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SURFACE BRIGHTNESSES IN THE U, B, V SYSTEM WITH APPLICATIONS ON M_V AND DIMENSIONS OF STARS

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SUMMARY

The surface brightnesses are expressed in the V system in terms of V_0 and d'' (the angular diameter) with the formula $s_V = V_0 + 5 \log d$. s_V is derived for 18 stars from observation and the relation between s_V and $(B-V_B)_0$ is obtained. The central problem is the determination of M_V from R (the linear radius) or vice versa; the formula $M_V - s_V + 5 \log R = +15 \cdot 15$ is basic for the solution thereof. The use of s_V is compared with a method based on BC and T_e currently employed in the same problem. The two methods are equivalent provided a certain condition is satisfied. Lines of equal radius have been drawn in a HR diagram having $(B-V)_0$ and M_V coordinates and the radii of stars along the zero age main sequence are given.

The mean absolute magnitude of δ Cephei has been derived in good accord with other modern determinations ($M_V = -3 \cdot 56$). The absolute magnitudes M_V for the components of some eclipsing variables are derived and a mass–luminosity relation is drawn with small scatter. The dimensions of some supersupergiants belonging to the Magellanic Clouds are calculated; HDE 268757 is one of the largest known stars with a radius of 10·3 a.u. A check is made on a published list of BC and T_e, using the results of this paper.

1. INTRODUCTION

The following problem is solved in the article:

To determine the visual absolute magnitude, M_V, given the linear dimensions and the intrinsic colour.

In the equation

$$M_V - s_V + 5 \log R = \text{const.} \tag{1}$$

M_V, s_V and R represent, respectively, the visual absolute magnitude, the surface brightness (on a magnitude scale) and the linear radius. The constant in equation (1) depends on the zero points of M_V and s_V and on the unit chosen for R; the constant is given below (Section 2).

s_V is derived in Section 2, using observed angular diameters and photometry. In the same section, the empirical relation between s_V and $(B-V)_0$ is obtained. It is clear that the problem stated above can be solved using this relation and equation (1). The inverse problem of finding R, given M_V and the intrinsic colour, is an obvious variant of the first. It is, of course, also solved using equation (1) with s_V derived from $(B-V)_0$. In the subsequent sections various applications are given.

It is customary to solve the problems otherwise, using a method depending on the bolometric correction BC and the effective temperature T_e. It is of interest to compare the s_V method, as advocated here, with the current bolometric method.

We may rewrite equation (1) in terms of bolometric quantities as follows:

$$M_b - s_b + 5 \log R = \text{const.} \tag{2}$$

M_b, s_b and R, respectively, represent the bolometric absolute magnitude, the bolometric surface brightness (on a magnitude scale) and the linear radius.

Stefan's law expresses s_b in T_e as follows:

$$s_b = -10 \log T_e + \text{const.} \tag{3}$$

Substitution of equation (3) in equation (2) gives:

$$M_b + 10 \log T_e + 5 \log R = \text{const.} \tag{4}$$

T_e is one of the most fundamental quantities in astrophysics, but unfortunately not directly observable. Considerable effort, made by many astronomers, has led to a fair knowledge of the quantity and tables are available giving T_e as a function of $(B-V)_0$ (**1**), (**2**). With such a table and using equation (4) M_b can be determined whenever $(B-V)_0$ and the radius are known.

The bolometric correction BC is also of great importance in astrophysics, but like T_e not directly observable. BC has been tabulated as a function of $(B-V)_0$ (**1**), (**2**).

Using such a table for BC we find M_V from the equation:

$$M_b = M_V + BC \tag{5}$$

which solves our problem with the bolometric method. If M_b is not required, M_V is more directly found from equation (6), which follows from equations (4) and (5):

$$M_V + BC + 10 \log T_e + 5 \log R = \text{const.} \tag{6}$$

but now using a tabulation of $BC + 10 \log T_e$ as a function of intrinsic colour. M_V follows directly from equation (6) without having evaluated M_b as an intermediary.

It is well known that both BC and T_e are still quite uncertain especially for stars with extreme blue and red colours. On the other hand, s_V can be observed directly for a small number of stars (see next section). The empirical relation between s_V and $(B-V)_0$, which follows from these observations, may be used to find s_V for almost any star with an error not exceeding $\pm 0^{m}\cdot 15$.

In view of these circumstances, one may well wonder whether our problem is not better solved by means of s_V than with the bolometric method.

However, we may write:

$$s_b = s_V + BC \tag{7}$$

which is equivalent to

$$BC + 10 \log T_e = -s_V + \text{const.} \tag{8}$$

after use of equation (3).

Substitution of equation (8) in equation (6) shows that equations (1) and (6) are identical provided BC and T_e agree with s_V according to equation (8). The two methods are, therefore, identical and the results equally accurate, on condition (8).

Studies of BC and T_e are usually made together (**1**) with implicit attention to equation (8). Although s_V is rarely evaluated explicitly, it is the relevant quantity for the problems which are the subject of the present article.

Here, no attempt is made to derive new values for BC or T_e, but it is suggested that s_V, or future improved determinations thereof, shall be used through equation (8) for their derivation whenever another independent relation comes available.

We remark that equation (8) is, of course, not new; the equation occurs in a different guise elsewhere (**3**). In Section 7 we illustrate the use of equation (8) as a check on a published list of BC and T_e.

