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Support-Neutrosophic Set: A New Concept in Soft Computing

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Abstract. Today, soft computing is a field that is used a lot in solving real-world problems, such as problems in economics, finance, banking... With the aim to serve for solving the real problem, many new theories and/or tools which were proposed, improved to help soft computing used more efficiently. We can mention some theories as fuzzy sets theory (L. Zadeh, 1965), intuitionistic fuzzy set (K Atanasov, 1986), neutrosophic set (F. Smarandache 1999). In this paper, we introduce a new notion of support-neutrosophic set (SNS), which is the combination a neutrosophic set with a fuzzy set. So, SNS set is a direct extension of fuzzy set and neutrosophic sets (F. Smarandache). Then, we define some operators on the support-neutrosophic sets, and investigate some properties of these operators.

Keywords: support-neutrosophic sets, support-neutrosophic fuzzy relations, support- neutrosophic similarity relations

1 Introduction

In 1998, Prof. Smarandache gave the concept of the *neutrosophic set* (NS) [3] which generalized fuzzy set [10] and intuitionistic fuzzy set [1]. It is characterized by a degree of truth (T), a degree of indeterminacy (I) and a degree of falsity (F). Over time, the sub-class of the neutrosophic set were proposed to capture more advantageous in practical applications. Wang et al. [5] proposed the interval neutrosophic set and its operators. Wang et al. [6] proposed a single-valued neutrosophic set as an instance of the neutrosophic set accompanied with various set theoretic operators and properties. Ye [8] defined the concept of simpli-fied neutrosophic set whose elements of the universe have a degree of truth, indeterminacy and falsity respectively that lie between [0, 1]. Some operational laws for the simplified neutrosophic set and two aggregation operators, including a simplified neutrosophic weighted arithmetic average operator and a simplified neutrosophic weighted geometric average operator were presented.

In 2015, Nguyen et al. [2] introduced a Support-intuitionistic fuzzy set, it combines a intuitionistic fuzzy set with a fuzzy set (the support of an intuitionistic). Apter, Young et al [9] applied support – intuitionistic in decision making.

Practically, lets' consider the following case: a customer is interested in two products A and B. The customer has one rating of good (i), indeterminacy (ii) or not good (iii) for each of the products. These ratings (i),(ii) and (iii) (known as neutrosophic ratings) will affect the customer's decision of which product to buy. However, the customer's financial capacity will also affect her decision. This factor is called the support factor, with the value is between 0 and 1. Thus, the decision of which product to buy are determined by truth factors (i), indeterminacy factors (ii), falsity factors (iii) and support factor (iv). If a product is considered good and affordable, it is the best situation for a buying decision. The most unfavorable situation is when a product is considered bad and not affordable (support factor is bad),in this case, it would be easy to refuse to buy the product.

Another example, the business and purchase of cars in the Vietnam market. For customers, they will care about the quality of the car (good, bad and indeterminacy, they are neutrosophic) and prize, which are considered as supporting factors for car buyers. For car dealers, they are also interested in the quality of the car, the price and the government's policy on importing cars such as import duties on cars. Price and government policies can be viewed as supporting components of the car business.

In this paper, we combine a neutrosophic set with a fuzzy set. This raise a new concept called support-neutrosophic set (SNS). In which, there are four membership functions of an element in a given set. The remaining of this paper was structured as follows: In section 2, we introduce the concept of support-neutrosophic set and study some properties of SNS. In section 3, we give some distances between two SNS sets. Finally, we construct the distance of two support-neutrosophic sets.

