

# UC Berkeley

## UC Berkeley Previously Published Works

### Title

Supply Chain Coordination Under Channel Rebates with Sales Effort Effects

### Permalink

<https://escholarship.org/uc/item/0762675h>

### Journal

Management Science, 48(8)

### ISSN

0025-1909

### Author

Taylor, Terry A

### Publication Date

2002-08-01

### DOI

10.1287/mnsc.48.8.992.168

Peer reviewed

# Supply Chain Coordination Under Channel Rebates with Sales Effort Effects

Terry A. Taylor

Graduate School of Business, Columbia University, New York, New York 10027

tat2002@columbia.edu

---

A channel rebate is a payment from a manufacturer to a retailer based on retailer sales to end consumers. Two common forms of channel rebates are linear rebates, in which the rebate is paid for each unit sold, and target rebates, in which the rebate is paid for each unit sold beyond a specified target level. When demand is not influenced by sales effort, a properly designed target rebate achieves channel coordination and a win-win outcome. Coordination cannot be achieved by a linear rebate in a way that is implementable. When demand is influenced by retailer sales effort, a properly designed target rebate and returns contract achieves coordination and a win-win outcome. Other contracts, such as linear rebate and returns or target rebate alone, cannot achieve coordination in a way that is implementable. Contrary to the view expressed in the literature that accepting returns weakens incentives for retailer sales effort, we find that the provision of returns strengthens incentives for effort. (*Channel Coordination; Supply Chain Management; Incentives; Rebates; Sales Effort*)

---

## 1. Introduction

A channel rebate is a payment from a manufacturer to a retailer (reseller) based on retailer sales to end consumers.<sup>1</sup> Channel rebates are important in the hardware, software, and auto industries. In the personal computer industry between mid-1996 and mid-1997, Compaq, Hewlett-Packard (HP), and IBM shifted the emphasis of their channel incentive formulas to rebates based on the volume of sales to end consumers. Channel rebates that, prior to mid-1997, were less than 3% jumped in some cases to more than 6% (Zarley 1997). "Nearly all" printer vendors offer channel rebates (Terdoslavich 1998), and rebates are "rampant" in the network hardware switching industry (Preston 1999). Channel rebates are also significant in the software industry. Microsoft has used rebates of 3% and 5.5%, and Novell has employed rebates of 3.25% and 5% (Kanellos 1996). Software makers

Lotus and Symantec have also used channel rebates (Moltzen 1997, Pender 1998).

Given the narrow margins of resellers in the computer industry—median net income as a percentage of sales is 4.5%—manufacturers and resellers identify rebates as a powerful channel policy (Roberts 1998). "Rebates are a key incentive for resellers," said Gordon Ball, IBM Canada's national channel sales executive (Bisby 1999). Merisel chairman and CEO, Dwight Steffensen, said vendor rebates significantly affect margins (Campbell 1999). In the auto industry, channel rebates are termed "dealer incentives." Edmunds reports that as of February 1999, 13 auto makers—including Chrysler, Ford, General Motors, Mazda, and Toyota—offered dealer incentives. At that time, 188 models had dealer incentives, and the median rebate was \$1,000 (Edmunds 1999).

Rebates may be initiated at various points in time over the product life cycle. Rebates initiated at the start of the product life cycle are important in practice. Beginning-of-life channel rebates are common in the software industry (Caborn 2001) and are also used

<sup>1</sup> A channel rebate is distinct from a consumer rebate where the manufacturer pays the *end consumer* via a rebate or coupon.

in the computer hardware and automotive industries. For example, Intel used channel rebates when it launched its Willamette Pentium 4 processor (Robertson 2001). Similarly, in late summer 1999, Nissan paid its dealers \$1,000 for every 2000 model year Frontier pickup sold (*Automotive News* 1999).

There are two common forms of channel rebates. Nissan's Frontier rebate is an example of a *linear rebate*: The manufacturer pays the retailer a fixed rebate for each unit sold. In the second form of rebate the size of the rebate is a function of the retailer's hitting specified sales volume levels. Intelligent Electronics president, Michael Norris, said much of reseller profit results from manufacturers' volume-based rebates (O'Heir and Pereira 1997). In a simple version, which we term a *target rebate*, the rebate is paid for each unit sold beyond the target level. An example of a target rebate is that HP paid its resellers a 10% rebate for sales of servers on volume that exceeded a target level (Zarley 1998). Typically, in order to receive a channel rebate, the retailer is required to provide evidence of sale to an end user, such as an invoice. Thus, retailer sales are observable to and verifiable (via an audit process) by the manufacturer.

A rebate is distinct from a reduction in the manufacturer's wholesale price in that the "reduction in price" caused by the rebate is only realized if the item is sold to an end user. The computer hardware and software industries, where rebates are frequently used, are characterized by short life cycles and high demand variability. Hence, there is often a mismatch between the quantity ordered by the retailer and the quantity sold to end users. If there is a possibility that the retailer may order more units than she sells, then a rebate is distinct from a reduction in wholesale price. In other contexts, rebates may be offered for a limited time. For example, Nissan's Frontier rebate was offered through September 30, 1999. For longer life-cycle products, time restrictions tend to create a distinction between wholesale price reductions and channel rebates.

Given the short life cycles of many products covered by channel rebates, we employ a one-period model to explore the role of rebates. This approach is consistent with the supply-chain contracting literature

where one-period models are widely used (see Lariviere 1999). This model may serve as an approximation for time-restricted rebates for longer life-cycle products. We show that when demand is not influenced by sales effort, a properly designed target rebate achieves channel coordination (i.e., maximizes the profitability of the entire supply chain of both manufacturer and retailer) and win-win (i.e., both parties are made better off). Coordination cannot be achieved by a linear rebate in a way that is implementable.

In many settings, retailer sales effort is important in influencing demand. Retailers can influence demand by merchandising, doing point-of-sale or other advertising, providing attractive shelf space, and guiding consumer purchases with sales personnel. We employ a simple model in which a retailer makes quantity and effort decisions and then observes demand. If retailer sales effort influences demand in a multiplicative fashion, then a properly designed target rebate and returns contract achieves coordination and win-win (shown analytically for uniform demand and consistent with a range of examples for normal demand). Other contracts, such as linear rebate and returns or target rebate alone, cannot achieve coordination in a way that is implementable. Contrary to the view expressed in the literature that accepting returns weakens incentives for retailer sales effort, we find that the provision of returns *strengthens* incentives for effort.

The paper is organized as follows. Section 2 provides a survey of related research. Section 3 presents a model for channel rebates when demand is not influenced by sales effort. Section 4 explores the setting in which sales effort influences demand. It describes prospects for channel rebates and returns to achieve coordination and examines the effect of endogenous and exogenous factors on the behavior and performance of the supply chain. Section 5 provides concluding remarks.

## 2. Literature Survey

One area of related research involves retailer sales-dependent payments. Consumer rebates and sales force compensation involve a firm making payments based on retailer sales to end consumers, but to parties other than a retailer. In a consumer rebate, the

payment is made by the manufacturer to the *end consumer* via a rebate or coupon. Consumer rebates have been explored in the economics and marketing literature (cf. Gerstner et al. 1994, Zhang et al. 2000). In the context of the internal workings of a firm, sales force compensation involves payments by a firm to its *employees* based on sales to consumers. An extensive body of research has examined a range of compensation schemes, including linear and quota-based schemes (cf. Basu et al. 1985, Rao 1990, Raju and Srinivasan 1996). Porteus and Whang (1991) and Chen (2000) extend this literature to consider incentive problems that arise within the firm between marketing and manufacturing functions. The literature on sales force compensation involving stochastic sales effort–demand relationships is built on agency theory, an extensive literature in economics regarding the provision of incentives for managerial effort. Foundational papers include Harris and Raviv (1978, 1979), Holmstrom (1979), Shavell (1979), and Grossman and Hart (1983). Coughlan (1993) provides a comprehensive review for sales force compensation, as does Baiman (1982) for agency theory.

A second area of research regards manufacturer–retailer contractual relationships. Revenue sharing contracts are a converse of linear rebates in that under revenue sharing, the *retailer* pays the *manufacturer* a portion of the retail price for each unit sold to an end consumer. Pasternack (1999) shows that a properly designed revenue sharing contract coordinates the channel in a single-period, stochastic demand (i.e., newsvendor) setting. Dana and Spier (2000) obtain a similar result when the downstream market consists of competing newsvendors under perfect competition. Cachon and Lariviere (2000) extend these results to the case of price-sensitive end-consumer demand and oligopolistic retail competition. Gerchak and Wang (2000) explore revenue sharing in assembly systems.

Accepting returns is a converse of offering a channel rebate in that under returns the manufacturer pays the retailer for each unit *not* sold. Pasternack (1985) shows that a properly chosen wholesale price and return rebate coordinate the channel in a newsvendor setting. Donohue (2000) and Taylor (2001) extend Pasternack by considering a second

buying opportunity. Kandel (1996), Padmanabhan and Png (1997), and Emmons and Gilbert (1998) incorporate price-sensitive end-consumer demand. Narayanan and Raman (1997) explore returns, vendor managed inventory, and retailer managed inventory when manufacturer sales effort influences demand. Tsay (1999) examines a quantity flexibility contract which is equivalent to a returns contract in which the quantity that can be returned is restricted (see Lariviere 1999). Webster and Weng (2000) also consider quantity-restricted returns.

Quantity discounts are related to volume-based channel rebates, but differ in that the discount is based on retailer *purchases* rather than retailer sales. One set of literature assumes that demand is deterministic and identifies schemes to increase manufacturer profits (cf. Lee and Rosenblatt 1986) or to achieve channel coordination (cf. Jueland and Shugan 1983). In contrast, Jucker and Rosenblatt (1985) explore the role of quantity discounts in a newsvendor setting. Comprehensive reviews of the supply chain contracting literature are provided by Lariviere (1999) and Tsay et al. (1999).

### 3. The Quantity-Only Model

#### 3.1. The Integrated Channel

First we introduce notation. Let  $p$  be the retail price,  $w$  the wholesale price,  $c$  the manufacturing cost,  $s$  the salvage value,  $T$  the target level, and  $u$  the channel rebate (i.e., the amount paid by the manufacturer to the retailer for each unit the retailer sells beyond the target). Let demand be given by the random variable  $\xi$  with density  $\phi(\cdot)$  and distribution  $\Phi(\cdot)$ . Assume that

ASSUMPTION A1.  $0 < c < w < p$ ,  $s < c$ ,  $u > 0$ ,  $T \geq 0$ .

ASSUMPTION A2.  $p, c, s$  are exogenous;  $w, u, T$  are endogenous.

ASSUMPTION A3. No lump sum side payment is allowed.

ASSUMPTION A4.  $\phi(\xi) > 0$  for all  $\xi \geq 0$ .

