Supporting Teachers in the Development of Young Children's Mathematical Thinking: Three Large Scale Cases[1]

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Recognition of the importance of the early childhood years in the development of numeracy is a significant characteristic of the New Zealand Numeracy Development Project, the Victorian Early Numeracy Research Project and the Count Me In Too program in New South Wales, Australia. This article outlines the background, key components and major impacts of these three innovative and successful professional development and research initiatives. Juxtaposing the three projects highlights important commonalities—research-based frameworks, diagnostic interviews, and whole-school approaches to professional development. Each program has been significant in rethinking what mathematics and how mathematics is taught to young children.

Increasingly researchers have demonstrated that children begin to think mathematically from an early age. There is considerable research to show that many children arrive at primary school already equipped with an extraordinary understanding of the number system and how it works (Aubrey, 1999; Wright, 1994; Young-Loveridge, 2004). However, even as early as age five, there is huge variation in the kinds of understanding children bring with them to school (Ginsburg, Klein, & Starkey, 1998). Evidence shows that differences among children at school entry are quite stable – students tend to remain at about the same rank order position with respect to their peers throughout the primary and secondary school years (Newman, 1984). Furthermore, disparities in mathematics achievement evident at school entry tend to increase as students advance through the school system; that is, the gap between the most and the least competent students increases over time (Young-Loveridge, 1991). Research shows that intervention in the

1 Please note that slightly different terminology is used in the NSW, Victoria, and New Zealand sections of this article e.g., Year 2 (NSW and NZ), Grade 2 (Vic); count-on (NSW), count on (Vic).

early childhood years can be effective in reducing disparities in mathematics achievement (Gervasoni, 2003; Wright, Martland, & Stafford, 2000; Young-Loveridge, Peters, & Carr, 1998). Tutoring of individuals or small groups of students by specially trained teachers can be very effective (Wright, Stanger, Cowper, Dyson, 1996; Young-Loveridge, 2004). The poor performance of many western countries on international comparisons of mathematics achievement in the 1990s (e.g., National Center for Educational Statistics, n.d.) reinforced the need for a radically different approach to be taken across the whole education system, but particularly in the early years of school.

In Australia and New Zealand, like many other western countries, major initiatives have been put in place to raise student achievement in numeracy and improve the professional capability of teachers. This article focuses on initiatives undertaken in New Zealand, and the Australian states of New South Wales and Victoria. Several key features characterise all three initiatives including the use of research-based frameworks to describe progressions in mathematics learning, and individual task-based interviews to assess children's mathematical thinking. This commonality is mostly due to the research-base from which the elements originated as well as the exchange of ideas among the projects. However, the ways in which the various elements have evolved are unique. This paper gives an overview of each initiative, outlining the key features and their linked theoretical basis, followed by a discussion of common themes and their implications for practitioners.

Count Me In Too-An Evolving Design Experiment

Background

The Count Me In Too (CMIT) numeracy program is an on-going initiative of the Department of Education and Training in New South Wales (NSW). This large school system caters for a population of about seven million people. The CMIT program melds findings from research about how children learn mathematics, with research on effective professional development. Hence, its two main aims are to help teachers understand children's mathematical development and to improve children's achievement in mathematics.

CMIT represents new directions from what characterises traditionally conceived professional development programs. There is a change in emphasis from the transmission of knowledge at one-shot workshops to long-term classroom-based learning. Teachers are viewed as active learners and are encouraged to examine their own teaching practices and to focus on the mathematics children *can* do and *how* they do it. The principles upon which the program is based support those of early childhood learning and teaching through the recognition and development of young children's intuitive numeracy strategies.

The development of CMIT has been cyclical over a period of approximately eight years. This cyclical approach is characterised by on-going phases of development, implementation and evaluation. Parallel with this cyclical approach has been an approach involving expansion and scaling up of the program.

The school system in which CMIT has been developed and implemented has approximately 1700 primary schools. The system is organised into geographic and administrative districts (40 districts prior to 2004), each with 40 or more primary schools. CMIT began in 1996 as a pilot program involving 13 schools across four of the 40 districts. In 1997, CMIT was expanded to all 40 districts, and involved 4 to 6 new schools in each district. This pace of implementation continued each year, and by 2003 almost every one of the 1700 schools had implemented CMIT.

A second aspect of the expansion of CMIT began in 1999, and involved extending the focus on the number strand of the curriculum, to the measurement strand (Outhred, Mitchelmore, McPhail, & Gould, 2003) and the space (geometry) strand (Owens, Reddacliff, Gould, & McPhail, 2001). In 2000, the focus of CMIT on number in the first three years of school (K-2) was extended to the third and fourth years of school (i.e. K-4). Since 2001, there have been two other significant expansions of the program. One of these involved the development of a related program called Counting On (Perry & Howard, 2001), focused specifically on low-attaining students in Year 6—the last year of primary school—and Year 7—the first year of secondary school. The other expansion involved an adaptation of the CMIT number program to more appropriately meet the needs of Indigenous students. Each of these extensions of the program had their own on-going phases of development, implementation and evaluation, as well as scaling up from an initial pilot to large numbers of schools in the system.

As stated earlier, CMIT was developed by and for the government school system of NSW. However, the program has had considerable impact beyond the NSW government school system. The program has been adopted by many Catholic schools and many independent schools in NSW. As well, government systems in other Australian states and territories have adopted the program. Aspects of the program have been used by school systems in the United States, the UK, and Papua New Guinea. In 2000, the New Zealand Ministry of Education conducted a pilot of CMIT (Thomas & Ward, 2001). This pilot informed the New Zealand Numeracy Development Project which was implemented from 2001.

Key Features of CMIT

The Learning Framework in Number. The key theoretical component of CMIT is the Learning Framework in Number (LFIN). This framework was initially developed by Wright (1994) for the purpose of researching and documenting progress in number learning of students in the first three years of school (five- to eight-year-olds). In the early 1990s, use of the framework was incorporated into a professional development program for teachers (Wright, 2000) focusing on an intervention program that involves intensive teaching of low-attaining students (Wright et al., 1996). The framework has since been used extensively with students of all levels of attainment, and has been used as a basis for classroom teaching, as well as for intensive intervention for low-attaining students. This intervention program—*Mathematics Recovery* (Wright, 2003; Wright, Martland, Stafford, & Stanger, 2002) has been used extensively by school systems in several countries including the United States and the UK. As well, *Mathematics Recovery* provided the basis for CMIT in several respects—the learning framework, the approach to interview-based assessment and teacher's use of video-taped interviews as learning tools.

A key aspect of the LFIN is Steffe's psychological model of the development of children's counting-based strategies (Steffe, 1992; Steffe, Cobb, & von Glasersfeld, 1988). In Steffe's model, and in the LFIN, the term counting is used in a special sense. A distinction is made between counting and saying a number word sequence because it is assumed that students' mental processes are quite different when they are counting, compared with when they are saying a number word sequence without intending to coordinate number words and items.

Steffe's model of the development of counting incorporates the notion of distinct progressions (or stages of development) in the ways children use counting to solve problems. The progressions relate to qualitative differences in the meanings of number words and are characterised by limitations in the ways students use counting to solve problems. In the LFIN these stages, in order of increasing sophistication, are: *emergent*, *perceptual*, *figurative*, *counting-on* and *counting-back*, and *facile*. Detailed descriptions of this model are beyond the scope of this article (see Wright et al., 2000).

The key components of the LFIN are:

- Building addition and subtraction through counting by ones;
- Building addition and subtraction through grouping;
- Building multiplication and division through equal counting and grouping;
- Building place value through grouping;
- Forward number word sequences;
- Backward number word sequences;
- Number word sequences by 10s and 100s; and
- Numeral identification.

Each of the key components of the LFIN is elaborated into a progression of up to six levels (or stages). Thus one of the goals of assessment using LFIN, is to profile the student's knowledge across the spectrum of the key components (Wright et al., 2000). Profiling of students' knowledge in this way forms a basis for instruction that is targeted to each student's current levels of knowledge and strategies.

The research by Steffe and colleagues is important in CMIT for several reasons. As well as providing the model of students' development of counting used in CMIT, Steffe pioneered an approach to research involving detailed and extended observation of students' learning via videotaped episodes of one-to-one assessment interviews and teaching sessions. Significant aspects of this approach have been incorporated into teachers' professional learning in CMIT.

Since its initial development in 1990, the LFIN has undergone further development and extension through the influence of a wide range of research in early number learning (e.g., Bobis, 1996; Cobb & Wheatley, 1988; Gravemeijer, 1994; McClain & Cobb, 2001; Mulligan & Mitchelmore, 1997).

