

Soliton transverse instabilities in nonlocal nonlinear media

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We analyze the transverse instabilities of spatial bright solitons in nonlocal nonlinear media, both analytically and numerically. We demonstrate that the nonlocal nonlinear response leads to a dramatic suppression of the transverse instability of the soliton stripes, and we derive the asymptotic expressions for the instability growth rate in both short- and long-wave approximations. © 2007 Optical Society of America

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Symmetry-breaking instabilities have been studied in different areas of physics, since they provide a simple means to observe the manifestation of strongly nonlinear effects in nature. One example is the transverse instabilities of spatial optical solitons [1] associated with the growth of transverse modulations of quasi-one-dimensional bright and dark soliton stripes in both focusing [2–4] and defocusing [5] nonlinear media. In particular, this kind of symmetry-breaking instability turns a bright-soliton stripe into a array of two-dimensional filaments [6] and bends a dark-soliton stripe creating pairs of optical vortices of opposite polarities [7]. Consequently, the transverse instabilities set severe limits on the observation of one-dimensional spatial solitons in bulk media [8].

Several different physical mechanisms for suppressing the soliton transverse instabilities have been proposed and studied, including the effect of partial incoherence of light [9, 10] and anisotropic nonlinear response [10] in photorefractive crystals, and the stabilizing action of the nonlinear coupling between the different modes or polarizations [11]. Recently initiated theoretical and experimental studies of nonlocal nonlinearities revealed many novel features in the propagation of spatial solitons including the suppression of their modulational [12] and azimuthal [13] instabilities. In this Letter we demonstrate that significant suppression of the soliton transverse instabilities can be achieved in nonlocal nonlinear media, and we derive analytical results for the instability growth rate in both long- and short-scale asymptotic limits.

We consider the propagation of an optical beam in a nonlocal nonlinear medium described by the normalized two-dimensional nonlinear Schrödinger (NLS) equation,

$$\begin{aligned} i\frac{\partial E}{\partial z} + \frac{1}{2}\Delta_{\perp}E + nE &= 0, \\ n - d\Delta_{\perp}n &= |E|^2, \end{aligned} \quad (1)$$

where $\Delta_{\perp} = \partial^2/\partial x^2 + \partial^2/\partial y^2$, $E = E(x, y; z)$ is the slowly varying electric field envelope, $n = n(x, y; z)$ is the

optical refractive index, and the parameter d stands for the strength of nonlocality. The model (1) describes the light propagation in different types of nonlocal nonlinear media including nematic liquid crystals [14].

We look for stationary solutions of Eq. (1) in the form of the bright soliton stripes, $E(x, y; z) = u(x)\exp(i\beta z)$, where $u(x)$ is a (numerically found) localized function, $u(\pm\infty) = 0$, and β is the (real) propagation constant.

The transverse instability of quasi-one-dimensional solitons in nonlocal nonlinear media is investigated by a standard linear stability analysis [1], by introducing the perturbed solution in the form: $n = n_0(x) + \epsilon\delta n$, and

$$E = e^{i\beta z} [u_0 + \epsilon(v + iw)e^{i\lambda z + ipy} + \epsilon(v^* - iw^*)e^{-i\lambda^* z - ipy}],$$

where (u_0, n_0) is the solution of Eqs. (1), $\epsilon \ll 1$ is a small perturbation, and $v(x)$, $w(x)$, and $\delta n(x)$ are perturbed amplitudes that are modulated in the transverse y direction with the wavenumber p . The instability growth rate is defined as an imaginary part of the eigenvalue λ .

Substituting these asymptotic expansions into Eq. (1), in the first order of ϵ we obtain a set of linear equations,

$$\begin{aligned} \lambda w &= (\beta + \frac{1}{2}p^2)v - \frac{1}{2}\frac{d^2v}{dx^2} - n_0v - u_0\delta n, \\ \lambda v &= (\beta + \frac{1}{2}p^2)w - \frac{1}{2}\frac{d^2w}{dx^2} - n_0w, \\ \delta n &= d\left(\frac{d^2}{dx^2} - p^2\right)\delta n + 2u_0v, \end{aligned} \quad (2)$$

which we then study numerically and analytically.