The retail price is assumed to be exogenous. Others in the supply chain contracting literature have used this assumption (cf. Pasternack 1985, Narayanan and

Raman 1997, Lariviere 1999). This assumption may be justified if the retail market is sufficiently competitive that retailers are essentially price takers. Alternately, the manufacturer may have strong control over the retail price through, for example, resale price maintenance (RPM) or minimum advertised price (MAP) restrictions. In the computer hardware and software industries, where rebates are commonly used, manufacturers also employ MAP restrictions (Patterson 1999, Peers and Ramstad 2000). The main results of the paper, Theorems 1 and 2, extend to the case in which the retail price is endogenous, provided that the contract can dictate the retail price (see Taylor 2002); Kandel (1996) provides an analogous result for returns.

Salvage value may be negative, indicating that the holding and/or disposal cost may exceed the nominal salvage value. Assumption A4 can be relaxed to allow any support  $[\alpha, \beta]$  where  $0 \leq \alpha < \beta \leq \infty$ . The firms are risk neutral, and the demand distribution and all parameters are commonly known. First the terms of trade (e.g., wholesale price, rebate) are specified, and then the retailer chooses her order quantity (and in §4 her effort level).

The integrated channel faces a newsvendor problem where her expected profit is

$$\pi(Q) = -cQ + pE \min(Q, \xi) + sE(Q - \xi)^+.$$

The optimal order quantity for the integrated channel is  $\bar{Q}_0 = \Phi^{-1}((p - c)/(p - s))$ , and the resulting profit to the integrated channel is  $\pi = (p - s)\Gamma(\bar{Q}_0)$  where  $\Gamma(Q) = \int_0^Q \xi d\Phi(\xi)$ .

The independent retailer's problem under a wholesale price-only contract is identical to that of the integrated channel, except that  $w$  replaces  $c$ . Therefore, the optimal order-up-to quantity is  $Q_0 = \Phi^{-1}((p - w)/(p - s))$ , and the resulting retailer profit is  $\underline{\pi} = (p - s)\Gamma(Q_0)$ . Note  $Q_0 < \bar{Q}_0$ . This is a form of quantity distortion driven by double marginalization (Spengler 1950). Because the retailer sees only a portion of the integrated channel margin, the retailer underorders relative to the integrated channel benchmark.

### 3.2. The Independent Retailer with Rebate

Under a target rebate, the retailer's profit function is

$$r(Q|T) = -wQ + pE \min(Q, \xi) + sE(Q - \xi)^+ + uE(\min(Q, \xi) - T)^+.$$

Let  $Q_1 = \Phi^{-1}((p + u - w)/(p + u - s))$ . Note  $Q_0 < Q_1$ . Define  $f_0(T) \equiv r(Q_0|T) - r(Q_1|T)$  on  $T \in [Q_0, Q_1]$ , and define  $\tau_0$  to satisfy  $f_0(\tau_0) = 0$ . Note

$$r(Q|T) = \begin{cases} (p-w)Q - (p-s) \int_0^Q (Q-\xi) d\Phi(\xi) & \text{if } Q \leq T \\ (p-w)Q - (p-s) \int_0^Q (Q-\xi) d\Phi(\xi) + u \left( \int_T^Q (\xi-T) d\Phi(\xi) + (Q-T)[1-\Phi(Q)] \right) & \text{if } Q > T. \end{cases}$$

Thus,

$$(\partial/\partial Q)r(Q|T) = \begin{cases} p-w - (p-s)\Phi(Q) & \text{if } Q \leq T \\ p+u-w - (p+u-s)\Phi(Q) & \text{if } Q > T. \end{cases}$$

It is easy to verify that  $r(\cdot|T)$  is concave on  $[0, T]$  and  $(T, \infty)$ . All functions described as concave, convex, increasing, or decreasing are strictly so. Although  $r(\cdot|T)$  is continuous,  $\lim_{Q \rightarrow T^-} (\partial/\partial Q)r(Q|T) < \lim_{Q \rightarrow T^+} (\partial/\partial Q)r(Q|T)$ .

LEMMA 1. (a)  $\tau_0$  exists, is unique, and satisfies  $\tau_0 \in (Q_0, Q_1)$ . (b) The optimal order quantity for the retailer under a target rebate,  $Q_0^*$ , is given by the following: If  $T < \tau_0$ , then  $Q_0^* = Q_1$ ; if  $T > \tau_0$ , then  $Q_0^* = Q_0$ ; if  $T = \tau_0$ , then  $Q_0^* = \{Q_0, Q_1\}$ .

The proof of Lemma 1, as well as the other main results, appears in the Appendix. The optimal order quantity is decreasing stepwise in  $T$ . To see the intuition behind the result, first consider extreme values of  $T$ . If  $T$  is extremely large, then the probability of selling units beyond the target level is very small. To have a shot at the rebate, the retailer must order a very large quantity (more than  $T$ ), and the resulting expected overage cost exceeds the expected revenue from the rebate. Consequently, the retailer behaves as if the rebate does not exist; i.e.; she orders the same quantity as in the no-rebate case. If  $T = 0$ , then the

rebate effectively increases the retail price the retailer faces because the retailer receives  $p + u$  rather than  $p$  for each unit she sells. Hence, the optimal order quantity is the newsvendor quantity where the retail price is  $p + u$ .

For intermediate values of  $T$ , the retailer has to choose whether she will “go for” the rebate (i.e., order more than  $T$ ) or not. In either case, the retailer’s optimal order quantity is a function of her cost and her marginal revenue from an incremental sale. If the retailer orders more than  $T$ , her marginal revenue is  $p + u$ ; if the retailer orders less than  $T$ , her marginal revenue is  $p$ . There is a candidate order quantity associated with each marginal revenue. As  $T$  increases, the attractiveness of going for the rebate diminishes. At the threshold  $\tau_0$  the retailer is indifferent between ordering the larger quantity and ordering the smaller quantity.

The retailer profit under a target rebate is

$$r = \begin{cases} (p + u - s)\Gamma(Q_1) - u(\Gamma(T) + T[1 - \Phi(T)]) & \text{if } T < \tau_0 \\ (p - s)\Gamma(Q_0) & \text{if } T \geq \tau_0. \end{cases}$$

Let  $m^*$  be the manufacturer profit and  $r^*$  the retailer profit under a channel-coordinated solution.

PROPOSITION 1. *If  $T = 0$ , then channel coordination requires  $m^* < 0$ .*

Proposition 1 indicates that while channel coordination is achievable under a linear rebate, this leaves the manufacturer with negative profit. To see why this may hold, suppose a rebate of  $w - c$  is offered. In this case the retailer’s underage cost is equated with that of the integrated channel, but the retailer’s overage cost is greater than the integrated channel’s. Hence, the retailer still underorders relative to the integrated channel benchmark. To align the retailer’s order quantity with that of the integrated channel, the rebate must exceed the manufacturer’s margin. Hence, the coordinating rebate has the manufacturer losing money on each unit the retailer sells. In the coordinating scheme, in expectation this loss exceeds the profit the manufacturer earns on units purchased but unsold by the retailer. It is easy to verify that the manufacturer profit is positive under a wholesale price-only contract. Hence, a channel-coordinating

linear rebate is not implementable, as it requires the manufacturer to be worse off. Define  $\hat{u}(w) \equiv (w - c)(p - s)/(c - s)$ .

THEOREM 1. *For any  $\kappa \in (0, \pi)$ , consider the target rebate contract  $(w^*, u^*, T^*)$ :*

- (1)  $w^*$  is set such that  $\underline{r} = \kappa - \varepsilon$  where  $\varepsilon \in (0, \kappa)$ ;
- (2)  $u^* = \hat{u}(w^*)$ ; and
- (3)  $T^*$  is set such that

$$(p + u^* - s)\Gamma(\bar{Q}_0) - u^*(\Gamma(T^*) + T^*[1 - \Phi(T^*)]) = \kappa. \quad (1)$$

Then for  $\varepsilon$  sufficiently small,

- (a)  $(w^*, u^*, T^*)$  exists and is unique;  $w^* \in (c, p)$ ,  $u^* > 0$ ,  $T^* > 0$ ;
- (b) the target rebate contract achieves channel coordination; and
- (c) the resulting profit to the manufacturer and retailer is  $m^* = \pi - \kappa$  and  $r^* = \kappa$ , respectively.

For any wholesale price  $w \in (c, p)$ , there exists a  $u$  and a  $T$  which achieve coordination. Thus, there exists a continuum of coordinating schemes. The target rebate addresses the quantity distortion by rewarding the retailer for achieving high sales. The coordinating rebate is increasing in the wholesale price: The retailer pays for her chance at a reward for high sales through her acquisition cost. Higher wholesale prices are associated with higher manufacturer profits.

Suppose the manufacturer has monopoly power over the retailer. The manufacturer offers a set of terms to the retailer as a take-it-or-leave-it offer, and the retailer accepts if and only if her expected profit under those terms is greater than or equal to her opportunity cost. By setting the terms of the target rebate properly (i.e., setting  $\kappa$  equal to the retailer’s opportunity cost and specifying  $w^*$ ,  $u^*$ , and  $T^*$  accordingly), the manufacturer ensures that the total chain’s profits are maximized and the retailer earns only her opportunity cost. Hence, the manufacturer maximizes her profit. It is interesting to note how this profit maximization for the manufacturer is achieved. Because  $u^* > w - c$ , the manufacturer incurs a loss on each unit the retailer sells beyond the target. However, the manufacturer earns profit on units sold to the retailer to which the rebate is not applied (i.e., the first  $T$  units sold and any overage quantity).

By setting the rebate at  $u^*$ , the manufacturer equates the critical fractile associated with the range where the marginal revenue is  $p + u$  with that of the integrated channel. By setting the wholesale price and target appropriately, the manufacturer ensures that it is optimal for the retailer to order in this range and that the retailer earns only her opportunity cost. (The retailer earns  $\kappa$  if she orders in this range and  $\kappa - \varepsilon$  if she orders in the range where the marginal revenue is  $p$ .)

Suppose the manufacturer does not have all the bargaining power, and instead the parties seek to identify channel rebate terms that will leave each party better off relative to her profit under a wholesale price-only contract. Under a no-rebate regime, the sum of the manufacturer and retailer profit is less than that of the integrated channel. Theorem 1 ensures that by properly specifying the target rebate terms, the total profit "pie" is enlarged and the pie can be split in any way. Consequently, the theorem guarantees that a target rebate can be constructed that results in an outcome which is both coordinated and a "win-win;" i.e., it leaves both parties strictly better off vis-a-vis any allocation of profit under a wholesale price-only contract.

Several alternatives exist for achieving coordination and arbitrary profit splitting in a newsvendor setting which involve retailer sales-contingent payments: paying the retailer to sell units beyond a target (target rebate), charging the retailer for selling units (revenue sharing), or paying the retailer not to sell units she had purchased (returns). Within this last category, the units eligible for the payment may be unrestricted, i.e., *full returns* (Pasternack 1985), limited to a percentage of the quantity purchased (Tsay 1999), or limited to units purchased beyond a target level (Webster and Weng 2000).