The Professional Development Model. The CMIT professional development model "creates a climate for learning for four linked groups within the project—academic facilitators, consultants, teachers and students" (NSW Department of Education and Training, 2003, p. 3). Due to this focus on collaborative learning, CMIT operates best when groups of teachers from the same school are involved in the program. The program normally involves a close relationship between the district mathematics consultant and a team of three to five teachers at each school. Teachers are viewed as active learners as they engage in observational assessment, diagnostic interviews, and reflection upon students' work and their teaching. The consultants, in their roles as co-learners, assist teachers as they individually assess the mathematical development of their children. They guide the teachers as they learn to plan their instruction based on the ideas inherent in the LFIN. Hence, the focus is on the learning of both children and teachers in often confusing and messy contexts, such as classrooms (Gould, 2000).

An important aspect inherent in the CMIT professional development model is the synergy of research and practice. Such an "intimate relationship between theory and practice is a defining characteristic of Design Research" (Cobb, 2003, p. 3). This design aspect clearly drives much of the program's innovative practices, since its purpose is not to simply affirm that the program's initial instructional design succeeds, but to improve it. Such development is guided not only by the on-going, formal evaluations that systematically address various aspects of CMIT, but also by informal, anecdotal evidence gathered from consultants, teachers and children on a daily basis. Consistent with a design research methodology, development occurs in small increments. Each development is followed by a cycle of implementation, evaluation and revision. Additionally, each cycle is viewed as an iterative process. Short cycles make the process more efficient as elements envisioned "behind the desk" (Gravemeijer, 1994, p. 449), are immediately put into practice and scrutinised as part of the on-going evaluation. Insights gained are used to *feed forward* into the theory development and instructional design loops of CMIT. In this way, the program remains dynamic, an essential component if professional development is to be long-term and a generative process.

As mentioned, on-going, systematic evaluation is inherent in design research methodology—the principles upon which CMIT is based. The formal evaluation component has resulted in over ten separate reports and numerous conference papers aimed at synthesising the findings. Parallel to these documents is a growing body of literature associated with the developmental research that feeds into the theoretical and instructional design of CMIT. The following section briefly summarises the major findings of this increasing body of literature and highlights the impact of CMIT on classroom practices and systemic changes at the curriculum development level.

Key Outcomes of CMIT

A research-based agenda of evaluations has systematically explored the impact of CMIT. Each evaluation has focused on specific aspects or anticipated outcomes of the program, with some aspects being re-evaluated at different times. The following overview will only focus on the impact of CMIT on teachers and student achievement.

Student achievement: In 1999, assessment interview data were collected from 162 primary schools involved in the program. Children (N = 15,176) in Kindergarten to Year 3 (ranging in age from 4 years 5 months to 7 years 11 months) were interviewed twice—at the start of the program and approximately 3 months later. Interviews were conducted by classroom teachers as part of the CMIT professional development program. Interview results were used to produce a set of matched information for 10,075 children. Students with incomplete data for either of the assessments were excluded from the study. Figures 1 and 2 present the percentage of students in age groups achieving each stage of Early Arithmetical Strategy at the time of the initial and final interviews. It can be seen in both figures that the percentage of emergent and perceptual counters across age groups tends to decrease with a corresponding increase in counting-on and facile strategy use. More importantly, a comparison of strategy use at the initial interview (Figure 1) with that at the time of the second interview (Figure 2) shows significant improvement. In particular, the percentage of emergent stage students aged less than 5 years at the start of the program fell from 61% to 16% at the final interview. The rate of progression through the various stages is far greater than could normally be expected within this population.

White and Mitchelmore (2001) surveyed the Basic Skills Test (BST) results of 71 schools that had been implementing CMIT for two or more years. The BST are standardised tests in literacy and numeracy implemented in Year 3 and Year 5 across the state of NSW.

Figure 3 shows the overall mean z-scores on literacy and numeracy BST results for Years 3 and 5 from 1996–2001 for each of the 71 schools. The graph shows a clear upward trend in BST results for Year 3 numeracy. White and Mitchelmore (2001) concluded that the implementation of CMIT in NSW public schools had caused a definite improvement in Year 3 BST numeracy performance.

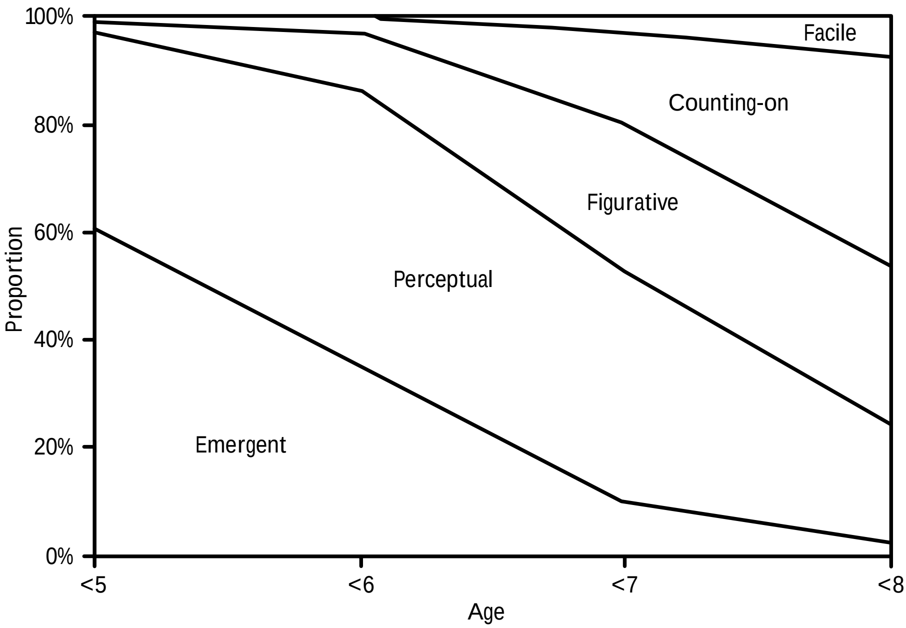

Figure 1. Strategy use by age on initial interview.

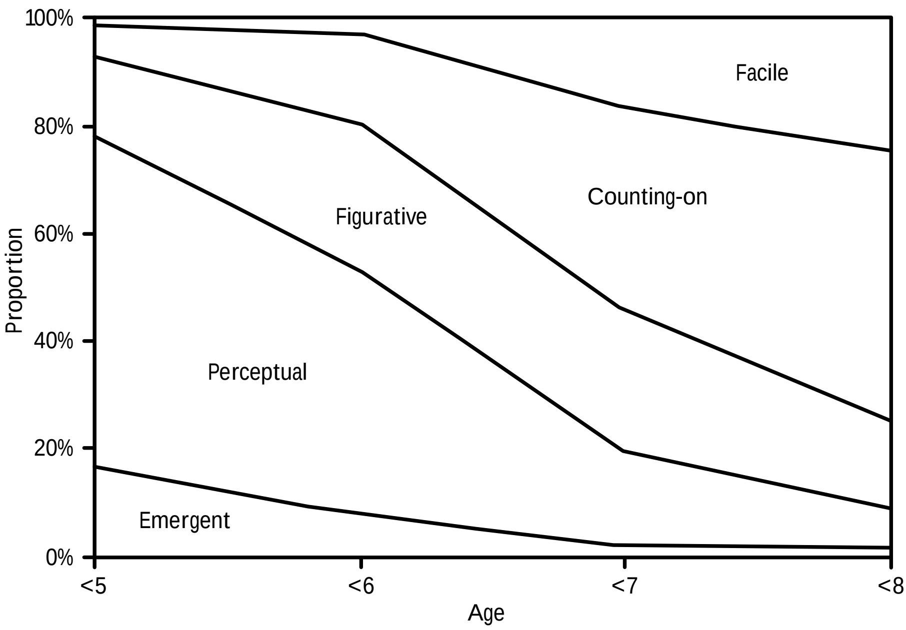

Figure 2. Strategy use by age on final interview.

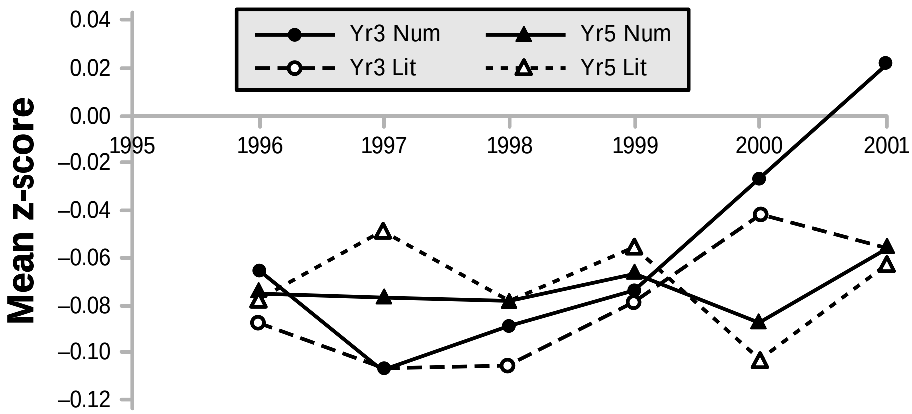

Figure 3. Mean BST scores of 71 schools 1996–2001.