Typical solutions for bright solitons of the model (1) are shown in Fig. 1(a) for local (solid curve) and nonlocal (dashed and dashed-dotted curves) media. Figure 1(b) shows the growth rate of the soliton transverse instabilities vs. the modulation wavenumber p , for local ($d = 0$) and nonlocal ($d = 0.5$ and $d = 1$) nonlinearities, respectively. It can be seen that nonlocality reduces the growth rate of the transverse instability of the soliton stripe. Moreover, the cutoff transverse wavenumber p_c

of the gain spectrum becomes smaller as the value of the nonlocality parameter d grows.

To describe the suppression of the soliton transverse instabilities quantitatively, we calculate the dependence of the cutoff transverse wavenumber p_c and the maximum growth rate on the strength of nonlocality, d , shown in Figs. 1(c,d). We observe that the maximum growth rate decreases significantly at large values of the nonlocality parameter, and eventually it approaches zero when $d \rightarrow \infty$. The cutoff wavenumber p_c of the transverse instability domain becomes smaller as the value of nonlocality grows. In the limit of very large values of d , the cutoff wavenumber vanishes as well. Consequently, the soliton stripes tend to become more stable when the degree of nonlocality increases.

Next, we analyze the transverse instability of bright solitons in nonlocal nonlinear media by applying a variation method, in accord with the following steps. First, we expand the nonlocal refractive index function into series in the nonlocality parameter [15] in the terms involving the refractive index in Eqs. (2). Second, we apply the method of Ref. 3 to construct the asymptotic expansions of the elliptical problem defined by Eqs. (2). We employ the corresponding ansatz defined as: $v = v_0 + \Gamma v_1$ and $w = w_0 + \Gamma w_1$, where Γ is an imaginary part of the eigenvalue of the linear stability problem.

In the long-scale expansion, we use the following solutions: $v_0 = 0$, $v_1 = d^2[\text{sech}(b_1 x)]/dx^2$, and $w_0 = u_0$, where the parameter b_1 is obtained by minimizing the

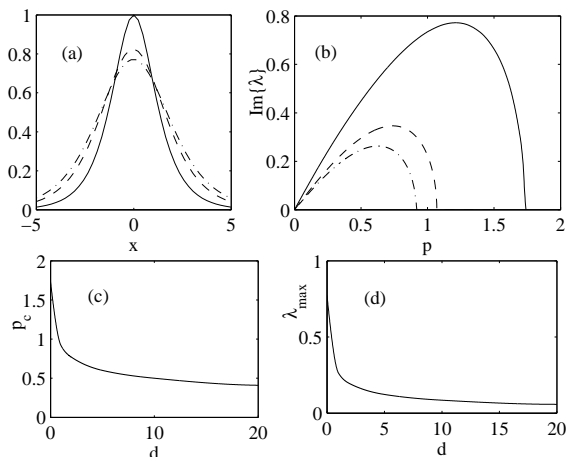


Fig. 1. (a) Intensity profiles of the bright solitons in local ($d = 0$, solid) and nonlocal ($d = 0.5$, dashed; $d = 1$, dashed-dotted) nonlinear media. (b) The instability growth rate for bright solitons vs. the transverse wavenumber p for local (solid) and nonlocal ($d = 0.5$, dashed; $d = 1$, dash-dotted) nonlinear media. (c,d) Cutoff value of the transverse wavenumber and the maximum growth rate vs. the nonlocality parameter d .

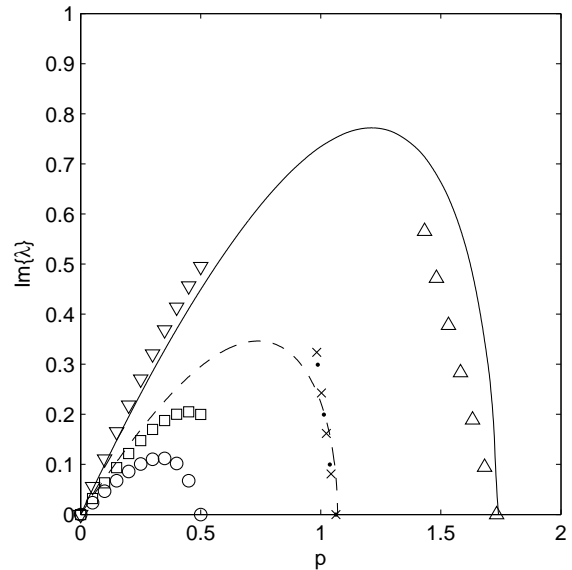


Fig. 2. Growth rate of the soliton transverse instabilities defined from numerical simulations and variational approach. Solid and dashed lines are numerical data for local ($d = 0$) and nonlocal ($d = 0.5$) nonlinearities. Upper and lower triangular marks: long-scale and short-scale asymptotic expansions for local nonlinearity. Squares and circles: long-scale asymptotic expansion based on the linearized nonlocal eigenvalue system expanded up to the first and second orders in d . Dots and crosses: the short-scale asymptotic expansion up to the first and second orders in d .