2 Support-Neutrosophic set

Throughout this paper, U will be a nonempty set called the universe of discourse. First, we recall some the concept about fuzzy set and neutrosophic set. Here, we use mathematical operations on real numbers. Let S_1 and S_2 be two real standard or non-standard subsets, then

$$S_1 + S_2 = \{x | x = s_1 + s_2, s_1 \in S_1, s_2 \in S_2\}$$

$$S_1 - S_2 = \{x | x = s_1 - s_2, s_1 \in S_1, s_2 \in S_2\}$$

$$\bar{S}_2 = \{1^+\} - S_2 = \{x | x = 1^+ - s_2, s_2 \in S_2\}$$

$$S_1 \times S_2 = \{x | x = s_1 \times s_2, s_1 \in S_1, s_2 \in S_2\}$$

$$S_1 \vee S_2 = [\max\{infS_1, infS_2\}, \max\{supS_1, supS_2\}]$$

$$S_1 \wedge S_2 = [\min\{infS_1, infS_2\}, \min\{supS_1, supS_2\}]$$

$$d(S_1, S_2) = \inf_{s_1 \in S_1, s_2 \in S_2} d(s_1, s_2)$$

Remark: $\overline{S_1 \wedge S_2} = \overline{S_1} \vee \overline{S_2}$ and $\overline{S_1 \vee S_2} = \overline{S_1} \wedge \overline{S_2}$. Indeed, we consider two cases:

+ if $infS_1 \leq infS_2$ and $\sup S_1 \leq \sup S_2$ then $1 - infS_2 \leq 1 - infS_1$, $1 - supS_2 \leq 1 - supS_1$ and $S_1 \wedge S_2 = S_1$, $S_1 \vee S_2 = S_2$. So that $\overline{S_1 \wedge S_2} = \overline{S_1} = \overline{S_1} \vee \overline{S_2}$ and $\overline{S_1 \vee S_2} = \overline{S_2} = \overline{S_1} \wedge \overline{S_2}$.

+ if $infS_1 \leq infS_2 \leq supS_2 \leq \sup S_1$. Then $S_1 \wedge S_2 = [infS_1, supS_2]$ and $\overline{S_1} \vee \overline{S_2} = [1 - supS_2, 1 - infS_1]$. Hence $\overline{S_1 \wedge S_2} = \overline{S_1} \vee \overline{S_2}$. Similarly, we have $\overline{S_1 \wedge S_2} = \overline{S_1} \vee \overline{S_2}$.

Definition 1. A fuzzy set A on the universe U is an object of the form

$$A = \{(x, \mu_A(x)) | x \in U\}$$

where $\mu_A(x) (\in [0,1])$ is called the degree of membership of x in A.

Definition 2. A neutrosophic set A on the universe U is an object of the form

$$A = \{(x, T_A(x), I_A(x), F_A(x)) | x \in U\}$$

where T_A is a truth –membership function, I_A is an indeterminacy-membership function, and F_A is falsity – membership function of A. $T_A(x), I_A(x)$ and $F_A(x)$ are real standard or non-standard subsets of $]0^-, 1^+[$, that is

$$T_A: U \to]0^-, 1^+[$$
$$I_A: U \to]0^-, 1^+[$$
$$F_A: U \to]0^-, 1^+[$$

In real applications, we usually use

$$T_A: U \to [0,1]$$
$$I_A: U \to [0,1]$$
$$F_A: U \to [0,1]$$

Now, we combine a neutrosophic set with a fuzzy set. That leads to a new concept called support-neutrosophic set (SNS). In which, there are four membership functions of each element in a given set. This new concept is stated as follows:

Definition 3. A support – neutrosophic set (SNS) A on the universe U is characterized by a truth –membership function T_A, an indeterminacy-membership function I_A, a falsity – membership function F_A and support-membership function s_A. For each $x \in U$ we have $T_A(x), I_A(x)$, $F_A(x)$ and $s_A(x)$ are real standard or non-standard subsets of $]0^-, 1^+[$, that is

$$T_A: U \to]0^-, 1^+[$$
$$I_A: U \to]0^-, 1^+[$$
$$F_A: U \to]0^-, 1^+[$$
$$s_A: U \to]0^-, 1^+[$$

We denote support – neutrosophic set (SNS)

$$A = \{(x, T_A(x), I_A(x), F_A(x), s_A(x)) | x \in U\}.$$

There is no restriction on the sum of $T_A(x), I_A(x)$, $F_A(x)$, so $0^- \leq supT_A(x) + supI_A(x) + sup\ F_A(x) \leq 3^+$, and $0^- \leq s_A(x) \leq 1^+$.

