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Suppressing the formation of dwarf galaxies via photoionization

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SUMMARY

In hierarchical clustering theories, some sort of feedback mechanism is required to prevent most of the baryonic material collapsing into subgalactic objects at high redshifts. We argue that a photoionizing background, of the magnitude suggested by applying the Gunn–Peterson constraint to high-redshift quasars, would strongly suppress the cooling of a hydrogen–helium plasma and so inhibit the formation of dwarf galaxies. The effectiveness of this mechanism depends on the spectrum of the photoionizing radiation and so galaxy formation could depend on the proximity of protogalactic perturbations to unusual sources of hard photons such as luminous quasars. This could introduce large-scale spatial variations in the galaxy distribution.

Key words: plasmas – galaxies: formation – quasars: general – diffuse radiation – large-scale structure of Universe.

1 INTRODUCTION

Explaining the masses and sizes of galaxies remains one of the most important problems in cosmology. In the context of hierarchical clustering models, the cooling arguments of Rees & Ostriker (1977) and others (e.g. Hoyle 1953; Binney 1977; Silk 1977) clearly play a key role in setting an *upper* bound to the mass of a galaxy. However, the mechanisms involved in the formation of dwarf galaxies and in determining the shape of the galaxy luminosity function (especially at low luminosities) are not well understood.

According to the standard folklore, gas in 'mini-haloes' (i.e. with circular speeds $\lesssim 100$ km s^{-1}) can cool from the virial temperature $kT_v \sim m_p v_c^2$ on a time-scale that is short compared to the dynamical time. Unless this cooling is suppressed, or there is a source of heating to balance radiative losses, most of the baryonic material would collapse into objects that are much smaller than a typical L_* galaxy like our own. Thus, various authors (White & Rees 1978; Dekel & Silk 1986; Cole 1991; White & Frenk 1991; Lacey & Silk 1991) have invoked heating by supernovae to suppress the formation of small galaxies, and even then the shape of the galaxy luminosity function turns out to be too steep at the faint end compared with observations of field galaxies. However, in all of these discussions of cooling, the role of photoionizing radiation at the time of galaxy formation has been ignored. This is questionable given recent evidence that the intergalactic medium is highly ionized out to redshifts $z \sim 5$ (Webb *et al.* 1992).

In this Short Communication, we calculate how the cooling of a hydrogen–helium plasma is affected by a photoionizing background and we sketch how this might influence galaxy formation. Where details of the fluctuation spectrum are required, we have assumed the power spectrum of the standard adiabatic Cold Dark Matter (CDM) model (Bond & Efstathiou 1984, equation 6) with total cosmological density $\Omega = 1$, baryon density $\Omega_b \sim 0.1$ and Hubble constant $h_{50} = 1$.* The amplitude of the power spectrum is normalized so that the variance of the mass fluctuations in spheres of radius 8 h^{-1} Mpc is $\sigma^2 = 0.35\ (1.7/b)^2$ at the present day, where b is a 'biasing factor' (e.g. Davis *et al.* 1985).

2 THE PHOTOIONIZING FLUX AT HIGH REDSHIFT

A uniform intergalactic medium of neutral hydrogen with density $\Omega_{\rm H_I} = \xi \Omega_{\rm I}$ gives an optical depth

$$\tau(z) \approx 1.9 \times 10^4\, \xi (\Omega_{\rm I}/0.1)\, h_{50} (1+z)^{3/2} \tag{1}$$

for the absorption of photons in a quasar spectrum shortward of Lyα (Gunn & Peterson 1965, hereafter GP). If the intergalactic ionizing flux is $J(\nu) = J_{-21}(z) \times 10^{-21} (\nu_{\rm L}/\nu)^{\alpha}$ erg cm^{-2} sr^{-1} Hz^{-1} s^{-1}, where $\nu_{\rm L}$ is the Lyman limit, and we assume that the neutral fraction ξ is fixed by ionization equilibrium at a temperature $T \sim 40\,000$ K, equation (1) gives

$$\tau(z) \approx 4.8 \times 10^{-5} J_{-21}(z)^{-1} (3+\alpha) (\Omega_{\rm I}/0.1)^2 h_{50}^3 (1+z)^{9/2}. \tag{2}$$

In the CDM model, a high fraction of the total baryon density could be nearly uniform at redshifts $z \gtrsim 5$, since a gas at 10^4 K (below which cooling by hydrogen line radiation is

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* $H_0 = 50\ h_{50}$ km s^{-1} Mpc^{-1}.