2. DISCUSSION OF OBSERVATIONS

2.1 *Determination of* s_V *and the relation between* s_V *and* $(B-V)_0$

We use Johnson's & Morgan's U, B, V system of photometry (**4**) not only for the apparent magnitudes and colour indices, but also for the absolute magnitudes and the surface brightness. Throughout this article the term, surface brightness indicates its value averaged over the stellar disc. The quantity is expressed in apparent magnitude per $\pi/4$ square seconds of arc. Therefore, on the scale chosen, a star with zero V magnitude, showing a diameter of one second of arc would have zero magnitude of surface brightness. With this definition we have

$$s_V = V_0 + 5 \log d''. \tag{9}$$

In equation (9), V_0 is the apparent V magnitude corrected for interstellar absorption, while d'' represents the angular diameter in seconds of arc. By adding $5+5 \log p$, where p is the parallax, to V_0 and subtracting the same quantity from $5 \log d$, we obtain the formula containing M_V, s_V and R

$$M_V - s_V + 5 \log R = +15{\cdot}15. \tag{10}$$

Here R is measured in terms of the radius of the Sun. The constant follows from the angular size of the Sun at the mean distance from the Earth.

To date, there are no more than 19 stars for which reliable values of d are available (including the Sun), if the two equal components of Castor C (YY Gem) are counted as a single star. Of these 19, the photometry of α Car (see below) is in doubt, leaving us 18 stars for which s_V can be measured, using equation (9).

The $(B-V)_0$ cover almost the entire range of intrinsic stellar colours $(-0{\cdot}30 < B-V < 1{\cdot}80)$, so that a fair study of the variation of s_V with colour is possible. Unfortunately, there is an extensive gap in the colours

$$(+0{\cdot}64 < B-V < 1{\cdot}45)$$

where no angular diameters are available. Within this interval measures of angular diameter are badly needed, but suitable stars may be difficult to find. The 18 stars of which s_V was determined, are listed in Table I, in order of intrinsic colour $(B-V)_0$. The diameters of the first 14 were taken from the paper by Hanbury-Brown *et al.* (**5**), whose results were obtained with the Narrabri intensity interferometer.

The 15th star is the Sun, the diameter of which has, of course, by far the smallest percentage error. The angular diameter of a component of the eclipsing variable Castor C follows from the known parallax, the radial velocity curves and an analysis of the lightcurve. The diameters of α Sco and α Ori were measured with the Michelson interferometer on the 100-in. Mt Wilson reflector by Pease (**6**). The interferometer results for α Sco agree well with the diameter derived from lunar occultations (**7**), (**8**). The diameters used in Table I were obtained with a variety of methods. We are not certain that the systematic errors depending on the technique were small, but we will assume that the homogeneity of the results is not seriously in doubt. Hanbury-Brown *et al.* (5) published two diameters for each star: θ_{UD} and θ_{LD}. θ_{UD} is the diameter derived on the assumption of a uniform disc, whereas θ_{LD} assumes a definite amount of limb-darkening. θ_{LD} is a few percent larger than θ_{UD} throughout, the difference increasing with $B-V$. Without exception, we took $d = \theta_{LD}$ as θ_{LD} is more likely to represent the true diameter than θ_{UD}. Hanbury-Brown *et al.* give standard errors for each stellar diameter; the percentage standard errors are all of the same order (± 6 per cent) corresponding to $\pm 0^{\text{m}}\cdot 13$ in 5 log d and in the resulting s_V. The percentage errors in d for Castor C, α Sco and α Ori are of the same order of magnitude, however, it is more difficult to give accurate estimates of these.

In their discussion of the temperature scale, Hanbury-Brown *et al.* use absolute measures of the energy radiation (9), (10). The present article does not attempt to establish a temperature scale, but rather intends to determine an accurate relative scale of surface brightness. Absolute measures could be used although the absolute character would not be required. Therefore, the accurate U, B, V photometry, available for all the stars has been used throughout. The photometry was taken either from a compilation by Cousins (11) or from the work of Johnson and his collaborators (12). In a single case the $U-B$ has been taken from Hogg (13).

Most stars of our list are sufficiently near that the interstellar absorption and the reddening can be neglected. For these stars the observed V and the observed colours were taken as the intrinsic values. The giants, however, suffer small amounts of absorption and reddening. The observed MK class was checked with the photometry according to Johnson & Morgan's 'Q' method (4). In all cases fair agreement was obtained between Sp (Q) and MK observed. Spectroscopic parallaxes were calculated using Blaauw's calibration of the luminosity criteria of the MK system (14). The distance and the colour excesses were considered in estimating the small amounts of absorption and reddening. Intrinsic colours as given by Johnson (15) and Arp (16) were used. The results have been collected in Table I.

2.2 *Remarks on individual stars*

β Cru. U, B, V photometry from Cousins (11); Sp (Q) is B0·5; MK is B0·5; $M_V = -4\cdot 2$; spectroscopic parallax 0″·008; $A_V = 0\cdot 10$; $E_{B-V} = 0\cdot 04$; $E_{U-B} = 0\cdot 02$.

γ Ori. U, B, V photometry from Cousins; Sp (Q) is B2; MK is B2 III; $M_V = -3\cdot 5$; spectroscopic parallax is 0″·010; $A_V = 0\cdot 06$; $E_{B-V} = 0\cdot 02$; $E_{U-B} = 0\cdot 01$.

ε Ori. V from Johnson *et al.* (12); $B-V$, $U-B$ from Cousins (11); Sp (Q) is O9; MK is O9 Ia; $M_V = -6\cdot 3$; spectroscopic parallax is 0″·0025; $A_V = 0\cdot 15$; $E_{B-V} = 0\cdot 05$; $E_{U-B} = 0\cdot 04$.

ε C Ma. U, B, V photometry from Cousins (11); Sp (Q) is B2; MK is B2 II; $M_V = -5\cdot 0$; spectroscopic parallax is 0″·005; $A_V = 0\cdot 06$; $E_{B-V} = 0\cdot 02$; $E_{U-B} = 0\cdot 02$.