system Lagrangian corresponding to Eq. (2),

$$L_{v1} = \int_{-\infty}^{\infty} dx \left\{ -w_0 |v_1|^2 + \frac{1}{2} \left| \frac{\partial v_1}{\partial x} \right|^2 + \left(\beta + \frac{p^2}{2} \right) |v_1|^2 - \left(|u_0|^2 + d \frac{\partial^2 |u_0|^2}{\partial x^2} \right) |v_1|^2 - \frac{2u_0^2 |v_1|^2}{1 + dp^2} - \frac{2d}{(1 + dp^2)^2} \left(u_0 \frac{\partial^2 u_0}{\partial x^2} |v_1|^2 - u_0^2 \left| \frac{\partial v_1}{\partial x} \right|^2 \right) \right\}. \quad (3)$$

Wavenumber β and solution $u_0 = A \text{sech}(ax)$ are obtained by minimizing the Lagrangian of the nonlocal NLS equation based on the same expansion technique. For a given power P and nonlocality parameter d , we obtain $\beta = (1/6)Pa + (1/6)a^2 + (2/15)dPa^3$, $A = (Pa/2)^{1/2}$, and $a = [-5 + (25 + 60dP^2)^{1/2}]/(12dP)^{-1}$. While evaluating the system Lagrangian, we use the functional expansion, $\sec(ax) \approx \text{sech}(b_1 x) - \text{sech}(b_1 x) \tanh(b_1 x) (a - b_1)x$ to deduce a close form for the Lagrangian in Eq. (3). The parameter b_1 can therefore be obtained as a function of both d and P .

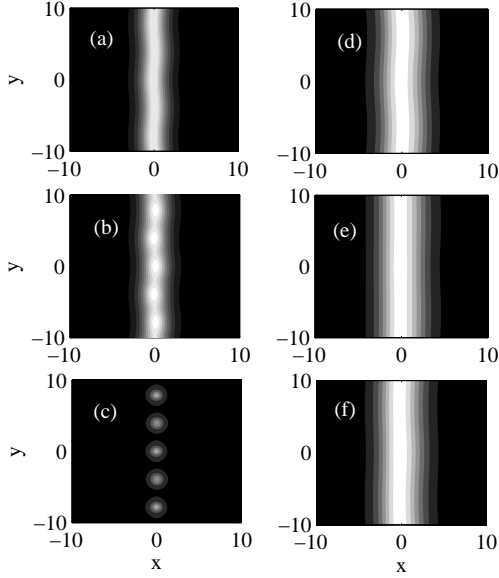


Fig. 3. Evolution of a modulated bright soliton stripe in (a-c) local and (d-f) nonlocal nonlinear media at the distances $z = 1.0, 6.0$ and 10.0 , respectively.

Then, the instability growth rate is obtained as

$$\Gamma^2 = \frac{(w_0, u_0 \Delta n(v_1; p=0))}{(w_0, v_1)} p^2 - \frac{1}{4} p^4,$$

$$\delta n(v_1; p=0) = \frac{u_0 v_1}{1 + dp^2} + d \frac{\left[\frac{\partial^2}{\partial x^2} (u_0 v_1) \right]}{(1 + dp^2)^2}, \quad (4)$$

where we define $(w_0, v_1) = \int_{-\infty}^{\infty} dx w_0^* v_1$.