Teacher data. Several studies have assessed the impact of various elements of CMIT on teachers. For example, Bobis and Gould (2000) investigated the anecdotal evidence provided by teachers in earlier reports more explicitly. Rather than relying on teachers' self-reports of changes to their knowledge, the study documented what knowledge changed, and how it changed, through the use of concept mapping and interviews with teachers. The study found that teachers' knowledge of pedagogy and children's cognition underwent a great deal of change, but the most significant change was to their knowledge of how children learn mathematics. While findings indicated little or no change had occurred to the mathematical content knowledge of teachers, it was evident that they started to see the connectedness of the different types of teacher knowledge, and how this knowledge impacted on the way they taught mathematics. For example, one teacher commented:

> I have learnt so much about how children learn and how it all links together. My knowledge links to what the children learn and how they learn. The strategies and the mathematical content-it is all interlinked.

The following points from an evaluation of the Stage 2 (Years 3 and 4) implementation of CMIT (Bobis, 2003) are indicative of previous findings concerning the impact on teachers:

- Generally, teachers' responses towards the overall impact of CMIT were very positive.
- High proportions (69.5%) of teachers involved in CMIT reported improvements to their attitude both towards mathematics and towards the teaching of mathematics.

- High proportions of the sample indicated that their understanding of how children learn mathematics (71.6%) and the way they taught mathematics (77.9%) had changed as a result of their involvement in CMIT.
- A smaller proportion of teachers indicated an increase or improvement to their mathematical content knowledge (48.3%) as a result of their involvement in CMIT.
- The most commonly cited changes to the way teachers taught mathematics included: more use of hands-on activities, increased emphasis on thinking strategies, reduced use of textbooks, and an increased use of ability groups.

Sustaining the Project

A determining factor in the long-term success of any professional development program is how it links to more comprehensive change. In 2002, a new *Mathematics K-6 Syllabus* (Board of Studies NSW, 2002) was released. This reshaped syllabus is closely aligned to the CMIT program. Like CMIT, the new syllabus emphasises the significance of mental computation and focuses on the development of increasingly more efficient thinking strategies. The introduction of standard pencil and paper algorithms has been delayed by around two years to accommodate the increased attention to mental computational strategies. Additionally, the expected level of number knowledge and counting skills for the early grades has been increased in line with CMIT outcomes.

An obstacle facing the implementation of CMIT in the past has been the perceived mismatch between the program's content and that of the syllabus. Having achieved close alignment with the syllabus, such an obstacle no longer exists.

Despite the mathematical content and focus on strategy development for mental calculation from CMIT being embodied in the new mathematics syllabus, the initial aims of the program are as relevant today as they were in the beginning. The improvement of learning outcomes and increasing teachers' knowledge of content, pedagogy, and how children learn mathematics are recognised as an on-going journey. While institutionalising much of the focus on students' solution strategies emerging from CMIT assists in aligning the program with syllabus implementation, other challenges remain. Issues of sustained teacher change and the generative growth of knowledge are challenges inherent in all aspects of professional development. Given the experimental research design of the CMIT model, revelations about new areas of content knowledge, namely fractions, have begun emerging since the new syllabus was released. New areas of knowledge such as this reinforce the need for professional development programs, such as CMIT, and the wealth of teaching support documents emerging from it, to continue.

The Victorian Early Numeracy Research Project

Background

The Victorian Early Numeracy Research Project (ENRP)[2] ran from January 1999 to February 2002 in 35 project ("trial") schools and 35 control ("reference") schools in the first three years of school (Prep to Year 2). It differed from CMIT in that it was conceptualised as a project that would inform ongoing programs and policy rather than an ongoing and evolving initiative.

The aims of the project included:

- to work with teachers to explore their beliefs and understandings about how students develop their understanding of mathematics in the early years of schooling, and how this could be supported through the teaching program;
- to evaluate the effect of the professional development program on student mathematical outcomes; and
- to describe effective practice in mathematics in the early years of schooling.

There were four main components of the ENRP:

- a research-based framework of *growth points* provided a means for understanding young children's mathematical thinking in general;
- a one-to-one, task-based, assessment interview provided a tool for assessing this thinking for particular individuals and groups;
- a multi-level professional development program was geared towards developing further such thinking; and
- case studies of particularly effective teachers and schools were conducted in the final year of the project, seeking to describe effective classroom practice.

Key Features of ENRP

Learning and assessment framework. The impetus for the ENRP was a desire to improve mathematics learning and so it was necessary to quantify such improvement. It would not have been adequate to describe, for example, the effectiveness of the professional development in terms of teachers' professional growth, or the children's engagement, or even to produce some success stories. It was decided to create a framework of key growth points in numeracy learning. Students' movement through these growth points in trial schools could then be compared to that of students in the reference schools, using statistical processes adapted by the team (e.g., Horne & Rowley, 2001).

[2] The Early Numeracy Research Project was supported by grants from the Victorian Department of Employment, Education and Training, the Catholic Education Office (Melbourne), and the Association of Independent Schools Victoria.

The project team studied available research on key *stages* or *levels* in young children's mathematics learning (e.g., Clements, Swaminathan, Hannibal, & Sarama, 1999; Fuson, 1992; Lehrer & Chazan, 1998; McIntosh, Bana, & Farrell, 1995; Wilson & Osborne, 1992), as well as frameworks developed by other authors and groups to describe learning. The Steffe and Wright work described earlier contributed significantly.

The decision was taken to focus upon the strands of Number (incorporating the domains of counting, place value, addition and subtraction strategies, and multiplication and division strategies), Measurement (incorporating the domains of length, mass and time), and Space (incorporating the domains of properties of shape, and visualisation and orientation).

Within each mathematical domain, growth points were stated with brief descriptors in each case. There are typically five or six growth points in each domain. To illustrate the notion of a growth point, consider the child who is asked to find the total of two collections of objects (with nine objects screened and another four objects). Many young children *count all* to find the total ("1, 2, 3, . . . , 11, 12, 13"), even once they are aware that there are nine objects in one set and four in the other. Other children realise that by starting at 9 and *counting on* ("10, 11, 12, 13"), they can solve the problem in an easier way. Counting all and counting on are therefore two important growth points in children's developing understanding of addition and exemplify the nature of the growth points. More information on the nature of the growth points and further examples are referenced elsewhere (Clarke, 2004; Clarke, Cheeseman, McDonough, & Clarke, 2003; Sullivan, Clarke, Cheeseman, & Mulligan, 2001).

One of the desired characteristics of the framework was that it needed to be in a form and language readily understood and, in time, retained by teachers. The aim was that teachers would use the framework as a kind of "lens" through which they could view interactions with children individually, in small group or whole class interactions, as well as during lesson planning.

In discussions with teachers, the team came to describe growth points as key "stepping stones" along paths to mathematical understanding. They provide collectively a kind of conceptual landscape (Fosnot & Dolk, 2001) or learning trajectory (Simon, 1995).

However, it is not claimed that all growth points are passed by every student along the way. For example, one of the growth points in addition and subtraction involves *count back*, *count down to* and *count up from* in subtraction situations, as appropriate. But there appears to be a number of children who view a subtraction situation (say, 12–9) as "What do I need to add to 9 to give 12?" and do not appear to use one of those three strategies in such contexts.

Also, the growth points should not be regarded as necessarily discrete. As with the CMIT framework, the extent of the overlap is likely to vary widely across young children, and "it is insufficient to think that all children's early arithmetical knowledge develops along a common developmental path" (Wright, 1998, p. 702).

Task-based assessment interview. The one-to-one interview was used with every child in trial schools and a random sample of around 40 children in each reference school at the beginning and end of the school year (February / March and November respectively), over a 30- to 40-minute period. The disadvantages of pen and paper tests were well established by Clements and Ellerton (1995) and others, and these disadvantages are particularly evident with young children, where reading issues are of great significance. The face-to-face interview was an appropriate response to these concerns. The interviews were conducted by the regular classroom teacher in trial schools, and by a trained team of interviewers in reference schools. A range of procedures was developed to maximise consistency among the 70 schools.

Although the full text of the ENRP interview involved around 60 tasks (with several sub-tasks in many cases), no child was posed all of these. The interview was of the form of a "choose your own adventure" story, in that the interviewer made a decision after each task, as instructed in the interview schedule. Given success with the task, the interviewer continued with the next task in the given mathematical domain as far as the child could go with success. Given difficulty with the task, the interviewer either abandoned that section of the interview and moved on to the next domain, or moved into a detour designed to elaborate more clearly the difficulty a child might be having with a particular content area.

All tasks were piloted with children of ages five to eight in non-project schools, in order to gain a sense of their clarity and their capacity to reveal a wide range of levels of understanding in children. This was followed by a process of refining tasks, further piloting and refinement and, where necessary, adjusting the framework of growth points. The final interview was published by the Department of Education and Training (2001).