Revenue sharing and full returns are equivalent instruments (if the retail price is exogenous) in that for any returns contract a revenue sharing contract can be devised which leads to the same realization of retailer profit for any realization of demand (see Cachon and Lariviere 2000). However, rebates are distinct from revenue sharing and returns. Consider a revenue sharing contract  $(w_1, \lambda)$  and a linear rebate scheme  $(w_2, u)$ . Under revenue sharing, the retailer receives  $\lambda$

of the revenue where  $\lambda \in [0, 1]$ . The retailer's profit under revenue sharing is  $\lambda p - w_1$  when a unit is sold and  $-w_1$  when a unit is not sold. The retailer's profit under a rebate is  $p + u - w_2$  when a unit is sold and  $-w_2$  when a unit is not sold. Equating the profit under these two schemes implies  $w_2 = w_1$  and  $u = -p(1 - \lambda)$ , which contradicts  $u > 0$ . Essentially, revenue sharing is equivalent to employing a negative channel rebate (i.e., taxing the retailer every time she makes a sale). Similarly, a target rebate is distinct from quantity-restricted returns.

If realized *demand* (as distinct from sales) is observable and contractible, then coordination can be achieved by penalizing the retailer for missing sales (Lariviere 1999). A rebate and a per-unit penalty each impose an additional cost for missing a sale, and the coordinating rebate and penalty are identical for any given wholesale price. Implementing a penalty scheme is difficult because it depends on the ability of the manufacturer to verify unsatisfied consumer demand. The retailer has every reason to report that demand did not exceed her stock and to frustrate efforts to verify the existence of lost sales. In contrast, under a rebate the manufacturer need only audit sales rather than unsatisfied demand, and the retailer has an incentive to cooperate because she is rewarded for sales.

We have assumed that the wholesale price is linear and have excluded the use of side payments. In the context of multiple retailers, the use of nonlinear (i.e., quantity-dependent) pricing or fixed fees might be curtailed by legal restraints or the prospect of arbitrage amongst retailers. If side payments or nonlinear wholesale prices are allowed, then coordination with arbitrary profit splitting is achieved without making the contract dependent on retailer sales. It is well known that a two-part tariff (i.e., the cost of purchasing is given by a fixed fee plus a linear cost per unit) in which the linear cost is the manufacturer's marginal cost, achieves coordination. By setting the fixed fee appropriately, arbitrary profit splitting is ensured. Such a scheme is equivalent to "selling the firm" for a fixed fee. It is also equivalent to a quantity discount (because the average cost per unit is decreasing in the order quantity). Hence, when nonlinear pricing is allowed, coordination and win-win

can be achieved by contracting on retailer purchases rather than retailer sales.<sup>2</sup>

Achieving coordination with a specified split in expected profit requires that the designer of the contractual terms know the demand distribution of the retailer. However, simply achieving coordination does not require precise knowledge of the demand distribution. Setting the rebate equal to  $\hat{u}(w)$  and employing a sufficiently small target ensures coordination. The information requirements for returns (Pasternack 1985) and revenue sharing (Cachon and Lariviere 2000) are similar: Achieving coordination does not require knowledge of the demand distribution, but achieving coordination and win-win does.

How effective is a target rebate in the presence of multiple, heterogeneous retailers? Again, a properly designed target rebate contract achieves coordination: For any wholesale price and rebate,  $(w, \hat{u}(w))$ , set the target sufficiently low that it is optimal for each retailer to order the integrated channel order quantity. The presence of "small" retailers (i.e., retailers with demand densities with probability mass concentrated toward low levels of demand) may require that the target be small. A scheme that employs a single target for all retailers may tend to favor larger retailers. Hence, while total system profits are maximized with a single target rebate contract, it may lead to an undesirable allocation of profit. In practice, the target level is often a function of retailer sales in a previous period—e.g., in HP's server rebate, the target was 115% of the retailer's sales in the previous quarter (Zarley 1998)—so larger retailers face larger targets. Employing retailer-specific targets may allow for a wider range of profit allocations: For any given  $(w, \hat{u}(w))$ , employing retailer-specific targets (instead of a single coordinating cross-retailer target) allows the manufacturer to capture a larger share of the profit. Alternately, for at least some problem parameters it is possible to offer a menu of  $(w, u, T)$  contracts such that different retailers select different coordinating contracts. Such a scheme may support a still broader allocation of profits.

<sup>2</sup>If side payments are allowed, then a properly specified linear rebate plus a side payment achieves coordination and win-win. However, such a scheme is more complicated than the simple marginal cost pricing two-part tariff.

Given that several instruments achieve coordination, it remains to consider whether additional considerations outside the model—such as a retailer's role in exerting sales effort or setting the retail price—favor one instrument over another. Because a channel rebate rewards retailer sales, it provides an incentive for retailer sales effort (see Proposition 3). In contrast, because revenue sharing "taxes" retailer sales, it provides a disincentive for sales effort: The retailer's optimal effort decreases as the manufacturer's share of the revenue increases (Cachon and Lariviere 2000). Because returns reward the retailer for having unsold units, one might conjecture that returns also provide a disincentive for effort. However, the effect of returns on the retailer's optimal effort depends on how effort affects demand, and returns may either increase or decrease retailer effort (see §4.4.2). Nonetheless, it is plausible that in many contexts, rebates may provide a stronger positive incentive for sales effort than returns. Hence, if retailer sales effort is important in determining demand and generating large sales is deemed attractive, channel rebates may be an attractive instrument. When the retailer sets the quantity and the retail price, a properly designed revenue sharing contract achieves coordination and win-win (Cachon and Lariviere 2000). Returns cannot achieve coordination in this environment (Emmons and Gilbert 1998). The potential for channel rebates, perhaps in conjunction with other instruments (besides RPM), to achieve coordination when the retail price is endogenous is left to future research.

In the context of multiple retailers, both target rebates and quantity-restricted returns schemes may have a potential advantage over revenue sharing and full returns schemes. Consider a manufacturer selling to multiple retailers, each with a distinct demand distribution and a positive opportunity cost. Suppose for simplicity that the manufacturer has monopoly power. The manufacturer's profit is maximized if total system profit is maximized and each retailer receives only her opportunity cost. There is at least the possibility of designing a menu of optimal self-selecting target rebate contracts: Under such a scheme, each retailer self-selects a coordinating contract which yields an expected profit equal to her opportunity

cost. This is possible because the relative attractiveness of the various target rebate contracts depends on the retailer's demand distribution. Similar remarks apply for the potential effectiveness of a menu of quantity-restricted returns contracts (Lariviere 1999, Webster and Weng 2000). In contrast, for both revenue sharing and full returns, because the coordinating contract is independent of the demand distribution, the relative attractiveness of the distinct coordinating contracts does not depend on the retailer's demand distribution (e.g., when faced with a menu of coordinating revenue sharing contracts, all the retailers will select the contract that offers the greatest revenue share to the retailer). Hence, it is not possible to construct a menu of either revenue sharing or full returns contracts such that different retailers select different coordinating contracts (Lariviere 1999).<sup>3</sup>

Because the firms are assumed to be risk neutral, they are concerned with the mean rather than the variance of their profit. The variability in the firms' profits is a second-order consequence of the retailer's maximizing her *expected* profit. While returns offers the retailer insurance, a target rebate essentially offers the retailer a lottery with "extreme payoffs." Consequently, the variance of the retailer's profit under any coordinating target rebate contract is greater than under any coordinating full returns (or revenue sharing) contract (see Taylor 2002).

## 4. The Quantity and Effort Model

### 4.1. The Integrated Channel

Consider the setting in which demand is stochastic and a multiplicative function of retailer sales effort. Specifically, let demand be given by  $e\xi$ , where  $e$  is the level of effort and  $\xi$  is a random variable. The cost to the retailer of exerting  $e$  units of effort is  $V(e)$  where  $e \geq 0$ . Assume:

ASSUMPTION A5.  $V(\cdot)$  is convex, increasing, and  $V(0) = 0$ .

<sup>3</sup> Cachon and Lariviere (2000) demonstrate that in the context of competing retailers, a single revenue sharing contract can guarantee coordination but not arbitrary profit splitting.

Thus, the marginal effectiveness of effort is constant, and the marginal cost of effort is increasing. Alternatively, one can assume that demand is  $z(e)\xi$ , where  $z(\cdot)$  is a concave, increasing function; i.e., the marginal effectiveness of effort is decreasing. To accommodate this, simply employ the change of variable  $x = z(e)$  and adapt the cost of effort accordingly. This type of effort–demand model is used in Gerchak and Parlar (1987) and Rao (1990). This modeling approach is consistent with the implication of sales response models having multiplicative error terms used in empirical studies of the relationship between advertising and sales (cf. Bass 1969, Ryans and Weinberg 1979, Rao et al. 1988). Assumption A5 implies  $V(\cdot)$  is continuous.

We assume that quantity and effort decisions are both made prior to observing the state of market demand. This is appropriate when the sales cycle is short, early information about demand is difficult/costly to obtain early in the life cycle (e.g., poor information systems) or the information is a poor indicator of demand later in the life cycle, and/or a lead time exists with respect to sales effort. The integrated channel's profit function is

$$\Pi(Q, e) = -cQ + pE \min(Q, e\xi) + sE(Q - e\xi)^+ - V(e).$$

Although effort is costly, the integrated channel's *revenue* is clearly increasing in  $e$ . It is straightforward to show that  $\Pi(Q, e)$  is concave in  $Q$  and  $e$ . The optimal solution, if it exists, satisfies the first-order conditions: The optimal effort level is given by the  $\bar{e}$  satisfying  $(\partial/\partial e)V(e)|_{e=\bar{e}} = (p-s)\Gamma(\bar{Q}_0)$ , and the optimal order quantity is  $\bar{Q} = \bar{e}\bar{Q}_0$  where, recall,  $\bar{Q}_0 = \Phi^{-1}((p-c)/(p-s))$ . Assume the cost of effort function and demand distribution are chosen such that the existence of an optimal solution is assured. The resulting integrated channel profit is  $\Pi = \Lambda(\bar{e})$ , where  $\Lambda(\gamma) = \gamma(\partial/\partial e)V(e)|_{e=\gamma} - V(\gamma)$ .

We conclude this section by noting two of the many possible alternate specifications for how effort could affect demand. First, effort could influence demand in an additive fashion; i.e., demand is  $e + \xi$ . In contrast to the multiplicative model, where the effect of effort on demand is stochastic (in that it depends on  $\xi$ ), in the additive model the effect of effort on demand is deterministic. To the extent that the effect

of effort on demand is, in fact, not deterministic, the multiplicative model may be more appealing. This paper focuses in exposition on the multiplicative model and discusses related results for the additive model. Second, the retailer may be able to postpone her sales effort decision until she sees a signal of market demand. This may be plausible when the sales effort lead time is short relative to the length of the product's sales cycle. The resulting model is discussed in §4.4.2.