1992MNRAS.256P..43E

ineffective) cannot clump into mini-haloes with circular speeds

$v_c \lesssim (2kT/\mu m_p)^{1/2} \sim 17(T/10^4\,\mathrm{K})^{1/2}\ \mathrm{km\,s^{-1}}$.

Adopting the Press-Schechter (1974) model relating halo formation to the linear Gaussian density field (see e.g. Efstathiou *et al.* 1988), we find that the fraction of baryonic material associated with haloes of circular speed $v_c \le 17$ km s^{-1} is

$$f_1(z) \approx \frac{2}{\sqrt{\pi}} \int_0^{s_{\mathrm{crit}}} \exp(-x^2)\,dx = \mathrm{erf}(s_{\mathrm{crit}}), \qquad (3a)$$

where

$$s_{\mathrm{crit}} \approx 0.17(1+z)^{0.91}(v_c/17\ \mathrm{km\,s^{-1}})^{0.175}, \qquad (3b)$$

and we have assumed that the comoving initial radius of a halo is related to its circular speed by

$x_h = 0.2(1+z)^{-1/2}(v_c/17\ \mathrm{km\,s^{-1}})\,h_{50}^{-1/3}$ Mpc.

For redshifts less than ~ 5, where $f_1(z) \lesssim 1$, equation (3b) can be approximated to reasonable accuracy by $f_1(z) \approx s_{\mathrm{crit}}$. The spectra of high-redshift quasars give $\tau(z) \lesssim 0.02$ at $z \sim 2.6$ (Steidel & Sargent 1987) and $\tau(z) \lesssim 0.1$ at $z \sim 4.1$–4.7 (Webb *et al.* 1992; Jenkins & Ostriker 1991). The GP limits at $z \sim 2.6$ are consistent with equations (2) and (3) provided that $J_{-21} \gtrsim 0.9$ for $\alpha = 1$, which is similar to the value $J_{-21} \sim 1$ expected from quasars at these redshifts (Bechtold *et al.* 1987) and deduced from the inverse effect (Bajtlik, Duncan & Ostriker 1988). However, to satisfy the GP constraint at $z \sim 5$ requires

$$J_{-21} \gtrsim 5(\Omega_b/0.1)^2[(1+z)/6]^{6.3}, \qquad [\tau(z) < 0.1, \quad \alpha = 1]. \qquad (4)$$

Notice first that the very steep dependence of J_{-21} on redshift in equation (4) implies a sharp onset of GP absorption unless the photoionizing flux rises rapidly with increasing redshift. Secondly, the value of J_{-21} required at $z \sim 5$ is well above the intensity expected from known high-luminosity quasars (Bechtold *et al.* 1987), and so other sources are required to supply the photoionizing flux, e.g. low-luminosity active galactic nuclei (AGN) or star-forming galaxies (e.g. Songaila, Cowie & Lilly 1990).

3 IMPLICATIONS FOR GALAXY FORMATION

3.1 Cooling rates

The cooling of a plasma with primordial abundance of helium (here assumed to be $Y = 0.24$ by mass) has been calculated by several authors on the assumption that the gas is collisionally ionized (see, e.g. Rees & Ostriker 1977; Fall & Rees 1985). The cooling curve for this case, computed from the rates given in Black (1981), is shown as the solid line in Fig. 1. In the presence of a photoionizing background, the cooling rate depends both on the spectrum of the radiation and on the density of the gas. The dashed lines in Fig. 1 show rates for various spectral indices and photoionizing fluxes for gas at an overdensity $\Delta = 1$ at $z = 5$ [where $\rho_b = \bar{\rho}_b(z)\Delta$ and $\bar{\rho}_b(z)$ is the mean baryon density at redshift z]. Clearly, reasonable values of the photoionizing flux can produce large reductions in the cooling function over a wide range of temperature (as has been noted by J. P. Ostriker, private communication). The two prominent peaks in the cooling curve for $J(\nu) = 0$ arise from collisional excitation of H and He$^+$; these sources of cooling are ineffective if the gas is highly photoionized. In the limit of a high photoionizing flux, cooling from radiative recombinations gives $\Lambda/n_H^2 \sim kT\alpha_R(T) \sim 10^{-24}$ erg cm^3 s^{-1} for temperatures of $T \sim 10^5$ K, where α_R is the recombination rate for H or He$^+$, which is of the same order as cooling from bremsstrahlung at these temperatures. This is why the curves for a 'hard' spectrum ($\alpha = 1$) in Fig. 1 are so similar, even though $J(\nu_L)$ differs by an order of magnitude, and why they have values of $\sim 10^{-24}$ and rise slowly with increasing temperature. The rates of heating (Γ) by photoionization are shown by the dot-dashed curves in Fig. 1.