Professional development program. The professional development program occurred (formally) at three levels. The 250 or so teachers from trial schools met with the research team each year for around five full days, spread across the year. The focus of the meetings was on understanding the framework and interview, and on appropriate classroom strategies, content, and activities for meeting identified needs of their students. On four or five occasions each year, the teachers met in regional cluster groups for two hours, usually after school. Each cluster contained from three to five school teams. One member of the university research team was responsible for each cluster group. The focus of these meetings was to complement the central professional development. There was usually a time of sharing, during which teachers discussed readings or particular activities or approaches that they had tried since last meeting together. This was followed by the content focus for the day, and further tasks were set that needed to be completed before the groups met again. The third level of professional development took place at the school and classroom level. School professional learning teams met every two weeks, and coordinators and members of the research team provided classroom support.

The cluster coordinator visited each school approximately three times per year, spending time in classrooms team teaching or observing, participating in planning meetings, jointly leading parent evenings, and acting as a "sounding board" for teachers, coordinators and principals. In addition, the Early Numeracy Coordinator at each school conducted weekly or fortnightly meetings of the "professional learning team" to maintain continuity, communication, team cohesion, and purpose.

Key Outcomes of the ENRP

Student data. The interview was conducted over 36,000 times, including two years of follow-up data in Grades 3 & 4 for children who were interviewed six times during the main phase of the project. This extensive data set provided the quantity and quality of reliable data on Victorian children's mathematical thinking that was previously unavailable to systems to inform curricula decisions. Table 1 provides data for the domain of addition and subtraction strategies for a variety of cohorts.

As shown in Table 1, over half of the Prep children arrive at school able to count all. Most of the rest gain this skill during the year, with nearly two-fifths of the students in trial schools counting on by the end of the Prep

Table 1
Percentage of Students Achieving Addition and Subtraction Strategies Growth Points Over Time (Prep to Grade 2 for Trial Schools, Grade 2 for Reference Schools)

Growth Points	Prep Mar 2000 (*n* = 1485)	Prep Nov 2000 (*n* = 1483)	Grade 1 Nov 2000 (*n* = 1262)	Grade 2 Nov 2001 (*n* = 1262)	Ref Grade 2 Nov 2001 (*n* = 296)
0 Not apparent	48	10	1	0	2
1 Count all	44	42	18	3	10
2 Count on	7	28	41	22	33
3 Count back / down / up	1	8	16	14	16
4 Basic strategies	0	3	17	29	21
5 Derived strategies	0	0	6	31	18
6 Extending and applying	0	0	0	2	1

year. By the end of Grade 2, there was a considerable spread of student performance on these strategies. Given the percentage of students who successfully used derived strategies (e.g., build to next ten, related doubles) by the end of Grade 2, particularly in reference schools, there are important implications for the appropriateness and timing of the teaching of formal algorithms in the early years (see Clarke, 2004, for a discussion of these issues).

Reference school data for Grade 2 children show a quite different distribution across the growth points with, for example, 19% of children demonstrating derived strategies or better, compared to 33% for trial school children. Students in trial schools outperformed significantly (at the .05 level) children in reference schools, at every grade level and in every mathematical domain. To examine this further, Figure 4 presents the percentage of the trial school students at each growth point on addition and subtraction strategies over the four interviews. For each point in time, by looking vertically, the reader can determine the approximate percentage of children who are at each growth point. By moving from the bottom left to the top right of the graph, the reader gains a sense of how much time is spent relatively at each growth point.

Figure 4 shows the way in which children progress over time, but also indicates that some children remain at the lower growth points at the end of

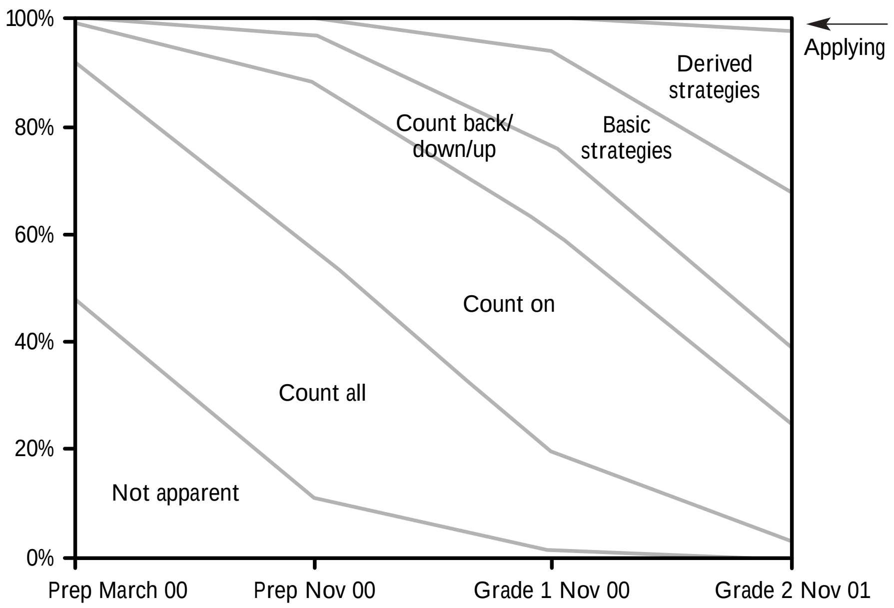

Figure 4. Trial school students (%) achieving Addition and Subtraction Strategies growth points over time.

Grade 2. The first three transitions all seem substantial. This is an important issue because it means that teacher awareness of such barriers or potential difficulties is an important prerequisite to facilitating student progress.

In 2000, teachers were also asked to comment on aspects of children's growth that they had observed which were not necessarily reflected in movement through the growth points. Although the research team had a great interest in cognitive growth as demonstrated by the response to interview tasks, growth can take other forms (e.g., productive disposition, as identified by Kilpatrick, Swafford, & Findell, 2001). It is important to document these other forms of growth.

Data from the 210 responses were categorised into themes. The five most common themes (in decreasing order of frequency) were the following:

- children are better at explaining their reasoning and strategies;
- children enjoy mathematics more, look forward to mathematics time, and expect to be challenged;
- the development of a "give it a go" mentality is evident, with greater overall persistence;
- children are thinking more about what they have learned and are learning; and
- all children are experiencing a level of success.

Teacher data. Given the clearly successful efforts of trial school teachers in developing children's mathematical skills and understandings in 1999 and 2000, it became important for the research project to study successful teachers' practice to try to discern those aspects of "what the teacher does" that make a difference.

All teachers facilitated growth in student learning over time, but the data for some teachers are particularly impressive. In 2001, the research team conducted detailed case studies of some of these teachers, as well as those school professional learning teams whose overall data are impressive (Clarke & Clarke, 2004). It should be emphasised that growth in student understanding was the main measure of success, not achievement at a given time. Although leadership and other school factors were of interest, the major focus of these studies was on what the teacher does in the classroom.

The ENRP had a significant impact on teachers' beliefs and understandings, in relation to their teaching of mathematics. In the final professional development session, teachers were asked to comment on how their teaching had changed due to their involvement in the ENRP. Two hundred and twenty teachers responded to the final questionnaire. These open responses were then grouped in themes. Six of the top ten themes or categories related to aspects of teachers' knowledge and beliefs. According to the teachers, as a result of their involvement in the ENRP they were more likely to:

- use growth points to inform planning (63 responses);
- use knowledge of individual understanding / better assess needs (49);
- challenge and extend children, and have higher expectations (42);
- have more confidence in teaching mathematics (28);
- enjoy mathematics more, have fun and make mathematics more interesting (27); and
- have greater knowledge of how children learn (24).

Teachers' specific responses reflected the importance placed on the knowledge and understandings that they had developed, and on how their beliefs, attitudes, and practices had changed. The growth points provided not only a way to discuss what the children already know but also the direction to move. The notions of trajectories of learning, or learning landscapes, are helpful here.

> Knowledge of mathematics must also be linked to knowledge of students' thinking, so that teachers have conceptions of typical trajectories of student learning and can use this knowledge to recognise landmarks of understanding in individuals. (Carpenter & Lehrer, 1999, p. 31)

Teachers in the ENRP gained the kinds of knowledge described above, and therefore developed a clearer picture of the typical trajectories of student learning, and a recognition of landmarks of understanding in individuals. Such a picture guided the decisions they made, in planning and in classroom interactions, as their knowledge of the understanding of individuals informed their practice.

In addition to these aspects of teachers' knowledge, a range of teaching strategies provided additional pedagogical tools, and it was the combination of these that appeared to empower the teachers. One teacher commented:

> [My teaching] hasn't changed that much but … I give children more time to think. I ask more relevant, thought-provoking questions. Some are even open-ended! Concrete materials are more prominent in my teaching. Most of all, the scenarios (through rich learning activities provided through ENRP PD) offered to children are more real and more interesting. Growth points and the interview have been great in identifying needs.