α Pav. U, B, V photometry from Cousins (11); Sp (Q) is B3; MK is B3 IV; $M_V = -2\cdot 5$; spectroscopic parallax is 0″·013. Absorption and reddening have been neglected.

TABLE I

Star	MK	V_0	$(B-V)_0$	$(U-B)_0$	s_V	s_B	s_U	(s.e.)	M_V	A_V
β Cru	B0·5 IV	+1·15	−0·28	−1·03	−14·54	−14·82	−15·85	±0^m·07	−4·2	0·10
γ Ori	B2 III	+1·58	−0·24	−0·88	−14·02	−14·26	−15·14	0^m·14	−3·5	0·06
ε Ori	B0 Ia	+1·54	−0·23	−1·08	−14·18	−14·41	−15·49	0^m·15	−6·3	0·15
ε CMa	B2 II	+1·44	−0·22	−0·95	−14·02	−14·24	−15·19	0^m·13	−5·0	0·06
α Pav	B3 IV	+1·94	−0·19	−0·73	−13·54	−13·73	−14·46	0^m·16	−2·5	—
α Eri	B5 IV	+0·48	−0·16	−0·60	−13·09	−13·25	−13·85	0^m·09	−1·5	—
α Gru	B5 V	+1·74	−0·14	−0·48	−13·22	−13·36	−13·84	0^m·16	−1·0	—
α Leo	B7 V	+1·35	−0·11	−0·36	−12·95	−13·06	−13·42	0^m·10	−0·5	—
β Ori	B8 Ia	0·00	−0·08	−0·68	−12·85	−12·93	−13·61	0^m·11	−7·2	0·15
α Lyr	A0 V	+0·03	0·00	0·00	−12·27	−12·27	−12·27	0^m·10	+0·5	—
α CMa	A1 V	−1·41	0·00	−0·02	−12·47	−12·47	−12·49	0^m·13	+0·8	—
α PsA	A3 V	+1·15	+0·09	+0·06	−12·25	−12·16	−12·10	0^m·14	+1·5	—
α Aql	A7 V	+0·77	+0·22	+0·08	−11·87	−11·65	−11·57	0^m·11	+2·1	—
α CMi	F5 IV	+0·36	+0·42	+0·03	−10·86	−10·44	−10·41	0^m·14	+3·2	—
Sun	G2 V	−26·73	+0·64	+0·16	−10·31	−9·67	−9·51	—	+4·8	—
YY Gem	M1 V	+9·82	+1·48	+1·21	−7·06	−5·58	−4·37	—	+9·1	—
α Sco	M1 Ib	+0·75	+1·78:	+1·29:	−5·99	−4·21	−2·92	—	−4·8	0·15
α Ori	M2 Iab	+0·55	+1·81:	+1·96:	−6·05	−4·24	−2·28	—	−5·8	0·15

α *Eri.* V, $B-V$ from Cousins (11); $U-B$ from Johnson *et al.* (12); Sp (Q) is B3; MK is B5 IV; $M_V = -1 \cdot 5$; spectroscopic parallax is $0'' \cdot 040$. Absorption and reddening have been neglected.

α *Gru.* U, B, V photometry from Cousins (11); Sp (Q) is B6; MK is B5 V; $M_V = -1 \cdot 0$; spectroscopic parallax is $0'' \cdot 029$. Absorption and reddening are negligible.

α *Leo.* U, B, V photometry by Johnson *et al.* (12); Sp (Q) is B7; MK is B7 V; $M_V = -0 \cdot 5$; spectroscopic parallax is $0'' \cdot 043$. Absorption and reddening are negligible.

β *Ori.* V, $B-V$ from Cousins (11); $U-B$ from Johnson *et al.* (12); Sp (Q) is B7·5; MK is B8 Ia; $M_V = -7 \cdot 2$; spectroscopic parallax is $0'' \cdot 0036$; $A_V = 0 \cdot 15$; $E_{B-V} = 0 \cdot 05$; $E_{U-B} = 0 \cdot 03$.

α *Lyr.* U, B, V photometry by Johnson *et al.* (12); MK is A0 V; $M_V = +0 \cdot 5$; spectroscopic parallax is $0'' \cdot 080$. Absorption and reddening are negligible.

α *C Ma.* U, B, V photometry from Cousins (11); no absorption and reddening in this star which is only 3 parsecs distant.

α *Ps A.* V, $B-V$ from Cousins (11); $U-B$ by Johnson *et al.* (12); MK is A3 V; $M_V = +1 \cdot 5$; spectroscopic parallax is $0'' \cdot 11$. No absorption and reddening in this star.

α *Aql.* U, B, V photometry from Cousins (11); MK is A7 V; $M_V = +2 \cdot 1$; spectroscopic parallax is $0'' \cdot 18$. Absorption and reddening are negligible.

α *C Mi.* V and $B-V$ from Cousins (11); $U-B$ by Johnson *et al.* (12); MK is F5 IV; $M_V = +3 \cdot 2$; spectroscopic parallax is $+0'' \cdot 29$. No absorption and reddening in this star.

α *Car.* V, $B-V$ from Cousins (11); $U-B$ from Hogg (13); The MK class is F0 Ib; $M_V = -4 \cdot 8$; spectroscopic parallax is $0'' \cdot 016$. At a distance of 60 parsecs not much absorption, or reddening can be expected.