When U is continuous, a SNS can be written as

$$A = \int_U \left< T_A(x), I_A(x), F_A(x), s_A(x) \right> / x$$

When $U = \{x_1, x_2, .., x_n\}$ is discrete, a SNS can be written as

$$A = \sum_{i=1}^{n} \frac{< T_A(x_i), I_A(x_i), F_A(x_i), s_A(x_i) >}{x_i}$$

We denote $SNS(U)$ is the family of SNS sets on U.

Remarks:

+ The element $x_* \in U$ is called "worst element" in A if $T_A(x_*) = 0, I_A(x_*) = 0, F_A(x_*) = 1, s_A(x_*) = 0$. The element $x^* \in U$ is called "best element" in A if

$T_A(x^*) = 1, I_A(x^*) = 1, F_A(x_*) = 0, s_A(x_*) = 1$

(if there is restriction $sup\mathrm{T_A}(\mathrm{x}) + sup\mathrm{I_A}(\mathrm{x}) + sup\ \mathrm{F_A}(\mathrm{x}) \leq 1$ then the element $x^* \in U$ is called "best element" in A if

$T_A(x^*) = 1, I_A(x^*) = 0, F_A(x_*) = 0, s_A(x_*) = 1$).

+ the support – neutrosophic set A reduce an neutrosophic set if $s_A(x) = c \in [0,1], \forall x \in U$.

+ the support – neutrosophic set A is called a support-standard neutrosophic set if

$\mathrm{T_A}(\mathrm{x}), \mathrm{I_A}(\mathrm{x}), \mathrm{F_A}(\mathrm{x}) \in [0,1]$ and

$\mathrm{T_A}(\mathrm{x}) + \mathrm{I_A}(x) + \mathrm{F_A}(\mathrm{x}) \leq 1$

for all $x \in U$.

+ the support – neutrosophic set A is a support-intuitionistic fuzzy set if $\mathrm{T_A}(\mathrm{x}), \mathrm{F_A}(\mathrm{x}) \in [0,1], \mathrm{I_A}(x) = 0$ and $\mathrm{T_A}(\mathrm{x}) + \mathrm{F_A}(\mathrm{x}) \leq 1$ for all $x \in U$.

+ A constant SNS set

$(\alpha, \widehat{\beta, \theta}, \gamma) = \{(x, \alpha, \beta, \theta, \gamma) | x \in U$

where $0 \leq \alpha, \beta, \theta, \gamma \leq 1\}$.

+ the SNS universe set is

$U = 1_U = (1, \widehat{1,0}, 1) = \{(x, 1,1,0,1) | x \in U\}$

+ the SNS empty set is

$U = 0_U = (0, \widehat{0,1}, 0) = \{(x, 0,0,1,0) | x \in U\}$

Definition 4. The complement of a SNS A is denoted by $c(A)$ and is defined by

$$T_{C(A)}(x) = F_A(x)$$
$$I_{C(A}(x) \quad \{1^+\} - I_A(x)$$
$$F_{C(A)}(x) = T_A(x)$$
$$s_{C(A)}(x) \quad \{1^+\} - s_A(x)$$

for all $x \in U$.

Definition 5. A SNS A is contained in the other SNS B, denote $A \subseteq B$, if and only if

$$inf T_A(x) \leq inf T_B(x), \quad sup T_A(x) \leq \sup T_B(x)$$
$$inf\ F_A(x) \geq inf\ F_B(x), \quad sup F_A(x) \geq sub\ F_B(x)$$
$$inf s_A(x) \leq inf s_B(x), \quad sup s_A(x) \leq \sup s_B(x)$$

for all $x \in U$.