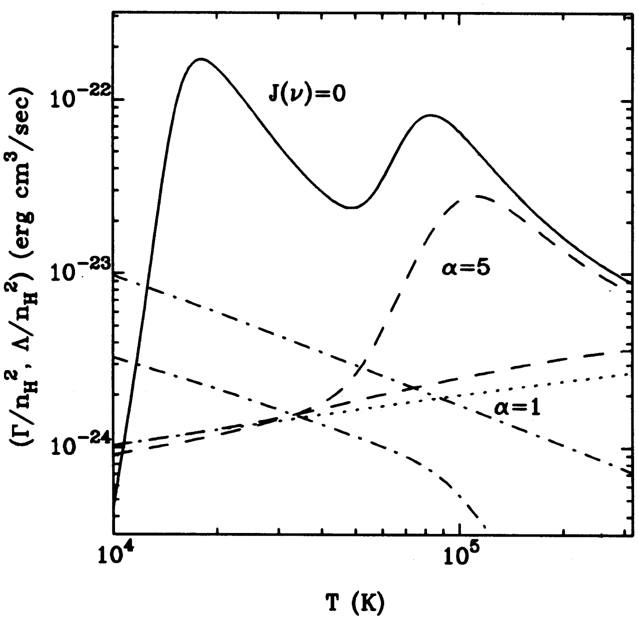

Figure 1. Heating and cooling rates as a function of temperature for a hydrogen–helium gas in which helium contributes 24 per cent by mass. The solid curve shows the cooling curve in the absence of a photoionizing background. The dotted and dashed curves show cooling curves for gas with density $n_H = 4.9 \times 10^{-5}$ cm^{-3} (i.e. $\Delta = 1$ at $z = 5$ for $\Omega_B = 0.1$, $h_{50} = 1$) for a photoionizing flux of $J(\nu) = J_{-21} \times 10^{-21}(\nu_L/\nu)^\alpha$ erg cm^{-2} sr^{-1} Hz^{-1} s^{-1}. The dashed lines show $J_{-21} = 1$ for $\alpha = 1$ and 5 and the dotted line shows $J_{-21} = 10$ for $\alpha = 1$. The dot–dashed lines show the rates of heating by photoionization for $J_{-21} = 1$ and $\alpha = 1$ (upper curve) and $\alpha = 5$ (lower curve).

3.2 Cooling times

Fig. 1 shows that a photoionizing background can significantly lengthen the cooling time of low-density gas and hence slow down, or prevent, the formation of dwarf galaxies. We define the cooling time t_{cool} for the gas:

$$t_{\mathrm{cool}} = E\left(\frac{dE}{dt}\right)^{-1} = \frac{3}{2}\,\frac{m_p}{\mu(1-Y)^2\rho_b\Delta}\,\frac{kT}{L(T,\Delta)}, \qquad (5)$$

where $L(T,\Delta) = \Lambda(T,\Delta)/n_H^2$ and n_H is the number density of hydrogen atoms [we use units of erg cm^3 s^{-1} for $L(T,\Delta)$ in the equations below] and μ is the mean molecular weight

(≈ 0.6). Dividing $t_{\rm cool}$ by the Hubble time $t_{\rm H} = (6\pi G\bar{\rho}_{\rm c})^{1/2}$, we obtain

$$\frac{t_{\rm cool}}{t_{\rm H}} \approx \frac{52}{\Delta(1+z)^{3/2}} \left(\frac{T}{10^4\ {\rm K}}\right) \left[\frac{10^{-24}}{L(T,\Delta)}\right] \left(\frac{0.1}{\Omega_{\rm b}}\right) h_{50}^{-1}. \qquad (6)$$