However, $(B-V)_0 = +0 \cdot 23$, which is *redder* than the observed colour $B-V = +0 \cdot 16$. Again, $(U-B)_0 = +0 \cdot 27$, whereas the observed $U-B = +0 \cdot 04$. The colours of α Car are therefore peculiar unless the intrinsic colours of a F0 Ib star are not that well known. Warner (17) finds the spectrum peculiar. For these reasons, we have excluded the star from our discussion of s_V although a good angular diameter was determined by Brown *et al.* (5).

The Sun. The Sun is the only star of our list of which the error in 5 log d is negligible, and the main contribution to the error in s_V comes from the photometry. The error in V (Sun) is likely to be somewhat larger than for any of the other stars. We used $V = -26 \cdot 73$ as determined by Stebbins & Kron (18). The magnitude is on the international system, but this is identical with the V system (19). We have adopted $(B-V)_0 = +0 \cdot 64$ and $(U-B)_0 = +0^{m} \cdot 16$, assuming G2V for the MK type of the Sun (20).

YY *Gem* = *Castor C*. The analysis of the lightcurve shows that the components are nearly alike. From Kron's photometry (21) we deduce $V_0 = 9 \cdot 82$ for each component. There are good radial velocity curves for both components. Combining spectroscopic and photometric analyses, we find for each component $R = 0 \cdot 62$ solar radii. Using the trigonometrical parallax $0'' \cdot 073$ we find for each star $d'' = 4'' \cdot 22 \times 10^{-4}$. This then leads to the s_V given in Table I with equation (9).

α *Sco.* For this star two independent determinations of d'' are available which agree well. The Michelson interferometer result is $d = 0'' \cdot 045$ (5) whereas the mean result from occultations, corrected for limbdarkening, is also $0'' \cdot 045$ (7), (8).

The photometry is by Cousins. The observed MK type is M1 Ib giving $M_V = -4{\cdot}8$. The spectroscopic parallax is $0''{\cdot}005$. $A_V = 0{\cdot}15$; $E_{B-V} = 0{\cdot}05$; $E_{U-B} = 0{\cdot}03$.

α Ori. The angular diameters of a number of bright red giants were measured by Michelson & Pease with the Michelson interferometer (**6**). The result $d = 0''{\cdot}048$ is used here. The *U*, *B*, *V* photometry has been taken from Cousins (**11**). There is some uncertainty in the photometry as the star is slightly variable. The observed MK is M2 Iab; $M_V = -5{\cdot}8$; the spectroscopic parallax is $0''{\cdot}005$. $A_V = 0{\cdot}15$; $E_{B-V} = 0{\cdot}05$; $E_{U-B} = 0{\cdot}03$.

2.3 *The relation between s_V and $(B-V)_0$*

In Fig. 1(a) and 1(b), s_V has been plotted against $(B-V)_0$. Table II gives the coordinates of the smooth curve drawn through the points. The scatter in the ordinates is $\pm 0^m{\cdot}11$, whereas the mean standard error in 5 log *d* is $\pm 0{\cdot}12$. This shows that the uncertainty in the $(B-V)_0$ must be quite small. With these conditions of the errors, it is most unlikely that systematic effects can be discovered in

TABLE II

Coordinates of s_V, $(B-V)_0$ relation

$(B-V)_0$	s_V	$(B-V)_0$	s_V
$-0^m{\cdot}28$	$-14^m{\cdot}65$	$+0^m{\cdot}24$	$-11^m{\cdot}69$
$-0^m{\cdot}26$	$-14^m{\cdot}40$	$+0^m{\cdot}28$	$-11^m{\cdot}55$
$-0^m{\cdot}24$	$-14^m{\cdot}14$	$+0^m{\cdot}32$	$-11^m{\cdot}40$
$-0^m{\cdot}22$	$-13^m{\cdot}90$	$+0^m{\cdot}36$	$-11^m{\cdot}26$
$-0^m{\cdot}20$	$-13^m{\cdot}66$	$+0^m{\cdot}40$	$-11^m{\cdot}10$
$-0^m{\cdot}18$	$-13^m{\cdot}47$	$+0^m{\cdot}50$	$-10^m{\cdot}72$
$-0^m{\cdot}16$	$-13^m{\cdot}30$	$+0^m{\cdot}60$	$-10^m{\cdot}38$
$-0^m{\cdot}14$	$-13^m{\cdot}15$	$+0^m{\cdot}70$	$-10^m{\cdot}00$
$-0^m{\cdot}12$	$-13^m{\cdot}01$	$+0^m{\cdot}80$	$-9^m{\cdot}63$
$-0^m{\cdot}10$	$-12^m{\cdot}89$	$+0^m{\cdot}90$	$-9^m{\cdot}27$
$-0^m{\cdot}08$	$-12^m{\cdot}79$	$+1^m{\cdot}00$	$-8^m{\cdot}90$
$-0^m{\cdot}06$	$-12^m{\cdot}69$	$+1^m{\cdot}10$	$-8^m{\cdot}51$
$-0^m{\cdot}04$	$-12^m{\cdot}59$	$+1^m{\cdot}20$	$-8^m{\cdot}14$
$-0^m{\cdot}02$	$-12^m{\cdot}50$	$+1^m{\cdot}30$	$-7^m{\cdot}77$
$0^m{\cdot}00$	$-12^m{\cdot}40$	$+1^m{\cdot}40$	$-7^m{\cdot}40$
$+0^m{\cdot}04$	$-12^m{\cdot}28$	$+1^m{\cdot}50$	$-7^m{\cdot}06$
$+0^m{\cdot}08$	$-12^m{\cdot}16$	$+1^m{\cdot}60$	$-6^m{\cdot}70$
$+0^m{\cdot}12$	$-12^m{\cdot}04$	$+1^m{\cdot}70$	$-6^m{\cdot}37$
$+0^m{\cdot}16$	$-11^m{\cdot}93$	$+1^m{\cdot}80$	$-6^m{\cdot}00$
$+0^m{\cdot}20$	$-11^m{\cdot}82$		

