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SURFACE RESONANCES OF METAL STRIPE GRATING ON THE PLANE BOUNDARY OF METAMATERIAL

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Abstract—The paper is devoted to the study of the interaction of the electromagnetic waves with the structure composed of perfectly conducting strip grating, situated on the plane boundary of metamaterial with effective permittivity, depending on the frequency of the wave of excitation. The rigorous solution to the relevant diffraction boundary value problem is developed. The extensive numerical experiments, performed with a help of corresponding algorithm constructed, allowed to establish several regularities in the complicated process of interaction of electromagnetic waves with grating on dispersive metamaterial. The efficient association of analytical and numerical study has provided the understanding of the nature of resonant phenomena appearing in this process.

1. INTRODUCTION

The technology aimed at the construction of artificial metamaterials, having in the microwave frequency range extraordinary electromagnetic properties, is intensively developing in recent years [1]. Various aspects of perspective applications of such materials in radioengineering are in agenda of numerous research groups all over the world [2–7].

The investigation of peculiarities of wave processes in open structures (open resonators and waveguides, diffraction gratings etc.) comprising metamaterials is of considerable interest for the development of new physical principles of electromagnetic waves generation, amplifying and transmission. The important stage in such research is the study of diverse resonant phenomena, arising in the interaction of plane monochromatic electromagnetic wave with periodic structures, containing metamaterials.

The present paper is devoted to the study of diffraction properties of the strip grating, situated on the surface of metamaterial with effective permittivity, depending on the frequency of the wave of excitation. The rigorous solution to the relevant boundary value problem is developed. The extensive numerical experiments, performed with a help of corresponding algorithm constructed, allowed to establish several regularities in the complicated process of interaction of electromagnetic waves with dispersive metamaterial with grating. The efficient association of analytical and numerical study has provided the understanding of the nature of resonant phenomena appearing in this process.

2. THE FORMULATION OF THE PROBLEM AND METHOD OF ITS SOLUTION

The process of interaction of monochromatic electromagnetic waves with stripe grating located on the plane surface of metamaterial is modelled with the following diffraction boundary value problem. The geometry of the problem is presented in Figure 1.

Let the half space $z < 0$ is filled with isotropic material with effective permittivity depending on frequency according the formula

$$\varepsilon(\omega) = 1 - \frac{\omega_p^2}{\omega(\omega + i\gamma)}, \tag{1}$$

where ω_p is the characteristic frequency, defined by the structure of the elements, composing metamaterial, $\gamma \geq 0$ is the frequency, describing active losses, the permeability of the material is supposed to be $\mu = 1$. Apparently for the first time such model of metamaterial has been suggested in [8]. The periodic perfectly conducting strip grating with period l and slot width d is placed on the plane boundary of metamaterial ($z = 0$). The grating is infinite and homogeneous along axis Ox. Suppose that incident wave and diffracted field are independent of coordinate x, the time dependence is chosen in the form $e^{-i\omega t}$. The incident wave is a plane H-polarized electromagnetic wave of unite amplitude, coming from the half space ($z > 0$), propagating along axis Oz

$$H_x^i = e^{-ikz}, \quad E_y^i = e^{-ikz}, \quad E_x^i = E_z^i = G_y^i = H_z^i = 0,$$

here $k = \omega/c$, c is the velocity of light in vacuum.

The field of diffraction $(\vec{E}^{\partial}, \vec{H}^{\partial})$ has to satisfy the homogeneous system of Maxwell's equations, conditions at edges; radiation condition at the infinity ($z \to \pm\infty$), the periodicity conditions, the boundary conditions at perfectly conducting strips and the transparency conditions in between them. It is easy to show, that under the present suggestions the components of diffraction field $(\vec{E}^{\partial}, \vec{H}^{\partial})$ can be expressed via the only component of magnetic field H_x^{∂}:

$$E_y^{\partial} = -\frac{1}{ik\varepsilon}\frac{\partial H_x^{\partial}}{\partial z}, \qquad E_z^{\partial} = \frac{1}{ik\varepsilon}\frac{\partial H_x^{\partial}}{\partial y}.$$

The rest components of the field $(\vec{E}^{\partial}, \vec{H}^{\partial})$ are equal to zero, thus the diffraction field is H-polarized.