#### 4.2. The Independent Retailer with Returns

When the retailer is independent, her effort level generally is not directly observable or verifiable by the manufacturer, and hence is not legally contractible. While effort is not contractible, it is possible to contract on retailer sales. A "good outcome" (high sales) is a signal of "good performance" for the retailer (high sales effort), and a rebate creates incentive for that performance.

In this section we expand our study to include returns. In the software industry, channel rebates and returns are commonly used simultaneously (Caborn 2001). Similarly, in the computer hardware industry, where channel rebates are frequently used, returns policies are also common (Padmanabhan and Png 1995). The independent retailer's problem under a wholesale price-only contract is identical to that of the integrated channel, except that  $w$  replaces  $c$ . Therefore, the retailer's optimal effort level is given by the  $\check{e}$  satisfying  $(\partial/\partial e)V(e)|_{e=\check{e}} = (p-s)\Gamma(Q_0)$ , and the optimal order quantity is  $\check{Q}_2 = \check{e}Q_0$  where, recall,  $Q_0 = \Phi^{-1}((p-w)/(p-s))$ . The resulting retailer profit is  $\check{R} = \Lambda(\check{e})$ . Note that  $\check{e} < \bar{e}$  and  $\check{Q}_2 < \bar{Q}$ : The retailer underexerts effort and underorders relative to the integrated channel benchmark.

Let the return credit for each unsold unit be given by  $b$  where  $b \in [s, w)$ . The retailer's problem under returns alone is identical to that of the retailer under a wholesale price-only contract, except that  $b$  replaces  $s$ . Define  $\underline{Q}_0 \equiv \Phi^{-1}((p-w)/(p-b))$  and  $\underline{Q}_2 \equiv e\underline{Q}_0$ . The optimal effort level is given by the  $\underline{e}$  satisfying  $(\partial/\partial e)V(e)|_{e=\underline{e}} = (p-b)\Gamma(\underline{Q}_0)$ , and the optimal order quantity is  $\underline{Q}_2$ . The resulting independent retailer profit is  $\underline{R} = \Lambda(\underline{e})$ .

#### 4.3. The Independent Retailer with Returns and Rebate

Under a target rebate and returns, the retailer's profit function is

$$R(Q, e|T) = -wQ + pE \min(Q, e\xi) + uE(\min(Q, e\xi) - T)^+ + bE(Q - e\xi)^+ - V(e).$$

The analysis of the retailer's optimal effort and quantity decisions proceeds in two parts. First, we characterize the optimal quantity decision for any given effort level (Lemma 2). Second, we embed the optimal quantity decision in the retailer's objective function and characterize the optimal effort decision (Lemmas 3 and 4). We begin by considering the order quantity decision for any given level of effort. It is easy to verify that we can restrict our attention to strictly positive effort. Note that

$$(\partial/\partial Q)R(Q, e|T) = \begin{cases} p - w - (p - b)\Phi(Q/e) & \text{if } Q \leq T \\ p + u - w - (p + u - b)\Phi(Q/e) & \text{if } Q > T, \end{cases}$$

and

$$(\partial^2/\partial Q^2)R(Q, e|T) = \begin{cases} -(p - b)\phi(Q/e)/e & \text{if } Q \leq T \\ -(p + u - b)\phi(Q/e)/e & \text{if } Q > T. \end{cases}$$

Thus,  $R(\cdot, e|T)$  is concave on  $[0, T)$  and  $(T, \infty)$ . Although  $R(\cdot, e|T)$  is continuous,

$$\lim_{Q \rightarrow T^-} (\partial/\partial Q)R(Q, e|T) < \lim_{Q \rightarrow T^+} (\partial/\partial Q)R(Q, e|T).$$

Define  $\underline{Q}_1 \equiv \Phi^{-1}((p+u-w)/(p+u-b))$  and  $Q_3 \equiv e\underline{Q}_1$ . Note  $\underline{Q}_2 < Q_3$ . Define  $\tau$  to be the quantity analogous to  $\tau_0$  when  $b$  replaces  $s$ . ( $\tau_0$  is defined at the beginning of §3.2.)

**LEMMA 2.** For any given  $e$ , the optimal order quantity for the retailer under a target rebate and returns,  $Q^*$ , is given by the following: If  $e < T/\tau$ , then  $Q^* = \underline{Q}_2$ ; if  $e > T/\tau$ , then  $Q^* = Q_3$ ; if  $e = T/\tau$ , then  $Q^* = \{\underline{Q}_2, Q_3\}$ .

Thus, the retailer profit under a target rebate can be expressed as a function of a single decision variable,  $e$ .

Let  $A(e|T) \equiv R(Q^*(e), e|T)$ , where  $Q^*(e)$  is the optimal quantity given effort  $e$ . Then

$$A(e|T) = \begin{cases} e(p-b)\Gamma(\underline{Q}_0) - V(e) & \text{if } e \leq T/\tau \\ e(p+u-b)\Gamma(\underline{Q}_1) - u(e\Gamma(T/e) + T[1-\Phi(T/e)]) - V(e) & \text{if } e > T/\tau. \end{cases}$$

Thus,

$$(\partial/\partial e)A(e|T) = \begin{cases} (p-b)\Gamma(\underline{Q}_0) - (\partial/\partial e)V(e) & \text{if } e \leq T/\tau \\ (p+u-b)\Gamma(\underline{Q}_1) - u\Gamma(T/e) - (\partial/\partial e)V(e) & \text{if } e > T/\tau, \end{cases}$$

and

$$(\partial^2/\partial e^2)A(e|T) = \begin{cases} -(\partial^2/\partial e^2)V(e) & \text{if } e \leq T/\tau \\ e^{-3}uT^2\phi(T/e) - (\partial^2/\partial e^2)V(e) & \text{if } e > T/\tau. \end{cases}$$

Although  $A(\cdot|T)$  is continuous,

$$\lim_{e \rightarrow (T/\tau)^-} (\partial/\partial e)A(e|T) < \lim_{e \rightarrow (T/\tau)^+} (\partial/\partial e)A(e|T).$$

Thus,  $T/\tau$  cannot be the optimal effort level.  $A(\cdot|T)$  need not be well-behaved.

To obtain managerial insights we begin by assuming  $\xi$  has a uniform distribution to obtain analytic results. We then show that key results are consistent with a range of examples for the normal distribution. For the remainder of this paper, unless otherwise noted, assume  $V(e) = ae^2/2$  where  $a > 0$ ;  $a$  can be interpreted as the costliness of effort. We now proceed to show that under the uniform assumption, the retailer's optimal policy is similar to that under no effort: There exists a unique threshold such that if the target exceeds the threshold, then the retailer behaves as if the rebate does not exist; i.e., the optimal effort and order levels are the same as in the no-rebate case. If the target is below the threshold, then there exists a distinct optimal effort-order quantity pair; both levels are greater than the levels associated with no rebate. If the target equals the threshold, then the retailer is indifferent between the high and low effort-quantity pairs (see Lemma 5).

The analysis of the retailer's optimal effort level proceeds in two parts. First, we show in Lemma 3 that the objective function,  $A(\cdot|T)$ , is concave on  $[0, T/\tau]$

and either convex and then concave or simply concave on  $(T/\tau, \infty)$ . Second, we use this result to specify the optimal effort level in Lemma 4. Assume  $\xi \sim \text{Uniform}(\theta, \theta + \delta)$ . Although for expositional purposes we use  $\theta = 0$  and  $\delta = 1$ , identical or analogous results hold for all  $\theta \geq 0$  and  $\delta > 0$ .

LEMMA 3. Suppose  $\xi \sim \text{Uniform}(0, 1)$ . If  $T < u\tau^3/a$ , then  $A(\cdot|T)$  is concave on  $[0, T/\tau]$  and  $([uT^2/a]^{1/3}, \infty)$  and convex on  $(T/\tau, [uT^2/a]^{1/3})$ ; if  $T \geq u\tau^3/a$ , then  $A(\cdot|T)$  is concave on  $[0, T/\tau]$  and  $(T/\tau, \infty)$ .

A consequence of Lemma 3 and  $\lim_{e \rightarrow \infty} A(e|T) = -\infty$  is that  $A(\cdot|T)$  has one maximizer on  $[0, T/\tau]$  and at most one maximizer on  $(T/\tau, \infty)$ . Denote the maximizer on  $[0, T/\tau]$  by  $\tilde{e}$ . Let  $\hat{e}$  be the maximizer on  $(T/\tau, \infty)$ , if it exists; let  $\hat{e} = T/\tau$  if no such maximizer exists. Let  $\underline{A}(\gamma|T) = A(\gamma|T)$  for  $\gamma \in [0, T/\tau]$  and  $\bar{A}(\gamma|T) = A(\gamma|T)$  for  $\gamma \in [T/\tau, \infty)$ . Note  $\tilde{e} = \min(\underline{e}, T/\tau)$ . Define  $j(T) \equiv \underline{A}(\tilde{e}|T) - \bar{A}(\hat{e}|T)$  on  $[T\underline{e}, \infty)$  and define  $\Upsilon$  to satisfy  $j(\Upsilon) = 0$ .

LEMMA 4. Suppose  $\xi \sim \text{Uniform}(0, 1)$ . (a)  $\Upsilon$  exists and is unique. (b) If  $T < \Upsilon$ , then  $e^* = \hat{e}$  and further  $\hat{e} > T/\tau$ ; if  $T > \Upsilon$ , then  $e^* = \underline{e}$  and further  $\underline{e} < T/\tau$ ; if  $T = \Upsilon$ , then  $e^* = \{\hat{e}, \underline{e}\}$  and further  $\underline{e} < T/\tau < \hat{e}$ .

Lemma 5, which specifies the retailer's optimal effort and quantity, follows directly from Lemmas 2 and 4.

LEMMA 5. Suppose  $\xi \sim \text{Uniform}(0, 1)$ . If  $T < \Upsilon$ , then  $e^* = \hat{e}$  and  $Q^* = Q_3$ ; if  $T > \Upsilon$ , then  $e^* = \underline{e}$  and  $Q^* = Q_2$ ; if  $T = \Upsilon$ , then  $(e^*, Q^*) = \{(\underline{e}, Q_2), (\hat{e}, Q_3)\}$ .

Having specified the retailer's optimal behavior, we now construct a coordinating, win-win target rebate and returns contract. Define

$$u(T) \equiv (w-c) \frac{(p-c)^5}{(p-c)^5 - \zeta(T)},$$

and

$$b(T) \equiv s + (w-c) \frac{(p-c)^6 - (p-s)\zeta(T)}{(p-c)^6 - (p-c)\zeta(T)},$$

where  $\zeta(T) = 4a^2(p-s)^3T^2$ . Define  $T_3 \equiv (p-c)^3/[2a(p-s)^2]$ . Define  $m_1(T) \equiv \underline{e}\tau$  and  $m_2(T) \equiv \bar{e}\tau$  for  $T \in [0, T_3]$ , where  $u = u(T)$  and  $b = b(T)$ ; hence,  $\underline{e}$  and  $\tau$  are functions of  $T$ . Define  $T_i$  to be a fixed point of  $m_i$ ; i.e.,  $T_i = m_i(T_i)$ ;  $i = 1, 2$ .