the residuals. In fact, a search of such an effect depending on luminosity was negative. It is true that the gradual variation of luminosity along the curve (due to the main sequence) makes easy detection of such an effect difficult, but the giants and supergiants do not obviously deviate systematically from neighbouring points representative of lower luminosity dwarfs. We will therefore assume that luminosity effects are absent from the s_V, $(B-V)_0$ relation, at least outside the accuracy with which the curve can be determined at the present time. Similarly, the s_B, $(B-V)_0$ relation is independent of luminosity. On the other hand, the precision in s_U is just high enough to show some significant luminosity effect. However, neither s_B or s_U will be considered further in the article.

There is a serious gap in the curve between $(B-V)_0 = +0{\cdot}64$ and $+1{\cdot}45$

which hopefully will be filled in the future. α Boo would fall in this gap and is bright enough for measurement with the intensity-interferometer at Narrabri. Although the angular diameter of this star was previously measured by Pease (**6**) with the Michelson interferometer, we omitted the star from our list because of the relative large percentage error in *d*.

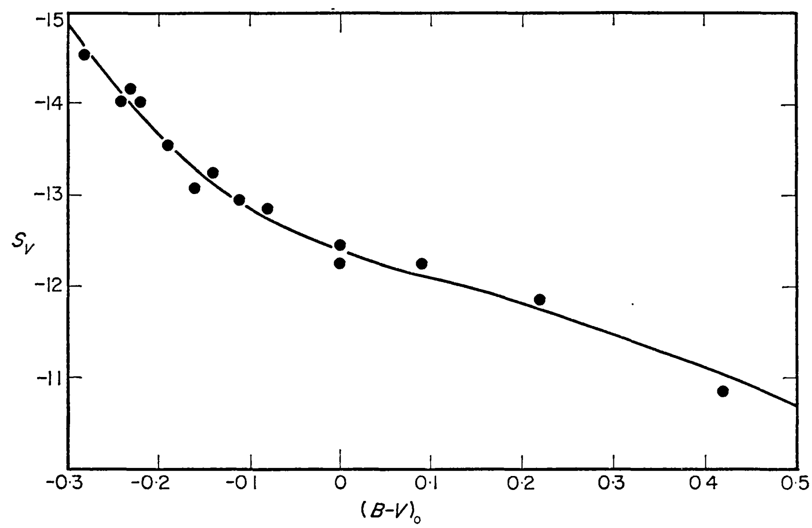

FIG. 1(a). *The magnitude of the surface brightness in the V-system, s_V as a function of* $(B-V)_0$. In the range $-0{\cdot}3<(B-V)_0<+0{\cdot}5$.

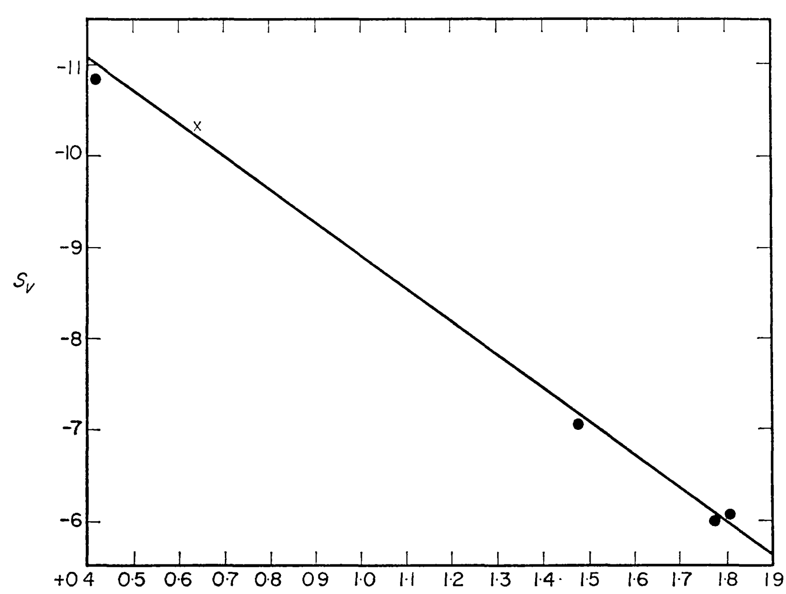

FIG. 1(b). *The magnitude of the surface brightness in the V-system, s_V as a function of* $(B-V)_0$. In the range $+0{\cdot}4<(B-V)_0<+1{\cdot}9$. *The cross represents the Sun.*

Table II and Fig. 1(a) and 1(b) allow the determination of s_V and angular diameters for all normal stars with intrinsic colours between $-0\cdot3$ and $+1\cdot80$ on the $B-V$ scale.