Some Concluding Comments

The requirement of teachers to participate in the assessment interviews meant that they were involved deeply in researching the understanding of their children, as individuals and as a group. Having access to data from a much larger group of students also enabled them to consider patterns or trends and to start to consider reasons for them. Ongoing assessment, and interviews in the latter part of each year provided an opportunity to evaluate the effectiveness of their teaching across different domains. This process proved very powerful in teachers' own professional development. They increased their knowledge of how children learn mathematics in general, they had a much clearer picture of their own children's understanding, and they had a repertoire of teaching approaches to enhance this understanding.

The research team noted with considerable pleasure, particularly in the third year of the project, the increasing fluency of trial school teachers with mathematics education research terminology, and the willingness to engage in complex ideas over extended periods. It appears that the "shared language" about young children's learning, so evident among teachers in the context of literacy in Victoria, was becoming a reality in mathematics as well.

The Early Numeracy Interview and the framework of growth points now provide the basis of much of the professional development in mathematics in the early years in Victoria, both in pre-service (e.g., McDonough, Clarke, & Clarke, 2002) and in-service settings.

The New Zealand Numeracy Development Project

Background

The New Zealand Numeracy Development Project (NDP)[3] is a major government initiative in mathematics education. Much of the impetus for this initiative came from the results of the Third International Mathematics and Science Study [TIMSS] which showed that New Zealand students performed poorly in mathematics compared to those of other education systems (National Center for Educational Statistics, n.d.). New Zealand, like other western countries, responded to its poor TIMSS results by focusing attention on mathematics teaching and learning in schools, with a particular emphasis on numeracy. Teachers' pedagogical content knowledge as well as teaching quality and confidence were recognised as being important for ensuring optimal mathematics learning. *Understanding* mathematics (instead of the mindless application of rote procedures) was seen as a key aspect in the preparation of students to participate fully within a democratic society.

The NDP came about as a result of extensive work by the Ministry of Education, which included the development of a comprehensive numeracy policy and strategy, and several pilot projects that provided teachers with professional development in mathematics (Ministry of Education, 2001). The NDP sits within the context of the New Zealand Ministry of Education's Literacy and Numeracy Strategy and reflects the key themes of that strategy: clarifying expectations, improving professional capability and involving the community (Ministry of Education, 2002). The focus of the project has been on improving student achievement in mathematics by improving the professional capability of teachers.

[3] The Numeracy Development Project was funded by the New Zealand Ministry of Education. The views expressed in this paper do not necessarily represent the views of the New Zealand Ministry of Education.

The NDP was first implemented in New Zealand schools in 2001. Since then, approximately 300,000 students and 11,500 teachers in approximately 1450 schools have participated in the project. It is intended that by 2007, almost every teacher of Year 1 to 6 students, and most teachers of Year 7 and 8 students, will have had the opportunity to participate in the project.

The project has been informed by annual evaluation reports that have examined the impact of the project on students' mathematics learning, as well as exploring the experiences of numeracy facilitators/consultants, classroom teachers, and school principals. Findings from the evaluations indicate that the project has had a significant impact on the quality of teaching and learning in mathematics (Christensen, 2003; Higgins, 2003; Irwin, 2003; Irwin & Niederer, 2002; Thomas, Tagg, & Ward, 2003; Thomas & Ward, 2001, 2002).

Key Features of the Numeracy Development Project

The Number Framework. At the core of the NDP is the Number Framework which consists of a sequence of global stages describing the mental processes students use to solve problems with numbers (Strategies), as well as the key pieces of knowledge that students need to learn in order to be able to use strategies effectively (Knowledge). The Number Framework has been informed by research showing that there are identifiable progressions in how children develop number concepts (Cobb, Gravemeijer, Yackel, McClain, & Whitenack, 1997; Jones et al., 1996; Steffe, 1992; Wright, 1998; Young-Loveridge & Wright, 2002). The framework has evolved since the 2001 implementation of the project, in response to student achievement information and feedback from project personnel and teachers. The Number Framework gives teachers "direction for responding effectively to children's learning needs" (Higgins, Parsons, & Hyland, 2003, p.166).

The Strategy section of the Framework consists of nine stages. The first five stages (0 to 4) focus on counting, with each stage involving increasingly sophisticated counting skills. The Framework begins with the Emergent stage (0), at which there is no counting, and progresses through counting a single collection (1), counting from one to join two collections of objects (2), counting from one mentally to solve addition/subtraction problems (3), to counting on to solve addition/subtraction problems (4). The four upper stages of the framework involve the use of increasingly complex *part-whole* strategies. These strategies are based on using knowledge of number properties to break numbers apart (partitioning) and recombine them in ways that make the problem solution easier. The first of the part-whole stages is Early Additive (5) involving a limited number of partitioning and recombining strategies, whereas the Advanced Additive stage (6) involves choosing from a wide range of strategies for solving addition and subtraction problems. The Additive part-whole stages are followed by the Advanced Multiplicative stage (7), in which the strategies chosen are for solving multiplication and division problems, and the Advanced Proportional stage

(8) for solving problems involving fractions, proportions, and ratios. Each of the nine stages contains three operational domains: addition and subtraction, multiplication and division, and proportions and ratios.

The Knowledge section describes the key items of knowledge that students need to learn, including numeral identification, number sequence and order, grouping and place value, basic facts, and written recording. The two sections are viewed as interdependent with "strategy creating new knowledge through use, and knowledge providing the foundation upon which new strategies are built" (Young-Loveridge & Wright, 2002, p. 722). It is considered important that students make progress in both sections concurrently:

> Strong knowledge is essential for students to broaden their strategies across a full range of numbers, and knowledge is often an essential prerequisite for the development of more advanced strategies (Ministry of Education, 2004, p 1).

The diagnostic interview. Another key factor in the success of the NDP is the diagnostic interview, an individual task-based interview designed to provide teachers with valuable information about their students' knowledge and mental strategies, and aligned with the Number Framework (Higgins et al., 2003).

The most recent version of the diagnostic interview (the Numeracy Project Assessment tool, or NumPA) has three overlapping forms at different difficulty levels. The teacher determines the appropriate form to use for each student, following his/her response to the "strategy window questions". Using the NumPA tool, teachers can develop for each student a number profile that has two main components: operational strategies (addition & subtraction, multiplication & division, proportions & ratios), and number knowledge (forwards & backwards number-word sequences, numeral identification, knowledge of fractions, decimals, percentages & basic facts, and understanding of place value).

Teachers participating in the project are expected to assess their students using the diagnostic interview on two occasions. The first is near the beginning of the project (after completing the professional development workshop on the use of the NumPA tool), and the second is at the conclusion of the project (after at least 15-20 weeks of the teaching program). Teachers are shown how to use information from assessments such as the diagnostic interview to make decisions regarding learning experiences necessary for students, both individually and in groups. Copies of the interview are available on the New Zealand Mathematics website (Ministry of Education, n.d.).

The professional development program. A third key factor in the success of the project is the professional development program, which requires the participation of the whole school over a one to three year period, and is predominantly situated in the classroom. The professional development approach adopted was informed by research that identified key elements of effective programs (e.g., Clarke & Cheeseman, 2000; Fullan & Hargreaves, 1992; Stephens, 2000). The recommended teaching model builds on a simplification of the theoretical models for the growth of students' understanding developed by Pirie and Kieran (1989) and the research of Fraivillig, Murphy, and Fuson (1999) which describes a framework for extending students' mathematical thinking. The project is delivered by a team of expert numeracy facilitators/consultants who have demonstrated expertise in mathematics curriculum, assessment, and pedagogy. Each full-time facilitator/consultant works with approximately 90 teachers, tailoring a series of workshops and in-class visits to meet the needs of individual schools and teachers in a particular cluster. The in-class visits by the facilitator/consultant provide feedback and support to the teacher in their implementation of the project.

Key Outcomes of the NDP

The impact of the project has been closely monitored through tightly focused evaluations conducted annually with each version of the professional development program for a particular age-group of students and medium of instruction (Christensen, 2003; Higgins, 2003; Irwin, 2003; Irwin & Niederer, 2002; Thomas, Tagg and Ward, 2003; Thomas & Ward, 2001, 2002). The outcomes of the project include quantitative data on student achievement, and qualitative data on teacher capability.

Student achievement. Data from the NumPA interview have been collected for more than 300,000 students since 2000. This extensive data set provides valuable information about what constitutes reasonable expectations for student achievement and progress at particular year levels. The following overview reports data from over 73,000 Year 0 to 3 students. Table 2 presents the percentage of students at each stage on the strategy section of the Framework, at the initial and final interviews. A comparison of the initial and final data for each year level shows significant improvement. For example, the percentage of Year 2 students at the advanced counting stage or higher increased from 23% to 55%. The percentage of Year 3 students at the early additive part-whole stage or higher increased from 14% to 39%.

Other findings related to the student achievement data include:

- All students benefited from participation in the NDP, regardless of ethnicity, gender, and socio-economic status.
- Asian and European/Pakeha students began the project at higher stages on the Number Framework, and benefited more from participation in the project, than did students of Maori and Pacific Islands descent.
- Girls who began the project at lower framework stages appeared to make slightly better progress than boys who began at the same stage, but the opposite pattern was found at higher framework stages, with more boys progressing to a higher stage than girls.