LEMMA 6. Suppose  $\xi \sim \text{Uniform}(0, 1)$ .  $T_i$  exists and is unique;  $i = 1, 2$ .  $0 < T_1 < T_2 < T_3$ .

Under the target rebate and returns contract  $(w, u(T), b(T), T)$ , let  $\bar{L}(T)$  be the retailer's profit when she exerts effort  $\bar{e}$  where  $\bar{e} > T/\tau$ , and let  $\underline{L}(T, w)$  be the retailer's profit when she exerts effort  $\underline{e}$  where  $\underline{e} < T/\tau$ .

**THEOREM 2.** *Suppose  $\xi \sim \text{Uniform}(0, 1)$ . For any  $\kappa \in (0, \Pi)$ , consider the target rebate and returns contract  $(w^*, u^*, b^*, T^*)$ :  $u^* = u(T^*)$ ;  $b^* = b(T^*)$ ;  $w^*$  and  $T^*$  are set such that  $T^* \in (T_1, T_2)$ ,  $\underline{L}(T^*, w^*) = \kappa - \varepsilon$  where  $\varepsilon \in (0, \kappa)$ , and  $\bar{L}(T^*) = \kappa$ . Then for  $\varepsilon$  sufficiently small,*

- (a)  $(w^*, u^*, T^*)$  exists;  $w^* \in (c, p)$ ,  $u^* > 0$ ,  $b^* \in (s, w^*)$ ,  $T^* > 0$ ;
- (b) the target rebate contract achieves channel coordination; and
- (c) the resulting profit to the manufacturer and retailer is  $M^* = \Pi - \kappa$  and  $R^* = \kappa$ , respectively.

Theorem 2 guarantees that if the terms of the target rebate and returns contract are properly chosen, then both channel coordination and win-win are guaranteed. To achieve coordination, the terms of trade must be designed to induce the retailer to exert additional effort and increase her order size. For a fixed wholesale price, a target rebate increases the retailer profit when sales are high, and a return credit increases her profit when demand is low. It is intuitive that a target rebate pushes the retailer's effort and quantity decisions in the desired direction: Exerting additional effort makes the realization of higher demand more likely, and converting demand into sales requires that the retailer order a sufficiently large quantity. More surprisingly, returns has the same positive effect on both of the retailer's decisions (see Proposition 3). When the two instruments are used together properly, the retailer's decisions are aligned with those of the integrated channel. To overcome double marginalization it is intuitive that as the wholesale price is increased, the manufacturer must compensate the retailer with more generous terms. Hence, the coordinating rebate and return credit are increasing in the wholesale price. Theorem 2 holds for  $\xi \sim \text{Uniform}(\theta, \theta + \delta)$  where  $\theta \geq 0$  and  $\delta > 0$ . For the additive demand case, i.e., when demand is given by  $e + \xi$ , we obtain a result that is similar to Theorem 2, but weaker: If the costliness of effort is sufficiently small (i.e.,  $a < \max((p - c)/\delta, (p - c)(p - s)/[(p - c)\delta + 2(p - s)\theta])$ ), then a properly specified target rebate and returns contract guarantees both channel

coordination and win-win. Because the approach is similar, we omit the details.

The uniform distribution has the advantage of being easily understood by managers. However, it is important to consider other possible distributions. Perhaps the most natural distribution that may arise is the normal distribution. Recall that for any demand distribution, the retailer's problem can be expressed as a function of effort alone (by Lemma 2). Suppose  $\xi \sim \text{Normal}(\mu, \sigma^2)$ . Using the identity  $\int_a^\infty y\psi(y) dy = \psi(a)$ , where  $\psi(\cdot)$  is the density of a standard unit normal, it is straightforward to show that  $\int_{-\infty}^x \xi d\Phi(\xi) = \mu\Phi(x) - \sigma^2\phi(x)$ , and hence numerical analysis can be performed without numerical integration.

Numerical analysis indicates that under normally distributed demand, channel coordination with arbitrary profit splitting can be achieved. To construct channel-coordinating contracts the approach is similar to that specified in Theorem 2. For the normal case,  $b(T) = s + [(p - s)(w - c) - u(T)(c - s)]/(p - c)$ , and  $u(T)$  is given implicitly by the  $u$  satisfying  $(u - w + c)(p - s) \int_{-\infty}^{\bar{Q}} \xi d\Phi(\xi)/(p - c) - u \int_{-\infty}^{T/\bar{e}} \xi d\Phi(\xi) = 0$ . Call the following set of problems Set A:  $\mu = 10$ ,  $\sigma = \{2, 4, 6\}$ ,  $a = \{1, 3, 5\}$ ,  $p = 10$ ,  $c = \{2, 4, 8\}$ , and  $s = \{-1, 1\}$ . Consider a base case problem  $\mu = 10$ ,  $\sigma = 4$ ,  $a = 1$ ,  $p = 10$ ,  $c = 4$ , and  $s = 1$ . The resulting integrated channel effort level, order quantity, and profit are  $\bar{e} = 46.9104$ ,  $\bar{Q} = 549.926$ , and  $\Pi = 1100.29$ . Table 1 specifies channel-coordinating target rebate and returns contracts which result in a range of profit splits. Similar tables can be constructed for the remaining problems in Set A. This suggests that target rebate and returns contracts may be able to achieve channel coordination with arbitrary profit splitting under demand distributions other than the uniform.

**Table 1** Channel-Coordinating Contracts

$w$	$u$	$b$	$T$	$R^*$
4.04	0.045	1.0375	310.020	0.99Π
4.45	0.480	1.4350	263.220	0.90Π
5.49	1.560	2.4550	239.536	0.70Π
6.69	2.780	3.6450	218.187	0.50Π
8.20	4.290	5.1550	192.557	0.30Π
9.60	5.705	6.5475	185.397	0.10Π
9.90	6.031	6.8345	194.501	0.01Π

Cachon and Lariviere (2000) show that if retailer sales effort influences demand, then revenue sharing cannot achieve coordination and allow positive manufacturer profit. Because the retailer bears the full cost of exerting effort but sees only a portion of the total chain margin, the retailer underexerts effort relative to the integrated channel benchmark. In contrast, under the target rebate and returns contract of Theorem 2, the retailer's margin on units sold beyond the target exceeds that of the total chain. As in the quantity-only case, a properly specified two-part tariff achieves coordination and win-win (this is a commonly suggested solution in the principal-agent literature when the agent (retailer) is risk neutral).

For the remainder of this section we relax the assumption that  $\phi(\cdot)$  and  $V(\cdot)$  have specific forms. What are the prospects for other contracts to achieve coordination?

**PROPOSITION 2.** *Channel coordination in effort and quantity cannot be achieved by returns alone, linear rebates alone, or target rebates alone.*

It is easy to verify that coordination under a linear rebate and returns is achieved only if  $u = w - c$  and  $b = s + w - c$ . This amounts to the manufacturer pricing at marginal cost; hence, under such a scheme, the retailer takes the total chain profit, and the manufacturer profit is zero. Because a wholesale price-only contract yields positive profit to the manufacturer, a channel coordinating linear rebate and returns contract is not implementable.

#### 4.4. Sensitivity Analysis

**4.4.1. Independent Retailer with Returns and Rebate.** This section explores the effect of exogenous and endogenous parameters on the behavior and performance of the supply chain. Although a properly designed target rebate and returns contract maximizes the total chain profit, there may be nontrivial costs associated with establishing and administering such a contract. Before pursuing a coordinating contract, firms should assess the financial benefit that coordination offers. Analytical and numerical results reported in Taylor (2002) indicate that the relative loss in system profit due to decentralized decision making under a wholesale price-only contract is larger when the coefficient of variation of demand and wholesale price

are high. Lariviere and Porteus (2001) report a similar result regarding the effect of demand variability in the no-effort case. This suggests that the relative profit increase that is obtained through a properly designed target rebate and returns contract is particularly large in industries where demand uncertainty is high and the manufacturer's share of the total chain margin is large. This is consistent with industry practice in that it suggests that powerful manufacturers (e.g., Cisco, IBM, Microsoft) facing markets with high demand uncertainty may be able to increase their profits substantially by employing target rebate and returns contracts rather than wholesale price-only contracts.

Proposition 3 summarizes the effect of the various contract parameters on the retailer's optimal behavior under target rebate and returns. If  $T = \mathcal{T}$ , then the optimal effort level and order quantity are not single valued; hence, we exclude this case. ( $\mathcal{T}$  is defined immediately before Lemma 4.)

**PROPOSITION 3.** *Suppose  $\xi \sim \text{Uniform}(0, 1)$  and  $T \neq \mathcal{T}$ . Then (a)  $\partial e^*/\partial u \geq 0$ , and  $\partial Q^*/\partial u \geq 0$ , where the equality is strict if and only if  $T < \mathcal{T}$ ; (b)  $\partial e^*/\partial b > 0$ , and  $\partial Q^*/\partial b > 0$ .*

Generally speaking, as the manufacturer's terms of trade become more generous, the retailer exerts greater effort and purchases a larger quantity. Either increasing the rebate—if the target is sufficiently small—or increasing the return credit increases the retailer's effort and order quantity. Call the following Set B:  $\mu = 10$ ,  $\sigma = 4$ ,  $a = \{1, 3, 5\}$ ,  $p = 10$ ,  $w = 6$ ,  $u = \{1, 3, 5\}$ ,  $b = \{1, 3, 5\}$ , and  $T = \{10, 200, 400\}$ . The claims in Proposition 3b hold under normal demand for Set B, and the claims in Proposition 3a are consistent with the numerical results for Set B.

**4.4.2. Independent Retailer with Returns.** Under uniform demand (and numerical evidence suggests under normal demand as well), the retailer's effort level under target rebate and returns is increasing in the return credit. In fact, under general demand and general cost of effort, where the effort decision is made prior to observing demand, the retailer's effort level under returns alone is increasing in the return credit (see Proposition 4). This runs counter to the view expressed in the literature that returns lessen incentives for retailer effort (cf. Chu 1992, Padmanab-

han and Png 1995, Kandel 1996, Lariviere and Padmanabhan 1997). Padmanabhan and Png (1995) point out that retailers stimulate demand by merchandising, providing attractive shelf space, and doing point-of-sale advertising. Padmanabhan and Png (1995, p. 70) state: "By reducing the risk of losses due to excess inventory, a returns policy lessens some of the retailer's incentive to invest in such efforts." Similarly, Kandel (1996, p. 348) argues that returns undercut incentives for retailer effort: "[I]f all the risk is on the manufacturer's side, as in a consignment contract, the retailer has less incentive to provide service or in-store promotions." Although for other effort-demand models it may be that effort is decreasing in the return credit, in the context of our model we obtain the opposite result.

