta(p-w) + \frac{a}{2} \overbrace{\geq}^{\rho^C \geq 1} \frac{\beta\left[m+\delta(4w-3p-c)\right]}{2} + \delta\beta(p-w) + \frac{a}{2} =$$

$$= \frac{\beta\left[m+\delta(p-c)-4\delta(p-w)\right]}{2} + \frac{2\delta\beta(p-w)}{2} + \frac{a}{2} = \frac{\beta\left[m+\delta(p-c)\right]-2\delta\beta(p-w)+a}{2} =$$

$$= \frac{a}{2}\left\{\frac{\beta\left[m+\delta(p-c)\right]}{a} + 1 - 2\frac{\delta\beta(p-w)}{a}\right\} = \frac{a}{2}\left\{(\rho^C)^3 + 1 - 2\frac{\delta\beta(p-w)}{a}\right\} \overbrace{\geq}^{(*)} \frac{a}{2}\left\{(\rho^C)^3 + 1 - 2\right\} =$$

$\frac{a}{2}\left\{(\rho^C)^3 - 1\right\} \overbrace{\geq}^{\rho^C \geq 1} 0$, where in (*) we have used the fact that if $\rho^D = \max\left\{\left(\delta(p-w)\beta/a\right)^{1/3}, 1\right\}$, and $\rho^D = 1$, then necessarily $\delta(p-w)\beta/a \leq 1$. This completes the proof. □

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