3. APPLICATIONS

3.1 *Dimensions of stars in the HR diagram*

Lines of constant radius are frequently drawn in the HR diagram, when the coordinates are log T_e and M_b (**22**), (**2**), (**23**). Such lines are easily constructed in such a diagram, when the directly-observable quantities $(B-V)_0$ and M_V are the coordinates, using Table II and equation (10). Fig. 2 has been adapted from a well-known diagram first given by Sandage (**23**) showing the M_V, $(B-V)_0$ arrays for a number of galactic clusters of different ages. The lines of constant

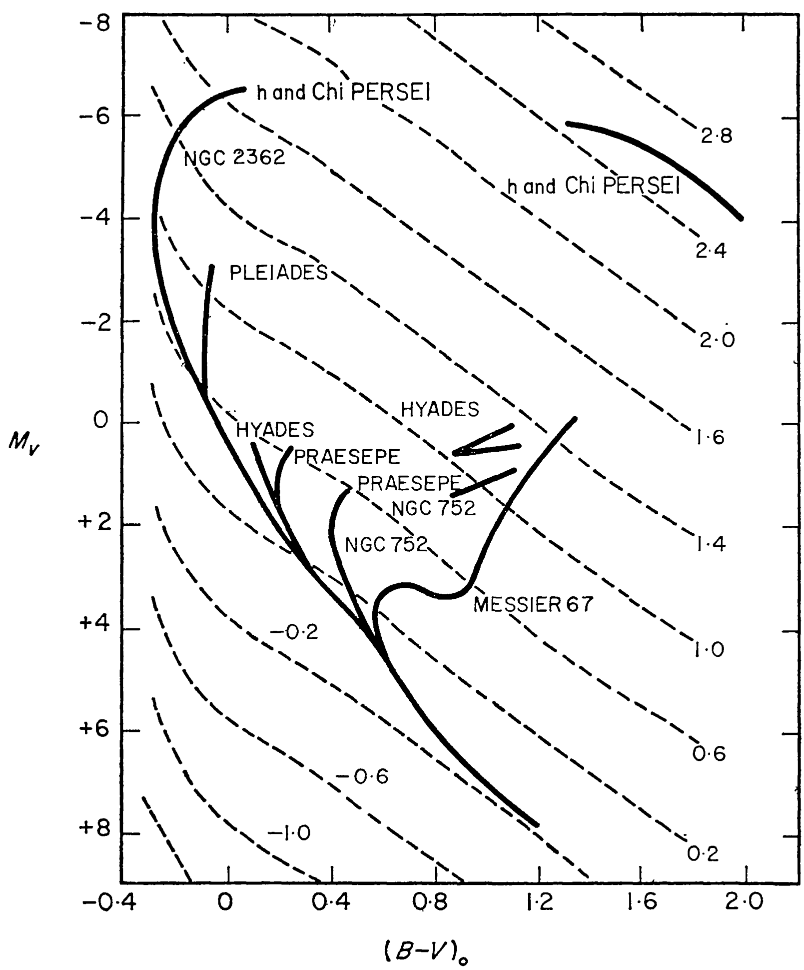

Fig. 2. *HR diagram with $(B-V)_0$ as abscissa and M_v as ordinate (adapted from Sandage). Lines of equal radius are shown.*

radius were drawn for the following values of log $R/R_\odot$: $-1\cdot4$, $-1\cdot0$, ... $+3\cdot0$ covering a range of 1 : 25 000. It appears that the brightest stars of the Hyades, Praesepe and NGC 752 have similar dimensions (log $R = +0\cdot6$). The brightest Pleiades are as large as the Hyades red giants. The red supergiants of h and χ Persei are comparable in size to some blue supersupergiants known to exist in the Magellanic Clouds (**24**).

3.2 *Stellar radii along the zero age main sequence*

As another application of equation (10) and Table II we calculated the linear radii of stars along the zero age main sequence. The corresponding values of $(B-V)_0$ and M_V of this sequence were taken from Table IV as given by Blaauw (**14**). Blaauw's Table is reproduced here, but with log R added in Table III.

TABLE III

Radius and zero age main sequence

$B-V$	M_V	s_V	log R
−0·30	−3·30	−15·00	+0·69
−0·25	−2·10	−14·25	+0·60
−0·20	−1·10	−13·66	+0·52
−0·15	−0·20	−13·21	+0·43
−0·10	+0·60	−12·89	+0·33
−0·05	+1·15	−12·64	+0·27
0·00	+1·55	−12·40	+0·24
+0·05	+1·80	−12·22	+0·23
+0·10	+2·00	−12·09	+0·21
+0·15	+2·25	−11·96	+0·19
+0·20	+2·50	−11·82	+0·17
+0·25	+2·70	−11·65	+0·16
+0·30	+2·95	−11·47	+0·15
+0·35	+3·20	−11·29	+0·13
+0·40	+3·55	−11·10	+0·10
+0·50	+4·20	−10·72	+0·05
+0·60	+4·80	−10·38	−0·01
+0·70	+5·40	−10·00	−0·05
+0·80	+5·90	−9·63	−0·08
+0·90	+6·30	−9·27	−0·09
+1·00	+6·80	−8·90	−0·10
+1·10	+7·20	−8·51	−0·11
+1·20	+7·65	−8·14	−0·13

4. THE ABSOLUTE MAGNITUDE OF δ CEPHEI

Fernie (**25**) using photoelectric photometry of δ Cephei by Mitchell *et al.* (**26**) finds $\langle B-V\rangle = +0{\cdot}72$. With a colour-excess also given by Fernie, we find a mean intrinsic colour index $\langle (B-V)_0\rangle = +0{\cdot}58$. Table II or Fig. 1 then gives $\langle s_V\rangle = -10{\cdot}31$ on the assumption that the s_V as determined in Section 2 are also valid for Cepheid variables.

Van Hoof (**27**) and Wesselink (**28**) determined the mean radius using a modified Baade method. Using Wesselink's result $\langle R\rangle = 48\ R_\odot$ equation (10) finally leads to $\langle M_V\rangle = -3{\cdot}56$ for δ Cephei.

Shapley's period luminosity relation (**29**) gives $\langle M_V\rangle = -2{\cdot}20$ for a Cepheid variable with the period of δ Cephei. The present result is therefore $1^{\mathrm{m}}{\cdot}36$ brighter then Shapley's value in good agreement with the corrections found by others using methods that are independent from the one used here (**30**).