Let us introduce the function $u(y, z)$, coinciding with magnetic component H_x^{∂}. This function has to satisfy the Helmholtz equation

$$\Delta u(y, z) + k^2(z) u(y, z) = 0,$$
$$k^2(z) = \begin{cases} k^2, & z > 0 \\ k^2\varepsilon, & z < 0, \end{cases} \tag{2}$$

everywhere except the strips of grating and boundary of metamaterial; it has to be periodic with period l along axis y; and it has also to satisfy the radiation conditions

$$u(y, z) = \begin{cases} \sum\limits_n x_n e^{i\frac{2\pi n}{l}y} e^{i\rho_{1n}\frac{2\pi}{l}z}, & z > 0, \\ \sum\limits_n y_n e^{i\frac{2\pi n}{l}y} e^{-i\rho_{2n}\frac{2\pi}{l}z}, & z < 0. \end{cases} \tag{3}$$

Here $\rho_{1n} = \sqrt{\kappa^2 - n^2}$, $\rho_{2n} = \sqrt{\kappa^2\varepsilon - n^2}$, $\kappa = \omega l/2\pi c$. The branches in formulas for ρ_{1n}, ρ_{2n} are chosen according the radiation conditions [9, 10]:

$$\kappa \operatorname{Re}(\rho_{1n}) \geq 0, \quad \operatorname{Im}(\rho_{1n}) \geq 0, \quad \kappa \operatorname{Re}(\rho_{2n}) \geq 0, \quad \operatorname{Im}(\rho_{2n}) \geq 0.$$

Besides, the functions $u(y, z)$ and $\frac{1}{k^2(z)}\frac{\partial u(y,z)}{\partial z}$ have to be continuous in the slots of grating

$$\left(z = 0, \left|\frac{2\pi y}{l} + n\right| > \pi\left(1 - \frac{d}{l}\right),\ n = 0, \pm 1, \pm 2, \ldots\right).$$

and have to satisfy boundary conditions

$$\left.\frac{\partial u(y, z)}{\partial z}\right|_{z=0+0} = ik, \quad \left.\frac{\partial u(y, z)}{\partial z}\right|_{z=0-0} = 0. \tag{4}$$

at the grating strips

$$\left(z=0,\left|\frac{2\pi y}{l}+n\right|<\pi\left(1-\frac{d}{l}\right),\ n=0,\pm1,\pm2,\ldots\right).$$

By means of the analytical regularization method that has been developed in [11, 12] this boundary value problem is reduced to the infinite system of linear algebraic equations with respect to unknown amplitudes $(x_n)_{n=-\infty}^{\infty}$ of diffraction field in half space $z > 0$ (see equation (3)) that has the form

$$(1+\varepsilon)x = Ax + b, \tag{5}$$

where $x=(x_n)_{n=-\infty}^{\infty},\ b=(b_n)_{n=-\infty}^{\infty},\ A=\|a_{mn}\|_{m,n=-\infty}^{\infty}$.

The entries of the matrix A and vector b are expressed by relations

$$a_{mn} = \begin{cases} \varepsilon+\dfrac{-i\kappa W_0(u)(1+\varepsilon)}{1+\sqrt{\varepsilon}}, & m=n=0, \\ \dfrac{\delta_n|n|}{(1+\sqrt{\varepsilon})\,n}V_{n-1}^{-1}(u), & m=0,\ n\neq0, \\ (1+\varepsilon)\dfrac{i\kappa}{m}V_{m-1}^{-1}(u), & m\neq0,\ n=0, \\ \dfrac{|n|\delta_n}{m}V_{m-1}^{n-1}+\varepsilon\delta_m^n\left(1-\dfrac{\rho_{1m}}{\rho_{2m}}\right), & m,n\neq0, \end{cases} \tag{6}$$

$$b_m = \begin{cases} \dfrac{\sqrt{\varepsilon}-1}{\sqrt{\varepsilon}+1}+\dfrac{i\kappa W_0(u)(1+\varepsilon)}{1+\sqrt{\varepsilon}}, & m=0, \\ -(1+\varepsilon)\dfrac{i\kappa V_{m-1}^{-1}(u)}{m}, & m\neq0, \end{cases}$$