- Students at schools in high socio-economic areas started the project at higher framework stages, and made larger gains over the course of the project, than did students at schools in low and medium socio-economic areas.
- Ethnicity, gender and socio-economic status had a combined effect on students' performance and progress. For example, being of Maori or Pasifika descent, being a girl, and attending a school in a low socio-economic area, was more disadvantageous than any one of those factors on its own.
- Analysis of the patterns of progress showed that even when starting point was taken into account, European / Pakeha and Asian students made greater progress on the Number Framework over the course of the project than did Maori or Pasifika students. Hence the project did not narrow the "achievement gap" as hoped but, instead, widened the gap slightly.

The NDP appears to have had a positive impact on students' mathematics achievement generally, not just on their number skills. This was evident in students' performance on tasks from the TIMSS study, which was better than that of their same-age peers in 1995, indicating that the project is doing more than simply accelerating students on the Number Framework.

Table 2
Percentage of Students at Each Stage on the Number Framework as a Function of Project Status (Initial or Final) and Year Level over 2002 and 2003.

Year level	0-1 (*n* = 20,207)		2 (*n* = 25,435)		3 (*n* = 27,908)	
Project Status	**Initial**	**Final**	**Initial**	**Final**	**Initial**	**Final**
0 Emergent	20.1	4.0	4.7	1.1	2.0	0.6
1 One-to-one counting	28.8	10.9	12.8	2.7	5.0	1.0
2 Counting from one on materials	40.7	44.4	44.6	21.7	21.6	7.1
3 Counting from one by imaging	7.0	20.3	14.6	18.4	11.6	8.5
4 Advanced counting	3.2	18.5	21.0	43.1	46.2	43.5
5 Early additive part-whole	0.1	1.9	2.2	12.4	12.7	34.7
6 Advanced additive part-whole	0.0	0.0	0.0	0.6	0.9	4.6

Teacher data. The professional development program was positively received by participating teachers, principals and facilitators in each year of the project since 2001. The teachers have reported developments in their professional knowledge as a result of their involvement in the project, and have noted changes in their classroom practices to accommodate their new knowledge and understandings (Thomas & Ward, 2001, 2002). They also noted increases in confidence and enthusiasm for mathematics teaching (Higgins, 2003; Thomas & Ward, 2001, 2002).

Sustaining the Project

The project is now focused on investigating issues related to the consolidation and maintenance of gains made within the project since its initial implementation. Aspects that are now being incorporated into the project include:

- the establishment of Numeracy Lead Teachers in each school;
- building communities of professional practice both within and outside the school context;
- aligning school management and classroom practice; and
- providing ongoing access to support from outside the school through, for example, facilitators' visits and access to online resources.

Concern about meeting the needs of teachers and students in remote and rural schools, as well as the issue of teacher mobility led to the development of a new (web-based) version of the professional development program which was trialed in 2003. The success of this web-based facilitation model led to its expansion in 2004 as a strategy for addressing a range of issues. In addition to this web-based resource, a web-based program for developing the professional practice of Numeracy Lead Teachers is to be trialed in 2004.

Common Themes and Implications

In this article, key components of three innovative and successful research and professional development programs in New Zealand and Australia have been outlined. The background contexts of the programs have been described, as have their key components.

It could be argued that most research in children's mathematical learning until the 1990s could be categorised as either large *n* and small *d* or small *n* and large *d*. By this, we mean that either the research involved large numbers of subjects, but the information gained on them was not particularly deep, or alternatively a considerable amount of information was gained about a small number of subjects. A feature of these three programs is that large numbers of students and teachers were involved, and yet the depth of the information on student learning and teacher growth was considerable. We now know so much more about what young children know and can do, and this information is already informing curriculum documents and classroom practices in New Zealand and Australia.

Although there are subtle but important differences between the programs in the grade level focus, the number of schools involved, the advocacy or otherwise of ability grouping, and the level of implementation across the three programs, several common features will be evident to the reader. These commonalities will now be discussed, as will the implications of these programs for other states, countries, and school systems that are considering implementing professional development and research programs in mathematics in the early years of schooling and beyond.

The Development and Use of Research-based Frameworks

The programs drew upon research in young children's mathematical learning to describe key stages or growth points in such development. Initially, the focus tended to be on the number domains, possibly for two reasons: the perceived importance of number in the curriculum, and the extent of the research base in this area (Kilpatrick, Swafford, & Findell, 2001).

In the case of number learning, the seminal work of Steffe and his colleagues provided a basis for these programs, and this has been complemented by the work of key researchers in Australia and New Zealand, such as Mulligan, Wright, and Young-Loveridge. In each program, the learning frameworks focus on a progression of increasingly sophisticated strategies in the particular mathematical domains. It is clear that, over time, teachers internalised and "owned" these frameworks, which provided a "lens" by which interactions with individuals, small groups and the whole class could be viewed.

The Use of Task-based, One-to-one Assessment Interviews

In New Zealand and Australia, the teaching of literacy in the early years of schooling has been characterised for a number of years by teachers working with individuals to assess their progress in reading. Count Me In Too in NSW was the first major systemic approach in Australia to provide an emphasis on the power of the assessment interview in numeracy. Increasingly widespread agreement on the limitations of pencil and paper testing in mathematics (e.g., Clements & Ellerton, 1995), particularly in the early years when issues of reading are particularly important, provided part of the impetus for the use of interviews with young children in mathematics.

Projects elsewhere, such as Cognitively Guided Instruction (Carpenter, Fennema, Franke, Levi, & Empson, 1999), also emphasised the benefits of sharing research with teachers, in a form that could be readily related to their classroom practices. Although one-to-one interviews are clearly demanding of teacher time, and therefore financially expensive, the experiences of teachers in these three programs have indicated that the benefits are considerable in terms of creating an understanding of what children know and can do in mathematics in general terms and for the teacher's own students, and in informing planning. The frameworks and interviews have also assisted to move the focus of professional development from the notion of children carefully reproducing taught procedures to an emphasis on children's thinking, with teachers as researchers.

Ongoing, Reflective Professional Development

We have known for a long time that the "one-shot" professional development program has little long-term effects (e.g., Clarke, 1994; Owen, Johnson, Clarke, Lovitt, & Morony, 1988). All three programs are based on a longer-term view of teacher growth and a view of teachers as reflective practitioners (Doyle, 1990; Schon, 1983) who can take research information from external sources and from their own children, reflect on it with colleagues, and make adjustments to planning for individuals and groups, with this iterative process continuing over an extended period of time. In this way teachers are viewed as "sense makers" constructing meaning in a social context, in the same way as their students do in their classrooms (Hiebert et al., 1997).

Another important feature of the professional development in these initiatives is the focus on whole school professional development, where the whole staff, at least for the relevant grade levels, is involved. Of course, it is not always the case that a program developed on a relatively small scale retains all of its key features when implemented more broadly. For example the Victorian ENRP (involving 70 schools) had this whole school focus, with all teachers involved in five days of central professional development each year. The implementation of the Victorian Early Numeracy Program by the Department of Education and Training, however, subsequently involved a "train-the-trainers" model, in which numeracy coordinators participated in a professional development program and were then expected to train the staff back at their schools. Such train the trainers programs can lead to a watering down of the professional development received by initial participants, with likely impacts on teacher professional growth and student learning.

Changes in Student Achievement

In each program, there are clear data showing that the knowledge and understanding of young children have been enhanced by the involvement of teachers and schools in the programs, and that such changes have persisted over time. These changes also have implications for curriculum and teaching in the middle and upper primary school. One such aspect is the traditional emphasis on the teaching of algorithms, which can have a negative impact on young children's thinking and number sense (e.g., Narode, Board, & Davenport, 1993).

Changes in Teacher Knowledge and Practice

For each program, evaluation data show that teachers identify considerable personal professional growth in their knowledge of children's learning in mathematics, and an understanding of how such growth can be facilitated. Teachers also commented on the growth in their personal mathematics understanding, and a greater sense of the connectedness of this understanding (Hiebert & Carpenter, 1992). Shulman (1986) claims that thinking properly about content knowledge "requires understanding the variety of ways in which the basic concepts and principles of the discipline are organized to incorporate its facts" (p. 9). This organization is important, for as Brophy (1991) indicated:

> Where (teacher) knowledge is more explicit, better connected, and more integrated, they will tend to teach the subject more dynamically, represent it in more varied ways, and encourage and respond fully to student comments and questions. Where their knowledge is limited, they will tend to depend on the text for content, de-emphasize interactive discourse in favor of seatwork assignments, and in general, portray the subject as a collection of static, factual knowledge. (p. 352)

Increased confidence in teaching mathematics and an enthusiasm for doing so are other important consequences of these projects. In terms of teaching practice, common themes include a greater focus on children's thinking, the greater use of hands-on approaches, and a capacity to cater more appropriately for the range of understanding evident in their children.