The solid curves in Fig. 2 show $t_{\rm cool}/t_{\rm H} = 1$ in the (T, Δ) plane for various values of J_{-21}, z and α. The dotted lines show the equilibrium temperature determined by thermal balance between cooling and heating by photoionization. These lie in the range $T_{\rm equ} \sim 10^4$–10^5 K for a wide range of parameters and so if the gas attains temperatures of this order, it will be prevented from collapsing into haloes with circular speeds less than $v_{\rm equ} \sim 20$–50 km s^{-1}. Haloes with circular speeds $\sim v_{\rm equ}$ could trap gas at moderate density contrasts and would be visible as Lyα absorption lines in quasar spectra (Rees 1986). For haloes with higher circular speeds, Fig. 2 shows that gas at low density contrasts will collapse quasi-statically if the photoionizing flux is sufficiently high and the spectrum sufficiently hard. As the density contrast rises, the optical depth to UV photons rises until the gas becomes self-shielding at a critical density contrast

$$\Delta(\tau=1) \approx 180 \left(\frac{1+z}{6}\right)^{-3} \left(\frac{J_{-21}}{3+\alpha}\right)^{3/5} \left(\frac{M_{\rm B}}{10^{11}\ M_\odot}\right)^{-1/5} \times \left(\frac{\Omega_{\rm B} h_{50}^2}{0.1}\right)^{-1}; \qquad (7)$$

the cooling rate then rises rapidly and the gas collapses on a free-fall time-scale.

It is possible that a photoionizing background could inhibit the formation of galaxies with circular speeds greater than $v_{\rm equ}$. In the limit of an adiabatic gas collapsing from temperature $T_1 \sim T_{\rm equ}$ and density $\Delta_1 \sim 1$, the temperature will rise as $T_2 \sim (\Delta_2/\Delta_1)^{2/3} T_1$, requiring a halo with circular speed $\sim (200)^{1/3} v_{\rm equ}$ if the gas is to collapse and form a neutral core. In reality, the gas will be able to cool and so the temperature will rise less steeply than suggested by this argument. A somewhat more realistic picture of how the gas behaves during collapse is given by the energy equation

$$\frac{d}{dt}[\ln(c_{\rm s}^2/\rho_{\rm b}^{2/3})] = \frac{2}{3}\,\frac{(\Gamma-\Lambda)}{\rho_{\rm b} c_{\rm s}^2}, \qquad (8)$$

where $c_{\rm s}^2 = (kT/\mu m_{\rm p})$ and we have assumed that $\rho_{\rm b}$ evolves according to the collapse of a uniform pressureless spherical perturbation in an $\Omega = 1$ universe:

$$\rho(\theta) = \frac{9\pi^2}{2}\,\frac{\bar{\rho}(t_{\rm m})}{(1-\cos\theta)^3}, \qquad \frac{t}{t_{\rm m}} = \frac{(\theta - \sin\theta)}{\pi} \qquad (9)$$

(Peebles 1980, section 19) where $t_{\rm m}$ is the time at which the sphere reaches maximum expansion. Fig. 3 shows the evolution of the temperature with density for two values of $\Omega_{\rm b}$