Definition 6. The union of two SNS A and B is a SNS $C = A \cup B$, that is defined by

$$T_C = T_A \vee T_B$$
$$I_C = I_A \vee I_B$$
$$F_C = F_B \wedge F_B$$
$$s_C = s_A \vee s_B$$

Definition 7. The intersection of two SNS A and B is a SNS $D = A \cap B$, that is defined by

$$T_D = T_A \wedge T_B$$
$$I_D = I_A \wedge I_B$$
$$F_D = F_B \vee F_B$$
$$s_D = s_A \wedge s_B$$

Example 1. Let $U = \{x_1, x_2, x_3, x_4\}$ be the universe. Suppose that

$$A = \frac{\langle [0.5, 0.8], [0.4, 0.6], [0.2, 0.7], [0.7, 0.9] \rangle}{x_1}$$
$$+ \frac{\langle [0.4, 0.5], [0.45, 0.6], [0.3, 0.6], [0.5, 0.8] \rangle}{x_2}$$
$$+ \frac{\langle [0.5, 0.9], [0.4, 0.5], [0.6, 0.7], [0.2, 0.6] \rangle}{x_3}$$
$$+ \frac{\langle [0.5, 0.9], [0.3, 0.6], [0.4, 0.8], [0.1, 0.6] \rangle}{x_4}$$

and

$$B = \frac{\langle [0.2, 0.6], [0.3, 0.5], [0.3, 0.6], [0.6, 0.9] \rangle}{x_1}$$
$$+ \frac{\langle [0.45, 0.7], [0.4, 0.8], [0.9, 1], [0.4, 0.9] \rangle}{x_2}$$
$$+ \frac{\langle [0.1, 0.7], [0.4, 0.8], [0.6, 0.9], [0.2, 0.7] \rangle}{x_3}$$
$$+ \frac{\langle [0.5, 1], [0.2, 0.9], [0.3, 0.7], [0.1, 0.5] \rangle}{x_4}$$

are two support –neutrosophic set on U.

We have

+ complement of A, denote $c(A)$ or $\sim A$, defined by

$$c(A)=\frac{\langle[0.2,0.7],[0.4,0.6],[0.5,0.8],[0.1,0.3]\rangle}{x_1}+\frac{\langle[0.3,0.6],[0.4,0.55],[0.4,0.5],[0.2,0.5]\rangle}{x_2}+\frac{\langle[0.6,0.7],[0.5,0.6],[0.5,0.9],[0.4,0.8]\rangle}{x_3}+\frac{\langle[0.4,0.8],[0.4,0.7],[0.5,0.9],[0.4,0.9]\rangle}{x_4}$$

+ Union $C = A \cup B$:

$$C=\frac{\langle[0.5,0.8],[0.4,0.6],[0.2,0.6],[0.7,0.9]\rangle}{x_1}+\frac{\langle[0.45,0.7],[0.45,0.8],[0.3,0.6],[0.4,0.9]\rangle}{x_2}+\frac{\langle[0.5,0.9],[0.4,0.8],[0.6,0.7],[0.2,0.7]\rangle}{x_3}+\frac{\langle[0.5,1],[0.3,0.9],[0.3,0.7],[0.1,0.6]\rangle}{x_4}$$

+ the intersection $D = A \cap B$:

$$D=\frac{\langle[0.2,0.6],[0.3,0.5],[0.3,0.7],[0.6,0.9]\rangle}{x_1}+\frac{\langle[0.4,0.5],[0.4,0.6],[0.9,1],[0.4,0.8]\rangle}{x_2}+\frac{\langle[0.1,0.7],[0.4,0.5],[0.6,0.9],[0.2,0.6]\rangle}{x_3}+\frac{\langle[0.5,0.9],[0.2,0.6],[0.4,0.8],[0.1,0.5]\rangle}{x_4}$$