Conclusion

The experiences of these three Australian and New Zealand projects indicate that research-based learning frameworks, together with carefully-constructed interview tasks, embedded in whole-school professional development programs that view teachers as researchers who reflect on the results of their teaching actions and their observations of children, provide important ingredients in developing confident and capable mathematics learners in the early years of schooling.

> The teacher's role will no longer be to dispense "truth" but rather to guide the student in the conceptual organization of certain areas of experience. Two things are required for the teacher to do this: on the one hand, an adequate idea of where the student is and, on the other, an adequate idea of the destination. (von Glaserfeld, 1987, p. 16)

References

Aubrey, C. (1999). Children's early learning of number in school and out. In I. Thompson (Ed.), *Teaching and learning early number* (pp. 20–29). London: Open University Press.

Board of Studies NSW. (2002). *Mathematics K-6 syllabus*. Sydney: Author.

Bobis, J. (1996). Visualisation and the development of number sense with kindergarten children. In J. Mulligan & M. Mitchelmore (Eds.), *Children's number learning* (pp. 17–34). Adelaide: Australian Association of Mathematics Teachers and MERGA.

Bobis, J. (2003). *Count Me In Too: Evaluation of Stage 2*. Sydney: NSW Department of Education and Training.

Bobis, J., & Gould, P. (2000). Changing the professional knowledge of teachers. In J. Bana & A. Chapman (Eds.), *Mathematics education beyond 2000* (Proceedings of the 23rd annual conference of the Mathematics Education Research Group of Australasia, Fremantle, pp. 47–54). Sydney: MERGA.

Brophy, J. E. (1991). Conclusion to advances in research on teaching, Vol. II: Teachers' knowledge of subject matter as it relates to teaching practice. In J. Brophy (Ed.), *Advances in research on teaching: Vol. 2 Teachers' subject matter knowledge and classroom instruction* (pp. 347–362). Greenwich, CT: JAI Press.

Carpenter, T. A., Fennema, E., Franke, M. L., Levi, L., & Empson, S. B. (1999). *Children's mathematics: Cognitively guided instruction*. Portsmouth, NH: Heinemann.

Carpenter, T., & Lehrer, R. (1999). Teaching and learning mathematics with understanding. In E. Fennema & T. Romberg (Eds.), *Mathematics classrooms that promote understanding* (pp. 19–32). Mahwah, NJ: Lawrence Erlbaum.

Christensen, I. (2003). *An evaluation of Te Putama Tau 2002: Exploring issues in mathematics education*. Wellington: Ministry of Education.

Clarke, D. M. (1994). Ten key principles from research for the professional development of mathematics teachers. In D. B. Aichele & A. F. Croxford (Eds.), *Professional development for teachers of mathematics.* (1994 Yearbook of the National Council of Teachers of Mathematics, pp. 37–48). Reston, VA: NCTM.

Clarke, D. M. (2004). Some issues in the teaching of algorithms in primary schools. In B. Clarke, D. M. Clarke, F. K. Lester, & D. Lambdin (Eds.), *International perspectives on mathematics teaching and learning* (pp. 21–36). Göteborg, Sweden: Nationellt Centrum för Matematikutbildning.

Clarke. D. M., & Cheeseman, J., (2000). Some insights from the first year of the early numeracy research project. *Improving numeracy learning: Proceedings of the ACER Research Conference 2000* (pp. 6–10), Brisbane: Australian Council for Educational Research.

Clarke, D. M., Cheeseman, J., McDonough, A., & Clarke, B. A. (2003). Assessing and developing measurement with young children. In D. H. Clements & G. W. Bright (Eds.), *Learning and teaching measurement* (65th Yearbook of the National Council of Teachers of Mathematics, pp. 68–80). Reston, VA: NCTM.

Clarke, D. M., & Clarke, B. A. (2004). Mathematics teaching in Grades K-2: Painting a picture of challenging, supportive, and effective classrooms. In R. N. Rubenstein & G. W. Bright (Eds.), *Perspectives on the teaching of mathematics* (66th Yearbook of the National Council of Teachers of Mathematics, pp. 67–81). Reston, VA: NCTM.

Clements, D. H., Swaminathan, S., Hannibal, M. A. Z., & Sarama, J. (1999). Young children's conceptions of space. *Journal for Research in Mathematics Education*, *30*(2), 192–212.

Clements, M. A., & Ellerton, N. (1995). Assessing the effectiveness of pencil-and-paper tests for school mathematics. In B. Atweh & S. Flavel (Eds.), *Galtha: MERGA 18*: (Proceedings of the 18th Annual Conference of the Mathematics Education Research Group of Australasia, pp. 184–188). Darwin, NT: University of the Northern Territory.

Cobb, P. (2003). Investigating students' reasoning about linear measurement as a paradigm case of design research. In M. Stephan, J. Bowers, P. Cobb, & K. Gravemeijer (Eds.), *Supporting students' development of measuring conceptions: Analyzing students' learning in social contexts* (pp. 1–16), Reston: NCTM.

Cobb, P., Gravemeijer, K., Yackel, E., McClain, K., & Whitenack, J. (1997). Mathematizing and symbolizing: The emergence of chains of signification in one first-grade classroom. In D. Kirshner & J. A. Whitson (Eds.), *Situated cognition theory: Social, semiotic, and neurological perspectives* (pp. 151–233). Mahwah: Lawrence Erlbaum.

Cobb, P., & Wheatley, G. (1988). Children's initial understandings of ten. *Focus on Learning Problems in Mathematics, 10*(3), 1–36.

Department of Education & Training. (2001). *Early numeracy interview booklet*. Melbourne: Author.

Doyle, W. (1990). Themes in teacher education research. In W. Hewson (Ed.), *Handbook of research on teacher education* (pp. 3–24). New York: Macmillan.

Fosnot, C., & Dolk, M. (2001). *Young mathematicians at work: Constructing number sense, addition and subtraction*. Portsmouth: Heinemann.

Fraivillig, J., Murphy, L., & Fuson, K. (1999). Advancing children's mathematical thinking in everyday mathematics classrooms. *Journal for Research in Mathematics Education, 30*(2), 148–170.

Fullan, M., & Hargreaves, A. (1992). *Teacher development and educational change*. London: Falmer Press.

Fuson, K. (1992). Research on whole number addition and subtraction. In D. Grouws (Ed.), *Handbook of research on mathematics teaching and learning* (pp. 243–275). New York: Macmillan.

Gervasoni, A. (2003). Key transitions in counting development for young children who experience difficulty. In N. Pateman, B. Dougherty, & J. Zilliox (Ed.), *Proceedings of the 2003 joint meeting of PME and PMENA* (Vol. 2, pp. 421–427). Hawaii: PME.

Ginsburg, H., Klein, A., & Starkey, P. (1998). The development of children's mathematical thinking: Connecting research with practice. In I. E. Sigel & K. A. Renninger (Eds.), *Handbook of child psychology, 5th ed., Vol. 4: Child psychology in practice* (pp. 401–476). New York: John Wiley & Sons.

Gould, P. (2000). Count Me In Too: Creating a choir in the swamp. In *Improving numerary learning: What does the research tell us?* (Proceedings of the ACER Research Conference 2000, pp. 23–26) Melbourne: Australian Council for Educational Research.

Gravemeijer, K. (1994). Educational development and developmental research in mathematics education, *Journal of Research in Mathematics Education, 25*(5) 443–471.

Hiebert, J., & Carpenter, T. (1992). Learning and teaching with understanding. In D. Grouws (Ed.), *Handbook of research on mathematics teaching and learning* (pp. 65–97). New York: Macmillan.

Hiebert, J., Carpenter, T., Fennema, E., Fuson, K., Wearne, D., Murray, H., et al. (1997). *Making sense: Teaching and learning mathematics with understanding*. Portsmouth: Heinemann.

Higgins, J. (2003). *An evaluation of the Advanced Numeracy Project 2002: Exploring issues in mathematics education*. Wellington: Ministry of Education.

Higgins, J., Parsons, R., & Hyland, M. (2003). The Numeracy Development Project: Policy to practice. In J. Livingstone (Ed.), *New Zealand annual review of education*. (pp 157–174). Wellington: Victoria University of Wellington.

Horne, M., & Rowley G. (2001). Measuring growth in early numeracy: Creation of interval scales to monitor development. In M. van den Heuvel-Panhuizen (Ed.), *Proceedings of the 25th conference of the International Group for the Psychology of Mathematics Education* (pp. 3–161–3–167). Utrecht, The Netherlands: PME.

Irwin, K. (2003). *An evaluation of the Numeracy Project for Years 7–10, 2002*. Wellington: Ministry of Education.

Irwin, K., & Niederer, K. (2002). *An evaluation of the Numeracy Exploratory Study (NEST) and the associated Numeracy Exploratory Study Assessment* (NESTA) years 7–10, 2001. Wellington: Ministry of Education.