The mean apparent magnitude $\langle V\rangle$ of δ Cephei is $+4{\cdot}0$. Assuming an absorption 3·0 times the colour excess, we find $A_V = 0^{\mathrm{m}}{\cdot}40$ and $\langle V_0\rangle = +3{\cdot}6$. Absolute and apparent magnitude then lead to a parallax of $0''{\cdot}0037$ and a corresponding distance of 270 parsecs. Mean absolute magnitudes for other Cepheids could be derived using the same method, provided precision photometry is available.

5. THE M_V OF SOME ECLIPSING VARIABLES AND THE MASS–LUMINOSITY RELATION

In this section we shall apply Table II for s_V to determine M_V for each of the components of a number of eclipsing variables with known radii.

TABLE IV

	$(B-V)_0$	$R/R_\odot$	s_V	M_V	log mass
VZ Hya	+0·42	1·25	−11·02	+3·6	+0·09
	+0·49	1·05	−10·76	+4·3	+0·05
WW Aur	+0·17	2·00	−11·90	+1·8	+0·26
	+0·22	1·9	−11·75	+2·0	+0·25
β Aur	+0·04	2·5:	−12·28	+0·9	+0·36
	+0·04	2·3:	−12·28	+1·1	+0·35
AR Aur	−0·09	1·9	−12·84	+0·9	+0·39
	−0·03	1·7	−12·55	+1·4	+0·36
WZ Oph	+0·54	1·36	−10·58	+3·9	+0·05
	+0·54	1·33	−10·58	+4·0	+0·05
UV Leo	+0·63	1·09	−10·27	+4·7	0·00
	+0·63	1·05	−10·27	+4·8	−0·04
RX Her	−0·04	2·4	−12·59	+0·7	+0·43
	+0·02	2·0	−12·34	+1·3	+0·36
Z Her	+0·47	1·6	−10·83	+3·3	+0·09
	+0·91	2·6	−9·23	+3·8	+0·04

For the eclipsing variables shown in the first column of Table IV, accurate photometric and spectroscopic data are available mostly due to the efforts of Popper (**31**)–(**35**). The second column gives $(B-V)_0$. The third column gives the radius in terms of the solar radius as it follows from a combined analysis of lightcurve and spectroscopic data, all these stars being double-lined spectroscopic binaries. Column 4 gives s_V using Table II. Column 5 gives M_V through application of equation (10). The shallow eclipses of the light curve of β Aur are the cause of less precision in radii and M_V for this star than for the other stars of the list. For all these stars good masses are available. The logarithms of the latest values as determined by Popper are shown in the last column.

In Fig. 3, M_V has been plotted against the logarithm of the mass. We have added the data for YY Gem ($M_V = +9{\cdot}8$ and log mass $= -0{\cdot}25$ for each component). The star was omitted from Table IV as the M_V was not derived through s_V from Table II. In fact, the photometric and spectroscopic data together with the known parallax were used to determine s_V.

Fig. 3 also contains the Sun ($M_V = +4{\cdot}84$ and log mass $= 0{\cdot}0$), indicated by a cross. The dots, representing the components of the same eclipsing system, have been connected by a line. The scatter in Fig. 3 around the smooth curve drawn through the points is remarkably small, which proves the accuracy of the data.

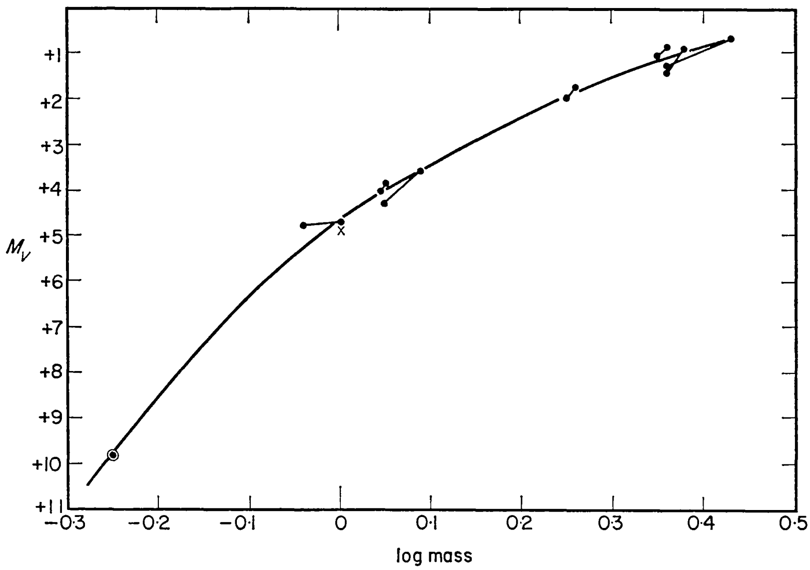

FIG. 3. *Mass–luminosity diagram with the logarithm of the mass as abscissa and M_V as ordinate, for some eclipsing variables. The cross represents the Sun.*

It further shows that the 'cosmical' scatter in the M_V log–mass relation is indeed small for the sample of stars selected. It would be interesting to see if this condition will remain true when accurate data become available for more stars.

6. THE DIMENSIONS OF SOME MAGELLANIC CLOUD SUPERSUPERGIANTS

We found in Section 2.3, that there is no significant observational evidence for a luminosity effect in the relation between s_V and $(B-V)_0$. Hence, on the assumption that the relation is valid for stars of higher luminosity than used in the determination of that relation (up to $M_V = -10$) we may estimate the dimensions of the stars with the largest known luminosity. As before, the calculation requires the knowledge of M_V and the intrinsic colour $(B-V)_0$; the radius is found from equation (10).