Proposition 1. For all $A, B, C \in SNS(U)$, we have

(a) If $A \subseteq B$ and $B \subseteq C$ then $A \subseteq C$,
(b) $c(c(A)) = A$,
(c) Operators $\cap$ and $\cup$ are commutative, associative, and distributive,
(d) Operators $\cap, \sim$ and $\cup$ satisfy the law of De Morgan. It means that $\overline{A \cap B} = \overline{A} \cup \overline{B}$ and $\overline{A \cup B} = \overline{A} \cap \overline{B}$

Proof.

It is easy to verify that (a), (b), (c) is truth.

We show that (d) is correct. Indeed, for each

$$T_{\sim(A\cap B)} = F_{A\cap B} = F_A \vee F_B = T_{\sim A} \vee T_{\sim B}$$
$$I_{\sim(A\cap B)} = \{1^+\} - I(A\cap B) = \overline{I(A) \wedge I(B)}$$
$$= \overline{I(A)} \vee \overline{I(B)} = I_{\sim A} \vee I_{\sim B}$$
$$F_{\sim(A\cap B)} = T_{A\cap B} = T_A \wedge T_B = F_{\sim A} \wedge F_{\sim B}$$
$$s_{\sim(A\cap B)} = \{1^+\} - s(A\cap B) = \overline{s(A) \wedge s(B)}$$
$$= \overline{s(A)} \vee \overline{s(B)} = s_{\sim A} \vee s_{\sim B}$$

So that $\overline{A \cap B} = \overline{A} \cup \overline{B}$. By same way, we have $\overline{A \cup B} = \overline{A} \cap \overline{B}$. □

3 The Cartesian product of two SNS

Let U, V be two universe sets.

Definition 8. Let A, B two SNS on U, V, respectively. We define the Cartesian product of these two SNS sets:

a)

$$A\times B=\left\{\left\langle\begin{matrix}(x,y), T_{A\times B}(x,y), I_{A\times B}(x,y),\\ F_{A\times B}(x,y), s_{A\times B}(x,y)\end{matrix}\right\rangle \mid x\in U, y\in V\right\}$$

where

$$T_{A\times B}(x,y) = T_A(x)T_B(y),$$
$$I_{A\times B}(x,y) = I_A(x)I_B(y),$$
$$F_{A\times B}(x,y) = F_A(x)F_B(y)$$

and

$$s_{A\times B}(x,y) = s_A(x)s_B(y), \forall x \in U, y \in V.$$

$$A\otimes B=\left\{\left\langle\begin{matrix}(x,y), T_{A\otimes B}(x,y), I_{A\otimes B}(x,y),\\ F_{A\otimes B}(x,y), s_{A\otimes B}(x,y)\end{matrix}\right\rangle \mid x\in U, y\in V\right\}$$

Where

$$T_{A\otimes B}(x,y) = T_A(x) \wedge T_B(y),$$
$$I_{A\otimes B}(x,y) = I_A(x) \wedge I_B(y),$$
$$F_{A\otimes B}(x,y) = F_A(x) \vee F_B(y)$$

and

$$s_{A\otimes B}(x,y) = s_A(x) \wedge s_B(y), \forall x \in U, y \in V\ .$$

Example 2. Let $U = \{x_1, x_2\}$ be the universe set. Suppose that

$$A = \frac{\langle[0.5,0.8],[0.4,0.6],[0.2,0.7],[0.7,0.9]\rangle}{x_1} + \frac{\langle[0.4,0.5],[0.45,0.6],[0.3,0.6],[0.5,0.8]\rangle}{x_2}$$

and

$$B = \frac{\langle[0.2,0.6],[0.3,0.5],[0.3,0.6],[0.6,0.9]\rangle}{x_1} + \frac{\langle[0.45,0.7],[0.4,0.8],[0.9,1],[0.4,0.9]\rangle}{x_2}$$