here δ_m^n is Kronecker delta, $\delta_n = 1+\frac{\varepsilon\rho_{1n}}{\rho_{2n}}+\frac{i(1+\varepsilon)}{|n|}\rho_{1n},\ u=-\cos(\pi d/l)$, functions $W_0(u),\ V_{m-1}^{n-1}(u)$ are defined in [13]. The amplitudes $(y_n)_{n=-\infty}^{\infty}$ of filed of diffraction in the domain of metamaterial are expressed via $(x_n)_{n=-\infty}^{\infty}$ according the formulas

$$\begin{aligned} y_0 &= \sqrt{\varepsilon}(1-x_0), \\ y_n &= -x_n\frac{\varepsilon\rho_{1n}}{\rho_{2n}},\ n=\pm1,\pm2,\ldots \end{aligned} \tag{7}$$

Using the asymptotic estimates for $\delta_n = O\left(\frac{\kappa^2}{n^2}\right)$ and for $V_{m-1}^{n-1}(u)$ (see [13]), one can prove that matrix A of the system (5) produces in the space $l_2=\left\{(x_n)_{n=-\infty}^{\infty}:\sum_n|x_n|^2<\infty\right\}$ the kernel type operator [14]. Thus, the solution to the system (5) can be obtained numerically by means of truncation method. Therefore, finding out $(x_n)_{n=-\infty}^{\infty}$ from the system (5) and using the relations (3) and (7), we obtain the solution of the required boundary value problem.

For the lossless metamaterial ($\gamma = 0$) the solution to the original boundary value problem can be obtained from (5) for all values of frequencies of the incident wave except $\omega = \omega_p/\sqrt{2}$, that is equivalent to $\varepsilon = -1$. This value of frequency is, in certain cense, the essential singular point for the system (5). In particular, when $\omega \to \omega_p/\sqrt{2}$ the operator $[(\varepsilon + 1)I - A]$ of the system (5) becomes a kernel one and, consequently, it has no bounded inverse operator. That is why in such case the straight forward application of truncation method for numerical solution of the system (5) appears to be impossible. In this situation we treat as a solution to the system (5) the limiting value of its solution $(x_n)_{n=-\infty}^{\infty}$ when the losses of metamaterial tend to zero, that is when $\gamma \to 0$.

3. SPECTRAL PROBLEM

In formulation of spectral problem we use the concept, according which such problem describes the singularities of the diffraction field $(\vec{E}^{\partial}, \vec{H}^{\partial})$, appearing in its analytical continuation into domain of complex valued frequencies [9, 10]. At that, the grating with metamaterial can be treated as open resonant structure, having complex eigen frequencies and relevant eigen oscillations (modes). In present paper we shall consider the case of spectral problem when losses in metamaterial are absent ($\gamma = 0$), that is the effective permittivity has the form

$$\varepsilon(\omega) = 1 - \frac{\omega_p^2}{\omega^2} \tag{8}$$

The frequency $\omega(\kappa = \omega l/2\pi c)$ is considered as a spectral parameter to be defined. The formulation of the spectral problem has two essential differences from diffraction problem (2)–(4). First, the incident wave vanishes, and second, the radiation condition (3) has to be analytically continued from the domain of real valued frequencies into infinitely sheeted Riemannian surface (for more details see [9, 10]).

Further on we restrict our consideration by so called "physical" sheet of Riemannian surface. It has to be defined in following way. Consider functions $\rho_{1n} = \sqrt{\kappa^2 - n^2}$ and $\rho_{2n} = \sqrt{\kappa^2\varepsilon - n^2}$, $n = 0, \pm 1, \pm 2, \ldots$, having branching points of the second order, correspondingly: $\hat{\kappa}_n = n$ and $\hat{\kappa}_n = \pm\sqrt{\kappa_p^2 + n^2}$, where $\kappa_p = \omega_p l/2\pi c$.