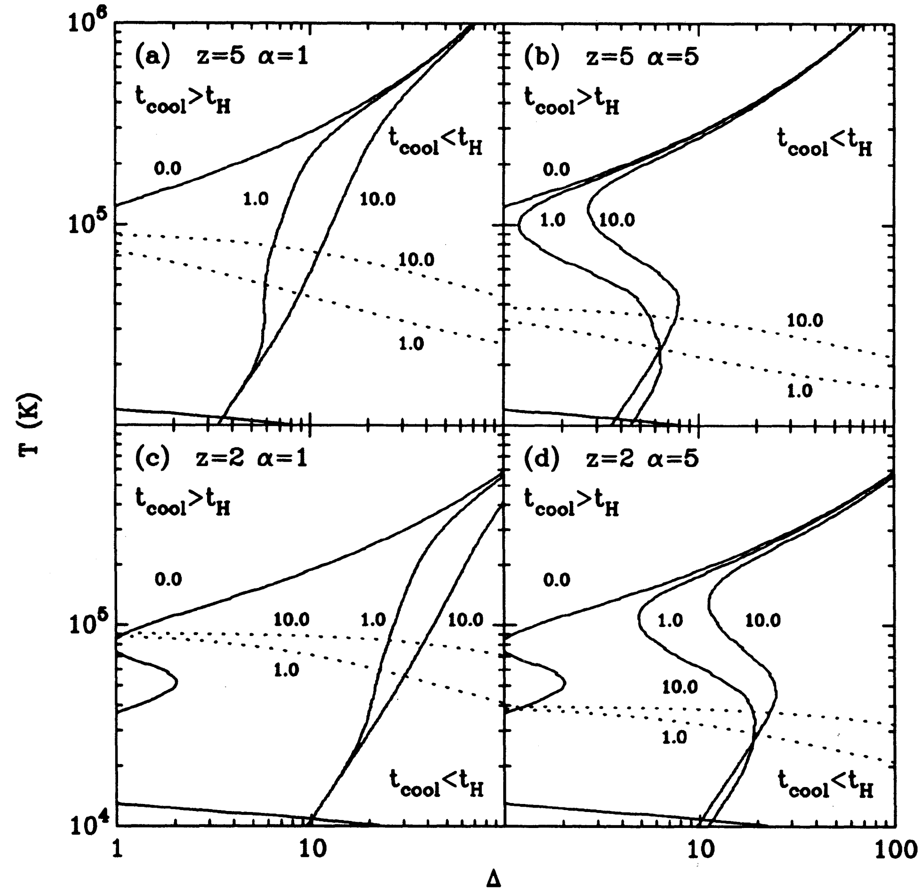

Figure 2. The solid lines show contours in the T–Δ plane where $t_{\rm cool}/t_{\rm H} = 1$ for various redshifts and spectral indices α (where we have ignored self-shielding of UV radiation). The dotted lines show the equilibrium temperature where cooling losses balance heating by photoionization. The numbers next to each curve give J_{-21}. We have assumed $\Omega_{\rm B} = 0.1$ and $h_{50} = 1$.

assuming that the photoionizing background evolves as

$$J_{-21}(z)=\frac{10}{1+[5/(1+z)]^4} \tag{10}$$

and that $T=10^5$ K when $\Delta=1$ (see e.g. Terasawa 1992). Fig. 3 shows that the temperature drops as clouds expand and then rises during collapse until cooling becomes important and the temperature drops. The tracks followed in Fig. 3 depend on the initial adiabat of the gas, because the cooling and heating time-scales are long compared to a Hubble time. This figure shows that the temperature can indeed rise to values well above $T_{\rm equ}$ if the redshift of maximum expansion is relatively low ($\lesssim 2$). These results are clearly sensitive to the mean temperature of the intergalactic medium, which could be $\sim 10^4$ K if it is photoionized slowly (*cf.* Miralda-Escudé & Ostriker 1990). Clearly, these models are highly simplified and should be taken only as an indication of what might happen. To check how effectively this mechanism can suppress the formation of low-mass galaxies we require more detailed numerical calculations of dissipative collapses (e.g. Katz & Gunn 1991) that include photoionizing radiation and radiative transfer (see Cen 1992).

3.3 Supernova-driven winds

As mentioned in the Introduction, energy injection from supernovae has been favoured as a mechanism for suppressing the formation of dwarf galaxies in hierarchical clustering models. The process has been described by Dekel & Silk (1986) and, more recently, by Babul & Rees (1992). The cooling arguments given above may have interesting implications for wind models, for they suggest that protogalaxies would develop strong density gradients during the early stages of collapse. Most of the star formation in a forming galaxy could therefore be confined to a high-density core while the bulk of the gas is at lower densities collapsing quasi-statically. From the analysis given in Babul & Rees (1992), a wind with speed $V_{100}\times 100$ km s^{-1} will propagate out to a radius $R_{\rm c}$ before it is confined by the pressure of the gas in the halo:

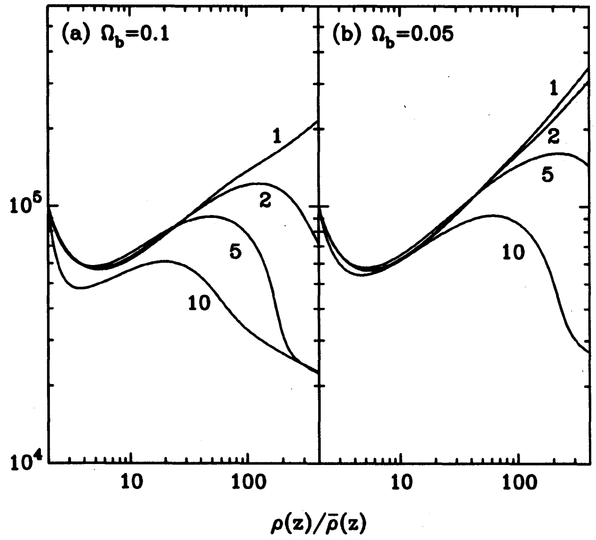

Figure 3. The evolution of the temperature with density in the spherical collapse models described in the text (computed from equations 8–10). The numbers next to each curve give the redshift of maximum expansion and in each case we assumed a temperature $T=10^5$ K when $\Delta=1$.