**PROPOSITION 4.** *Suppose  $\phi(\cdot)$  satisfies Assumption A4 and  $V(\cdot)$  satisfies Assumption A5. Then*

$$\partial e / \partial b > 0.$$

The "conventional wisdom" is not correct because it is based on a faulty logic that ignores the effect of returns on the order quantity: If one supposes, incorrectly, that the order quantity is unaffected by the return credit, then one obtains the result that, for any fixed quantity, returns decrease effort. However, when the effect of returns on the order quantity is properly accounted for, the result that returns increase effort is obtained. The intuition is that returns make it attractive for the retailer to increase her order quantity, which makes incremental effort more likely to convert demand into sales. If effort influences demand in an additive fashion (i.e., demand is given by  $e + \xi$ ), then returns have no effect on effort. Although it need not be that returns increase effort for all effort-demand relationships, these results demonstrate that for two important effort-demand relationships, returns do not decrease effort.

We conclude this section by considering the effect of returns on effort when quantity and effort decisions are made in a sequential fashion. The demand-effort model presented in §4.1 assumes that quantity and effort decisions are made contemporaneously: The retailer commits to her sales effort level before observing the state of market demand. However, in some settings the life cycle may be sufficiently long

that the retailer is able to postpone her sales effort decision until she sees a signal of market demand. If production lead times are sufficiently long and sales effort lead times are sufficiently short, then a sequential order-effort model is appropriate. Consider the following sequence of events. First, the retailer orders  $Q$ . Second, she observes the demand signal  $\xi$ . Third, she exerts effort  $e$ , incurs cost  $V(e) = ae^2/2$ , and experiences demand  $e\xi$ . Because the optimal effort level is a function of the demand signal, the effect of returns on effort depends on the realization of the demand signal. Taylor (2002) shows that the provision of returns impacts effort in the following way: If the demand signal is large, then the effort level is increasing in the return credit. If the demand signal is small, then the effort level is decreasing in the return credit. Loosely speaking, making generous terms (low wholesale price, high return credit) even more generous through a larger return credit will tend to be associated with decreased retailer effort; making ungenerous terms more generous via a larger return credit will tend to be associated with increased effort. This observation may provide helpful guidance to a manufacturer that desires to stimulate retailer sales effort via the terms of trade.

## 5. Discussion

This paper demonstrates the superiority of a target rebate over a linear rebate in achieving coordination in a way that is attractive to the firms involved. Under a linear rebate, the manufacturer induces the retailer to exert additional effort and order a larger quantity by increasing the retailer's marginal revenue. However, the manufacturer fully bears the financial burden of increasing the retailer's marginal revenue. A target rebate offers an advantage to the manufacturer. By setting the target properly, the manufacturer can induce the retailer to behave in a way that reflects the marginal revenue of the rebate while shielding the manufacturer from the full cost of doing so.

In our setup we assumed that the manufacturer offers the rebate at the start of the life cycle. In practice, rebates are at times initiated during the life cycle. For example, in March 1996 Apple faced a \$2 billion glut of inventory in its distribution channel and the prospect of accepting substantial and costly returns

from distributors. Apple extended channel rebates to its distributors with the objective of clearing the channel and reducing its financial exposure to returns (Kannellos and Zarley 1996). In the auto industry the use of channel rebates is particularly intense at the end of the model year, when rebates are used to clear inventory to make way for new models. Exploring the role of midlife and/or end-of-life channel rebates may be a fruitful direction for future research.

An essential element to capture in a model that examines retailer sales-dependent payments is that the retailer may be able to influence the demand she experiences. This paper captures a fundamental way that the retailer can influence her demand—by exerting sales effort. However, we have taken the retail price to be exogenous. Exploring the manufacturer’s use of rebates as an instrument that influences the retailer’s pricing decision (e.g., to stimulate demand by driving down retail prices) may be a promising area for research. Finally, our analysis suggests that quota or threshold schemes which are commonly used within organizations in, for example, salesforce compensation, can be used productively in cross-organization transactions. This suggests that it may be fruitful for researchers to explore other target-type schemes in interfirm relationships that extend beyond channel rebates and quantity discounts.

**Appendix**

**PROOF OF LEMMA 1.** It is straightforward to show that if  $T \leq Q_0$ , then  $Q_1$  maximizes  $r(\cdot|T)$ , and if  $T \geq Q_1$ , then  $Q_0$  maximizes  $r(\cdot|T)$ . If  $Q_0 \leq T \leq Q_1$ , then  $r(Q_0|T) = (p - s)\Gamma(Q_0)$  and  $r(Q_1|T) = (p + u - s)\Gamma(Q_1) - u(\Gamma(T) + T[1 - \Phi(T)])$ . Because  $f_0(Q_0) < 0 < f_0(Q_1)$  and  $f_0(\cdot)$  is continuous and increasing, there exists a single-valued inverse function  $f_0^{-1}$  and a unique  $\tau_0$ ; further,  $\tau_0 \in (Q_0, Q_1)$ . If  $Q_0 < T < Q_1$ , then  $\lim_{Q \rightarrow T^-} (\partial/\partial Q)r(Q|T) < 0 < \lim_{Q \rightarrow T^+} (\partial/\partial Q)r(Q|T)$ . Because  $Q_0$  maximizes  $r(\cdot|T)$  on  $[0, T]$  and  $Q_1$  maximizes  $r(\cdot|T)$  on  $(T, \infty)$ ,  $Q_0^* = \arg \max_{Q \in [Q_0, Q_1]} r(Q|T)$ . If  $T < \tau_0$ , then  $f_0(T) < 0$  and  $r(Q_1|T) > r(Q_0|T)$ . If  $T > \tau_0$ , then  $f_0(T) > 0$ .  $\square$

**PROOF OF PROPOSITION 1.** If  $T = 0$ , then  $Q_0^* = Q_1$ . Further,  $Q_1 = \bar{Q}_0$  if and only if  $u = (w - c)(p - s)/(c - s)$ . Under this rebate,  $m^* = -(w - c)(p - s)\Gamma(\bar{Q}_0)/(c - s) < 0$ .  $\square$

**PROOF OF THEOREM 1.** Because  $\underline{r} = \pi$  if  $w = c$ ,  $\underline{r} = 0$  if  $w = p$ , and  $\underline{r}$  is continuous and decreasing in  $w$ , there exists a single-valued inverse function  $\underline{r}^{-1}$  and a unique  $w^*$  which satisfies  $\underline{r} = \kappa - \varepsilon$ ; further,  $w^* \in (c, p)$ . Under  $u^*$ ,  $Q_1 = \bar{Q}_0$ . Let  $\alpha = \Gamma(\bar{Q}_0) - \Gamma(Q_0) - \varepsilon/(p - s)$  and  $g(T) = [\Gamma(T) + T[1 - \Phi(T)] - \Gamma(\bar{Q}_0)][\Phi(\bar{Q}_0) - \Phi(Q_0)]^{-1}$ .  $T^*$  satisfies (1) if and only if  $g(T^*) = \alpha$ . It is

easy to verify  $g(Q_0) < \Gamma(\bar{Q}_0) - \Gamma(Q_0)$ . Select  $\varepsilon < (p - s)[\Gamma(\bar{Q}_0) - \Gamma(Q_0) - g(Q_0)]$ . Hence,  $g(Q_0) < \alpha$ . Further,  $g(\bar{Q}_0) = \bar{Q}_0[\Phi(\bar{Q}_0) - \Phi(Q_0)] > \Gamma(\bar{Q}_0) - \Gamma(Q_0) > \alpha$ . Because  $g(Q_0) < \alpha < g(\bar{Q}_0)$  and  $g(\cdot)$  is continuous and increasing, there exists a single-valued inverse function  $g^{-1}$  and a unique  $T^*$  such that  $g(T^*) = \alpha$ ; further,  $T^* \in (Q_0, \bar{Q}_0)$ . Because  $f_0(T^*) < 0$  and  $f_0(\cdot)$  is increasing,  $T^* < \tau_0$ , and thus  $Q_0^* = \bar{Q}_0$  (by Proposition 1). Because  $\kappa - \varepsilon > 0$ ,  $Q_0 > 0$  and hence  $T^* > 0$ .  $\square$

**PROOF OF LEMMA 2.** Define  $f_1(T) \equiv R(Q_2, e|T) - R(Q_3, e|T)$  on  $T \in [Q_2, Q_3]$  and define  $\tau_1$  to satisfy  $f_1(\tau_1) = 0$ . It is straightforward to show that if  $T \leq Q_2$ , then  $Q_3$  maximizes  $R(\cdot, e|T)$ , and if  $T \geq Q_3$ , then  $Q_2$  maximizes  $R(\cdot, e|T)$ . If  $Q_2 \leq T \leq Q_3$ , then  $R(Q_2, e|T) = e(p - b)\Gamma(\underline{Q}_0) - V(e)$  and  $R(Q_3, e|T) = e(p + u - b)\Gamma(\underline{Q}_1) - u e\Gamma(T/e) + T[1 - \Phi(T/e)] - V(e)$ . Because  $f_1(Q_2) < 0 < f_1(Q_3)$  and  $f_1(\cdot)$  is continuous and increasing, there exists a single-valued inverse function  $f_1^{-1}$  and a unique  $\tau_1$ ; further,  $\tau_1 \in (Q_2, Q_3)$ , and  $\tau_1 = e\tau$ . If  $Q_2 < T < Q_3$ , then  $\lim_{Q \rightarrow T^-} (\partial/\partial Q)R(Q, e|T) < 0 < \lim_{Q \rightarrow T^+} (\partial/\partial Q)R(Q, e|T)$ . Because  $Q_2$  maximizes  $R(\cdot, e|T)$  on  $[0, T]$  and  $Q_3$  maximizes  $R(\cdot, e|T)$  on  $(T, \infty)$ ,  $Q^* = \arg \max_{Q \in [Q_2, Q_3]} R(Q, e|T)$ . If  $T < e\tau$ , then  $f_1(T) < 0$  and  $R(Q_3, e|T) > R(Q_2, e|T)$ . If  $T > e\tau$ , then  $f_1(T) > 0$ .  $\square$

**PROOF OF LEMMA 3.** For  $e \in [0, T/\tau]$ ,  $(\partial^2/\partial e^2)A(e|T) = -a < 0$ . For  $e \in (T/\tau, \infty)$ ,  $(\partial^2/\partial e^2)A(e|T) = uT^2e^{-3} - a$ . If  $T \geq u\tau^3/a$ , then  $uT^2e^{-3} - a < u\tau^3/T - a \leq 0$ . Suppose  $T < u\tau^3/a$ . Note  $uT^2e^{-3} - a > 0$  if and only if  $e < (uT^2/a)^{1/3}$ .  $\square$