Let us in complex plane $\kappa = \mathrm{Re}\,\kappa + i\,\mathrm{Im}\,\kappa$ make the cuts

$$(\mathrm{Re}\,\kappa)^2 - (\mathrm{Im}\,\kappa)^2 - \hat{\kappa}_n^2 = 0, \quad n = 0, \pm 1, \pm 2, \ldots, \quad \mathrm{Im}\,\kappa \leq 0, \tag{9}$$

The functions $\rho_{1n}(\kappa)$ and $\rho_{2n}(\kappa)$, that have been defined for the real valued frequency parameter κ, now have to be analytically continued, according to [9, 10], into the complex plane with cuts made along the lines (9):

- for $\operatorname{Im}\kappa > 0$, for all $n = 0, \pm 1, \pm 2, \ldots$;
  $\operatorname{Im}\rho_{1n} > 0$, $\operatorname{Im}\rho_{2n} > 0$, and $\operatorname{Re}\rho_{1n} \geq 0$, $\operatorname{Re}\rho_{2n} \geq 0$, when κ is from $0 < arg\ \kappa \leq \pi/2$
  and
  $\operatorname{Re}\rho_{1n} \leq 0$, $\operatorname{Re}\rho_{2n} \leq 0$ when κ is from $-\pi/2 < arg\ \kappa \leq \pi$,
- for κ from $3\pi/2 < arg\ \kappa \leq 2\pi$, the values of finite number of functions $\rho_{1n}(\kappa)$ and $\rho_{2n}(\kappa)$ (with the numbers n, such that $(\operatorname{Re}\kappa)^2 - (\operatorname{Im}\kappa)^2 - \hat{\kappa}_n^2 > 0$) are defined by the relations
  $\operatorname{Im}\rho_{1n} < 0$, $\operatorname{Im}\rho_{2n} < 0$, $\operatorname{Re}\rho_{1n} > 0$, $\operatorname{Re}\rho_{2n} > 0$,
  for the rest number of functions $\rho_{1n}(\kappa)$ and $\rho_{2n}(\kappa)$:
  $\operatorname{Im}\rho_{1n} > 0$, $\operatorname{Re}\rho_{1n} \leq 0$, $\operatorname{Im}\rho_{2n} > 0$, $\operatorname{Re}\rho_{2n} \leq 0$
- for κ from $\pi < arg\ \kappa \leq 3\pi/2$ the situation is similar to the previous one, but the signs of $\operatorname{Re}\rho_{1n}$, $\operatorname{Re}\rho_{2n}$ are to be changed to opposite ones.

The solution to the spectral problem can be obtained from (5) if we put $b = 0$ (there is no incident wave). Besides, the matrix entries in (6) have to be treated as functions of spectral parameter κ, which is varying in "physical" sheet of Riemannian surface. It is proved, relying on the results [14], that matrix operator $A(\kappa)$ is a kernel analytical operator function of a complex variable κ, except the points $\kappa = 0$ and $\kappa = \pm\kappa_p/\sqrt{2}$, and also except the branching points $\hat{\kappa}_n$, $n = 0, \pm 1, \pm 2, \ldots$. Thus, the required complex eigen frequencies are the roots of the equation

$$\det(I - B(\kappa)) = 0, \tag{10}$$

where $\det(\ldots)$ is the infinite determinant of the operator $I - B(\kappa)$, and $B(\kappa) = (1 + \varepsilon)^{-1}\ A(\kappa)$. This equation ban be solved numerically in efficient way. Indeed, let $B_N(\kappa)$ is finite operator function, obtained as a truncation of matrix $B(\kappa)$ to the finite matrix of the dimensions $N \times N$. As $B(\kappa)$ is a kernel operator, for any arbitrarily small $\delta > 0$ in arbitrarily bounded domain of variation κ (the points $\kappa = 0$, $\pm\kappa_p/\sqrt{2}$ are excluded) one can guarantee the existence of the number N such that

$$\|B(\kappa) - B_N(\kappa)\| < \delta, \tag{11}$$

where $\|\ldots\|$ is the operator norm in the space l_2. The eigen frequencies of finite operator function $I - B_N(\kappa)$ can be found out as the roots of the determinant-function $\det(I - B_N(\kappa))$. As follows from (11), each solution κ_m to the spectral problem (10) can be approximated by the solution κ_m^N of the finite spectral problem for $I - B_N(\kappa)$ with sufficiently large N. As operator-function $A(\kappa)$ is a kernel one, one can show [14], that the procedure, described above, is numerically stable with N increasing.