The best values for M_V and $(B-V)_0$ for supersupergiants are known for stars belonging to the Magellanic Clouds. The stars collected in Table V may be considered typical for the stars of highest luminosity generally; they were selected from a list given by Feast, Thackeray & Wesselink (**24**).

TABLE V

Dimensions of some Magellanic Cloud supersupergiants

R	HD/HDE	$(B-V)_0$	V_0	s_V	M_V (distance modulus of 18·8)	$R/R_\odot$ (from equation (10))	R (a.u.)
45	7583	+0·04	9·83	−12·28	−8·97	233	1·1
76	33579	+0·08	8·81	−12·16	−9·99	395	1·8
92	271182	+0·50	9·53	−10·72	−9·27	550	2·6
150	269953	+0·76	9·42	−9·78	−9·38	892	4·2
117	269723	+0·92	9·49	−9·20	−9·31	1130	5·3
59	268757	+1·34	9·62	−7·62	−9·18	2200	10·3

The largest star of Table V, HDE 268757, is larger than the orbit of Saturn. However, definite evidence of even larger stars in our own Galaxy has recently been announced through the application of infra red photometry.

7. COMPARISON OF s_V AS CALCULATED FROM B_C AND T_e WITH s_V AS DETERMINED BY OBSERVATION IN THIS ARTICLE

In equation (8) the constant was left undetermined. With the following values for the Sun:

$$T_e = 5760°\text{K} \tag{2}$$

$$BC = -0\cdot07 \tag{36}$$

$$s_V = -10\cdot25 \tag{37}$$

equation (8) may be written:

$$s_V = -10 \log T_e - BC + 27\cdot28. \tag{8'}$$

TABLE VI

$B-V$	BC	$\log T_e$	$(s_V)_c$ (from equation (8′))	s_V	Δ
−0·31	−3·34	+4·504	−14·42	−15·10	+0·68
−0·30	−3·17	4·477	−14·32	−14·90	+0·42
−0·26	−2·50	4·384	−14·06	−14·40	+0·34
−0·24	−2·23	4·344	−13·93	−14·14	+0·21
−0·20	−1·77	4·274	−13·69	−13·66	−0·03
−0·16	−1·39	4·215	−13·48	−13·30	−0·14
−0·14	−1·21	4·188	−13·39	−13·15	−0·24
−0·12	−1·04	4·161	−13·29	−13·01	−0·28
−0·09	−0·85	4·127	−13·14	−12·82	−0·32
−0·06	−0·66	4·093	−12·99	−12·69	−0·30
0·00	−0·40	4·033	−12·65	−12·40	−0·25
+0·03	−0·32	4·009	−12·49	−12·29	−0·20
+0·06	−0·25	3·988	−12·35	−12·20	−0·15
+0·09	−0·20	3·967	−12·19	−12·13	−0·06
+0·15	−0·15	3·936	−11·93	−11·96	+0·03
+0·20	−0·12	3·913	−11·73	−11·82	+0·09
+0·33	−0·08	3·860	−11·14	−11·37	+0·23
+0·38	−0·06	3·841	−11·07	−11·18	+0·11
+0·45	−0·04	3·816	−10·84	−10·91	+0·07
+0·47	−0·04	3·810	−10·78	−10·82	+0·04
+0·50	−0·04	3·801	−10·69	−10·72	+0·03
+0·53	−0·05	3·792	−10·59	−10·62	+0·03
+0·60	−0·06	3·772	−10·48	−10·38	−0·10
+0·64	−0·07	3·762	−10·27	−10·23	−0·04
+0·68	−0·10	3·749	−10·11	−10·08	−0·03
+0·72	−0·15	3·740	−9·97	−9·93	−0·04
+0·81	−0·19	3·719	−9·72	−9·59	−0·13
+0·92	−0·25	3·679	−9·26	−9·20	−0·06
+0·98	−0·35	3·662	−8·99	−8·97	−0·02
+1·18	−0·71	3·599	−8·00	−8·21	+0·21
+1·38	−1·02	3·547	−7·17	−7·47	+0·30

With equation (8′) s_V can be calculated from any list giving BC and T_e. Comparison of the result with s_V, as observed, constitutes a check on the scales of BC and T_e rather than on the observed s_V. As an illustration, we have calculated s_V using equation (8′) using the data as given by Harris III (1) in his Table 7. For T_e his values as given under 'revised' have been used. The results are shown in Table VI.

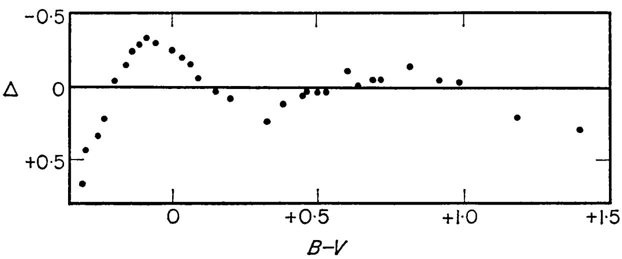

FIG. 4. *The difference* $\Delta = (s_V)_c - s_V$ *as a function of* $(B-V)_0$. *The quantity* $(s_V)_c$ *was calculated from a table by Harris giving BC and* T_e *(see text).*

The differences $(s_V)_c - s_V = \Delta$ have been plotted against $B-V$ in Fig. 4. It is seen that the residuals Δ show a considerable variation especially at the blue end. It is likely that the discrepancy will be solved in the future from theoretical studies of model atmospheres. An independent relation between BC, T_e and s_V or between any two of these combined with an equation like (8′), should give more reliable values for BC and T_e.

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