**PROOF OF LEMMA 4.** It is easy to verify  $\bar{A}(\hat{e}|\cdot)$  is decreasing, and  $\underline{A}(\hat{e}|\cdot)$  is weakly increasing. Therefore  $j(\cdot)$  is increasing. If  $T \leq \tau e$ , then  $0 \leq \lim_{e \rightarrow (T/\tau)^-} (\partial/\partial e)A(e|T) < \lim_{e \rightarrow (T/\tau)^+} (\partial/\partial e)A(e|T)$ . Because  $A(\cdot|T)$  is concave on  $[0, T/\tau)$  and  $\lim_{e \rightarrow (T/\tau)^-} (\partial/\partial e)A(e|T) \geq 0$ ,  $(\partial/\partial e)A(e|T)|_{e \in [0, T/\tau]} > 0$ . Because  $\lim_{e \rightarrow (T/\tau)^+} (\partial/\partial e)A(e|T) > 0$ ,  $A(\cdot|T)$  has one stationary point on  $(T/\tau, \infty)$ , and the second-order condition is satisfied at that point. Therefore,  $\bar{A}(\hat{e}|T) > \underline{A}(\hat{e}|T)$  and  $j(\tau e) < 0$ . Clearly,  $\lim_{T \rightarrow \infty} \underline{A}(\hat{e}|T) = \lim_{T \rightarrow \infty} \underline{A}(e|T) = \underline{R} < \infty$ . It is easy to verify that  $j(\cdot)$  is continuous. It is easy to verify  $\lim_{T \rightarrow \infty} \bar{A}(\hat{e}|T) = -\infty$ ; thus,  $\lim_{T \rightarrow \infty} j(T) = +\infty$ . Because  $j(\cdot)$  is continuous and increasing,  $j(\tau e) < 0$ , and  $\lim_{T \rightarrow \infty} j(T) = +\infty$ , there exists a single-valued inverse function  $j^{-1}$  and a unique  $\Upsilon$ ; further  $\Upsilon \in (\tau e, \infty)$ . Recall  $e = T/\tau$  cannot be the optimal effort level. Thus, if  $T < \Upsilon$ , then  $\bar{A}(\hat{e}|T) > \underline{A}(\hat{e}|T)$  and  $e^* = \hat{e} > T/\tau$ ; if  $T > \Upsilon$ , then  $\bar{A}(\hat{e}|T) < \underline{A}(\hat{e}|T)$  and  $e^* = \underline{e} < T/\tau$ ; if  $T = \Upsilon$ , then  $\bar{A}(\hat{e}|T) = \underline{A}(\hat{e}|T)$  and  $e^* = \{\hat{e}, \underline{e}\}$ .  $\square$

**PROOF OF LEMMA 6.** We write  $\tau(T)$  and  $\underline{e}(T)$  to indicate the quantities’ dependence on  $T$ . It is straightforward to verify  $(\partial/\partial T)\tau(T) \leq 0$  and  $(\partial/\partial T)\underline{e}(T) \leq 0$ . Thus,  $(\partial/\partial T)m_i(T) \leq 0$ ;  $i = 1, 2$ . Because  $\underline{e}(0) < \bar{e}$ ,  $\underline{e}(T) < \bar{e}$  for  $T \in [0, T_3]$  and consequently  $m_2$  strictly dominates  $m_1$ , i.e.,  $m_2(T) > m_1(T)$  for  $T \in [0, T_3]$ . Because  $m_2(0) < T_3$  and  $m_1(T_3) > 0$ ,  $m_i$  is a mapping from  $[0, T_3]$  into  $[0, T_3]$ ;  $i = 1, 2$ . Because  $m_i(\cdot)$  is continuous, by Brouwer’s Fixed Point Theorem,  $T_i$  exists. Let  $n_i(T) = m_i(T) - T$ ;  $i = 1, 2$ . Hence,  $n_2$  strictly dominates  $n_1$ . Because  $n_i(\cdot)$  is continuous,  $(\partial/\partial T)n_i(T) < 0$ , and  $n_i(T_3) < 0 < n_i(0)$ , there exists a single-valued inverse function  $n_i^{-1}$ ,  $T_i$  is unique, and  $0 < T_i < T_3$ ;  $i = 1, 2$ . Because  $n_1(T_1) = n_2(T_2) > n_1(T_2)$  and  $(\partial/\partial T)n_1(T) < 0$ ,  $T_1 < T_2$ .  $\square$

**PROOF OF THEOREM 2.** Let  $\underline{L}(T)$  be  $\underline{L}(T, w)$  where the second argument is suppressed. Define  $y(T) \equiv \underline{L}(T) - \bar{L}(T) + \varepsilon$  on

$T \in [T_1, T_2]$  and define  $T^*$  to satisfy  $y(T^*) = 0$ . Recall  $0 < T_1 < T_2 < T_3$  (by Lemma 6). Because  $T < T_3$ ,  $u(T) > 0$  and  $b(T) > s$ . It is easy to verify  $(\partial/\partial T)b(T) \leq 0$  and  $b(0) < w$ . Therefore,  $b(T) < w$ . It is easy to verify  $(\partial/\partial w)\underline{L}(T, w) < 0$ ,  $\underline{L}(T, c) = \Pi$ , and  $\underline{L}(T, p) = 0$ . Because  $\underline{L}(T, \cdot)$  is decreasing and continuous, for any  $T$  there exists a single-valued inverse function with respect to  $w$ ,  $\underline{L}^{-1}$ , and a unique  $w^*(T)$  such that  $\underline{L}(T, w^*(T)) = \kappa - \varepsilon$ ; further,  $w^*(T) \in (c, p)$ . For  $T < T_2$ , we have  $\bar{e} > T/\tau(T)$ , and hence  $(\partial/\partial e)\bar{A}(e|T)|_{e=\bar{e}} = 0$  because  $u^* = u(T)$  and  $b^* = b(T)$ . For  $T \geq T_1$ , we have  $\bar{A}(\bar{e}|T) = \underline{L}(T)$ . Note  $\lim_{e \rightarrow \bar{e}^+} (\partial/\partial e)A(e|T_1) > \lim_{e \rightarrow \bar{e}^-} (\partial/\partial e)A(e|T_1) = 0$ . Since  $\lim_{e \rightarrow \bar{e}^+} (\partial/\partial e)A(e|T_1) > 0$ ,  $\bar{A}(\cdot|T)$  is convex and then concave or simply concave, and  $\bar{e} > T_1/\tau(T_1)$ , therefore  $\bar{A}(\bar{e}|T_1) = \bar{L}(T_1)$  and further  $\bar{L}(T_1) > \underline{L}(T_1)$ . Select  $\varepsilon < \bar{L}(T_1) - \underline{L}(T_1)$ . Hence,  $y(T_1) < 0$ . Note  $\lim_{e \rightarrow \bar{e}^-} (\partial/\partial e)A(e|T_2) < \lim_{e \rightarrow \bar{e}^+} (\partial/\partial e)A(e|T_2) = 0$ . Thus,  $\bar{A}(\bar{e}|T_2) = \underline{L}(T_2) > \bar{L}(T_2)$ . Hence,  $y(T_2) > 0$ . Because  $y(T_1) < 0 < y(T_2)$  and  $y(\cdot)$  is continuous, by the intermediate value theorem there exists  $T^* \in (T_1, T_2)$ . Thus,  $\underline{e}(T^*) < T^*/\tau(T^*) < \bar{e}$ . Because  $T^* < T_2$ ,  $\bar{e}$  is a stationary point of  $\bar{A}(\cdot|T^*)$  on  $(T^*/\tau(T^*), \infty)$ . Because  $\bar{A}(\cdot|T^*)$  may be convex and then concave, there may a second stationary point on  $(T^*/\tau(T^*), \infty)$ ; if it exists, call it  $e_0$ . Because this holds only if  $(\partial/\partial e)A(e|T^*)|_{e=(T^*/\tau(T^*))^+} < 0$ ,  $\bar{A}(e_0|T^*) < \bar{A}(\bar{e}|T^*) = \bar{A}(\underline{e}|T^*)$ . Because  $\bar{A}(\bar{e}|T^*) = \bar{L}(T^*) > \underline{L}(T^*) = \bar{A}(\underline{e}|T^*) > \bar{A}(e_0|T^*)$ ,  $\bar{e} = \arg \max A(e|T^*)$ . (Clearly, the conclusion holds if  $\bar{e}$  is the solitary stationary point.) Because  $\bar{e} > T^*/\tau(T^*)$ ,  $Q^* = Q_3$  (by Lemma 2). Because  $u^* = u(T^*)$  and  $b^* = b(T^*)$ ,  $Q_1 = \bar{Q}_0$ , and thus  $Q^* = \bar{Q}$ .  $\square$

**PROOF OF PROPOSITION 2.** Channel coordination requires  $Q^*/e^* = \bar{Q}/\bar{e}$  and  $e^* = \bar{e}$ . Under returns alone,  $e^* = \underline{e}$  and  $Q^* = Q_2$ . Note  $Q^*/e^* = \bar{Q}/\bar{e}$  only if  $b = s + (w - c)(p - s)/(p - c)$ . However, this  $b$  implies  $e^* < \bar{e}$ . To see that coordination cannot be achieved under a target (or linear) rebate alone, consider the magnitude of the retailer's optimal effort level relative to  $T/\tau_0$ . As argued previously,  $T/\tau_0$  cannot be the optimal effort level. Two possibilities remain. If  $e^* < T/\tau_0$ , then  $Q^* = e^*Q_0$  (by Lemma 1). Because  $Q^*/e^* < \bar{Q}/\bar{e}$ , coordination cannot be achieved. If  $e^* > T/\tau_0$ , then  $Q^* = e^*Q_1$ . Again,  $Q^*/e^* = \bar{Q}/\bar{e}$  only if  $u = (w - c)(p - s)/(c - s)$ . The optimal effort level must satisfy the first-order condition:  $(\partial/\partial e)V(e)|_{e=e^*} = (p + u - s)\Gamma(Q_1) - u\Gamma(T/e^*)$ . Under the specified  $u$ ,  $e^* = \bar{e}$  only if  $e^* = T/Q_1$ , which implies  $e^* < T/\tau_0$ , a contradiction.  $\square$

**PROOF OF PROPOSITION 3.** If  $T > \Upsilon$ , then  $e^* = \underline{e}$  and  $Q^* = Q_2$  (by Lemma 5). Note  $\partial \underline{e}/\partial u = \partial Q_2/\partial u = 0$ ,  $\partial \underline{e}/\partial b = (p - w)^2/[2a(p - b)^2] > 0$ , and  $\partial Q_2/\partial b = (p - w)^3/[a(p - b)^3] > 0$ . If  $T < \Upsilon$ , then  $e^* = \hat{e}$  (by Lemma 5). By the implicit function theorem  $\partial \hat{e}/\partial u = -[(\partial^2/\partial e \partial u)A]/[(\partial^2/\partial e^2)A]|_{e=\hat{e}}$ . It is easy to verify  $(\partial^2/\partial e \partial u)A|_{e=\hat{e}} > 0$ . Thus,  $\partial \hat{e}/\partial u > 0$ , and by similar argument  $\partial \hat{e}/\partial b > 0$ . If  $T < \Upsilon$ , then  $Q^* = Q_3$  (by Lemma 5) where  $Q_3 = e^*Q_1$ . Because  $\partial Q_1/\partial u = (w - b)/(p + u - b)^2 > 0$  and  $\partial Q_1/\partial b = (p + u - b)^{-1} > 0$ ,  $\partial Q_3/\partial u > 0$  and  $\partial Q_3/\partial b > 0$ . Because  $\Upsilon$  is continuous in  $u$  and  $b$ , the result is immediate.  $\square$