4. DISCUSSION OF THE RESULTS

Let us analyze the solution to the spectral problem (10). We shall perform our consideration within the domain of spectral parameter variation $\mathrm{Re}\,(\kappa) > 0$. From (6) we can obtain the equality

$$\det(I - B(\kappa)) = \overline{\det(I - B(-\overline{\kappa}))},$$

where the overline means the complex conjugation. That is why if κ is an eigen frequency with $\mathrm{Re}\,(\kappa) > 0$, then $-\overline{\kappa}$ is an eigen frequency with $\mathrm{Re}\,(\kappa) < 0$ (see more details in [9]). Moreover, the symmetry of the structure grating +metamaterial with respect to the plane $y = 0$ (see Figure 1), allows to extract and separate two classes of eigen oscillations: even and uneven with respect to the variable y. Further on we shall consider the even eigen oscillations (uneven may be considered in similar way).

In Figure 2 the behaviour or real and imaginary parts of the first three eigen frequencies $\kappa = \omega l/2\pi c$ with normalized width of grating slots d/l varying is presented. When $d/l = 1$ (the grating is absent) the imaginary part of eigen frequencies vanishes (the lossless metamaterial $\gamma = 0$). In this limit case the eigen frequencies may be defined in explicit form. Actually, when $d/l \to 1$ functions $W_0(u)$ and $V_{m-1}^{n-1}(u)$

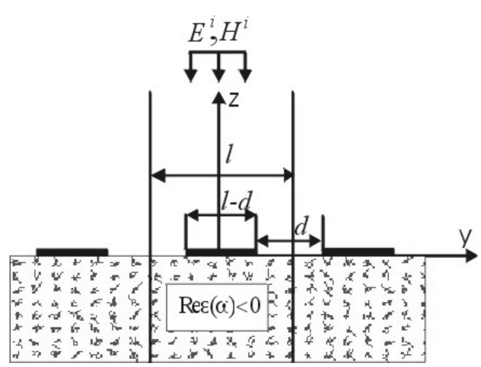
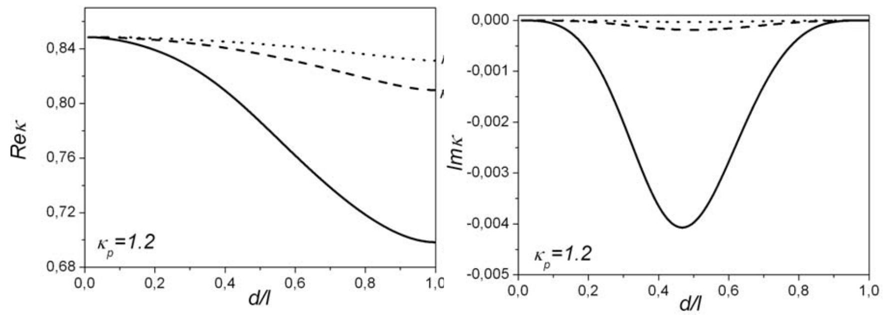


Figure 1. Geometry of the problem.



(a) (b)

Figure 2. The behaviour of eigen frequencies with d/l varying (a) real part of eigen frequencies; (b) imaginary parts of eigen frequencies.

tend to zero [13]. Then the equation (10) for $d/l = 1$ takes the form

$$\det(I - B(\kappa)) = \prod_{n=1}^{\infty} \left[1 - \frac{\varepsilon}{\varepsilon + 1}\left(1 - \frac{\rho_{1n}}{\rho_{2n}}\right)\right]^2 = 0. \tag{12}$$

It is easy to show, that the equation (12) is equivalent to the equations

$$\varepsilon\rho_{1n} + \rho_{2n} = 0, \quad n = 1, 2, \ldots, \tag{13}$$

from which we can obtain the following expression for eigen frequencies for this limit case ($d/l = 1$)

$$\kappa_n = \frac{\kappa_p}{\sqrt{1 + \frac{\kappa_p^2}{2n^2} + \sqrt{1 + \frac{\kappa_p^4}{4n^4}}}}, \quad n = 1, 2, \ldots, \tag{14}$$

where $\kappa_p = \omega_p l/2\pi c$.