For the asymptotic expansion near the cutoff wavenumber p_c , we use the ansatz $v_0 = \text{sech}(b_0 x)$, $w_0 = 0$, and $w_1 = \text{sech}(c_1 x)$. Again, to obtain b_0 we minimize the Lagrangian L_{v_1} as a function of the wavenumber p . Then the cutoff wavenumber p_c is found from the condition $L_{v_1} = 0$ with an optimized value of b_0 . The resulting parameters are deemed as constant in the Lagrangian for w_1 ,

$$L_{w_1} = \int_{-\infty}^{\infty} dx \left\{ -v_0 |w_1|^2 + \frac{1}{2} \left| \frac{\partial w_1}{\partial x} \right|^2 + \left(\beta + \frac{p^2}{2} \right) |w_1|^2 - \left(|u_0|^2 + d \frac{\partial^2 |u_0|^2}{\partial x^2} \right) |w_1|^2 \right\}, \quad (5)$$

to deduce parameter c_1 as a function of P and d . Similarly, in the short-scale expansion the amplitude growth rate can be calculated as

$$\Gamma^2 = \frac{(v_0, u_0 \Delta n(w_1; p=0))}{(v_0, w_1)} p^2 - \frac{1}{4} p^4,$$

$$\delta n(w_1; p=0) = \frac{u_0 w_1}{1 + dp^2} + d \frac{\left[\frac{\partial^2}{\partial x^2} (u_0 w_1) \right]}{(1 + dp^2)^2}. \quad (6)$$

Figure 2 compares the analytical results with the results of our direct numerical calculations. For nonlocal

media, the long-scale expansion to the first order in d (circles) is insufficient, but the second-order expansion (squares) gives a good result. In the short-scale expansion, both the first- (dots) and second-order (crosses) expansions are in a good agreement with the numerics.

To observe the consequence of the predicted instability suppression, we study numerically the evolution of the soliton stripe described by Eqs. (1). Figures 3(a-f) compare the transverse instability of a bright-soliton stripe with the power density 2 in local and nonlocal media, with the initial field modulated transversally with the maximum growth rate. In Fig. 3(a-c) we show the snapshots of the soliton stripe evolution in a local nonlinear medium at $z = 1.0, 6.0$ and 10.0 , respectively. We observe that at $z = 10.0$ [Fig. 3(c)], the bright-soliton stripe decays into a sequence of filaments due to the modulation in the y direction. In a sharp contrast, there is no visible decay of the soliton stripe in a nonlocal medium ($d = 1$) [Fig. 3(d-f)], therefore confirming our major conclusion that the soliton transverse instabilities are suppressed substantially in nonlocal nonlinear media.

In conclusion, we have analyzed the transverse instabilities of spatial solitons in nonlocal nonlinear media. By employing the linear stability analysis and numerical simulations, we have demonstrated that nonlocal nonlinear response can suppress significantly the transverse instabilities allowing experimental observations of the stable propagation of the soliton stripes in nonlocal media.

References

1. Yu.S. Kivshar and D.E. Pelinovsky, Phys. Rep. **331**, 117 (2000), and references therein.
2. V.E. Zakharov and A.M. Rubenchik, Sov. Phys. JETP **38**, 494 (1974).
3. S.J. Han, Phys. Rev. A **20**, 2568 (1979).
4. P.A.E.M. Janssen, Phys. Fluids **26**, 1279 (1983).
5. E.A. Kuznetsov and S.K. Turitsyn, Sov. Phys. JETP **67**, 1583 (1988).
6. A.V. Mamaev, M. Saffman, D.Z. Anderson, and A.A. Zozulya, Phys. Rev. A **54**, 870 (1996).
7. V. Tikhonenko, J. Christou, B. Luther-Davis, and Yu.S. Kivshar, Opt. Lett. **21**, 1129 (1996).
8. Yu. S. Kivshar and G. P. Agrawal, *Optical Solitons: from Fibers to Photonic Crystals* (Academic, San Diego, Calif., 2003), and references therein.
9. J.P. Torres, C. Anastassiou, M. Segev, M. Soljacic, and D.N. Christodoulides, Phys. Rev. E **65**, 015601 (2001).
10. K. Motzek, F. Kaiser, W.-H. Chu, M.-F. Shih, and Yu.S. Kivshar, Opt. Lett. **29**, 280 (2004).
11. Z.H. Musslimani and J. Yang, Opt. Lett. **26**, 1981 (2001).
12. W. Królkowski, O. Bang, N.I. Nikolov, D. Neshev, J. Wyller, J.J. Rasmussen, and D. Edmundson, J. Opt. B: Quant. Semiclass. Opt. **6**, S288 (2004).
13. S. Lopez-Aguayo, A.S. Desyatnikov, and Yu.S. Kivshar, Opt. Exp. **14**, 7903 (2006).
14. C. Conti, M. Peccianti, and G. Assanto, Phys. Rev. Lett. **91**, 073901 (2003).
15. Y. Lin and R.-K. Lee, Opt. Exp. **15**, 8781 (2007).