Jones, G., Thornton, C., Putt, I., Hill, K., Mogill, A., Rich, B., & Van Zoest, L. (1996). Multidigit number sense: A framework for instruction and assessment. *Journal for Research in Mathematics Education, 27*(3), 310–36.

Kilpatrick, J., Swafford, J., & Findell, B. (2001). *Adding it up: Helping children learn mathematics*. Washington: National Academy Press.

Lehrer, R., & Chazan, D. (1998). *Designing learning environments for developing understanding of geometry and space*. Mahwah: Lawrence Erlbaum.

McClain, K., & Cobb, P. (2001). An analysis of development of sociomathematical norms in one first–grade classroom. *Journal for Research in Mathematics Education, 32*(3), 236–266.

McDonough, A., Clarke, B. A., & Clarke, D. M. (2002). Understanding, assessing and developing young children's mathematical thinking: The power of the one-to-one interview for preservice teachers in providing insights into appropriate pedagogical practices. *International Journal of Education Research, 37*, 107–112.

McIntosh, A., Bana, J., & Farrell, B. (1995). *Mental computation in school mathematics: Preference, attitude and performance of students in years 3, 5, 7, and 9* (MASTEC Monograph Series No. 1). Perth: Mathematics, Science & Technology Education Centre, Edith Cowan University.

Ministry of Education (2001). *Curriculum update: The numeracy story*. No. 45. Wellington: Learning Media.

Ministry of Education. (2002). *Curriculum update: Literacy. No. 50*. Wellington: Learning Media.

Ministry of Education. (2004). *The Number Framework: Teachers' material*. Wellington: Author.

Ministry of Education. (n.d.). *New Zealand Numeracy Project material*. Retrieved May 30, 2004, from http://www.nzmaths.co.nz/Numeracy/project_material.htm

Mulligan, J. T., & Mitchelmore, M. C. (1997). Young children's intuitive models of multiplication and division. *Journal for Research in Mathematics Education, 28*, 309–330.

Narode, R., Board, J., & Davenport, L. (1993). Algorithms supplant understanding: Case studies of primary students' strategies for double-digit addition and subtraction. In J. R. Becker & B. J. Preece (Eds.), *Proceedings of the 15th Annual Meeting of the North American chapter of the International Group for the Psychology of Mathematics Education* (Vol. 1, pp. 254–260). San Jose: Center for Mathematics and Computer Science Education, San Jose State University.

National Center for Educational Statistics. (n.d.). *TIMSS results*. Retrieved November 12, 2004, from http://nces.ed.gov/timss/results.asp

Newman, R. S. (1984). Children's achievement and self-evaluations in mathematics: A longitudinal study. *Developmental Psychology, 76*, 857–873.

NSW Department of Education and Training (2003). *Count Me In Too: Professional development package,* Sydney: Author.

Outhred, L., Mitchelmore, M., McPhail, D., & Gould, P. (2003). Count Me into Measurement: A program for the early elementary school. In D. Clements & G. Bright (Eds.), *Learning and teaching measurement*. (65th Yearbook of the National Council of Teachers of Mathematics, p. 81–99), Reston, VA: NCTM.

Owen, J., Johnson, N., Clarke, D. M., Lovitt, C., & Morony, W. (1988). *Guidelines for consultants and curriculum leaders*. Melbourne: Curriculum Corporation.

Owens, K., Reddacliff, C., Gould, P., & McPhail, D., (2001). Changing the teaching of space mathematics. In J. Bobis, B. Perry, & M. Mitchelmore (Eds). *Numeracy and beyond*. (Proceedings of the 24th annual conference of the Mathematics Education Research Group of Australasia, pp. 402–409). Sydney: MERGA.

Perry, B., & Howard, P. (2001). Counting On: A systemic program for year 7 students who have experienced difficulty with mathematics. In J. Bobis, B. Perry, & M. Mitchelmore (Eds.), *Numeracy and beyond*. (Proceedings of the 24th annual conference of the Mathematics Education Research Group of Australasia, pp. 410–417). Sydney: MERGA.

Pirie, S., & Kieran, T. (1989). A recursive theory of mathematical understanding. *For the Learning of Mathematics, 9*, 7–11.

Schon, D.(1983). *The reflective practitioner: How professionals think in action*. New York: Basic Books.

Shulman, L. (1986). Those who understand: Knowledge growth in teaching. *Educational Researcher, 15*, 4–14.

Simon, M. (1995). Reconstructing mathematics pedagogy from a constructivist perspective. *Journal for Research in Mathematics Education, 26*, 114–145.

Steffe, L. (1992). Learning stages in the construction of the number sequence. In J. Bideaud, C. Meljac, & J. Fischer (Eds.), *Pathways to number: Children's developing numerical abilities* (pp. 83–88). Hillsdale: Lawrence Erlbaum.

Steffe, L., Cobb, P., & von Glasersfeld, E. (1988). *Construction of arithmetical meanings and strategies*. New York: Springer-Verlag.

Stephens, M. (2000). *Identification and evaluation of teaching practices that enhance numeracy achievement*. Retrieved May 30, 2004 from http://www.aamt.edu.au/projects/numeracy_papers/stephens.pdf

Sulllivan, P., Clarke, D. M., Cheeseman, J., & Mulligan, J. (2001). Moving beyond physical models in learning multiplicative reasoning. In M. van den Heuvel-Panhuizen (Ed.), *Proceedings of the 25th annual conference of the International Group for the Psychology of Mathematics Education* (Vol. 4, pp. 233–240). Utrecht, The Netherlands: PME.

Thomas, G., Tagg, A., & Ward, J. (2003). *An evaluation of the Early Numeracy Project 2002: Exploring issues in mathematics education*. Wellington: Ministry of Education.

Thomas, G., & Ward, J. (2001). *An evaluation of the Count Me In Too Pilot Project*. Wellington, New Zealand: Ministry of Education.

Thomas, G., & Ward, J. (2002). *An evaluation of the Early Numeracy Project 2001: Exploring issues in mathematics education*. Wellington: Ministry of Education.

Von Glasersfeld, E. (1987). Learning as a constructive activity. In C. Janvier (Ed.), *Problems of representation in the teaching and learning of mathematics* (pp. 3–17). Hillsdale: Lawrence Erlbaum.

White, P., & Mitchelmore, M. (2001). *The impact of Count Me In Too on year 3 Basic Skills Test numeracy scores*. Sydney: NSW Department of Education and Training.

Wilson, P. S., & Osborne, A. (1992). Foundational ideas in teaching about measure. In T. R. Post (Ed.), *Teaching mathematics in grades K-8: Research-based methods* (pp. 89–122). Needham Heights, MA: Allyn & Bacon.

Wright, R. J. (1994). A study of the numerical development of 5-year-olds and 6-year-olds. *Educational Studies in Mathematics, 26*, 25–44.

Wright, R. J. (1998). An overview of a research-based framework for assessing and teaching early number learning. In C. Kanes, M. Goos, & E. Warren (Eds.), *Teaching mathematics in new times* (Proceedings of the 21st annual conference of the Mathematics Education Research Group of Australasia, pp. 701–708). Brisbane: MERGA.

Wright, R. J. (2000). Professional development in recovery education. In L. Steffe & P. Thompson (Eds.), *Radical constructivism in action: Building on the pioneering work of Ernst von Glasersfeld* (pp. 135–151). London: Falmer.

Wright, R. J. (2003). Mathematics Recovery: A program of intervention in early number learning. *Australian Journal of Learning Disabilities, 8*(4), 6–11.

Wright, R. J., Martland, J., & Stafford, A. (2000). *Early numeracy: Assessment for teaching and intervention*. London: Paul Chapman Publications / Sage.

Wright, R. J., Martland, J., Stafford, A., & Stanger, G. (2002). *Teaching number: Advancing children's skills and strategies*. London: Paul Chapman Publications / Sage.

Wright, R. J., Stanger, G., Cowper, M., & Dyson, R. (1996). First-graders' progress in an experimental mathematics recovery program. In J. Mulligan & M. Mitchelmore (Eds.), *Children's number learning* (pp. 55–72). Adelaide: AAMT and MERGA.

Young-Loveridge, J. (1991). *The development of children's number concepts from ages five to nine. Early Mathematics Learning Project: Phase II. Volume I: Report of findings*. Hamilton, New Zealand: University of Waikato.

Young-Loveridge, J. (2004). Effects on early numeracy of a program using number books and games. *Early Childhood Research Quarterly, 19*, 82–98.

Young-Loveridge, J., Peters, S., & Carr, M. (1998). Enhancing the mathematics of four-year-olds: An overview of the EMI-4s study. *Journal of Australian Research in Early Childhood Education, 1*, 82–93.

Young-Loveridge, J., & Wright, V. (2002). Validation of the New Zealand Number Framework. In B.Barton, K. Irwin, M. Pfannkuch, & M. Thomas (Eds.), *Mathematics Education in the South Pacific* (Proceedings of the 25th annual conference of the Mathematics Education Research Group of Australasia, Auckland, pp 722–729). Sydney: MERGA.

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