**PROOF OF PROPOSITION 4.** Let  $\underline{R}(e)$  be the retailer profit under returns and effort  $e$ . Let  $\underline{R}_i, \underline{R}_i(\cdot)$ , and  $\underline{e}_i$  be the quantities under return credit  $b_i$ ;  $i = 1, 2$ . Let  $b_1 > b_2$ . Note  $\Lambda(\underline{e}_1) = \underline{R}_1 = \underline{R}_1(\underline{e}_1) > \underline{R}_1(\underline{e}_2) > \underline{R}_2(\underline{e}_2) = \underline{R}_2 = \Lambda(\underline{e}_2)$ . Because  $\Lambda(\underline{e}_1) > \Lambda(\underline{e}_2)$  and  $(\partial/\partial e)\Lambda(e) > 0$ ,  $\underline{e}_1 > \underline{e}_2$ .  $\square$

## Acknowledgments

The author thanks the associate editor and reviewers as well as seminar participants at Carnegie Mellon University, Columbia University, Cornell University, Duke University, Georgetown University, Georgia Tech, Harvard University, New York University, Northwestern University, Tulane University, University of Chicago, University of Colorado, University of Pennsylvania, Washington University, and Yale University for helpful comments. The author is particularly grateful to Hau Lee for thoughtful, valuable discussions and suggestions. An earlier version of this paper was titled "Coordination Under Channel Rebates with Sales Effort Effects."

## References

- Automotive News*. 1999. Dealer incentives. (August 23) 53.
- Baiman, S. 1982. Agency research in managerial accounting: A survey. *J. Accounting Literature* 1 155–213.
- Bass, F. M. 1969. A simultaneous equation regression study of advertising and sales of cigarettes. *J. Marketing Res.* 6 291–300.
- Basu, A. K., R. Lal, V. Srinivasan, R. Staelin. 1985. Salesforce-compensation plans: An agency theoretic perspective. *Marketing Sci.* 4(4) 267–291.
- Bisby, A. 1999. VARs stung by rebates. *Computer Dealer News* 15(48) (December 17) 1, 6.
- Caborn, K. 2001. Private conversation with Intuit executive.
- Cachon, G., M. A. Lariviere. 2000. Supply chain coordination with revenue-sharing contracts: Strengths and limitations. Working paper, University of Pennsylvania, Philadelphia, PA.
- Campbell, S. 1999. The numbers. *Computer Reseller News* (July 26) 6.
- Chen, F. 2000. Sales-force incentives and inventory management. *Manufacturing Service Oper. Management* 2(2) 186–202.
- Chu, W. 1992. Demand signalling and screening in channels of distribution. *Marketing Sci.* 11(4) 327–347.
- Coughlan, A. T. 1993. Salesforce compensation: A review of MS/OR advances. J. Eliashberg, G. L. Lilien, eds. *Handbooks in OR & MS*. Vol. 5. Elsevier Science Publishers, Amsterdam, The Netherlands, 611–651.
- Dana, J. D., K. E. Spier. 2000. Revenue sharing, demand uncertainty, and vertical control of competing firms. Working paper, Northwestern University, Evanston, IL.
- Donohue, K. L. 2000. Efficient supply contracts for fashion goods with forecast updating and two production modes. *Management Sci.* 46(11) 1397–1411.
- Edmunds. 1999. (<http://www.edmunds.com/edweb/incentives.html>).
- Emmons, H., S. M. Gilbert. 1998. Note: The role of returns policies in pricing and inventory decisions for catalogue goods. *Management Sci.* 44(2) 276–283.
- Gerchak, Y., M. Parlar. 1987. A single period inventory problem with partially controllable demand. *Comput. Oper. Res.* 14(1) 1–9.
- , Y. Wang. 2000. Revenue-sharing vs. wholesale-price contracts in assembly systems with random demand. Working paper, Case Western Reserve University, Cleveland, OH.

- Gerstner, E., J. D. Hess, D. M. Holthausen. 1994. Price discrimination through a distribution channel: Theory and evidence. *Amer. Econom. Rev.* **84**(5) 1437–1445.
- Grossman, S. J., O. D. Hart. 1983. An analysis of the principal-agent problem. *Econometrica* **51**(1) 7–45.
- Harris, M., A. Raviv. 1978. Some results on incentive contracts with applications to education and employment, health insurance, and law enforcement. *Amer. Econom. Rev.* **68**(1) 20–30.
- , ———. 1979. Optimal incentive contracts with imperfect information. *J. Econom. Theory* **20**(2) 231–259.
- Holmstrom, B. 1979. Moral hazard and observability. *Bell J. Econom.* **10**(1) 74–91.
- Jucker, J. V., M. J. Rosenblatt. 1985. Single-period inventory models with demand uncertainty and quantity discounts: Behavioral implications and a new solution procedure. *Naval Res. Logist. Quart.* **32**(4) 537–550.
- Jueland, A. P., S. M. Shugan. 1983. Managing channel profits. *Marketing Sci.* **2**(3) 239–272.
- Kandel, E. 1996. The right to return. *J. Law Econom.* **39**(1) 329–356.
- Kanellos, M. 1996. Novell regroups. *Computer Reseller News* (Sep 9) 127.
- , C. Zarley. 1996. Apple products rotting in channel. *Computer Reseller News* (Mar 25) 3.
- Lariviere, M. A. 1999. Supply chain contracting and coordination with stochastic demand. S. Tayur, M. Magazine, R. Ganeshan, eds. *Quantitative Models of Supply Chain Management*. Kluwer Academic Publishers, Boston, MA, 233–268.
- , V. Padmanabhan. 1997. Slotting allowances and new product introductions. *Marketing Sci.* **16**(2) 112–128.
- , E. Porteus. 2001. Selling to the newsvendor: An analysis of price-only contracts. *Manufacturing Service Oper. Management* **3**(4) 293–305.
- Lee, H. L., M. J. Rosenblatt. 1986. A generalized quantity discount pricing model to increase supplier's profits. *Management Sci.* **32**(9) 1177–1185.
- Moltzen, E. F. 1997. Lotus targets VARs to beef up support. *Computer Reseller News* (February 3) 721.
- Narayanan, V. G., A. Raman. 1997. Contracting for inventory in a distribution channel with stochastic demand and substitute products. Working paper, Harvard University, Boston, MA.
- O'Heir, J., P. Pereira. 1997. Future of midsize distributors cloudy. *Computer Reseller News* (June 30) 5, 203.
- Padmanabhan, V., I. P. L. Png. 1995. Returns policies: Making money by making good. *Sloan Management Rev.* **37**(1) 65–72.
- , ———. 1997. Manufacturer's returns policies and retail competition. *Marketing Sci.* **16**(1) 81–94.
- Pasternack, B. A. 1985. Optimal pricing and return policies for perishable commodities. *Marketing Sci.* **4**(2) 166–176.
- . 1999. Using revenue sharing to achieve channel coordination for a newsboy type inventory model. Working paper, California State University, Fullerton, CA.
- Patterson, P. 1999. E-commerce: The basics. *Wall Street Journal* (November 22) R20.
- Peers, M., E. Ramstad. 2000. Price of CDs likely to drop, thanks to FTC. *Wall Street Journal* (May 11) B1.
- Pender, L. 1998. Symantec woos VAR channel. *Computer Reseller News* (December 7) 159–160.
- Porteus, E., S. Whang. 1991. On manufacturing/marketing incentives. *Management Sci.* **37**(9) 1166–1181.
- Preston, F. 1999. Consolidated industry seeks out top VARs. *Computer Reseller News* (February 15) 41.
- Raju, J., V. Srinivasan. 1996. Quota-based compensation plans for multi-territory heterogeneous salesforces. *Management Sci.* **42**(10) 1454–1462.
- Rao, R. C. 1990. Compensating heterogeneous salesforces: Some explicit solutions. *Marketing Sci.* **9**(4) 319–341.
- Rao, V. R., J. Wind, W. S. DeSarbo. 1988. A customized market response model: Development, estimation, and empirical testing. *J. Acad. Marketing Sci.* **16**(1) 128–140.
- Roberts, J. 1998. Reseller's profits rise, but so do salary costs. *Computer Reseller News* (October 26) 46–48.
- Robertson, J. 2001. Intel readies less-expensive 1.3-GHz Pentium for mainstream market. *Electronic Buyers' News* (January 1) 3.
- Ryans, A. B., C. B. Weinberg. 1979. Territory sales response. *J. Marketing Res.* **16** 453–465.
- Shavell, S. 1979. Risk sharing and incentives in the principal and agent relationship. *Bell J. Econom.* **10**(1) 55–73.
- Spengler, J. 1950. Vertical integration and antitrust policy. *J. Political Econom.* **58** 347–352.
- Taylor, T. A. 2001. Channel coordination under price protection, mid-life returns, and end-of-life returns in dynamic markets. *Management Sci.* **47**(9) 1220–1234.
- . 2002. Supply chain coordination under channel rebates with sales effort effects (unabridged). (<http://www.gsb.columbia.edu/divisions/dro>).
- Terdoslavich, W. 1998. Incentive ideas: Using a big carrot to get a VAR to bite. *Computer Reseller News* (January 12) 771.
- Tsay, A. A. 1999. The quantity flexibility contract and supplier-customer incentives. *Management Sci.* **45**(10) 1339–1358.
- , S. Nahmias, N. Agrawal. 1999. Modeling supply chain contracts: A review. S. Tayur, M. Magazine, R. Ganeshan, eds. *Quantitative Models of Supply Chain Management*. Kluwer Academic Publishers, Boston, MA, 299–336.
- Webster, S., K. Weng. 2000. A risk-free perishable item returns policy. *Manufacturing Service Oper. Management* **2**(1) 100–106.
- Zarley, C. 1997. Vendors' volume-based incentives. *Computer Reseller News* (June 16) 333.
- . 1998. HP expands rebates. *Informationweek* (February 2) 93.
- Zhang, Z. J., A. Krishna, S. K. Dhar. 2000. The optimal choice of promotion vehicles: Front-loaded or rear-loaded incentives. *Management Sci.* **46**(3) 348–362.

Accepted by Fangruo Chen; received September 2000. This paper was with the author 7 months for 2 revisions.