As follows from (14) all eigen frequencies are real valued and are less than $\kappa_p/\sqrt{2}$. When $n \to \infty$ the values of eigen frequencies tend to $\kappa_p/\sqrt{2}$, that means that the value of frequency $\kappa = \kappa_p/\sqrt{2}$ is the accumulation point. Using operator extension of Rouche theorem [15], this result can be obtained also for sufficiently small values of $1 - d/l$. Therefore, at least the qualitative behaviour of eigen frequencies of grating with metamaterial is close to the considered above limit case $d/l = 1$. The numerical solution to the equation (10) confirms this conclusion. One can see from Figure 2(a) that the real parts of eigen frequencies tend to the value $\kappa_p/\sqrt{2}$ when d/l is fixed. It is worth to be noted that when $d/l \to 0$ (the grating turns into perfectly conducting plane) the real parts of eigen frequencies tend also to $\kappa = \kappa_p/\sqrt{2}$, and their imaginary parts vanish.

Thus the eigen frequencies of open structure considered are located within the region $0 < \mathrm{Re}\,\kappa < \kappa_p/\sqrt{2}$ and their imaginary parts are negative (see Figure 2(b)). That is why the corresponding eigen oscillations decay exponentially with a time. As follows from (3), the eigen oscillation are the superposition of infinite number of harmonics of the type

$$x_n e^{i\frac{2\pi n}{l}y} e^{i\rho_{1n}\frac{2\pi}{l}z}, \quad z > 0,$$
$$y_n e^{i\frac{2\pi n}{l}y} e^{-i\rho_{2n}\frac{2\pi}{l}z}, \quad z < 0.$$

Thus, in the half space, filled with metamaterial the eigen oscillations decay exponentially when $z \to -\infty$, as in this case $\mathrm{Im}\,\rho_{2n} > 0$ for all values of index n. In the half space $z > 0$ only the finite number of exponentially increasing with $z \to \infty$ harmonics may exist. They have such numbers n, for which $\mathrm{Re}\,\kappa^2 - \mathrm{Im}\,\kappa^2 - n^2 > 0$. The rest harmonics decay exponentially with $z \to \infty$. In particular, for $\kappa_p/\sqrt{2} \leq 1$ there is only one exponentially increasing harmonic with number $n = 0$.

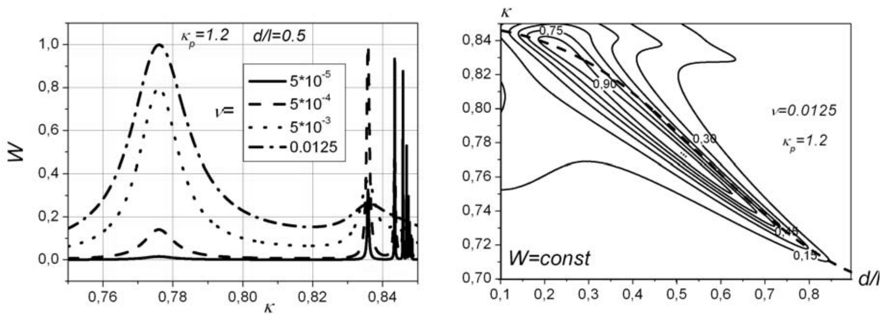
Note, that as follows from (8), the real part of permittivity of metamaterial for all eigen frequencies satisfies the condition $\mathrm{Re}\,\varepsilon < -1$.