$$\frac{R_{\rm c}}{R_{\rm H}}\approx(\eta_{0.1}V_{100}\dot M_*)^{1/2}\left(\frac{10^2\ {\rm cm^{-3}\,K}}{n_{\rm b}T}\right)^{1/2}\left(\frac{1+z}{6}\right)^{3/4} \times\left(\frac{20\ {\rm km\,s^{-1}}}{v_{\rm c}}\right)h_{50}^{-1}, \tag{11}$$

where $R_{\rm H}$ is the radius of the halo where $\Delta=200$, $\dot M_*$ is the star formation rate in $M_\odot$ yr^{-1} and $n_{\rm b}T$ is the pressure of the gas in the halo. In deriving (11) we have assumed one supernova of energy 10^{51} erg per 100 $M_\odot$ of star formation and that a fraction $\eta_{0.1}=0.1$ of the supernova energy goes into the wind (η is likely to be a complex function of $v_{\rm c}$, z, $\dot M_*$ and other parameters). Equation (11) suggests that low rates of star formation ($\lesssim 1$ $M_\odot$ yr^{-1}) could drive off the low-pressure quasi-statically collapsing gas in a dwarf galaxy halo. Thus, dwarf galaxies driving winds could be extremely faint [e.g. a galaxy with $\dot M_*\sim 1$ $M_\odot$ yr^{-1} at $z\sim 2$ has a B-band magnitude $m_{\rm AB}\lesssim 28$ (Cowie 1988)], and only a small fraction of the protogalactic gas need collapse to a high-density star-forming core to drive the rest of the gas from the halo.

3.4 Spatial modulations

There are few constraints on the spectrum of the photoionizing radiation. At redshifts $z\sim 2$, quasars may account for most of the photoionizing flux and so the spectrum could be hard, with $\alpha\approx 1$ (but see Madau 1992). We have even less idea of the form of the UV spectrum at higher redshifts, though Steidel & Sargent (1989) have argued from the ionization states of metal line absorption systems that the photoionizing spectrum must be hard, suggesting that AGN continue to make a significant contribution to the UV flux at $z\gtrsim 3$. As Figs 1 and 2 show, the cooling rates depend sensitively on whether He$^+$ is ionized strongly and hence on the UV background shortward of 228 Å. Luminous quasars would introduce spatial variations in the UV background. These could be extremely large if the mean UV background has a soft spectrum, and could influence the evolution of protogalactic clouds. Quasars could thus introduce spatial modulations in the galaxy distribution over scales

$$R_{\rm Q}\approx 5(1+z)(J_{-21})^{-1/2}(L_{\rm Q}/10^{46}\ {\rm erg\,s^{-1}})^{1/2}\ {\rm Mpc}, \tag{12}$$

which is the radius at which the flux from a quasar of luminosity $L_{\rm Q}$ equals the intergalactic photoionizing flux at the Lyman limit. The formation of dwarf galaxies may thus be inhibited in the vicinity of a quasar relative to a random point in space via the mechanisms described in Sections 3.2 and 3.3.

It is possible to think of other ways in which galaxy formation could be spatially modulated. For example, high gas pressures in developing clusters and groups of galaxies could strip gas from the outer regions of dwarf galaxies, and intracluster gas could prevent winds from expelling the gas in dwarf galaxies (Babul & Rees 1982). Babul & White (1991) describe schematic models of how quasars might alter the distribution of galaxies relative to the mass, and how this might explain observations of large-scale structure in the galaxy distribution.

4 CONCLUSIONS

We have argued that a UV background could have played a critical role in galaxy formation. In hierarchical clustering models, the first structures would form at high redshift ($z \sim 20$ in the CDM model, Couchman & Rees 1986). We envisage that star formation and/or nuclear activity in dwarf galaxies would photoionize the intergalactic medium and so regulate the galaxy formation process. Observations of $z \sim 5$ quasars imply a photoionizing flux of $J_{-21} \gtrsim 1$ and hence that a significant UV background is already in place before large galaxies formed according to the CDM model. A photoionizing flux of this magnitude would lengthen the cooling times of gas in haloes with low circular speeds and so inhibit the formation of dwarf galaxies. Gas in the outer parts of dwarf galaxy haloes with a long cooling time and a low pressure could be removed easily by a galactic wind. Variations in the photoionizing flux caused, for example, by nearby luminous quasars, could introduce spatial modulations in the galaxy distribution.

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