are two SNS on U. Then we have

$$A \times B \quad \frac{\langle[0.25,0.72],[0.16,0.3],[0.12,.49],[0.14,0.54]\rangle}{(x_1, x\ )} + \frac{\langle[0.225,0.56],[0.16,0.48],[0.18,0.7],[0.28,1]\rangle}{(x_1, x\ )} + \frac{\langle[0.2,0.45],[0.18,0.3],[0.18,0.42],[0.1,0.48]\rangle}{(x_2, x\ )} + \frac{\langle[0.2,0.45],[0.135,0.36],[0.12,0.48],[0.05,0.48]\rangle}{(x_2, x\ )}$$

and

$$A \otimes B \quad \frac{\langle[0.5,0.8],[0.4,0.5],[0.6,0.7],[0.2,0.6]\rangle}{(x_1, x\ )} + \frac{\langle[05,0.8],[0.3,0.6],[0.4,0.8],[0.1,0.6]\rangle}{(x_1, x\ )} + \frac{\langle[0.4,0.5],[0.4,0.5],[0.6,0.7],[0.2,0.6]\rangle}{(x_2, x\ )} + \frac{\langle[0.4,0.5],[0.3,0.6],[0.4,0.8],[0.1,0.6]\rangle}{(x_2, x\ )}$$

Proposition 2. For every three universes U, V, W and three universe sets A on U, B on V, C on W. We have

a) $A \times B = B \times A$ and $A \otimes B = B \otimes A$
b) $(A \times B) \times C = A \times (B \times C)$
and $(A \otimes B) \otimes C = A \otimes (B \otimes C)$

Proof. It is obvious.

4 Distance between support-neutrosophic sets

In this section, we define the distance between two support-neutrosophic sets in the sene of Szmidt and Kacprzyk are presented:

Definition 9. Let $U = \{x_1, x_2, \ldots, x_n\}$ be the universe set. Given $A, B \in SNS(U)$, we define

a) The Hamming distance
$$d_{SNS}(A,B) = \frac{1}{n}\sum_{i=1}^{n}[d(T_A(x_i), T_B(x_i)) + d(I_A(x_i), I_B(x_i)) + d(F_A(x_i), F_B(x_i)) + d(s_A(x_i), s_B(x_i))]$$
b) The Euclidean distance
$$e_{SNS}(A,B) = \frac{1}{n}\sum_{i=1}^{n}\left[d^2(T_A(x_i), T_B(x_i)) + d^2(I_A(x_i), I_B(x_i)) + d^2(F_A(x_i), F_B(x_i)) + d^2(s_A(x_i), s_B(x_i))\right]^{\frac{1}{2}}$$

Example 3. Let $U = \{x_1, x_2\}$ be the universe set. Two SNS $A, B \in SNS(U)$ as in example 2 we have $d_{SNS}(A,B) = 0.15$; $e_{SNS}(A,B) = 0.15$.

If

$$C = \frac{\langle[0.5,0.7],[0.4,0.6],[0.2,0.7],[0.7,0.9]\rangle}{x_1} + \frac{\langle[0.4,0.5],[0.45,0.6],[0.3,0.6],[0.5,0.8]\rangle}{x_2}$$

and

$$D = \frac{\langle[0.2,0.4],[0.3,0.5],[0.3,0.6],[0.6,0.9]\rangle}{x_1} + \frac{\langle[0.6,0.7],[0.4,0.8],[0.9,1],[0.4,0.9]\rangle}{x_2}$$

then $d_{SNS}(C,D) = 0{,}25$ and $e_{SNS}(C,D) = 0{,}2081$.

Conclusion

In this paper, we introduce a new concept: support-neutrosophic set. We also study operators on the support-neutrosophic set and their initial properties. We have given the distance and the Cartesian product of two support – neutrosophic sets. In the future, we will study more results on the support-neutrosophic set and their applications.

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