Let us consider now the results, obtained from the solution to the problem of diffraction of H-polarized electromagnetic wave by strip grating with metamaterial. We shall consider the situation when the normalized frequency κ of incident wave is less than $\kappa_p/\sqrt{2}$. We can always get into this parameter range if we choose properly the period of grating ($\kappa_p = \omega_p l/2\pi c$). As follows from (3), the diffraction field in zone of reflection ($z > 0$) is a superposition of a propagating 0-th harmonic and of infinite number of surface harmonics, exponentially decaying when $z \to -\infty$. So at large distance from grating in zone of reflection ($z > 0$) the diffraction field is defined only by 0-th order propagating harmonic with amplitude x_0 (the reflection coefficient). As the amplitude of incident wave equals to unite, then when the losses in metamaterial are absent ($\mathrm{Im}\,\varepsilon = 0$) from the energy conservation law follows $|x_0| = 1$.

In the presence of losses in metamaterial ($\mathrm{Im}\,\varepsilon \neq 0$) the absorption coefficients becomes an important characteristic of the structure, as it shows the part of energy of incident wave, that is transferred into metamaterial. In our problem the absorption coefficient is defined as

$$W = 1 - |x_0|^2. \tag{15}$$

Let us study the behaviour of the absorption coefficient W when the frequency of the incident wave κ, geometrical parameters of grating



(a) (b)

Figure 3. (a) The behaviour of absolute value of absorption coefficient when normalized frequency is varying for various values of v; (b) lines if equal values of absorption coefficient in coordinates κ, d/l.

d/l and frequency parameter $v = \gamma l/2\pi c$, (see (1)) characterizing the active losses of the material are varying. Rather typical pictures of the behaviour of absorption coefficient as function of normalized frequency κ for various values of v are presented in Figure 3(a). It was natural to expect $W(\kappa)$ to have pronounced resonant character. The frequencies where the resonances of absorption occur coincide with real values of complex eigen frequencies of spectral problem (10). The frequency positions of these resonances are almost independent from the values of $v\,(v/\kappa_p < 1)$. In contrary, the value of v (active losses in the material) influences considerably the magnitude of amplitudes at resonance frequencies. With v decreasing the absorption coefficient W tends to zero, see Figure 3(a). That means that the incident wave is practically totally reflected from the grating. With v increasing, the situation changes to the opposite one. In this case the absorption coefficient W at certain resonance frequencies can take values close to the unit; hence, the considerable part of the energy of the incident wave is absorbed in the material. The further growth of $v\,(v/\kappa_p \gg 1)$ results in the disappearing of the resonances. These type of resonances we shall call as an "absorption resonances". Note, for the same values of parameter v the amplitudes of absorption resonances at various resonant frequencies may differ considerably. For example, for $v = 5 \cdot 10^{-4}$ the fist resonance appears at $\kappa \approx 0.775$ with the value of absorption coefficient $W \approx 0.1$; the second one appears at $\kappa \approx 0.836$, with $W \approx 0.98$ (see Figure 3(a)). These facts are apparently connected with the decrease of the values of imaginary parts of eigen frequencies approaching the accumulation point $\kappa = \kappa_p/\sqrt{2}$ (see Figure 2(b)).

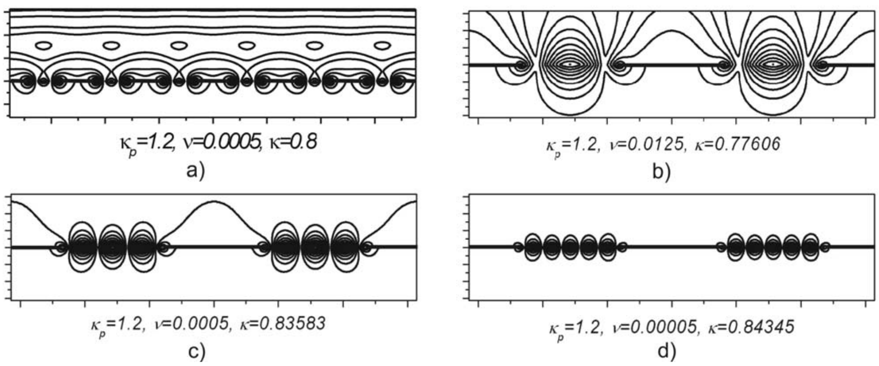


Figure 4. The patterns of diffraction fields $H_x(y,z) = const$ for several eigen frequencies κ, found from the solution to (10).

Let us analyse the influence of geometrical parameter of grating d/l on the character of the resonances. Consider Figure 3(b), where the lines of constant values of absorption coefficient are depicted in coordinates $(\kappa, d/l)$. The maximum in resonant absorption appears in local way in the vicinity of certain points $(\kappa, d/l)$. The values of resonant frequencies κ are close to real parts of eigen frequencies. The curve for real part for one of eigen frequencies with d/l changing is presented in Figure 3(b), dashed line. See also Figure 2a. The numerical experiments have shown also that the magnitudes of the parameter d/l, providing maximal absorption ($W \approx 1$), depend on the values of characteristic frequency κ_p of the material. With exception of the point $d/l = 0.5$. In this case the maximal value of absorption coefficient is independent of $\kappa_p\,(0 < \kappa_p < \sqrt{2})$, at least for the first eigen frequency. Consider now the configurations of diffraction field when grating is excited at resonant frequencies. In Figure 4 the patterns for magnetic components of total electromagnetic field $H_x(y,z)$ are presented for the first three frequencies $\kappa_1 \approx 0.77606$, $\kappa_2 \approx 0.83583$, $\kappa_3 \approx 0.84345$ providing absorption resonances (see Figure 3(a)). As we consider the excitation by plane wave directed normally to the grating, then only even type of eigen oscillations can be excited. The oblique incident wave is necessary to excite uneven eigen oscillations. From Figure 4(b), (c), (d) one can see that the resonant diffraction field $H_x(y,z)$

- is concentrated in the grating's slots and this concentration is the more the higher is the resonant frequency;
- decreases exponentially while moving away from grating along the

axis Oz.

The similar configuration of the field patterns can be seen at other resonance frequencies.

When the frequency of an incident wave moves away from the resonant one, the field pattern has the structure, that is characteristic for the reflection from the half space with finite impedance $\sigma = \frac{\omega_p^2}{4\pi\gamma}$ (see Figure 4(a), $\kappa = 0.8$). In such case for the reflection coefficient by means of method of successive approximations from (5) one can obtain following expression:

$$x_0 = \frac{\sqrt{\varepsilon} - 1 + i\kappa W_0(u)(1+\varepsilon)}{\sqrt{\varepsilon} + 1 + i\kappa W_0(u)(1+\varepsilon)}. \quad (16)$$

It is clear that for $d/l = 1$ the formula (16) is identical to that one for the half space, filled with metamaterial. For $\varepsilon = 1$ relation (16) corresponds to the well known Lamb approximation [13] for strip grating in vacuum.

Basing on the numerical experiments, described above, we can conclude that the resonant phenomena considered are caused by the excitation (by means of incident wave) of diffraction fields that are close by their structure to the fields of eigen oscillations of open electromagnetic structure grating +metamaterial.

5. CONCLUSION

In this paper in the base of mathematically rigorous solution to the boundary value and eigen value problems for open structure, composed be strip grating placed on the plane boundary of metamaterial with frequency dispersion of effective permittivity taken in the form $\varepsilon(\omega) = 1 - \frac{\omega_p^2}{\omega(\omega+i\gamma)}$, the following principal results are presented:

- It is demonstrated that such structure has to be treated as open oscillating system with infinite number of complex eigen frequencies, with finite accumulation points $\omega = \pm\omega_p/\sqrt{2}$, where ω_p is the characteristic parameter of metamaterial, defined by structural elements that is composed of. Their real parts are located within the interval $(-\omega_p/\sqrt{2}, \omega_p/\sqrt{2})$, and imaginary parts are negative, that is the relevant eigen oscillations exponentially decay with time (damped resonances).
- It is shown that in the frequency interval $(0, \omega_p/\sqrt{2})$, where the real part of permittivity of the metamaterial becomes negative, the almost total reflection of incident polarized wave takes place,

with exception of finite countable set of frequencies, in vicinity of which the reflection coefficient is close to zero. At these frequencies the resonant growth of amplitudes of surface waves (that are non propagating along the direction normal to the grating) appears. The surface resonances are caused by the excitation (by means of the incident wave) of eigen oscillations of open structure strip grating with metamaterial.

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